

**AUTOMATIC EVALUATION AND ERROR
IDENTIFICATION OF SOLUTIONS TO SINGLE-
VARIABLE ALGEBRAIC QUESTIONS**

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DECLARATION

“I declare that this is my own work and this dissertation does not incorporate without acknowledgement any material previously submitted for a Degree or Diploma in any other University or institute of higher learning and to the best of my knowledge and belief, it does not contain any material previously published or written by another person except where the acknowledgement is made in the text.

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Abstract

There are two types of single-variable equation solving questions that are present in the Ordinary Level mathematics curriculum in Sri Lanka: linear equations with fractions and quadratic equations. Answers to these questions are open-ended and multi-step in nature. This thesis describes a mechanism that evaluates answers to these two types of questions and awards full/partial credit.

It is quite common that students make mistakes in their answers, which results in partial credit. They may repeat the same errors if they do not receive feedback on their mistakes. Therefore feedback in student errors is important for any subject. This thesis introduces a method to automatically identify the errors that the students make in their answers for the aforementioned two types of questions. To the best of our knowledge, this is the first work on automatically identifying student errors in complex multi-step solutions to single-variable equation solving questions.

Our evaluations show that the system we have implemented is capable of awarding full/ partial credit to student answers according to a marking scheme and also to identify errors in student answers with minimal teacher intervention. These evaluations were carried out using student answers from different sources.

Keywords- Computer Aided Assessment, Error Identification, Computer Algebra Systems, Partial Credit, Multi Step

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TABLE OF CONTENTS

DECLARATION	i
Abstract	ii
ACKNOWLEDGEMENT	iii
TABLE OF CONTENTS	iv
LIST OF TABLES	viii
LIST OF FIGURES	viii
LIST OF ALGORITHMS	ix
LIST OF ABBREVIATIONS	ix
1. INTRODUCTION	1
1.1. Problem Definition	2
1.2. Motivation	3
1.3. Objectives	4
1.4. Contributions	4
1.4.1. Articles	5
1.5. Organization of the Thesis	5
2. LITERATURE SURVEY	6
2.1. Overview	6
2.2. Computer Aided Assessment (CAA)	6
2.3. Computer Aided Assessment Systems	7
2.3.1. Systems that Grade Summaries/ Essays/ Texts	7

2.3.2. Systems that Grade Computer Programs	9
2.3.3. Systems that Grade Answers to Mathematics Questions	10
2.3.3.1. Systems that Evaluate Structured Answers	11
2.3.3.2. Systems that evaluate free response answers	13
2.4. Intelligent Tutoring Systems	15
2.5. Computer Algebra Systems (CAS)	17
2.5.1. Computer Algebra Systems that Can be Incorporated into Computers	17
2.5.2. Handheld Devices that are Capable of Doing Symbolic Computations	19
2.5.3. CASs Used in Education	19
2.6. Error Identification in Student Answers	20
2.6.1. Bugs (Systematic Errors)	20
2.6.2. Slips (Unsystematic Errors)	21
2.6.3. Error Identification	21
2.7. Rubrics/ Marking Schemes in Grading	25
2.8. Discussion	26
3. AUTOMATIC GRADING OF MULTI-STEP ANSWERS	29
3.1. Overview	29
3.2. Types of Questions Being Evaluated	30
3.3. Grading Answers	30
3.3.1. Grading Answers to Linear Equation Solving Questions with Fractions	31
3.3.1.1. Nature of Question, Answer, and Marking Criteria	31

3.3.2. Grading Answers to Quadratic Equation Solving Questions	34
3.3.2.1. Nature of Question, Answer, and Marking Criteria	34
3.3.2.2. Grading Process	35
3.3.3. Marking Scheme for Answer Grading	37
3.3.4. Awarding Partial Marks	38
3.4. Discussion	38
4. ERROR IDENTIFICATION IN STUDENT ANSWERS	40
4.1. Overview	40
4.2. Types of Errors	40
4.3. Error Identification in Answers	44
4.3.1. Replicating Errors	44
4.3.2. Identifying the Error Made by the Student	44
4.4. Discussion	46
5. EVALUATION AND DEMONSTRATION	47
5.1. Overview	47
5.2. Gathering Data for the Study	47
5.3. Evaluation of the Grading Modules	49
5.3.1. Evaluating Linear Equation Solving Questions with Fractions	50
5.3.2. Evaluating Quadratic Equation Solving Questions	51
5.4. Evaluation of the Error Identification Module	52

5.4.1. Error Identification in Erroneous Answers for Linear Equation Solving Questions with Fractions.....	53
5.4.2. Error Identification in Partially-Correct Answers for Quadratic Equation Solving Questions	56
5.4.2.1. Comments by Teachers	56
6. DISCUSSION	58
6.1. Contributions.....	58
6.2. Usability	60
6.3. Accuracy	60
7. CONCLUSION AND FUTURE WORK.....	61
REFERENCES.....	63
Appendix A: Sample Marking Scheme for Linear Equation Solving Question	69
Appendix B: Sample Marking Scheme for Quadratic Equation Solving Question	70

LIST OF TABLES

Table 2.1: Different types of Computer Algebra Systems that can be used inside other systems	18
Table 3.1 XML schema tags of the marking scheme.....	38
Table 4.1: Error codes and associated feedback	43
Table 5.1 Details of the gathered datasets.....	50
Table 5.2 Evaluation Results for Linear Equation Solving Questions.....	51
Table 5.3 Evaluation Results for Quadratic Equation Solving Questions	52
Table 5.4 Evaluation Results for Error Identification in Linear Equation Solving Questions.....	53
Table 5.5 Evaluation Results for Error Identification in Quadratic Equation Solving Questions.....	56

LIST OF FIGURES

Figure 2.1 A portion of the procedural network for addition [38]	23
Figure 3.1 Process of awarding full/ partial credit.....	30
Figure 3.2 Answers by two different students for a linear equation solving question.....	32
Figure 3.3 Two different approaches to reach a single answer in quadratic equation solving.....	36
Figure 4.1 Process for identifying the error in student's answer	40
Figure 5.1 An erroneous answer marked as correct by the teacher	54
Figure 5.2 First steps of a sample answer	55

LIST OF ALGORITHMS

Algorithm 3.1 Algorithm for evaluating linear equation solving question.....34

Algorithm 4.1 Algorithm to identify errors in student answers44

LIST OF ABBREVIATIONS

A/L – Advanced Level

ATPM – Approximate Tree Pattern Matching

CAA – Computer Aided Assessment

CAS – Computer Algebra System

CM – Cross Multiplication

GCE – General Certificate of Education

ITS – Intelligent Tutoring System

LAESA – Linear Approximation and Elimination Search Algorithm

LCD – Least Common Denominator

LSA – Latent Semantic Analysis

MCQ – Multiple Choice Questions

MOOC – Massive Open Online Course

O/L – Ordinary Level