

**STOCHASTIC DIFFERENTIAL EQUATION
APPROACH FOR DAILY GOLD PRICES IN SRI LANKA**

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(148914G)

Degree of Master of Science

Department of Mathematics

University of Moratuwa
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Dissertation submitted in partial fulfillment of the requirements for
the Degree Master of Science in Financial Mathematics

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Sri Lanka

May 2018

Declaration of the Candidate

“I declare that this is my own work and this thesis/dissertation does not incorporate without acknowledgement any material previously submitted for a Degree or Diploma in any University or other institute of higher learning and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text”

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Declaration of the Supervisor

I have supervised and accepted the thesis titled “Stochastic Differential Equation Approach for Daily Gold Prices in Sri Lanka” for the submission of the degree.

Signature of the supervisor:

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ABSTRACT

In our day to day life, predictability of gold prices is significant in many domains such as economic, financial and political environment. The objectives of this research are to study the behavior of the gold price in Sri Lanka, to forecast the daily gold prices making use of four Stochastic Differential Equation (SDE) models, Brownian motion, Geometric Brownian motion, Cox-Ingersoll-Ross (CIR) model and Vasicek model and compare the results with an ARIMA (2,1,2) model which is used to forecast the Sri Lankan gold prices in a previous research. The daily gold prices per troy ounce in Sri Lanka are obtained from 01st of October 2015 to 14th of October 2016 from the website http://www.cbsl.gov.lk/htm/english/_cei/er/g_1.asp on 1st of November, 2016. The gold prices from 01st of October 2015 to 07th of October 2016 are used to estimate the parameters of the four models and the parameter estimation is done using maximum likelihood estimation method. The gold prices from 10th of October 2016 to 14th of October 2016 are used to forecast the gold price. By taking the gold price on 10th of October 2016 as the initial value, daily gold prices from 11th of October 2016 to 14th of October 2016 are forecasted. Numerical approximations are carried out using Euler-Maruyama approximation method and the Monte Carlo simulation technique is used to simulate the daily gold prices. After evaluating forecasting accuracy of estimated models and existing ARIMA (2,1,2) model by root mean square error (RMSE) and mean absolute percentage error (MAPE), it turns out that the Vasicek model has the minimum RMSE and MAPE values for the given data set. The price of the gold may change rapidly because of some economic factors such as inflation, currency exchange rates etc. In these situations the best SDE model to forecast the daily gold price in Sri Lanka may be changed to another model. Hence this method is suitable for short runs only.

Keywords: Gold Price, Stochastic differential equations, Maximum likelihood estimation, Monte Carlo method, Euler-Maruyama method

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LIST OF ABBREVIATIONS

Abbreviation	Description
AIC	Akaike Information Criterion
ARFIMA	Auto Regressive Fractionalized Integrated Moving Average
ARIMA	Auto Regressive Integrated Moving Average
ARMA	Auto Regressive Moving Average
BIC	Bayesian Information Criterion
BMA	Bayesian Model Averaging
CIR	Cox Ingersoll Ross Model
CRB	Commodity Research Bureau
DMA	Dynamic Model Averaging
DMS	Dynamic Model Selection
ERC	Earnings Response Coefficients
ETF	Exponential Smoothing
INF	Inflation
MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
MLR	Multiple Linear Regression
RMSE	Root Mean Square Error
RW	Random Walk
SDE	Stochastic Differential Equations
TBATS	Trend and Seasonal components
VAR	Vector Auto Regressive

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