# TRAJECTORY PLANNING FOR 6-DOF ROBOT MANIPULATOR BASED ON OFFLINE ROBOT PROGRAMMING APPROACH 

Malnydelage Dimithri Maliyos Fernando

(148456D)

Degree of Master of Science

Department of Electronic and Telecommunication Engineering

University of Moratuwa
Sri Lanka

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Malnydelage Dimithri Maliyos Fernando

(148456D)

Thesis submitted in partial fulfillment of the requirements for the degree Master of Science in Electronics and Automation

Department of Electronic and Telecommunication Engineering

University of Moratuwa

Sri Lanka

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#### Abstract

ABTRACT

Industrial robot manipulators are highly involved in modern manufacturing industries. Robot programming is the procedure to carry out generating a sequence of robot instruction. Teaching method is highly applied where a teach pendent is used to generate the robot programme by teaching one point at a time. This process tends to consume more time and the accuracy can be varied depends on the application. Several other methods are used to program robot movement nevertheless industrial applications of these systems are still developing. Programming tends to be difficult and restricts the productivity and industrial application. Hence, requirement of flexible programming methods is still challenging for inexpert robot operators. Trajectory planning for a robot system is still a developing area where the accuracy, productivity and high quality on various operations are highly concerned. To address these limitations, off-line programming systems can be used where computer systems with realistic graphics, interfaces and features can be used to plan and program robot motions without using robot hardware. The research is aimed to present methods for finding a better mathematical way of optimized trajectory planning of 6-DOF industrial robot manipulator. Computer Aided Design software systems are used to implement off-line programming technique by developing human robot interface in order to create robot moving sequence and achieve required data for further calculations. Welding process of machine head cover using a 6 DOF robot manipulator is used to demonstrate and evaluate the proposed method. Methods for Point allocation along the robot moving path and data extraction are presented. Inverse kinematic model for the 6 DOF manipulator is developed and implemented in order to get joint space data represented by joint angles. Derived data is studied to analyze the manipulator motion behavior while moving along predefined path via points allocated. Robot path planning and trajectory planning with CAD system involvement as off-line programming technique is analyzed by comparing results in order to evaluate the performance of the proposed method.


Keywords: Offline robot programming, Computer Aided Design, 6 DOF robot manipulator, inverse kinematics, Human robot interface

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Malnydelage Dimithri Maliyos Fernando
B.Sc. Eng. (Moratuwa)

Department of Mechanical Engineering, University of Moratuwa, Katubedda, Sri Lanka, January 2019.

## TABLE OF CONTENTS

DECLARATION, COPYRIGHT STATEMENT AND THE STATEMENT OF THE SUPERVISOR .....  i
ABTRACT ..... ii
ACKNOWLEDGEMENT ..... iii
LIST OF FIGURES ..... vi
LIST OF TABLES ..... viii
LIST OF ABBREVIATIONS ..... viii
LIST OF APPENDICES ..... ix
INTRODUCTION ..... 1
1.1 Background and Motivation ..... 1
1.2. Problem Definition ..... 5
1.2.1 Thesis Objectives ..... 6
1.2.2 Goals ..... 6
1.3 Limitations ..... 6
1.4 Influences to the industry ..... 7
1.5. Report Summary ..... 7
LITERATURE REVIEW ..... 9
2.1 Some Literature ..... 9
ROBOT AND APPLICATION SYSTEM ..... 13
3.1 Robot system ..... 13
3.2 Automated welding operation ..... 18
3.3 Robotic welding Robot cell ..... 20
SYSTEM AND PROCESS DESIGN IDENTIFICATION ..... 24
4.1 Mathematical approach for closed form solution of inverse kinematic of 6 DOF robot manipulator ..... 24
4.1.1 Robot manipulator forward kinematics ..... 26
4.1.2 Robot manipulator inverse kinematics ..... 28
4.2 The Human Robot interface. ..... 33
4.3 Path planning and information extraction ..... 38
4.4 Mathematical approach for mapping trajectories ..... 40
SYSTEM DESIGN AND SIMULATION ..... 47
5.1 Design and Implementation of Inverse kinematic model ..... 47
5.2 Algorithms for developing trajectory planning schemes in section 4.4 ..... 57
5.3 Simulation model ..... 61
5.3.1 Creating Robot cell ..... 61
RESULTS AND CONCLUSION ..... 65
6.1 Various point cloud simulation Results ..... 65
6.2 Conclusion ..... 82
6.3 Future works ..... 83
BIBLIOGRAPHY ..... 84
Algorithms for inverse kinematic solution of 6 DOF robot manipulator and trajectory planning in Matlab ..... 87
Dimensions and the configuration of the Denso VP 6242 ..... 105
Machine head cover dimensions ..... 105
Robotic cell layout ..... 105
Scenario 1 joint angles calculation results ..... 108
Scenario 3 joint angles calculation results ..... 111
Scenario 4 joint angles calculation results ..... 114

## LIST OF FIGURES

Page
Figure 1.1 Denso robotics six DOF robot manipulator model: VP 6242 ..... 02
Figure 2.1 Hyper flexible robotic cell ..... 10
Figure 3.1 Denso VP 6242 mini sized vertical articulated robot ..... 13
Figure 3.2 External Dimensions and Workable Space (VP-6242) ..... 14
Figure 3.3 VP 6242 robot arm ceiling or ceiling mounted ..... 15
Figure $3.4 \quad$ RC8A controller for VP 6242 robot system ..... 16
Figure 3.5 RC8A controller System configuration ..... 16
Figure 3.6 3D arm view - Wincaps III software ..... 17
Figure 3.7 Data location data - Wincaps III software ..... 18
Figure $3.8 \quad$ Robotic welding operation -ABB robotics ..... 18
Figure 3.9 Arc welding defects due to speed changes ..... 19
Figure 3.10 Differences of welding torch unit between Manual welding and Robotic welding ..... 20
Figure 3.11 Robotic welding application - 3D model ..... 20
Figure 3.12 Welding application - Layout ..... 21
Figure 3.13 Machine Head Cover - Dimensions ..... 21
Figure 3.14 Industrial Robotic welding torch ..... 22
Figure 3.15 Robotic Welding Torch - Dimensions ..... 22
Figure 3.16 Robot End travelling path ..... 23
Figure 4.1 Coordinate assignment - Denso VP 6242 ..... 25
Figure $4.2 \quad$ Robot links 3D design models ..... 34
Figure 4.3 Robot manipulator assembly in Solidworks ..... 35
Figure 4.4 Coordinate system assignment in 3D model ..... 36
Figure 4.5 Tool design - Welding torch ..... 36
Figure 4.6 Machine head cover - 3D design ..... 37
Figure 4.7 Machine Head cover location ..... 37
Figure 4.8 Complete 3D design for the robot cell ..... 38
Figure 4.9 Spline feature for robot moving path definition ..... 38
Figure 4.10 Point allocation along the robot path ..... 39
Figure 4.11 VB based macro for data extraction ..... 39
Figure 4.12 Data extracted values in Microsoft excel ..... 40
Figure 4.13 Typical trajectory profile for position variation ..... 43
Figure 4.14 Typical trajectory profile for velocity variation ..... 43
Figure 4.15 Typical trajectory profile for acceleration variation ..... 44
Figure 4.16 Typical trajectory profile for jerk variation ..... 44
Figure 5.1 Path and point allocation along the robot moving path ..... 47
Figure 5.2 Tool orientation along the path ..... 48
Figure 5.3 Tool orientation with respect to work 0 coordinate ..... 48
Figure 5.4 Tool dimensions ..... 50
Figure 5.5 Orientation defined by manually ..... 51
Figure 5.6 Maximum velocity mapping ..... 59
Figure 5.7 Robot Arm selection in Wincaps III ..... 61
Figure 5.8 Default Robot configuration in Wincaps III ..... 62
Figure 5.9 Machine head cover and Tool placement in the simulation module 62
Figure 5.10 Work and Tool coordinate assignment ..... 63
Figure 5.11 Robot programs for simulating robot movement via points ..... 63
Figure 5.12 Import Point location data into the program ..... 64
Figure 5.13 Joint angles of the robot arm at point 1 position ..... 64
Figure 6.1 Robot path length ..... 66
Figure 6.2 Point allocation scenario 1 ..... 67
Figure 6.3 Scenario 1 simulation results ..... 68
Figure 6.4 Maximum velocity of joint rotation scenario ..... 69
Figure 6.5 Point allocation scenario 2 ..... 70
Figure 6.6 Scenario 2 simulation results ..... 72
Figure 6.7 Maximum velocity of joint rotation scenario 2 ..... 73
Figure 6.8 Point identification of the velocity variance begins ..... 74
Figure 6.9 Point allocation scenario 3 ..... 74
Figure 6.10 Scenario 3 simulation results ..... 76
Figure 6.11 Maximum velocity of joint rotation scenario 3 ..... 77
Figure 6.12 Point allocation scenario 4 ..... 78
Figure 6.13 Scenario 4 simulation results ..... 80
Figure 6.14 Maximum velocity of joint rotation scenario 4 ..... 81
LIST OF TABLES
Table 1 Joint movement and speed limitation (VP 6242) ..... 15
Table 2 Denso VP 6242 robot manipulator D-H parameters ..... 26
Table 3 Scenario 2 joint angles calculation results ..... 71

## LIST OF ABBREVIATIONS

Abbreviation Description
DOF Degree Of Freedom
CAD Computer Aided Design
3D Three Dimensional
OLP Off Line Programming

## LIST OF APPENDICES

Appendix Description ..... Page
Appendix A Algorithms for inverse kinematic solution of 6 DOF robot manipulator and trajectory planning in Matlab ..... 87
Appendix B Dimensions and the configuration of the Denso VP 6242
Machine head cover dimensions
Robotic cell layout ..... 105
Appendix C Scenario 1 joint angles calculation results ..... 108
Appendix D Scenario 3 joint angles calculation results ..... 111
Appendix E Scenario 4 joint angles calculation results ..... 114

## CHAPTER 1

## INTRODUCTION

Chapter 1 explains about background and motivation of the research. It outlines the foundation of the research initialization as a master thesis. The research is aimed for finding optimized solutions for trajectory planning for 6 DOF robot manipulator based on offline programming approach. Section 1.1 discusses the thesis background and motivation. Problem definition and the objectives of the thesis are defined in section 1.2. Section 1.3 describes research limitations. Research influences over the industry is discussed in section 1.4. Finally, Section 1.5 summarizes the remaining chapters of the report.

### 1.1 Background and Motivation

Industrial robots are highly involved and utilized in modern manufacturing industries due to its efficiency, productivity and programmability. The demand for the usage of robots in automated manufacturing systems is increasing especially in automotive systems. Performing high precision tasks with higher repeatability and quality makes them more utilized in the industries. Industrial robots can be operated continuously without taking a break. This supports manufacturers to increase the productivity and efficiency.

There are several types of industrial robots and can be simplified to five major types.

1. Cartesian
2. cylindrical
3. Delta
4. SCARA
5. Articulated robots

Each robot type has different features and specific elements which utilize them more appropriate for different applications. Main differentiators among them are speed, size and workspace.

Articulated robots which resemble a human arm in its mechanical configuration are the most commonly utilized industrial robots since the design and the configuration offer most flexibility. Articulated robot has connecting links which are connected to the base with rotary joints. Figure 1.1 shows an illustration of six degree of freedom industrial robot manipulator manufactured by Denso robotics.


Figure 1.1: Denso robotics six degree of freedom robot manipulator model: VP 6242

6 DOF robot manipulator has many advantages such as high speed operation, large work area for minimum floor space and ability to align to multiple planes. Even though it has significant advantages, challenges are being confronted since it is complicated in programming and kinematics.

Robot programming is the procedure to carry out creating a sequence of robot working instruction and work location/points that achieves the required task. Various methods and techniques are used to plan and program robot operations. Teaching method is highly applied where a teach pendent is used to generate the robot programme by teaching one point at a time. This process tends to consume more time and the accuracy
can be varied depends on the application. In some applications, technical expertise may be required to perform the task like welding, spraying etc.

Several other methods are used to program robot movement such as,

- software dedicated to a particular industrial process.
- human body attached sensors to capture arm movements
- CAD based solutions.
- vision-based interfaces.

Due to several reasons including reliability issues, applications of these systems are still developing. Thus, the teach pendant is considered as a common robot input device which grant access to robotic functionalities. The difficulty of this robot programming process limits the productivity of the robot and more widespread use of robot technology. Hence, requirement of flexible programming methods is still challenging for inexpert robot operators.

Currently CAD systems can provide more design and modelling capabilities having high precision and standard, simulation can be also done with great accuracy. Therefore, CAD systems can be utilized to give effective engineering solutions for preparing robot path and trajectories in robot programming.

Off-line programming (OLP) is introduced as a revolutionary robot programming method where the robot program can be created without considering the actual robot cell which represent the robot system and the application. The created robot program can be imported or uploaded to the robot controller for the robot manipulator execution. Simulator can be used to create the robot cell by creating comprehensive graphical three-dimension(3D) model. Currently, robot integrators use robotic simulators and off-line programming techniques to create efficient program paths for a particular robot manipulation to perform a specific task. Simulators are beneficial where robot movement and reachability analysis, collision detection and cycle time calculation can be done when robot programs are simulated.

Off-line robot programming is used to develop the robot program on an external computer system without interfering the production or the robot operation. This method improves the on-line programming abilities where programmers use teach pendants to program the robot manually. Robot programming change over time can be minimized by applying off-line robot programming techniques.

The capability to perform different operations and task is more important when considering the flexibility of industrial robot manipulators. Robot system is subjected to deal with higher degree of problems to resolve with respect to human flexibility. When considering an object to move between two space points, several factors are needed to be concerned.

1. Finding Optimum path or route.
2. Avoidance of obstacles and collisions.
3. Improving task productivity.
4. Maintaining higher efficiency.

Path planning is more important where planning of entire path from one point to another point in work space including stopping in predefined path points is executed. Path planning is a geometrical description of robot motion. Obstacles and path constraints can restrict the motion of a robot.

Considering obstacle constraints, Path planning can be done assuming that robot has to be moved by planning not moving through that obstacle takes place. Path constraints are needed to be concerned where there can be reference points that the robot must move through that points.

In the case of 6 DOF robot manipulator, physical limit of motion and mechanics constraints are appeared in most cases. Trajectory planning is planning of desired motion of a robot manipulator and depicts how well the robot manipulator operates. For optimal solution, actuator positions, velocity, acceleration, jerk and the limit of joints are needed to be considered. Energy expenditure while running the robot is more concerned. In every movement, robot has to accelerate, hold and brake hence energy is consumed. Therefore, unnecessary energy dissipation should be reduced in order to increase the energy efficiency of the manipulator. Speed is more concerned where
productivity of the robot manipulator is highly focused. Cycle time for the task is important in a high production rate hence integrator try to minimize as much as possible. Therefore, time taken for a path is more important to be considered. Path planning and trajectory planning should be highly concerned in order to get higher productivity while maintaining better robot performance.

### 1.2. Problem Definition

Robot manipulators are highly involved in current manufacturing industries and for planning and programming of the robot manipulator, teaching method is highly used. It is more suitable for simple operations where programmers need less time for take necessary point locations in robot moving path. In some critical operations such as welding, painting etc. time taken for teaching is much higher than usual since there can be higher number of task points hence programmer need significant time to teach and the experience and expertise is more important. If any modification has to be done, programmers face numerus problem where in some cases, the teaching of the robot path has to be done from the beginning. Offline robot programming is applied to overcome this kind of issue. Dedicated software solutions are available but can be expensive and limited to particular operations.

Computer Aided Design software is highly used in manufacturing industries since higher accuracy level can be achieved. User friendliness is the other factor that users involve CAD systems in their operations. CAD systems can be involved to develop human robot interface hence programmer can define robot path while analyzing the system graphically. Path planning can be done effectively and relevant data extraction from CAD system can be used.

Inverse kinematics of 6 DOF robot manipulator is needed to find joint space values for particular robot position in work space. Inverse kinematics is studied for many decades and solving of inverse kinematic for a particular robot system is difficult computationally and time to solve is higher. Effective inverse kinematic solution is more important to find joint angles since the actuator works in joint space. Trajectory
planning is important to improve the productivity of the robot manipulator operation. Optimization of the trajectory planning enhance the production while maintain the robot work cycle smoothly.

### 1.2.1 Thesis Objectives

Thesis main objective is to research and develop methods to involve CAD systems as offline robot programming technique to improve and optimize the trajectory planning of 6 DOF robot manipulator by implementing enhanced inverse kinematic model for the manipulator. In this research, methods for path planning and relevant data extraction in CAD systems, inverse kinematic model for the robot manipulator and trajectory planning optimization with CAD systems are investigated. Proposed method is validated by implementing for a practical robot application and comparing the results and its performance.

### 1.2.2 Goals

In this research, it is intended to present methods for optimizing trajectory planning using offline robot programming techniques in order to increase the productivity and accuracy. This research intends to find a better mathematical way of optimized trajectory planning of 6-DOF industrial robot manipulator.

### 1.3 Limitations

Proposed method is based on the controlling and operation of 6 DOF robot manipulator. Therefore, the development of the inverse kinematics model is limited to the type of 6 DOF manipulator which is intended to research on. Welding operation of a machine head cover using Denso VP 6242 robot manipulator is evaluated in order to apply the proposed method and analysis the result and performance. Inverse kinematic
model is subjected to change with types of robot manipulator and the work space where the robot end effector operates.

### 1.4 Influences to the industry

This research work demonstrates an approach to develop methods to optimize trajectory planning in order to increase the efficiency and productivity with offline programming techniques. The proposed method can be implemented for most common available 6 DOF manipulators in the industry and can be applied for other types of robot manipulators. The method uses for path planning including path creation, task point allocation and data extraction in CAD systems can be used for any kind of industrial operation for further development. Trajectory planning can be analyzed and path planning methods can be optimized according to trajectory planning requirements in order to reduce task cycle time while maintaining robot smooth movement. Integrators can develop other methods based on this proposed method for further development.

### 1.5. Report summary

The remaining sections is arranged as follows.
In Chapter 2,some literature is reviewed about existing methods for solving kinematics of robot manipulators, optimizing of trajectory planning for different applications and influence of current offline programming methods for best robot operation. Chapter 3 illustrates the behavior of the robot and the application which is evaluated. It also extends the background of the system that will support to develop methods for optimum solutions. In Chapter 4, an approach for developing appropriate methods for optimizing trajectory planning of 6 DOF robot manipulator is proposed. Path planning with CAD system support, mathematical model for inverse kinematic solution of 6 DOF robot manipulator and analysis of trajectory planning from derived joint space data in order to optimize of the robot programmability and operability are proposed.

Simulation model is developed in a separate robot programming software tool in order to verify the developed mathematical model results and debug for further development in chapter 5. The final results of the research are described in chapter 6 and it validates the results over the results of the simulation. It also suggests improvements as forthcoming developments. The development procedure of the mathematical model and algorithms for inverse kinematic solution of 6 DOF robot manipulator and trajectory planning are illustrated in appendix 'A'. Appendix 'B' shows the dimensions and the configuration of the Denso VP 6242. It also presents the dimensions of the work piece and the robotic cell layout. Appendix ' C ', ' D ' and ' $E$ ' illustrate calculated joint angles values tabulated for experiment scenario 1,3 and 4 .

## CHAPTER 2

## LITERATURE REVIEW

Existing approaches developed for inverse kinematic solutions, path planning and trajectory planning of robot manipulators and other similar applications are discussed in this chapter. Robot manipulators are highly involved in modern manufacturing industries specially in automotive [1]. Robot programming is critical significantly in order to operate productively and various programming techniques are used. Currently sophisticated robot controllers and software interfaces are involved [2] and most of these applications are expensive in use thus teaching method is highly used. Teaching method can be imprecise, time consuming process in some applications where technical expertise of the application should be concerned in order to perform robot teaching [3]. Offline robot programming methods are introduced as a revolutionary solution for the drawbacks of current online programming techniques including teaching where the robot programs are created in separate interface independent of actual robot cell prior to upload to the robot manipulator for execution [4]. Importance of the Path planning and trajectory planning is more valuable for robot operations and maintenance in order to increase the productivity and efficiency.

### 2.1 Some Literature

Even though there are many types of industrial robot manipulators and systems which are capable of offline robot programming, it is still very expensive and limited to a particular industrial application which tends to cost more when changing the application. In addition, the applications where the accuracy is highly concerned need advanced robot systems with controllers [1][2].
A. Paulo Moreira ,Pedro Neto and J. Norberto Pires researchers in University of Coimbra, Portugal and University of Porto, Portugal have presented CAD-Based OffLine Robot Programming approach in order to optimize robot programmability [3].

Currently CAD systems can provide more design and modelling capabilities having high precision and standard, simulation can be also done with great accuracy. Therefore, CAD systems can be utilized to give effective engineering solutions for preparing robot path and trajectories in robot programming.
L. Alonso Ferreira, M. Álvarez Souto, I. Fernández Iglesias and Y. Lapido Figueira, researchers present solution for offline robot programming method with CAD support to support ship building fabrications and supports [5]. They proposed a solution for a hyper-flexible welding cell with 6 DOF robot manipulator mounted on 3 axis gantry system which is programmed in a CAD environment [6] in order to prepare robot task sequences and extract relevant data for further processes. Figure 2.1 shows the application area of hyper flexible robotic cell they have implemented.


Figure 2.1 Hyper flexible robotic cell
It also emphasizes the capability of creating accurate robotic cell for complex robotic system arrangements in a CAD system which gives unexperienced programmers to program robot tasks easily.

Kinematics of a robot manipulator is a very critical problem when we deal with automatic control of the robot operation. A. Khan, C. Xiangming, Z. Xingxing and W.

Quan present method and solutions of closed form inverse kinematic modelling for a 6 DOF robot manipulator [7]. The manipulator is based on Puma560 robot which is intended to move in underwater area. They have developed a closed-form solution of inverse kinematics model for a 6 -DOF manipulator and were able to validate the developed algorithm using simulation in Robotic toolbox.

Mustafa Jabbar Hayawi from Thi-Qar University has presented an Analytical Inverse kinematics Algorithm of A 5-DOF Robot Arm. Forward kinematics and a closed form inverse kinematic solution for the educational 5 DOF robot manipulator TR 4000 are presented to overcome the high number of iterative numerical solution [8].

Trajectory planning is very essential to maintain Smoothness and Ease of accurate tracking by the manipulator. Non-smooth trajectories cause problems like high torque in actuators, Vibrations, Error in path tracking, manipulator wear and low level of quality output [9]. Jerk controlling is very important in order to minimize these problems since the nature of the jerk profile predicts the behavior of the motion. Studies shows that a trajectory with controlled jerk profile is essential in order to achieve the desired features. Controlled jerk trajectory can be achieved by,

1. Maintaining zero bound starting and ending profile which reserve the smoothness at start and end motion.
2. Maintaining continuous jerk profile to avoid potential 'Infinite jerk' of noncontinuous jerk profiles.
3. Limiting jerk between known values where user can compare with manufacturer's recommendations.

Trajectory planning can be executed in joint space or operational space. In joint space trajectory planning, the motion is described by joint values whereas in operational space trajectory planning, the motion is described in Cartesian space in many cases. The motion between two points is unpredictable in joint space trajectory planning while it is easy to visualize the path and the motion between the two points is known in Cartesian space trajectory planning. Collision can be prevented by proper trajectory planning in operational space but it is computationally expensive since we need to
solve inverse kinematics between discretized points along initial point to final point at each step. In joint space trajectory planning, inverse kinematics is needed to be calculated only once and constraints like joint angle and velocity can be concerned [10].

## CHAPTER 3

## ROBOT AND APPLICATION SYSTEM

This chapter explains the behavior of robot system and the application. Section 3.1 presents information about the robot system with relevant sub systems required for the operation. Section 3.2 presents information about the application which is evaluated by implementing proposed methods in order to optimize the output. Section 3.3 expresses the necessary details about the total system represented the entire robot working cell for the application.

### 3.1 Robot system

Robot system is consisted of two major sub systems.

1. Denso VP 6242 robot manipulator
2. Robot controller system

Denso VP 6242 robot [Figure 3.1] is a high speed, high accuracy robot unit produced by DENSO Robotics. It is categorized as mini sized vertical articulated robot.


Figure 3.1: Denso VP 6242 mini sized vertical articulated robot

Denso VP 6242 arm is a 6-axis type robot arm built with joints similar to a human arm, great flexibility can be achieved. 6-axis freedom of movement makes them suitable to handle a much wider application range such as assembly, Dispensing, Grinding, Laser welding, material handling, material removal, packaging, pick and place, ultrasonic welding, polishing, spot welding.

It is suitable for installations where operational space is limited. It has excellent repeatability of $\pm 0.02 \mathrm{~mm}$. It can handle payloads up to 3 kg and can reach up to 432 mm maximum [Figure 3.2]. It can be ceiling or ceiling mounted with no special hardware needed [Figure 3.3].


Figure 3.2: External Dimensions and Workable Space (VP-6242)


Figure 3.3: VP 6242 robot arm ceiling or ceiling mounted
VP 6242 robot arm is consisted of six joints operated with AC servomotors and brakes for all axes. Each joint has motion and speed limitation and table 1 presents the range of motion and speed limitation.

Table 1: Joint movement and speed limitation [VP 6242]

| Joint | Range of motion ( ${ }^{\circ}$ ) | Maximum joint speed ( ${ }^{\circ}$ sec.) |
| :---: | :---: | :---: |
| J1 | $\pm 160$ | 250 |
| J2 | $\pm 120$ | 187 |
| J3 | $+19,+160$ | 250 |
| J4 | $\pm 160$ | 300 |
| J5 | $\pm 120$ | 300 |
| J6 | $\pm 360$ | 300 |

Source: Denso VP 6242 user manual [11]

Model RC8A and RC8 are the controller compatible with the VP 6242 robot system. It features and supports the Safety Motion function [Figure 3.4].


Figure 3.4: RC8A controller for VP 6242 robot system
Robot controller can be connected many sub systems such as control devices, software and other peripherals for further expansions [Figure 3.5].


Figure 3.5: RC8A controller System configuration
The controller is the main controlling device of the industrial robotic arm which allows robot system and parts to function together. It also allows other systems to be connected with the current systems. The controller runs a set of instructions written in code named a program. The program is inputted or entered with a teach pendant [Figure 07] or software interface that built on most of operating systems. It converts the commands in the source code of the program to motion or motor drivers which are connected with robot arm joints. Internal model of kinematic structure of the robot
manipulator is inbuilt with the controller. It can coordinate the motions precisely which are commanded by each individual robot arm motor drives. Thousands of parameters are referred by the controller to ensure the robot operating precision of the required task enrolled with the application.

Wincaps III software interface is a programming tool developed by Denso corporation and it is used for various application including program developments, parameter settings, transferring relevant data between robot arm and the controller and robot posture checking on a 3D screen. Robot can be programmed in a separate area called program editing window and functions such as line number display, command color display, indentation, comment block and bookmark can be implemented. Simulation capability is more advantageous where programmer can run the program on a computer system. Cycle time measurement, interference can be checked by user prior to transfer to the controller. Program start/stop and break points with robot motion with robot trajectory display are beneficial in order to increase the productivity. 3D arm view is the capability of the displaying robots and peripheral devices three dimensionally when robot motion is simulated [Figure 3.6]. 3D graphic data, format of VRML, DirectX can be imported in to Wincaps interface. Programmer can obtain location data information [Figure 3.7] of the robot end easily in order understand and get relevant data for further calculations.


Figure 3.6: 3D arm view - Wincaps III software


Figure 3.7: Data location data - Wincaps III software

### 3.2 Automated welding operation

Robot manipulators are highly involved in modern manufacturing and fabrication industries such as arc welding and spot-welding operations are included [Figure 3.8].


Figure 3.8: Robotic welding operation - ABB robotics [12]

For welding operation, smooth movement with constant speed of the welding torch is highly important to get higher quality welding output. Due to higher temperature at the end of the welding torch, even a small jerk at a point in the moving line causes a huge impact to the output [Figure 3.9].


Figure 3.9: Arc welding defects due to speed changes [13]
Robotic welding operated system consists of two major parts.

1. Robot manipulator and controller system
2. Welding torch for robotic operations

Robot manipulator is used to handle the welding torch and operated as a human arm when the welding operation is done manually. Specially developed welding torch is now available for automated robotic operation since the conventional welding equipment cannot be used for robotic operation due to complexity of system components. Robotic welding torch can be attached to the robot end as an end effector without need of much effort. Figure 3.10 illustrates the differences of the design of conventional welding torch used for manual welding and robotic welding torch.


Figure 3.10: Differences of welding torch unit between Manual welding and Robotic welding [14]

### 3.3 Robotic welding Robot cell

Robotic welding operation done for fabricating a machine head cover by welding the edge of the head cover is evaluated as a research model in order to apply and test developed methods for trajectory planning with CAD based offline programming approach for a robot manipulator. Three-dimensional (3D) design of the overall system and dimensions are illustrated in Figure 3.11 and Figure 3.12.


Figure 3.11: Robotic welding application - 3D model


Figure 3.12: Welding application - Layout

Figure 3.13 illustrates the dimensions of the machine head cover which is confronted for welding.


Figure 3.13: Machine Head Cover - Dimensions

Figure 3.14 shows a sample industrial Robotic welding torch used in the industry [15]. Figure 3.15 illustrates the dimensions of the welding torch attached to end of the robot end as an end effector.


Figure 3.14: Industrial Robotic welding torch [15]


Figure 3.15: Robotic Welding Torch - Dimensions
Figure 3.16 shows the robot end moving path along the cover edge which is followed by the robot end effector to get proper welding operation.


Figure 3.16: Robot End travelling path (marked in blue color)

## CHAPTER 4

## SYSTEM AND PROCESS DESIGN IDENTIFICATION

This chapter describes development approach over optimized methods of finding inverse kinematics of 6 DOF robot manipulator and trajectory planning done according to path planning using CAD systems. Section 4.1 illustrates the methods and mathematical approach for finding optimized inverse kinematic solution for the robot manipulator in order to find accurate joint space data of the robot while operating along the path. Section 4.2 illustrates the process of creating the robotic cell or the human robot interface in CAD environment where programmer can develop precise virtual model of the robot and the application in a software. Section 4.3 proposes methods to path planning for the particular application and information extraction in order to create robot moving sequences. Section 4.4 proposes the mathematical approach for mapping trajectories according to speed limitation with respect to path planning.

### 4.1 Mathematical approach for closed form solution of inverse kinematic of 6 DOF robot manipulator

Inverse kinematics problem of robot manipulator is very essential to solve to find joint angles of each joints of the robot locating at a position and orientation of the robot end effector. Robot users work in the cartesian space but the robot operates in the joint space. Therefore, inverse kinematics is defined as transformation from cartesian space to joint space. Finding joint angles is more important as robot end position is needed to be located precisely. Solving inverse kinematic is the practical complication of manipulator control and need numerical methods to solve. Inverse kinematics is more complex comparatively and numerous solutions may exist for the identical robot manipulator posture. Closed form solutions cannot be derived always since trigonometric nonlinear simultaneous equations are consisted with inverse kinematics of robot manipulator. There may not always exist solutions for inverse kinematics for a particular range of robot end effector posture. Numerical methods are used to derive
inverse kinematic solutions to find joint space values when kinematic equations are not possible to solve analytically [16].

Denso VP 6242 Robot is a 6 DOF robot manipulator consists of six revolute joints at each links. Representation of the Denavit-Hartenberg (D-H) model can be used to model connections of the robot links and joints [17]. It is needed for finding solutions for forward kinematics and inverse kinematics.

The base of the manipulator is link 0 and not considered one of the six links generally. Link 1 is connected to the base link by joint 1 . Links are maintaining a fixed relationship with joints at each link end. The common normal distance is $a_{i}$ (length) and $\alpha_{i}$ (twist) is the angle between the axes in a plane perpendicular to $\mathrm{a}_{\mathrm{i}}$. The distance between each joint is denoted as $d_{i}$ and the angle between normal of each joint is denoted as $\theta_{\mathrm{i}}$. Coordinates can be assigned for the VP 6242 robot as illustrated in Figure 4.1.


Figure 4.1: Coordinate assignment - Denso VP 6242

According the coordinates assigned for the robot manipulator, relevant parameters can be assigned [Table 2].

Table 2: Denso VP 6242 robot manipulator D-H parameters

| Joint <br> $\mathbf{i}$ | $\boldsymbol{\alpha}_{\mathbf{i} \mathbf{- 1}}$ | $\mathbf{a}_{\mathbf{i} \mathbf{- 1}}$ | $\mathbf{d}_{\mathbf{i}}$ | $\boldsymbol{\theta}_{\mathbf{i}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 0 | 0 | 0 | $\theta_{1}$ |
| $\mathbf{2}$ | $90^{\circ}$ | 0 | 0 | $90^{\circ}+\theta_{2}$ |
| $\mathbf{3}$ | 0 | 210 | 0 | $90^{\circ}+\theta_{3}$ |
| $\mathbf{4}$ | $90^{\circ}$ | 75 | 210 | $\theta_{4}$ |
| $\mathbf{5}$ | $-90^{\circ}$ | 0 | 0 | $\theta_{5}$ |
| $\mathbf{6}$ | $90^{\circ}$ | 0 | 70 | $\theta_{6}$ |

### 4.1.1 Robot manipulator forward kinematic

Determining forward kinematic problem is finding robot end effector position, orientation given by joint angles. Every joint is consisted of position, orientation values relative to its previous joint values. Transformation matrices denote these relations. Following equation represents a general formulation for transformation matrix calculation.
$T_{i}^{i-1}=R_{x}\left(\alpha_{i-1}\right) D_{x}\left(a_{i-1}\right) R_{z}\left(\theta_{i}\right) D_{z}\left(d_{i}\right)$

Where
$R_{X}\left(\alpha_{i-1}\right)=$ rotation matrix about the $X$ axis by $\alpha_{i-1}$
$D_{X}\left(a_{i-1}\right)=$ translation matrix along the $X$ axis by $a_{i-1}$
$\mathrm{R}_{\mathrm{z}}\left(\theta_{\mathrm{i}}\right)=$ rotation matrix about the Z axis by $\theta_{\mathrm{i}}$
$\mathrm{D}_{\mathrm{z}}\left(\mathrm{d}_{\mathrm{i}}\right)=$ translation matrix along the Z axis by $\mathrm{d}_{\mathrm{i}}$
a, $\alpha, \theta, d$ are manipulator $\mathrm{D}-\mathrm{H}$ parameters.
$\mathrm{R}_{\mathrm{x}}\left(\alpha_{\mathrm{i}-1}\right)=\left[\begin{array}{cccc}1 & 0 & 0 & 0 \\ 0 & \cos \alpha_{i-1} & -\sin \alpha_{i-1} & 0 \\ 0 & \sin \alpha_{i-1} & \sin \alpha_{i-1} & 0 \\ 0 & 0 & 0 & 1\end{array}\right]$
$D_{x}\left(a_{i-1}\right)=\left[\begin{array}{cccc}1 & 0 & 0 & a_{i-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]$
$\mathrm{R}_{\mathrm{z}}\left(\theta_{\mathrm{i}}\right)=\left[\begin{array}{cccc}\cos \theta_{i} & -\sin \theta_{i} & 0 & 0 \\ \sin \theta_{i} & \cos \theta_{i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]$
$\mathrm{D}_{\mathrm{z}}\left(\mathrm{d}_{\mathrm{i}}\right)=\left[\begin{array}{cccc}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \mathrm{~d}_{i} \\ 0 & 0 & 0 & 1\end{array}\right]$

And T ${ }_{i}^{i-1}$ is
$\left[\begin{array}{cccc}\cos \theta_{i} & -\sin \theta_{i} & 0 & \mathrm{a}_{i-1} \\ \cos \alpha_{i-1} \sin \theta_{i} & \cos \alpha_{i-1} \cos \theta_{i} & -\sin \alpha_{i-1} & -\mathrm{d}_{i} \sin \alpha_{i-1} \\ \sin \alpha_{i-1} \sin \theta_{i} & \sin \alpha_{i-1} \cos \theta_{i} & \cos \alpha_{i-1} & \mathrm{~d}_{i} \cos \alpha_{i-1} \\ 0 & 0 & 0 & 1\end{array}\right]$
$\mathrm{T}_{1}^{0}=\left[\begin{array}{cccc}C_{1} & -S_{1} & 0 & a_{1} \\ S_{1} & C_{1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]$
$\mathrm{T}_{2}^{1}=\left[\begin{array}{cccc}C_{2} & -S_{2} & 0 & a_{2} \\ 0 & 0 & -1 & 0 \\ S_{2} & C_{2} & 0 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]$
(8)

$$
\begin{align*}
\mathrm{T}_{3}^{2} & =\left[\begin{array}{cccc}
C_{3} & -S_{3} & 0 & 210 \\
S_{3} & C_{3} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right] \\
\mathrm{T}_{4}^{3} & =\left[\begin{array}{cccc}
C_{4} & -S_{4} & 0 & 75 \\
0 & 0 & -1 & -210 \\
S_{4} & C_{4} & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{10}\\
\mathrm{T}_{5}^{4} & =\left[\begin{array}{cccc}
C_{5} & -S_{5} & 0 & 0 \\
0 & 0 & -1 & 0 \\
S_{5} & C_{5} & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{11}\\
\mathrm{T}_{6}^{5} & =\left[\begin{array}{cccc}
C_{6} & -S_{6} & 0 & 0 \\
0 & 0 & -1 & -70 \\
S_{6} & C_{6} & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right] \tag{12}
\end{align*}
$$

Hence, multiplication of matrices $(7,8,9,10,11,12)$ calculates the transformation matrix $\mathrm{T}_{6}^{0}$ that gives end effector position, orientation with respect to frame 0 (13).
$\mathrm{T}_{6}^{0}=\mathrm{T}_{1}^{0} \mathrm{~T}_{2}^{1} \mathrm{~T}_{3}^{2} \mathrm{~T}_{4}^{3} \mathrm{~T}_{5}^{4} \mathrm{~T}_{6}^{5}$

### 4.1.2 Robot manipulator inverse kinematic

Determining robot manipulator inverse kinematic is finding robot joint angles when end effector position, orientation location known.

Considering Denso VP 6242 robot manipulator, last three joints axes intersect at one point and it is referred as point A [figure 19]. The point A position can be considered as independent of the consecutive last three joints $\theta_{4}, \theta_{5}, \theta_{6}$. Hence, previous three joints are considered when determining Point A position.

End effector position and orientation $=\left[\mathrm{P}_{\mathrm{x}}, \mathrm{P}_{\mathrm{y}}, \mathrm{P}_{\mathrm{z}}, \gamma, \beta, \alpha\right]^{\mathrm{T}}$

Where
$\alpha$ is the angle between $\mathrm{Z}_{6}$ and $\mathrm{Z}_{0}\left(\mathrm{Z}\right.$ axis of $6^{\text {th }}$ and $1^{\text {st }}$ joint $)$
$\beta$ is the angle between $Y_{6}$ and $Y_{0}\left(Y\right.$ axis of $6^{\text {th }}$ and $1^{\text {st }}$ joint) $\gamma$ is the angle between $\mathrm{X}_{6}$ and $\mathrm{X}_{0}\left(\mathrm{X}\right.$ axis of $6^{\text {th }}$ and $1^{\text {st }}$ joint $)$

The Point A position is denoted as $\mathrm{P}_{\mathrm{a}}=\left[\mathrm{P}_{\mathrm{ax}}, \mathrm{P}_{\mathrm{ay}}, \mathrm{P}_{\mathrm{az}}\right]^{\mathrm{T}}$
$P_{a}$ can be described as,

$$
\begin{aligned}
& P_{a x}=P_{x}-d_{6} * a_{x} \\
& P_{a y}=P_{y}-d_{6} * a_{y} \\
& P_{a z}=P_{z}-d_{6} * a_{z}
\end{aligned}
$$

Where

$$
\mathrm{a}_{\mathrm{x}}=\overline{\mathrm{Z}_{6}} \cdot \overline{\mathrm{X}_{0}} \quad \mathrm{a}_{\mathrm{y}}=\overline{\mathrm{Z}_{6}} \cdot \overline{\mathrm{Y}_{0}} \quad \mathrm{a}_{\mathrm{z}}=\overline{\mathrm{Z}_{6}} \cdot \overline{\mathrm{Z}_{0}}
$$

Solution for the $\theta_{1}, \theta_{2}$ and $\theta_{3}$ can be derived as follows.
$\mathrm{P}_{\mathrm{a}}$, Point A position can be derived from homogeneous transformation matrix $\mathrm{T}_{4}^{0}$ derived from $\mathrm{T}_{1}^{0} \mathrm{~T}_{2}^{1} \mathrm{~T}_{3}^{2}$ and $\mathrm{T}_{4}^{3}$.
$\mathrm{T}_{4}^{0}=\mathrm{T}_{1}^{0} \mathrm{~T}_{2}^{1} \mathrm{~T}_{3}^{2} \mathrm{~T}_{4}^{3}=\left[\begin{array}{cccc}r_{11} & r_{12} & r_{13} & \mathrm{P}_{a x} \\ r_{21} & r_{22} & r_{23} & \mathrm{P}_{a y} \\ r_{31} & r_{32} & r_{33} & \mathrm{P}_{a z} \\ 0 & 0 & 0 & 1\end{array}\right]$
Where

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\(r_{11}=\mathrm{S}_{1} \mathrm{~S}_{4}-\mathrm{C}_{4}\left(\mathrm{C}_{1} \mathrm{C}_{2} \mathrm{C}_{3}-\mathrm{C}_{1} \mathrm{~S}_{2} \mathrm{~S}_{3}\right)\)
\(r_{12}=\mathrm{C}_{4} \mathrm{~S}_{1}+\mathrm{S}_{4}\left(\mathrm{C}_{1} \mathrm{C}_{2} \mathrm{C}_{3}-\mathrm{C}_{1} \mathrm{~S}_{2} \mathrm{~S}_{3}\right)\)
\(r_{13}=-\mathrm{C}_{1} \mathrm{C}_{2} \mathrm{~S}_{3}-\mathrm{C}_{1} \mathrm{C}_{3} \mathrm{~S}_{2}\)
\(r_{21}=-\mathrm{C}_{1} \mathrm{~S}_{4}-\mathrm{C}_{4}\left(\mathrm{C}_{2} \mathrm{C}_{3} \mathrm{~S}_{1}-\mathrm{S}_{1} \mathrm{~S}_{2} \mathrm{~S}_{3}\right)\)
\(r_{22}=\mathrm{S}_{4}\left(\mathrm{C}_{2} \mathrm{C}_{3} \mathrm{~S}_{1}-\mathrm{S}_{1} \mathrm{~S}_{2} \mathrm{~S}_{3}\right)-\mathrm{C}_{1} \mathrm{C}_{4}\)
\(r_{23}=-\mathrm{C}_{2} \mathrm{~S}_{1} \mathrm{~S}_{3}-\mathrm{C}_{3} \mathrm{~S}_{1} \mathrm{~S}_{2}\)
\(r_{31}=-\mathrm{C}_{4}\left(\mathrm{C}_{2} \mathrm{~S}_{3}+\mathrm{C}_{3} \mathrm{~S}_{2}\right)\)
\(r_{32}=\mathrm{S}_{4}\left(\mathrm{C}_{2} \mathrm{~S}_{3}+\mathrm{C}_{3} \mathrm{~S}_{2}\right)\)
\(r_{33}=\mathrm{C}_{2} \mathrm{C}_{3}-\mathrm{S}_{2} \mathrm{~S}_{3}\)
\(\mathrm{P}_{a x}=75 \mathrm{C}_{1} \mathrm{~S}_{2} \mathrm{~S}_{3}-75 \mathrm{C}_{1} \mathrm{C}_{2} \mathrm{C}_{3}-210 \mathrm{C}_{1} \mathrm{C}_{2} \mathrm{~S}_{3}-210 \mathrm{C}_{1} \mathrm{C}_{3} \mathrm{~S}_{2}-210 \mathrm{C}_{1} \mathrm{~S}_{2}\)
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$\mathrm{P}_{a y}=75 \mathrm{~S}_{1} \mathrm{~S}_{2} \mathrm{~S}_{3}-75 \mathrm{C}_{2} \mathrm{C}_{3} \mathrm{~S}_{1}-210 \mathrm{C}_{2} \mathrm{~S}_{1} \mathrm{~S}_{3}-210 \mathrm{C}_{3} \mathrm{~S}_{1} \mathrm{~S}_{2}-210 \mathrm{~S}_{1} \mathrm{~S}_{2}$
$\mathrm{P}_{a z}=210 \mathrm{C}_{2}+210 \mathrm{C}_{2} \mathrm{C}_{3}-75 \mathrm{C}_{2} \mathrm{~S}_{3}-75 \mathrm{C}_{3} \mathrm{~S}_{2}-210 \mathrm{~S}_{2} \mathrm{~S}_{3}$

Where $S_{i}=\sin \theta_{i}$ and $C_{i}=\cos \theta_{i}$
$(14) \times\left(\mathrm{S}_{1}\right)-(15) \times\left(\mathrm{C}_{1}\right)=0$
Therefore,
$\mathrm{P}_{\mathrm{ax}} \times \mathrm{S}_{1}=\mathrm{P}_{\mathrm{ay}} \times \mathrm{C}_{1}$
$\frac{\mathrm{S}_{1}}{c_{1}}=\frac{\mathrm{P}_{a y}}{\mathrm{P}_{a x}}$
$\tan \theta_{1}=\frac{\mathrm{P}_{a y}}{\mathrm{P}_{a x}}$ hence $\theta_{1}=\tan ^{-1}\left(\frac{\mathrm{P}_{a y}}{\mathrm{P}_{a x}}\right)$
by $(14) \times\left(\mathrm{C}_{1}\right)+(15) \times\left(\mathrm{S}_{1}\right)$, the following is obtained.
$\mathrm{P}_{\mathrm{ax}} \mathrm{C}_{1}+\mathrm{P}_{\mathrm{ay}} \mathrm{S}_{1}=75 \mathrm{~S}_{2} \mathrm{~S}_{3}-75 \mathrm{C}_{2} \mathrm{C}_{3}-210 \mathrm{C}_{2} \mathrm{~S}_{3}-210 \mathrm{C}_{3} \mathrm{~S}_{2}-210 \mathrm{~S}_{2}$
By taking $\mathrm{Pax}_{\mathrm{ax}} \times \mathrm{C}_{1}+\mathrm{P}_{\mathrm{ay}} \times \mathrm{S}_{1}=\mathrm{m}$, the following can be obtained.
Therefore from (18) : $-75 \mathrm{C}_{23}-210 \mathrm{~S}_{23}-210 \mathrm{~S}_{2}=\mathrm{m}$
Where $\mathrm{C}_{23}=\operatorname{Cos}\left(\theta_{2}+\theta_{3}\right)$

$$
\begin{gather*}
\mathrm{S}_{23}=\operatorname{Sin}\left(\theta_{2}+\theta_{3}\right) \\
-75 \mathrm{C}_{23}-210 \mathrm{~S}_{23}=\mathrm{m}+210 \mathrm{~S}_{2} \tag{19}
\end{gather*}
$$

From (16): $210 \mathrm{C}_{23}-75 \mathrm{~S}_{23}=\mathrm{P}_{\mathrm{az}}-210 \mathrm{C}_{2}$
From (19) and (20), the following is obtained.
$\mathrm{C}_{23}=\frac{14 \mathrm{P}_{a z}}{3315}-\frac{\mathrm{m}}{663}-\frac{70 \mathrm{~S}_{2}}{221}-\frac{196 \mathrm{C}_{2}}{221}$
$S_{23}=-\frac{\mathrm{P}_{a z}}{663}-\frac{14 \mathrm{~m}}{3315}-\frac{196 \mathrm{~S}_{2}}{221}+\frac{70 \mathrm{C}_{2}}{221}$
Further simplifies by substituting following,
$\mathrm{n}=\frac{14 \mathrm{P}_{a z}}{3315}-\frac{\mathrm{m}}{663}$ and $\mathrm{o}=\frac{-\mathrm{P}_{a z}}{663}-\frac{14 m}{3315}$
then equations (21) and (22) can be simplified as follows.
$\mathrm{C}_{23}=\mathrm{n}-\frac{70 \mathrm{~S}_{2}}{221}-\frac{196 \mathrm{C}_{2}}{221}$
$S_{23}=0-\frac{196 S_{2}}{221}+\frac{70 C_{2}}{221}$
substituting the equations (23) and (24) to $\mathrm{S}_{23}{ }^{2}+\mathrm{C}_{23}{ }^{2}=1$
then the following can be obtained.
$\frac{196 c_{2}^{2}}{221}-\frac{392 \mathrm{C}_{2} n}{221}+\frac{140 \mathrm{C}_{2} o}{221}+\mathrm{n}^{2}-\frac{140 \mathrm{~S}_{2} n}{221}+\mathrm{o}^{2}-\frac{392 \mathrm{~S}_{2} o}{221}+\frac{196 s_{2}^{2}}{221}=1$

Equation (25) can be simplified by taking $\mathrm{p}=\mathrm{n}^{2}+\mathrm{o}^{2}+\frac{196}{221}$
Substituting p then
$-\frac{392 \mathrm{c}_{2} n}{221}+\frac{140 \mathrm{c}_{2} o}{221}-\frac{140 \mathrm{~s}_{2} n}{221}-\frac{392 \mathrm{~s}_{2} o}{221}=1-\mathrm{p}$
$a S_{2}+b C_{2}=c$
by substituting $\mathrm{a}=-\frac{140 n}{221}-\frac{392 o}{221}, \mathrm{~b}=-\frac{392 n}{221}+\frac{140 o}{221}$ and $\mathrm{c}=1-\mathrm{p}$ for equation (26)
then the joint 2 angle $\theta_{2}$ can be calculated as follows.
$\theta_{2}=\operatorname{Atan} 2(a, b) \pm \operatorname{Atan} 2\left(\left(a^{2}+b^{2}-c^{2}\right)^{1 / 2}, c\right)$
from equation (19): $\quad-75 C_{23}-210 S_{23}=m+210 S_{2}$

Considering equation $\quad e_{23}+\mathrm{f}_{23}=\mathrm{g}$
by substituting $e=-210, f=-75$ and $g=m+210 S_{2}$ for equation (28)
then, $\theta_{2}+\theta_{3}=A \tan 2(e, f) \pm A \tan 2\left(\left(e^{2}+\mathrm{f}^{2}-\mathrm{g}^{2}\right)^{1 / 2}, \mathrm{~g}\right)$

Hence the joint 3 angle value can be derived as follows.
$\theta_{3}=\operatorname{Atan} 2(e, f) \pm \operatorname{Atan} 2\left(\left(\mathrm{e}^{2}+\mathrm{f}^{2}-\mathrm{g}^{2}\right)^{1 / 2}, \mathrm{~g}\right)-\theta_{2}$

## Solutions for $\boldsymbol{\theta}_{4}, \boldsymbol{\theta}_{5}$ and $\boldsymbol{\theta}_{6}$

End effector orientation is defined by $\mathrm{R}_{6}^{0}$. Since robot orientation is described by rotation matrix, position A orientation is defined by $R_{3}^{0}$.
$\mathrm{R}_{6}^{0}$ can be described as $\mathrm{R}_{6}^{0}=\mathrm{R}_{3}^{0} \mathrm{R}_{6}^{3}$
Matrix $R{ }_{6}^{3}$ can be described as
$\mathrm{R}_{6}^{3}=\mathrm{R}_{4}^{3} \mathrm{R}_{5}^{4} \mathrm{R}_{6}^{5}=\left[\begin{array}{ccc}c_{4} c_{5}^{2}-s_{4} s_{6} & -s_{4} c_{5}-c_{4} c_{5} s_{6} & c_{4} s_{5} \\ s_{5} c_{5} & -s_{5} s_{6} & -c_{5} \\ s_{4} c_{5}^{2}+c_{4} s_{6} & c_{4} c_{5}-s_{4} c_{5} s_{6} & s_{4} s_{5}\end{array}\right]=\left[\begin{array}{lll}n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33}\end{array}\right]$

Where $\mathrm{S}_{\mathrm{i}}=\sin \left(\theta_{\mathrm{i}}\right)$ and $\mathrm{C}_{\mathrm{i}}=\cos \left(\theta_{\mathrm{i}}\right)$
Then the joint 4 angle can be calculated as follows.
$\theta_{4}=\tan ^{-1} \frac{n_{33}}{n_{13}}$ when $\theta_{5} \neq 0$
when $\theta_{5}=0$, the link axes are in collinear which the arms are at singular position. In this condition, there is only one motion of robot end effector orientation which can be calculated by sum or difference of $\theta_{4}$ and $\theta_{6}$. In most cases, current $\theta_{4}$ value is considered.
$\mathrm{T}_{6}^{4}$ Can be derived as follows.
$\mathrm{T}_{6}^{4}=\mathrm{T}_{4}^{0}{ }^{-1} \mathrm{~T}_{6}^{0}$ since $\theta_{1}, \theta_{2}, \theta_{3}, \theta_{4}$ and $\mathrm{T}_{6}^{0}$ are known.
$\mathrm{T}_{6}^{4}$ can be described as

$$
\mathrm{T}_{6}^{4}=\mathrm{T}_{5}^{4} \mathrm{~T}_{6}^{5}=\left[\begin{array}{cccc}
c_{5} c_{6} & -c_{5} s_{6} & s_{5} & 70 s_{5} \\
s_{6} & c_{6} & 0 & 0 \\
-s_{5} c_{6} & s_{5} s_{6} & c_{5} & 70 c_{5} \\
0 & 0 & 0 & 1
\end{array}\right]
$$

Where $\mathrm{S}_{\mathrm{i}}=\sin \left(\theta_{\mathrm{i}}\right)$

$$
\mathrm{C}_{\mathrm{i}}=\cos \left(\theta_{\mathrm{i}}\right)
$$

Therefore joint 5 angle $\theta_{5}$ can be calculated as follows.
$\left.\theta_{5}=\sin ^{-1} \mathrm{~T}_{6}^{4}{ }^{4}, 3\right)$
$\mathrm{T}_{6}^{5}$ Can be derived as follows.
$\mathrm{T}_{6}^{5}=\mathrm{T}_{5}^{0^{-1}} \mathrm{~T}_{6}^{0}$ since $\theta_{1}, \theta_{2} \theta_{3}, \theta_{4}, \theta_{5}$ and $\mathrm{T}_{6}^{0}$ are known.
$\mathrm{T}_{6}^{5}$ can be described as follows.
$\mathrm{T}_{6}^{5}=\left[\begin{array}{cccc}c_{5} & -s_{5} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_{5} & c_{5} & 0 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]$

Therefore, joint 6 angle $\theta_{6}$ can be calculated as,

$$
\begin{equation*}
\theta_{6}=\sin ^{-1} \mathrm{~T}_{6}^{5}(1,2) \tag{33}
\end{equation*}
$$

### 4.2 The Human Robot interface

Denso VP 6242 robot manipulator can be virtually developed in a three dimensional graphical user interface using available CAD software interfaces which is high accurate and user friendly. Solidworks [18] is used to develop the CAD models. Model for each links of the robot manipulator can be designed and modelled [Figure 4.2]. 3D design is developed on $1: 1$ scale since it is used for creating simulation model in the simulation interface.


Figure 4.2: Robot links 3D design models

Created 3D link models can be assembled in Solidworks in order to create the complete robot assembly [Figure 4.3].


Figure 4.3: Robot manipulator assembly in Solidworks
Coordinate systems can be assigned to the relevant joint axes which are needed to describe the relative position and orientation of each link [Figure 4.4]. Work coordinate system can be assigned relative to developed mathematical model in section 4.1.


Figure 4.4: Coordinate system assignment in 3D model

Tool which is attached to the robot end can be modelled on 1:1 scale [Figure 4.5].


Figure 4.5:
Tool design - Welding torch

The machine head cover can be modelled on 1:1 scale [Figure 4.6].


Figure 4.6: Machine head cover - 3D design
The head cover can be located according the actual robot cell dimensions [Figure 4.7].


Figure 4.7: Machine Head cover location
Complete robot cell is illustrated in Figure 4.8.


Figure 4.8: Complete 3D design for the robot cell

### 4.3 Path planning and information extraction

Robot end moving line can be defined using a spline which is accurate to describe the path efficiently [Figure 4.9].


Figure 4.9: Spline feature for robot moving path definition

Points are located along the line which assigned to represent robot moving sequence [Figure 4.10].


Figure 4.10: Point allocation along the robot path
Position values of these points relative to origin can be used for calculating forward and inverse kinematics [Section 4.1]. $\mathrm{X}, \mathrm{Y}$ and Z coordinate values are extracted from the CAD spline. VB (Visual Basic) based macro is developed and run in Solidworks to generate the point coordinate values [Figure 4.11]. VB based macro is also capable to generate excel sheet with x y z point values [Figure 4.12].


Figure 4.11: VB based macro for data extraction


Figure 4.12: Data extracted values in Microsoft excel

### 4.4 Mathematical approach for mapping trajectories

Joint space trajectory generation in robotic field is commonly using for arranging smooth motion between one set of joint angles with another set such as for travelling between two specific cartesian postures having two joint angles sets for each posture. This has to be done simultaneously while generating for all joints independently.

Generally initial assumptions are considered as two discrete joint value sets are known and the requirement is to move between those two joint angles sets smoothly in joint space. The velocity and acceleration need to vary smoothly in order to maintain optimized robot movement without existing infinite jerks which make robot manipulators inaccurate operations, high vibration and wear.

Each of joints has a motor drive which is connected to rotate the joint according to control input by the robot controller system. The motor should be controlled under technical specifications such as maximum angular velocity, joint motion range, maximum inertia movement [11]. Each motor is limited with maximum angular velocity and controllers should manipulate the robot arm under these limitations.

A trajectory can be specified by assigning initial and final conditions on a time period, position, velocity, acceleration etc. Then, trajectory planning can be determined as a function so that the required conditions are satisfied. This is considered as a boundary condition problem which can be explained by considering polynomial functions such as:
$q(t)=a_{0}+a_{1} t+a_{2} t^{2}+\ldots+a_{n} t^{n}$
The degree n of the polynomial depends on the number of boundary conditions which should be determined by the trajectory smoothness we require. Given an initial and a final time period i.e. $t_{i}$ and $t_{\mathrm{f}}$, a trajectory segment can be specified by assigning initial and final conditions:
initial position and velocity $\mathrm{q}_{\mathrm{i}}, \dot{\mathrm{q}}_{\mathrm{i}}$
final position and velocity $\mathrm{q}_{\mathrm{f}}, \dot{\mathrm{q}}_{\mathrm{f}}$
A polynomial of degree 3 can be considered since there are four boundary conditions. From the studies [10], it may be noticed that position and velocity profiles are continuous functions of time but cannot be true for the acceleration so that discontinuities among different segments can be occurred. Besides, there is no possibility to specify initial and final values for each segment. In some cases, this is not a major problem and it is enough to have smooth trajectories. But for most cases, acceleration initial and final values for obtaining acceleration profiles is required for avoiding possible infinite jerks of non-continuous jerk profiles. Therefore, fifth order polynomial functions should be considered.
$q(t)=a_{0}+a_{1} t+a_{2} t^{2}+a_{3} t^{3}+a_{4} t^{4}+a_{5} t^{5}$

Boundary conditions are defined as follows:
$q\left(t_{i}\right)=q_{i}$ : initial position at time $t_{i} \quad q\left(t_{f}\right)=q_{f}$ : final position at time $t_{f}$
$\dot{\mathrm{q}}\left(\mathrm{t}_{\mathrm{i}}\right)=\dot{\mathrm{q}}_{\mathrm{i}}:$ initial velocity at time $\mathrm{t}_{\mathrm{i}} \quad \dot{\mathrm{q}}\left(\mathrm{t}_{\mathrm{f}}\right)=\dot{\mathrm{q}}_{\mathrm{f}}:$ final velocity at time $\mathrm{t}_{\mathrm{f}}$
$\ddot{\mathrm{q}}\left(\mathrm{t}_{\mathrm{i}}\right)=\ddot{\mathrm{q}}_{\mathrm{I}}$ : initial acceleration at time $\mathrm{t}_{\mathrm{i}} \quad \ddot{\mathrm{q}}\left(\mathrm{t}_{\mathrm{f}}\right)=\ddot{\mathrm{q}}_{\mathrm{f}}$ : final acceleration at time $\mathrm{t}_{\mathrm{f}}$ The coefficient of the polynomial can be derived.

In this case, the coefficients of the polynomial are derived as follows.
$\mathrm{a}_{0}=\mathrm{q}_{\mathrm{i}}$
$\mathrm{a}_{1}=\dot{\mathrm{q}}_{\mathrm{i}}$
$\mathrm{a}_{2}=\frac{1}{2} \ddot{\mathrm{q}}_{\mathrm{i}}$
$\mathrm{a}_{3}=\frac{1}{2 \mathrm{~T}^{3}}\left[20\left(\mathrm{q}_{\mathrm{f}}-\mathrm{q}_{\mathrm{i}}\right)-\left(8 \dot{\mathrm{q}}_{\mathrm{f}}+12 \dot{\mathrm{q}}_{\mathrm{i}}\right) \mathrm{T}-\left(3 \ddot{\mathrm{q}}_{\mathrm{f}}-\ddot{\mathrm{q}}_{\mathrm{i}}\right) \mathrm{T}^{2}\right]$
$\mathrm{a}_{4}=\frac{1}{2 \mathrm{~T}^{4}}\left[30\left(\mathrm{q}_{\mathrm{i}}-\mathrm{q}_{\mathrm{f}}\right)+\left(14 \dot{\mathrm{q}}_{\mathrm{f}}+16 \dot{\mathrm{q}}_{\mathrm{i}}\right) \mathrm{T}+\left(3 \ddot{\mathrm{q}}_{\mathrm{f}}-2 \ddot{\mathrm{q}}_{\mathrm{i}}\right) \mathrm{T}^{2}\right]$
$\mathrm{a}_{5}=\frac{1}{2 \mathrm{~T}^{5}}\left[12\left(\mathrm{q}_{\mathrm{f}}-\mathrm{q}_{\mathrm{i}}\right)-6\left(\dot{\mathrm{q}}_{\mathrm{f}}+\dot{\mathrm{q}}_{\mathrm{i}}\right) \mathrm{T}-\left(\ddot{\mathrm{q}}_{\mathrm{f}}-\ddot{\mathrm{q}}_{\mathrm{i}}\right) \mathrm{T}^{2}\right]$
where $T=t_{f}-t_{i}$
Velocity, acceleration and jerk profile are derived as follows.
$\dot{q}(t)=a_{1}+2 a_{2} t+3 a_{3} t^{2}+4 a_{4} t^{3}+5 a_{5} t^{4}$ (fourth order polynomial)
$\ddot{\mathrm{q}}(\mathrm{t})=2 \mathrm{a}_{2}+6 \mathrm{a}_{3} \mathrm{t}+12 \mathrm{a}_{4} \mathrm{t}^{2}+20 \mathrm{a}_{5} \mathrm{t}^{3}$ (third order polynomial)
$\ddot{\mathrm{q}}(\mathrm{t})=6 \mathrm{a}_{3}+24 \mathrm{a}_{4} \mathrm{t}+60 \mathrm{ast}^{2}($ second order polynomial $)$
Robot position is intended to travel point to point movement along the path via assigned positions. Boundary conditions of the trajectories are defined as follows.
$\dot{\mathrm{q}}_{\mathrm{i}}=0$ : initial velocity at time $\mathrm{t}_{\mathrm{i}} \quad \dot{\mathrm{q}}_{\mathrm{f}}=0:$ final velocity at time $\mathrm{t}_{\mathrm{f}}$
$\ddot{q}_{\mathrm{i}}=0$ : initial acceleration at time $\mathrm{t}_{\mathrm{i}} \quad \quad \ddot{\mathrm{q}}_{\mathrm{f}}=0:$ final acceleration at time $\mathrm{t}_{\mathrm{f}}$
Figure 4.13 illustrates Typical trajectory profile for position variation.


Figure 4.13: Typical trajectory profile for position variation
Figure 4.14 illustrates typical trajectory profile for velocity variation.


Figure 4.14: Typical trajectory profile for velocity variation
Figure 4.15 illustrates typical trajectory profile for acceleration variation.


Figure 4.15: Typical trajectory profile for acceleration variation
Figure 4.16 illustrates typical trajectory profile for Jerk variation.


Figure 4.16: Typical trajectory profile for jerk variation

From the equations (34) to (39) with boundary conditions assigned, following expressions can be derived.
$\mathrm{a}_{0}=\mathrm{q}_{\mathrm{i}}$
$\mathrm{a}_{2}=0$
$\mathrm{a}_{3}=\frac{1}{2 \mathrm{~T}^{3}}\left[20\left(\mathrm{q}_{\mathrm{f}}-\mathrm{q}_{\mathrm{i}}\right)\right]$
$\mathrm{a}_{4}=\frac{1}{2 \mathrm{~T}^{4}}\left[30\left(\mathrm{q}_{\mathrm{i}}-\mathrm{q}_{\mathrm{f}}\right)\right]$
$\mathrm{a}_{5}=\frac{1}{2 \mathrm{~T}^{5}}\left[12\left(\mathrm{q}_{\mathrm{f}}-\mathrm{q}_{\mathrm{i}}\right)\right]$
Maximum velocity of a particular joint is reached in the middle of the time duration [Figure 32]. Therefore, maximum velocity can be derived as follows.

Applying solutions to the equation (40):
$\dot{q}(t)=3 \frac{1}{2 T^{3}}\left[20\left(q_{f}-q_{i}\right)\right] t^{2}+4 \frac{1}{2 T^{4}}\left[30\left(q_{i}-q_{f}\right)\right] t^{3}+5 \frac{1}{2 T^{5}}\left[12\left(q_{f}-q_{i}\right)\right] t^{4}$
when $\mathrm{t}=\frac{\mathrm{T}}{2}$
Then,
$\dot{\mathrm{q}}(\mathrm{t})_{\max }=3 \frac{1}{2 \mathrm{~T}^{3}}\left[20\left(\mathrm{q}_{\mathrm{f}}-\mathrm{q}_{\mathrm{i}}\right)\right]\left(\frac{\mathrm{T}}{2}\right)^{2}+4 \frac{1}{2 \mathrm{~T}^{4}}\left[30\left(\mathrm{q}_{\mathrm{i}}-\mathrm{q}_{\mathrm{f}}\right)\right]\left(\frac{\mathrm{T}}{2}\right)^{3}+5 \frac{1}{2 \mathrm{~T}^{5}}\left[12\left(\mathrm{q}_{\mathrm{f}}-\mathrm{q}_{\mathrm{i}}\right)\right]\left(\frac{\mathrm{T}}{2}\right)^{4}$
$\dot{\mathrm{q}}(\mathrm{t})_{\max }=\frac{15\left(\mathrm{q}_{\mathrm{f}}-\mathrm{q}_{\mathrm{i}}\right)}{8 \mathrm{~T}}$
therefore, T (time period from one position to another) can be calculated for a particular speed limit.
$\mathrm{T} \geq \frac{15\left(\mathrm{q}_{\mathrm{f}}-\mathrm{q}_{\mathrm{i}}\right)}{8 \dot{\mathrm{q}}(\mathrm{t})_{\text {max }}}$
Maximum velocity should be considered when the robot manipulator is operated with a significant operating speed such that when robot is moving from one cartesian location to another position in a limited time period. Variation of each joint speed is varied depends on the location and robot arm configuration and speed limitation is
defined in order protect from possible breakdowns and enhance the robot operation. The robot user can realize the possible speed limitations by mapping maximum speed of each joint while moving along the path via assigned position points. Then the position points can be allocated according to the observations in order to minimize the joint speed variation and optimize the system in order to increase the productivity.

## CHAPTER 5

## SYSTEM DESIGN AND SIMULATION

Mathematical models developed for inverse kinematics are described in this chapter and trajectory planning solutions proposed in Chapter 4. Section 5.1 presents the procedure to implement inverse kinematic solution for the 6 DOF robot manipulator which is needed to derive joint space data for further development of trajectory planning. Section 5.2 presents the procedure to implement trajectory planning scheme developed for moving robot arm effectively. Development of robot cell to trial in a Simulation environment is illustrated in Section 5.3 which clarifies the accuracy and the effectiveness of the mathematical solutions and schemes developed.

### 5.1 Design and Implementation of Inverse kinematic model

Algorithms can be developed using derived mathematical models developed in section 4. Matlab is a software tool used for analyzing data, developing algorithms, or creating models [19].

The inverse kinematic model for the Denso VP 6242 robot arm has been developed considering without end effector attachment. But we can see tools attached to the robot arm end for many industrial uses such as painting nozzle, welding torch, cutter unit. In this application, welding torch has been applied as an end effector. Therefore, the tool end is subjected to move along the path. Path and Points can be assigned according to user's requirements [Figure 5.1].


Figure 5.1: Path and point allocation along the robot moving path

Robot end tool position and orientation are needed to define for calculating and perform developed algorithm. Tool position is same as the point location of the defined path. Orientation is defined as follows and the orientation is needed to persist the same along the path [Figure 5.2 and Figure 5.3].


Figure 5.2: Tool orientation along the path


Figure 5.3: Tool orientation with respect to work 0 coordinate

The end-effector(tool) position, orientation with respect to base, known as $\mathrm{T}_{\mathrm{Tool}}^{0}$ is defined as follows.
$\mathrm{T}_{\mathrm{Tool}} \begin{gathered}0 \\ =\end{gathered}\left[\begin{array}{cc}R_{\text {Tool }}^{0} & P_{\text {Tool }}^{0} \\ 0 & 0\end{array} 0\right.$
Where $P_{\text {Tool }}^{0}=\left[\mathrm{X}_{\text {tool }}, \mathrm{Y}_{\text {tool }}, \mathrm{Z}_{\text {tool }}\right]^{\mathrm{T}}$ position coordinate and $R_{\text {Tool }}^{0}$ can be derived as follows.
$R_{\text {Tool }}^{0}=\mathrm{R}_{\mathrm{Z}}\left(180^{0}\right) \mathrm{R}_{\mathrm{Y}}\left(-90^{0}\right) \mathrm{RX}_{\mathrm{X}}\left(0^{0}\right)$

Then $\mathrm{T}_{\mathrm{Tool}} \stackrel{0}{\text { can be derived as } \mathrm{T}_{\text {Tool }}} \stackrel{0}{ }=\mathrm{T}_{6}^{0}{ }_{\mathrm{T}}^{\mathrm{T}}{ }_{\mathrm{Tool}}^{6}$
${ }_{\mathrm{T}}^{\mathrm{Tool}}{ }^{6}$ Tool position, orientation with respect to joint coordinate 6 is derived as follows.
$\mathrm{T}_{\mathrm{Tool}}^{6}=\left[\begin{array}{cccc}1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 130 \\ 0 & 0 & 0 & 1\end{array}\right]$ [Figure 5.4]


Figure 5.4: Tool dimensions
From equation (53), $\mathrm{T}_{6}^{0}$ can be derived as $\mathrm{T}_{6}^{0}=\mathrm{T}_{\mathrm{Tool}}{ }^{0} \mathrm{~T}_{\mathrm{Tool}}{ }^{6}$
$\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ cartesian point data can be derived from the Solidworks CAD environment by running a Visual Basic based macro [Section 4.3]. This macro exports cartesian point values of each point location only and we need to manually input the $\mathrm{Rx}, \mathrm{Ry}, \mathrm{Rz}$ rotation angles (around axes $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ ) for each point location or sequence which will
be needed for finding joint angles of the robot arm for each point location by developing algorithms in Matlab software. This rotation angles are the angles values defined in equation (52) [Figure 5.5].



Figure 5.5: Orientation defined by manually

The position of point A in the robot arm is needed to find in order to implement the developed algorithms in section 4.1.2. Matlab codes can be written to implement this procedure in order to find the $\mathrm{P}_{\mathrm{a}}$ location.

## Importing data extracted from the CAD features

```
C = xlsread('C:\Users\pcadmin\research\simulation1.xlsx','Sheet1');
tam = size(C);
Pa = [];
for i = 1 : tam
    T6EF = [11 0 0 10;0 1 0 0;0 0 1 130;0 0 0 1];
    X(i,1) = C(i,1);%X value
    Y(i,2) = C(i,2);%Y value
    Z(i,3) = C(i,3);%Z value
    al = C(i,6); %angle arond Z axis
    be = C(i,5);%angle around Y axis
    ga = C(i,4);%angle around X axis
    %Eular angle Z,Y,X
    Rz = [ cosd(al) -sind(al) 0;
        sind(al) cosd(al) 0;
        0 0 1;];
    Ry = cosd(be) 0 sind(be);
        0 1 0;
        -sind(be) 0 cosd(be);];
    Rx = [ 1 0 0;
        0 cosd(ga) -sind(ga);
        0 sind(ga) cosd(ga);];
    %Rotation Matrix
    A_R_B = Rz*Ry*Rx;
```

```
TT(:, :, i) = [A R B (1,1) A R B (1, 2) A R B(1, 3) X(i,1); A R B (2,1)
```

TT(:, :, i) = [A R B (1,1) A R B (1, 2) A R B(1, 3) X(i,1); A R B (2,1)
A_R_B(2,2) A_R_B (2,3) Y(i,2); A_R_B(3,1) A_R_B (3,2) A_R_B ( 3, 3)
A_R_B(2,2) A_R_B (2,3) Y(i,2); A_R_B(3,1) A_R_B (3,2) A_R_B ( 3, 3)
Z(i,3);
Z(i,3);
0 0 0 1];
0 0 0 1];
T6EFT = ((T6EF)^(-1));
T6EFT = ((T6EF)^(-1));
T(:, :, i) = TT(:, :, i)* T6EFT;
T(:, :, i) = TT(:, :, i)* T6EFT;
%Pa position
%Pa position
x = T(1,4,i)- (70)*T(1,3,i);%Pa x position
x = T(1,4,i)- (70)*T(1,3,i);%Pa x position
y = T(2,4,i)- (70)*T(2,3,i);%Pa y position
y = T(2,4,i)- (70)*T(2,3,i);%Pa y position
z = T(3,4,i)- (70)*T(3,3,i);%Pa z position
z = T(3,4,i)- (70)*T(3,3,i);%Pa z position
end

```
end
```


## Algorithms for finding joint angles $\boldsymbol{\theta}_{1}, \boldsymbol{\theta}_{2}$ and $\boldsymbol{\theta}_{3}$

## Implementing equation (17) from section 4.1.2

```
%Calculating theta 1
theta1 = atan2d(y,x);
if thetal == 180
    thetal = 0;
end
if (90<theta1) && (thetal<180)
    thetal = -(180 - theta1);
end
if (-180>theta1) && (theta1>-90)
    thetal = 180 + theta1;
end
%}
```

Implementing equation (27) and (30)

```
%Calculating theta 2 and theta 3
c1 = cosd(theta1);
s1 = sind(thetal);
m = (x* c1) + (y* s1);
n = (-1/663) + (14*z/3315);
o = ((-14*l)/3315) - (z/663);
p = (196/221) + (n^2) + (o^2);
a_=((-140*n)-(392*o))/221;
b_}=((140*o)-(392*n))/221
c = 1 - p;
theta2 = atan2(a_,b_) + atan2(((a_^2 + b_^^2 - c^2 ) )}\mp@subsup{)}{}{\wedge}(1/2),c)
theta2 = (theta2*180)/pi;
if theta2 ==360
    theta2 = 0;
end
if (180<theta2) && (theta2<360)
theta2 = 360 - theta2;
end
%Theta 3
e = -210.00;
f = -75.00;
g = l+(210*(sind(theta2)));
```

```
h = ((e^2 + f^2 - g^2) )^(1/2);
if isreal(h) == 1
    theta23_1 = atan2(e,f)+ atan2(h,g);
    theta23_1 = (theta23_1*180)/pi;
    theta_23_2 = atan2(e,f)- atan2(h,g);
    theta_23_2 = (theta_23_2*180)/pi;
    if (theta_23_2>-360) && (theta_23_2<-180)
        theta_23_2 = theta_23_2 + 360;
    end
    theta_3_1 = - theta2 + theta23_1;
    theta_3_2 = - theta2 + theta_23_2;
theta2_ = atan2(a_,b_) - atan2((a_^2 + b_^2 - c^2)^(1/2),c);
theta2_ = (theta2_*180)/pi;
if ( theta2_ > -360) && (theta2_ < -180)
    theta2_- = 360 + theta2_;
end
e = -210.00;
f = -75.00;
g_ = l+(210*(sind(theta2_)));
h_}=(\mp@subsup{e}{}{\wedge}2 + f^2 - g_^ 2)^(1/2)
if isreal(h_) == 1
    theta23_3 = atan2(e,f)+ atan2(h_,g_);
    theta23_3 = (theta23_3*180)/pi;
    theta23_4 = atan2(e,f)- atan2(h_,g_);
    theta23_4 = (theta23_3*180)/pi;
    theta_3_3 = - theta2_ + theta23_3;
    theta_3_4 = - theta2_ + theta23_4;
else
            theta_3_3 = 0 - theta2_;
    theta_3_4 = 0 - theta2_;
end
```


## Algorithms for finding joint angles $\boldsymbol{\theta}_{4}, \boldsymbol{\theta} 5$ and $\boldsymbol{\theta}_{6}$

```
%DH parameters
al_0=0;al_1=90,al2=0;al_3=90;al_4=-90;al_5=90;
a 0=0;a 1=0;a 2=210;a 3=75;a 4=0;a 5=0;
d_1 =0; d_}2=0;\mp@subsup{d}{_}{-}3=0;\mp@subsup{d__ 4}{=}{=}210;\mp@subsup{d}{_}{5}=0;\mp@subsup{d}{-}{\prime}=70
t_1=theta_1;t_2=90+theta_2;t_3=90+theta_3;
T1_0 [ cosd(t_1), -sind(t_1), 0,
a 0 ;
        sind(t_1)*}\operatorname{cosd}(al_0),\operatorname{cosd}(t_1)*\operatorname{cosd}(al_0)
        -sind(al_0)
-sind(al_0)*d_1;
        sind(t_1)*sind(al_0), cosd(t_1)*sind(al_0),
cosd(al_0)*d_1 ;
        0, 0
        0,
; ]
T2_1 = [ cosd(t_2)
-sind(t_2),
a_1
        sind(t_2)*}\operatorname{cosd}(al_1), cosd(t_2)*\operatorname{cosd}(al_1)
    -sind(al_1)
-sind(al_1)*d_2;
        sind(t_2)*sind(al_1), cosd(t_2)*sind(al_1)
cosd(al_1)*d_2;
    0, 0
                                    0,
; ]
T3_2 [ cosd(t_3), -sind(t_3),
        sind(t_3)*cosd(al_2), cosd(t_3)*cosd(al_2),
-sind(al_2)*d_3;
        sind(t_3)*sind(al_2), cosd(t_3)*sind(al2)
cosd(al 2)
cosd(al_2)*d_3 ;
        0, 0,
```

; ]

Implementing equation (31)

```
%Calculating theta4
T3_0 = T1_0*T2_1*T3_2;
R03 = [T3_0(1,1:3);
    T\overline{3}0(2,1:3);
    T3_0(3,1:3);];
R03T = transpose(R03);
R06 = [T(1,1:3);
    T(2,1:3);
    T(3,1:3);];
R6_3 = R03T*R06;
theta4 = atan2d(R6_3(3,3),R6_3(1,3));
```

```
if theta4 == 180
    theta4 = 0;
end
```


## Implementing equation (32)

```
%Calculating theta 5
T_4 = theta4;
T4_3 [ cosd(t_4), -sind(t_4), 0,
a \overline{3}
    sind(t_4)*}\operatorname{cosd}(al_3), cosd(t4)*\operatorname{cosd(al_3), -sind(al_3),
-sind(al_3)*d_4;
    sind(t_4)*sind(al_3), cosd(t_4)*sind(al_3), cosd(al_3),
cosd(al_3)*d_4;
    0, 0,
; ]
T4_0 = T3_0 * T4_3;
T4_0T = transpose(T4_0);
T4\overline{6}=T4_0T * T;
theta5 = asind(T46(1,3));
```


## Implementing equation (33)

```
%Calculating theta 6
t_4 = theta4;
t_5 = theta5;
T4_}\mp@subsup{\mp@code{a}}{3}{3}=[\operatorname{cosd(t_4),
    sind(t_4)*}\operatorname{cosd(al_3), cosd(t_4)*cosd(al_3),
    -sind(al_3)
    cosd(al_3),
cosd(al_3)*d_4-
    0, 0,
; ]
T5_4 [ [ cosd(t_5), -sind(t_5), 0,
        sind(t_5)*}\operatorname{cosd(al_4), cosd(t_5)*cosd(al_4),
    -sind(al_4),
    cosd(al_4),
cosd(al_4)*d_5
        0, 0,
; ]
T5_0 = T3_0*T4_3*T5_4;
R05 = [T5 0(1,1:3);
    T5 0(2,1:3);
    T5_0(3,1:3);];
```

```
R5_0T = transpose(R05);
R6_0 = [T(1,1:3);
    T (2,1:3);
    T(3,1:3);];
R6_5 = R5_0T*R6_0;
%Calculating theta6
theta6 = asind(-R6_5(1,2));
end
```


### 5.2 Algorithms for developing trajectory planning schemes in section 4.4

```
initial_time=0;
final_time=time;
timestep=.05;
x=initial_time:timestep:final_time;%time 0 to 20 seconds
t = size(x,2);
jtable(t,7)=0;%joint table
vtable(t,7)=0;%velociy table
atable(t,7)=0;%angular table
qtable(t,7)=0;%jerk table
jtable(:,1)=x;%table with time
vtable(:,1)=x;%table with time
atable(:,1)=x;%table with time
qtable(:,1)=x;%table with time
c = jtable(1:t,1);%time from 0 to 10seconds
t1 = size(c,1);
%joint parameters
    if line == 1
    jointi=[51.84,12.298,-119.428,-103.03,-53.81,111.40];%home
position [230,140,215,0,-90,180]
jointf=[t_t(line,1),t_t(line,2),t_t(line, 3),t_t(line,4),t_t(line,5),
t_t(line,}\overline{6})]
    else
    jointi=[t_t(line-1,1),t_t(line-1,2),t_t(line-1,3),t_t(line-
1,4),t_t(line-1,5),t_t(line-1,6)];
jointf=[t_t(line,1),t_t(line, 2),t_t(line, 3),t_t(line,4),t_t(line,5),
t_t(line,6)];
    end
%Joint boundary conditions
for k=1:6
ji=jointi(k); %Initial position
jf=jointf(k); % Final position
```

```
j_vi=0; % Initial Angular velocity is assumed as zero
j_vf=0; % Final Angular velocity is assumed as zero
j_ai=0;% Initial acceleration is assumed as zero
j_af=0; % Final acceleration is assumed as zero
% Assume arm comes to the end position position with time
t i=0;%initial time
t_f=x(1,t);%final time
T=t_f-t_i;%time difference
%Coefficients calculation
for i = 1:t
a0=ji;
al=j_vi;
a2=j_ai/2;
a3=[\overline{20*(jf-ji)-(8*j_vf+12*j_vi)*T-(3*j_af-j_ai)*T.^2]/(2*T.^3);}
a4=[30*(ji-jf)+(14*\overline{j}_vf+16*\overline{j}_vi)*T+(3*\overline{j}_af-\overline{2}*j_ai)*T.^2]/(2*T.^4);
a5=[12*(jf-ji)-6*(j_vf+j_vi)*T-(j_af-j_ai)*T.^2]/(2*T.^5);
Coefficients_J1=[a_0 a_1 a_2 a_3 a_4 a_5]
jtable(i,k+1)=
a_0+(a_1*x(1,i))+(a_2*(x(1,i))^2)+(a_3*(x(1,i) ^^3)+(a_4*(x(1,i))^4)+
(a_5*(x(1,i))^5 );
vt\overline{able(i,k+1)=}
a_1+(2*a_2*x(1,i))+(3*a_3*(x(1,i)^2))+(4*a_4*(x(1,i)^3))+(5*a_5*(x(1
,i)^4));
atable(i,k+1)=
(2*a_2)+(6*a_3*x(1,i))+(12*a_4* (x(1,i)^2))+(20*a_5* (x(1,i)^3));
qtab\overline{le}(i,k+1)= 6*a_3+(24*a_4*x(1,i))+(60*a_5*(x(\overline{1},i)^2))
end
```


## Implementation of maximum velocity mapping

```
for i = 1:tam
[jtable vtable atable qtable] = trajectory(i,1);%line number,time
vsize = size(vtable);
    for q = 2:7
            vmax(i,q-1) = max(abs(vtable(:,q)))
    end
end
vmaxtable =
array2table(vmax (2:tam,:),'VariableNames',{'J1','J2','J3','J4','J5',
'J6'});
figure;
plot(2:tam,vmax(2:tam,1:6))
```

```
ylabel('Maximum velocity(deg/sec)');xlabel('Line
number');set(gca,'XTick',[0:1:line]);
legend('joint 1','joint 2','joint 3','joint 4','joint 5','joint 6')
```

sample graph derived for the maximum velocity mapping is shown in Figure 5.6.


Figure 5.6: Maximum velocity mapping
Time calculation algorithm for reaching maximum velocity developed in section 4.4.

```
initial_time=0;
final_time=2;
timestep=.05;
x=initial_time:timestep:final_time;otime 0 to 20 seconds
t = size(x,2);
if joint == 1
v_max = 250;
else
    if joint == 2
        v_max = 187;
    else
        if joint == 3
            v_max = 250;
            else
```

```
        if joint == 4
            v max = 300;
            else
                if joint == 5
                    v_max = 300;
        else
                        if joint == 6
                                    v_max = 300;
                    end
                end
            end
        end
    end
end
if line == 1
jointi=[51.84,12.298,-119.428,-103.03,-53.81,111.40];
jointf=[t_t(line,1),t_t(line, 2),t_t(line, 3),t_t(line,4),t_t(line,5),
t_t(line,6)];
    else
    jointi=[t_t(line-1,1),t_t(line-1,2),t_t(line-1,3),t_t(line-
1,4),t_t(line-1,5),t_t(line-1,6)];
jointf=[t_t(line,1),t_t(line,2),t_t(line,3),t_t(line,4),t_t(line,5),
t_t(line,}\overline{6})]
end
ji = jointi(joint);
jf = jointf(joint);
dj = jf - ji;
j_vi=0; % Initial Angular velocity is assumed as zero
j_vf=0; % Final Angular velocity is assumed as zero
j_ai=0;% Initial acceleration is assumed as zero
j_af=0; % Final acceleration is assumed as zero
```

\%Coefficients calculation

```
a_0=ji;
a_1=j_vi;
a_2=j_ai/2;
a_3=[20*(dj)-(8*j_vf+12*j_vi)*T-(3*j_af-j_ai)*T.^2]/(2*T.^3);
a_4=[30*(-dj)+(14*j_vf+16*j_vi)*T+(3*j_af-2*j_ai)*T.^2]/(2*T.^4);
a_5=[12*(dj)-6*(j_vf}+j_vi)*T-(j_af-j_a\overline{i})*T.^2]/(2*T.^5)
Coefficients_J1=[\overline{a_0 a_1 a_2 a_3 a_4 a_5]}
t_max = (15*dj)/(v_max*8);
time = abs(t_max);
end
```


### 5.3 Simulation model

Results taken by running developed algorithms are required to be verified in order to troubleshoot the system and its outcome. Verification of Algorithms developed for forward kinematics and inverse kinematics can be done by performing simulations in a separate simulating software tool. Wincaps III robot programming tool from Densowave corporation is used here for developing robot cell graphically and simulating the experiments to get better results [20].

### 5.3.1 Creating Robot cell

Robot working cell can be developed virtually in Wincaps III and the summarized procedure is illustrated below.

In Wincaps III, Robot arm can be selected and imported to the simulation module [Figure 5.7].


Figure 5.7: Robot Arm selection in Wincaps III
Default configuration is shown in Figure 5.8.


Figure 5.8: Default Robot configuration in Wincaps III
Designs of Machine head cover and the tool can be imported to Wincaps III working module and located according to actual working dimensions [Figure 5.9].


Figure 5.9: Machine head cover and Tool placement in the simulation module

Work and Tool coordinates are defined according to the developed coordinate assignment [Figure 5.10].


Figure 5.10: Work and Tool coordinate assignment
Robot program is written and executed according to robot movement along the robot end points positioned in the line [Figure 5.11].


Figure 5.11: Robot programs for simulating robot movement via points
Position and orientation data can be imported to the simulation module in order to run the simulation [ Figure 5.12].


Figure 5.12: Import Point location data into the program
Execution of the program and the simulation can be done according to the user's speed requirement. Joint angle values of each joint can be observed at each robot end point while moving via that point [Figure 5.13]. By observing these joint angles values, experiments can be done to troubleshoot the algorithms and developed methods. Optimization of the solution can be done in order to increase the accuracy.


Figure 5.13: Joint angles of the robot arm at point 1 position

## CHAPTER 6

## RESULTS AND CONCLUSION

Validations of implemented algorithms are discussed in this chapter and systems in chapter 4 and 5 . Section 6.1 illustrates simulations done for various point cloud allocation along the robot moving path. Optimization of path planning with trajectory planning is validated by performing several trials. Suitability of using Computer Aided Design features used as an offline robot programming approach for optimizing trajectory planning of 6 DOF robot manipulator in order to enhance the productivity is discussed in the conclusion in section 6.2. The thesis suggests additional developments of the proposed trajectory and path planning scheme which can be implemented in other robotic applications as the upcoming work in section 6.3.

### 6.1 Various point cloud simulation Results

Simulations for various point cloud of the path validate the efficiency of using offline robot programming techniques for path planning and trajectory planning. By trialing several scenarios with various point to point locations along the path, programmer can conclude an optimized path planning scenario for the application which enhance the productivity of the robot operation.

## Simulation objectives:

- Observe and analyze on various cartesian point allocation along a predefined path.
- Identification of Velocity behavior and limits.
- Identification of maximum velocity along task points/sequence.
- Optimization of point allocation in order to get smooth velocity variations along the task sequence.


## Procedure

1. Point allocation done as per the user's parameters.
2. Point located data is extracted by running developed VB based macro.
3. Find joint angles for each point location.
4. Verify scenario joint angles results with simulation in Wincaps III.
5. Mapping maximum velocity variation of robot arm joint movement.

## Robot task and operation requirements

Path length is 405.33 mm and the target cycle time for the operation is needed to be maintained below 60 seconds. [Figure 6.1]

Variation of the robot joint speeds should be low in order to get smooth operation while optimizing the cycle time.


Figure 6.1: Robot path length

## Scenario 1

Initial point to point distance can be calculated by taking the average point to point distance which is 6.75 mm ( $405.33 \mathrm{~mm} / 60$ point to point intervals) considering point to point time duration as 1 second. These parameters can be considered as base parameters to commence the simulation.

## Parameters:

| Point to point distance | $: 6.75 \mathrm{~mm}$ |
| :--- | :--- |
| Number of points | $: 61$ |
| Point to point time duration | $: 1$ Second |



Figure 6.2: Point allocation scenario 1

## Scenario 1 results

1. Joint angles can be calculated using developed algorithms as depicted in Appendix C.
2. Simulation verifies that the resulted joint angles values are accurate and set in order [Figure 6.3].

Maximum velocity of joint movement is generated and mapped in order to analyze the robot movement [Figure 6.4].


Figure 6.3: Scenario 1 simulation results


Figure 6.4: Maximum velocity of joint rotation scenario 1

## Observations:

- Smooth velocity profiles can be observed.
- Significant variation between joint 4, 6 and other joints can be observed. Maximum velocity is $32.18 \mathrm{deg} / \mathrm{sec}$ (joint 4) and the maximum velocity of joint 1 is $7.196 \mathrm{deg} / \mathrm{sec}$. Joint 2, 3 and 5 velocities are varied below joint 1 maximum velocity. Maximum velocity difference between those joints is $24.984 \mathrm{deg} / \mathrm{sec}$.


## Scenario 2

In order to reduce the cycle time, number of points has to be reduced by increasing point to point distance. Point to point distance is increased to 9.88 mm from 6.75 mm . Parameters:

| Point to point distance | $: 9.88 \mathrm{~mm}$ (10mm approximately) |
| :--- | :--- | :--- |
| Number of points | $: 42$ |
| Point to point time duration | $: 1$ Second |



Figure 6.5: Point allocation scenario 2

## Scenario 2 results

3. Joint angles can be calculated using developed algorithms as depicted in Table 3.
4. Simulation verifies that the resulted joint angles values are accurate and set in order [Figure 6.6].
5. Maximum velocity of joint movement is generated and mapped in order to analyze the robot movement [Figure 6.7].

Table 3: Scenario 2 joint angles calculation results

| Line No: | Joint |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | J1 | J2 | J3 | J4 | J5 | J6 |
| 1 | -54.462 | -14.039 | 123.366 | 103.300 | 56.737 | 66.684 |
| 2 | -54.462 | -13.962 | 120.765 | 101.667 | 56.191 | 69.640 |
| 3 | -54.462 | -13.766 | 118.067 | 100.006 | 55.721 | 72.606 |
| 4 | -54.352 | -13.581 | 115.373 | 98.339 | 55.215 | 75.591 |
| 5 | -53.836 | -13.742 | 113.023 | 96.723 | 54.382 | 78.557 |
| 6 | -52.891 | -14.259 | 111.135 | 95.175 | 53.202 | 81.402 |
| 7 | -51.510 | -15.112 | 109.795 | 93.714 | 51.662 | 84.026 |
| 8 | -49.688 | -16.271 | 109.066 | 92.370 | 49.746 | 86.336 |
| 9 | -47.432 | -17.690 | 108.984 | 91.188 | 47.445 | 88.244 |
| 10 | -44.772 | -19.312 | 109.552 | 90.242 | 44.772 | 89.659 |
| 11 | -41.774 | -21.034 | 110.626 | 89.543 | 41.775 | 90.612 |
| 12 | -38.464 | -22.665 | 111.647 | 88.720 | 38.476 | 91.635 |
| 13 | -34.821 | -24.170 | 112.549 | 87.671 | 34.854 | 92.837 |
| 14 | -30.824 | -25.532 | 113.329 | 86.315 | 30.895 | 94.293 |
| 15 | -26.542 | -26.546 | 113.443 | 83.815 | 26.709 | 96.917 |
| 16 | -22.324 | -27.005 | 112.498 | 79.167 | 22.751 | 101.723 |
| 17 | -18.630 | -26.882 | 110.572 | 71.942 | 19.634 | 109.095 |
| 18 | -15.867 | -26.233 | 107.829 | 62.790 | 17.905 | 118.383 |
| 19 | -12.626 | -25.553 | 105.180 | 51.206 | 16.288 | 129.945 |
| 20 | -8.149 | -25.108 | 103.260 | 34.894 | 14.347 | 145.953 |
| 21 | -2.819 | -24.889 | 102.251 | 12.683 | 12.944 | 167.629 |
| 22 | 2.819 | -24.889 | 102.251 | -12.683 | 12.944 | 192.371 |
| 23 | 8.149 | -25.108 | 103.260 | -34.894 | 14.347 | 214.047 |
| 24 | 12.626 | -25.553 | 105.180 | -51.206 | 16.288 | 230.055 |
| 25 | 15.867 | -26.233 | 107.829 | -62.790 | 17.905 | 241.617 |
| 26 | 18.630 | -26.882 | 110.572 | -71.942 | 19.634 | 250.905 |
| 27 | 22.324 | -27.005 | 112.498 | -79.167 | 22.751 | 258.277 |
| 28 | 26.542 | -26.546 | 113.443 | -83.815 | 26.709 | 263.083 |
| 29 | 30.824 | -25.532 | 113.329 | -86.315 | 30.895 | 265.707 |
| 30 | 34.821 | -24.170 | 112.549 | -87.671 | 34.854 | 267.163 |
| 31 | 38.464 | -22.665 | 111.647 | -88.720 | 38.476 | 268.365 |
| 32 | 41.774 | -21.034 | 110.626 | -89.543 | 41.775 | 269.388 |
| 33 | 44.772 | -19.312 | 109.552 | -90.242 | 44.772 | 270.341 |
| 34 | 47.432 | -17.690 | 108.984 | -91.188 | 47.445 | 271.756 |
| 35 | 49.688 | -16.271 | 109.066 | -92.370 | 49.746 | 273.664 |
| 36 | 51.510 | -15.112 | 109.795 | -93.714 | 51.662 | 275.974 |
| 37 | 52.891 | -14.259 | 111.135 | -95.175 | 53.202 | 278.598 |
| 38 | 53.836 | -13.742 | 113.023 | -96.723 | 54.382 | 281.443 |
| 39 | 54.352 | -13.581 | 115.373 | -98.339 | 55.215 | 284.409 |
| 40 | 54.462 | -13.766 | 118.067 | -100.006 | 55.721 | 287.394 |
| 41 | 54.462 | -13.962 | 120.765 | -101.667 | 56.191 | 290.360 |
| 42 | 54.462 | -14.039 | 123.366 | -103.300 | 56.737 | 293.316 |



Figure 6.6: Scenario 2 simulation results


Figure 6.7: Maximum velocity of joint rotation scenario 2
Observations:

- Smooth velocity profiles can be observed.
- Significant variation between joint 4, 6 and other joints can be observed. Maximum velocity is $47.56 \mathrm{deg} / \mathrm{sec}$ in joint 4 increased from $32.18 \mathrm{deg} / \mathrm{sec}$ and the maximum velocity of joint 1 is $10.57 \mathrm{deg} / \mathrm{sec}$ from $7.196 \mathrm{deg} / \mathrm{sec}$. Joint 2 , 3 and 5 velocities are varied below joint 1 maximum velocity. Maximum velocity difference between those joints is $36.99 \mathrm{deg} / \mathrm{sec}$ increased from 24.984 deg/sec.


## Scenario 3

It is realized that the after point 14 up to 30 , there is a significant variance in velocities of joint 4 and joint 6 [Figure 6.8]. This variance can be reduced by reducing the distance between points within that area. Therefore, point to point distance between that area has been reduced up to 5 mm .

Figure 6.8: Point identification of the velocity variance begins

Parameters:
Point to point distance $: 5$ and 10 mm
Number of points :57
Point to point time duration : 1 Second
[Figure 6.9]


Figure 6.9: Point allocation scenario 3

## Scenario 3 results

1. Joint angles can be derived by running the algorithms developed [Appendix D].
2. Simulation verifies that the resulted joint angles values are accurate and set in order [Figure 6.10].
3. Maximum velocity of joint movement is generated and mapped in order to analyze the robot movement [Figure 6.11].


Figure 6.10: Scenario 3 simulation results


Figure 6.11: Maximum velocity of joint rotation scenario 3

## Observations

- Smooth velocity profiles can be observed.
- Sudden changes of the joint 1 and 5 profile can be observed.
- Variation between joint 4, 6 and other joints is reduced than scenario 2. Maximum velocity is 23.78 deg/sec in joint 4 decreased from 47.56 deg/sec and the maximum velocity of joint 1 is $7.492 \mathrm{deg} / \mathrm{sec}$ decreased from 10.57 deg/sec. Joint 2, 3 and 5 velocities are varied below joint 1 maximum velocity. Maximum velocity difference between those joints is $16.288 \mathrm{deg} / \mathrm{sec}$ decreased from $36.99 \mathrm{deg} / \mathrm{sec} \mathrm{deg} / \mathrm{sec}$.


## Scenario 4

It is realized that the variance of maximum velocities in joint 4 and joint 6 has been reduced and velocities in joint 1 and joint 5 between position 14 to 15 and 44 to 45 are subjected to sudden changes which make robot arm to sudden movement. This incident can be occurred as the point to point distance changes from 10 mm to 5 mm between
those point locations. This effect can be minimized by allocating points which are located its point to point distances from 10 mm to 5 mm gradually.

Parameters:
Point to point distance $: 10 \mathrm{~mm}$ (point 1 to 13 and 44 to 56 )
5 mm (point 17 to 40 )
Number of points : 56
Point to point time duration : 1 Second
[Figure 6.12]


Figure 6.12: Point allocation scenario 4

## Scenario 4 results

1. Joint angles can be derived by running the algorithms developed [Appendix E].
2. Simulation verifies that the resulted joint angles values are accurate and set in order [Figure 6.13].
3. Maximum velocity of joint movement is generated and mapped in order to analyze the robot movement [Figure 6.14].


Figure 6.13: Scenario 4 simulation results


Figure 6.14: Maximum velocity of joint rotation scenario 4

## Observations:

- Smooth velocity profiles can be observed.
- Sudden changes of the joint 1 and 5 has been improved. Maximum velocity is $23.6 \mathrm{deg} / \mathrm{sec}$ in joint 4 and the maximum velocity of joint 1 is $6.831 \mathrm{deg} / \mathrm{sec}$ decreased from $7.492 \mathrm{deg} / \mathrm{sec}$. Joint 2,3 and 5 velocities are varied below joint 1 maximum velocity. Maximum velocity difference between those joints is $16.769 \mathrm{deg} / \mathrm{sec}$ nearly same to scenario 3 results.

Total cycle time is 55 seconds which is below than target cycle time 60 seconds. Maximum velocity variation between joints has been minimized up to $16.769 \mathrm{deg} / \mathrm{sec}$ which is a significant improvement from the results of initial simulation and scenarios.

From the above simulations, it is clear that trajectory planning for the robot arm moving along the path can be optimized by realizing the joint speed variations and allocating task points with proper evaluation in order to increase the productivity. This method is introduced as an offline programming technique where programmer can generate the robot programs with optimum output in a different environment separately and the robot operation is not needed to stop for programming.

### 6.2 Conclusion

SolidWorks features with computer aided design capabilities were used to create the robot cell and generate programs for robot operation with significant accuracy. Path planning of a robot movement can be done efficiently using Solidworks design features and proposed method proves its capability to create the robot working cell virtually. Developed design models can be used frequently for different type of robot cells unless the robot, tool and work piece are not changed.

The proposed method for finding inverse kinematic solutions of the Denso VP 6242 robot manipulator can be used in many working areas in the robot cell and implemented for other types of robot manipulators available in the industry.
Various point clouds with different point to point distances were tested and simulated in Denso Wincaps III software. Optimized point cloud allocation along the robot moving path, was derived gradually by simulating various point to point distances while maintaining smooth robot joint movement. Total cycle time is reduced to 55 seconds which is below required maximum cycle time ( 60 seconds). Maximum velocity difference between joint 1 and joint 4 has been reduced to $16.769 \mathrm{deg} / \mathrm{sec}$ from $24.984 \mathrm{deg} / \mathrm{sec}$ which is a significant improvement from the results of initial simulation and scenarios.

Mapping maximum velocity of joint variation between robot moving from one point to another, is very effective to understand overall behavior of the robot motion along the whole path. Simulation 3 Result demonstrates possible changes which are not suitable for better robot operation can be identified and user can improve the path planning according to its outcome needed. Simulation 4 result validates the possibility of arranging robot smooth movement by allocating proper task point cloud. Simulations validate that the point cloud allocation along the robot moving path can be optimized by running several simulations with various point to point distances.

### 6.3 Future works

Computer Aided Design software and tools are still developing gradually with the development of current manufacturing industries and its requirements hence the efficiency of its utilization can be further improved. Developed inverse kinematic model can be further developed for other types of robot manipulators and can be used for other types of robotic applications.

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## APPENDICES

Appendix A

- Algorithms for inverse kinematic solution of 6 DOF robot manipulator and trajectory planning in Matlab
- Machine head cover dimensions
- Robotic cell layout

Appendix B - Dimensions and the configuration of the Denso VP 6242
Appendix C - Scenario 2 joint angles calculation results
Appendix D - Scenario 3 joint angles calculation results

## Appendix A


#### Abstract

Algorithms for inverse kinematic solution of 6 DOF robot manipulator and trajectory planning in Matlab


## The main program

```
clear all;
clc;
syms c1 s1 d3 c23 s23 s2 c2 al be t l
syms m n p
syms s1 s2 s3 s4 s5 s6 c1 c2 c3 c4 c5 c6;
global tt;
```

```
%position read
C = xlsread('C:\Users\pcadmin\Desktop\MSc\research
matlab\simulation3.xlsx','Sheet1');
tam = size(C);
Pa = [];
for i = 1 : tam
    T6EF = [11 0 0 10;0 1 0 0;0 0 1 130;0 0 0 1];
    X(i,1) = C(i,1);%X value
    Y(i,2) = C(i,2);%Y value
    Z(i,3) = C(i,3);%Z value
    al = C(i,6); %angle arond Z axis
    be =C(i,5);%angle around Y axis
    ga = C(i,4);%angle around X axis
    %Eular angle Z,Y,X
Rz = [ cosd(al) -sind(al) 0;
    sind(al) cosd(al) 0;
        0 0 1;];
Ry = [ cosd(be) 0 sind(be);
    0 1 0;
    -sind(be) 0 cosd(be);];
Rx = [ 1 0 0;
    0 cosd(ga) -sind(ga);
    0 sind(ga) cosd(ga);];
%Rotation Matrix
ARB = Rz*Ry*Rx;
```

TT (: , : i) $=$ [A_R B(1,1) A R_B(1, 2) A_R_B(1,3) X(i,1); A_R_B(2,1)
A_R_B $(2,2) \quad A_{-} R \_B(2,3) Y(i, 2) ; A_{-} R \_B(3, \overline{1}) \mathcal{A}_{-} R \_B(3,2)$ A_R_B $(3,3)$
Z $(i, 3)$;
$0001]$;
T6EFT $=\left((T 6 E F)^{\wedge}(-1)\right)$;
$\mathrm{T}(:, \quad:, i)=\mathrm{TT}(:, \quad:, i) * \mathrm{~T} 6 \mathrm{EFT}$;
\%Pa position
$\mathrm{x}=\mathrm{T}(1,4, i)-(70) * T(1,3, i) ; \% \mathrm{~Pa} \mathrm{x}$ position
$y=T(2,4, i)-(70) * T(2,3, i) ; \% P a y$ position

```
z = T(3,4,i)- (70)*T(3,3,i);%Pa z position
distance = (x^2 + y^2 + z^2)^(1/2);
if distance > 420
    disp('error');
    pause;
end
%Calculating theta 1
theta1 = atan2d(y,x);
if thetal == 180
    theta1 = 0;
end
if (90<theta1) && (theta1<180)
    theta1 = -(180 - theta1);
end
if (-180>thetal) && (thetal>-90)
    theta1 = 180 + theta1;
end
%}
%Calculating theta 2 and theta 3
c1 = cosd(thetal);
s1 = sind(thetal);
l = (x*c1) + (y*s1);
m = (-1/663) + (14*z/3315);
n = ((-14*l)/3315) - (z/663);
p = (196/221) + (m^2) + (n^2);
a_ = ((-140*m) - (392*n))/221;
\mp@subsup{b}{-}{-}}=((140*\textrm{n})-(392*\textrm{m}))/221
c = 1 - p;
theta2 = atan2(a_,b_)+ atan2(((a_^2 + b_^2 - c^2))^(1/2),c);
theta2 = (theta2*180)/pi;
if theta2 ==360
        theta2 = 0;
end
if (180<theta2) && (theta2<360)
theta2 = 360 - theta2;
end
%Theta 3
```

```
e = -210.00;
f = -75.00;
g = l+(210*(sind(theta2)));
h = ((e^2 + f^2 - g^2) )^(1/2);
if isreal(h) == 1
    theta_23_1 = atan2(e,f)+\operatorname{atan2(h,g);}
    theta_23_1 = (theta_23_1*180)/pi;
    theta_23_2 = atan2(e,f)- atan2(h,g);
    theta_23_2 = (theta_23_2*180)/pi;
    if (theta_23_2>-360) && (theta_23_2<-180)
            theta_23_2 = 360 + theta_2\overline{3}_2;
    end
    theta_3_1 = - theta2 + theta_23_1;
    theta_3_2 = - theta2 + theta_23_2;
else
        theta_3_1 = 0 - theta2;
    theta_3_2 = 0 - theta2;
end
theta2_ = atan2(a,b)- atan2((a_^2 + b_^2 - c^2)^(1/2),c);
theta2_ = (theta2_*180)/pi;
if ( theta2_ > -360) && (theta2_ < -180)
    theta2_ = 360 + theta2_;
end
e = -210.00;
f = -75.00;
g_ = l+(210*(sind(theta2_)));
h_}=(\mp@subsup{e}{}{\wedge}2+f^2 - g_^2)^\overline{(1/2);
if isreal(h_) == 1
    theta_23_3 = atan2(e,f)+ atan2(h_, g_);
    theta_23_3 = (theta_23_3*180)/pi;
    theta_23_4 = atan2(e,f)- atan2(h_,g_);
    theta_23_4 = (theta_23_3*180)/pi;
    theta_3_3 = - theta2_ + theta_23_3;
    theta_3_4 = - theta2_ + theta_23_4;
else
    theta_3_3 = 0 - theta2_;
    theta_3_4 = 0 - theta2_;
```

end
\%calculating theta 4,5,6 - four solutions
[theta 41 theta 51 theta 6 1] =
theta456calcalternātive(thétā1, theta2, theta_3_1,T(:, :, i))
[Orientation1 position1] = forwardkinamatic(theta1, theta2, theta_3_1, theta_4_1, theta_5_1, theta_6_1,T(:,:,i))
[theta 4_2 theta_5 2 theta_6_2] =
theta4 $\overline{5} 6 \bar{c}$ alcalte $\bar{r} n \bar{a} t i v e(t h \bar{e} t \bar{a} 1$, theta2, theta_3_2,T(:, :,i))
[Orientation2 position2] = forwardkinamatic(theta1, theta2, theta_3_2, theta_4_2, theta_5_2, theta_6_2,T(:,:,i))
[theta_4_3 theta_5_3 theta_6_3] = theta4 $\overline{5} 6 \bar{c}$ calcalte $\bar{r} n \bar{a} t i v e(t h \bar{e} t \bar{a} 1$, theta2_, theta_3_3,T(:, :, i))
[Orientation3 position3] = forwardkinamatic(theta1, theta2_, theta_3_3, theta_4_3, theta_5_3, theta_6_3,T(:,:,i))
[theta_4_4 theta_5_4 theta_6_4] = theta4 $\overline{5} 6 \bar{c}$ alcalte $\bar{r} n \bar{a} t i v e(t h \bar{e} t \bar{a} 1$, theta2_, theta_3_4,T(:, :, i) )
[Orientation4 position4] = forwardkinamatic(theta1, theta2_, theta_3_4, theta_4_4, theta_5_4, theta_6_4,T(:,:,i))
\%four solutions in table
[table(:,1,i)] =
[position1; theta1; theta2; theta_3_1; theta_4_1; theta_5_1; theta_6_1]; [table(:,2,i)] =
[position2; theta1; theta2; theta_3_2; theta_4_2; theta_5_2; theta_6_2]; [table(:,3,i)] =
[position3; theta1; theta2_; theta_3_3;theta_4_3;theta_5_3; theta_6_3]; [table(:,4,i)] = [position4; theta1; theta2_; theta_3_4; theta_4_4; theta_5_4; theta_6_4];
for $k=1: 4$

$$
\begin{aligned}
& d x(i, k)=\operatorname{table}(1, k, i)-T(1,4, i) ; \\
& d y(i, k)=\operatorname{table}(2, k, i)-T(2,4, i) ; \\
& d z(i, k)=\operatorname{table}(3, k, i)-T(3,4, i) ;
\end{aligned}
$$

```
if \((a b s(d x(i, k))>0) \& \&(a b s(d x(i, k))<0.001)\)
    \(d x(i, k)=0 ;\)
else
    \(d x(i, k)=d x(i, k) ;\)
end
```

```
if (abs(dy(i,k))>0) && (abs(dy(i,k))< 0.001)
    dy(i,k)= 0;
else
    dy(i,k)= dy(i,k);
end
if (abs(dz(i,k))>0) && (abs(dz(i,k))< 0.001)
    dz(i,k)= 0;
else
    dz(i,k)= dz(i,k);
end
end
end;
%Selecting correct solution
tt(1,1:6)=[0 0 0 0 0 0}
for o = 1:tam
for j = 1:4
if dx(o,j)== 0 && dy(0,j) == 0 && dz(0,j) == 0
    tt(0,1) = table(4,j,0);
    tt(0,2) = table(5,j,0);
    tt(0,3) = table(6,j,0);
    tt (0,4) = table(7,j,0);
    tt (0,5) = table(8,j,0);
    tt (0,6) = table(9,j,0);
end
end
end;
for aa = 1:tam
    for bb = 1:6
        if (abs(tt(aa,bb))>0) && (abs(tt(aa,bb))< 0.001)
                tt (aa,bb) =0;
            else
                tt(aa,bb) = tt(aa,bb);
            end
    end
end;
for ii = 1:tam
    if Y(ii,2)<0
    tt(ii,4) = 180+tt(ii,4);
    else
        tt(ii,4) = -180+tt(ii,4);
    end
    if tt(ii,4) > 270
```

```
        tt(ii,4) = tt(ii,4) - 360;
    end
    if tt(ii,4) < -270
        tt(ii,4) = tt(ii,4) + 360;
    end
    tt(ii,5) = -tt(ii,5);
    tt(ii,6) = -180+tt(ii,6);
end
for i = 1:tam
    if 90<tt(i,4) && tt(i,4)<180
        tt(i,6) = tt(i,6);
    else if 0<tt(i,4) && tt(i,4)<90
                tt(i,6) = -90 - (tt(i,6)+90);
                else if 0>tt(i,4) && tt(i,4)>-90
                    tt(i,6) = -90 - (tt (i,6) +90);
                        else if -90>tt(i,4) && tt(i,4)>-180
                                    tt(i,6) = 90 + (270 + tt(i,6));
                                    end
            end
        end
    end
end
for i= 1:tam
    tt(i,6) = 180 + tt(i,6);
end
tttable =
array2table(tt,'VariableNames',{'J1','J2','J3','J4','J5','J6'});
%Denso simulation 1
Densot(:,1) = tt(:,1);
Densot(:,2) = -tt(:,2);
Densot(:,3) = -tt(:,3);
Densot(:,4) = tt(:,4);
Densot(:,5) = -tt(:,5);
Densot(:,6) = tt (:, 6);
Densottable =
array2table(Densot,'VariableNames',{'J1','J2','J3','J4','J5','J6'});
for i = 1:tam
[jtable vtable atable qtable] = trajectory(i,1);\%line number,time
vsize = size(vtable);
    for q = 2:7
    vmax(i,q-1) = max(abs(vtable(:,q)))
    end
```

end

```
vmaxtable =
array2table(vmax(2:tam,:),'VariableNames',{'J1','J2','J3','J4','J5',
'J6'});
figure;
plot(2:tam,vmax(2:tam,1:6))
ylabel('Maximum velocity(deg/sec)');xlabel('Line
number');set(gca,'XTick',[1:1:tam]);
legend('joint 1','joint 2','joint 3','joint 4','joint 5','joint 6')
%maxx = max(abs(vtable(:,3)));
for ii = 1:tam
    for jj=1:6
        if vmax(ii,1)>250
            vmaxd(ii,1)=vmax(ii,1)-250;
        else
            vmaxd(ii,1)= 0;
        end
        if vmax(ii,2)>187
            vmaxd(ii,2)=vmax(ii,2)-187;
        else
            vmaxd(ii,2)= 0;
        end
        if vmax(ii,3)>250
            vmaxd(ii,3)=vmax(ii,3)-250;
        else
            vmaxd(ii,3)= 0;
        end
        if vmax(ii,4)>300
            vmaxd(ii, 4)=vmax(ii, 4)-300;
        else
            vmaxd(ii,4)= 0;
        end
        if vmax(ii,5)>300
            vmaxd(ii,5)=vmax(ii,5)-300;
        else
            vmaxd(ii,5)= 0;
        end
        if vmax(ii,6)>300
            vmaxd(ii,6)=vmax(ii,6)-300;
        else
            vmaxd(ii,6)= 0;
        end
    end
end
for ii=1:tam
    vd(ii,1) = max(vmaxd(ii,:));
    for jj=1:6
        if vd(ii,1)== vmaxd(ii,jj)
```

```
        vd(ii,2) = jj;
        end
            if vd(ii,1) == 0
            vd(ii,2) = 0;
            end
    end
end
%Displaying velocity max values in same vmax table
for i = 1:tam
    if vmax(i,1)>250
            vmax(i,7)=vmax(i,1);
        end
        if vmax(i,2)>187
            vmax(i,8)=\operatorname{max}(i,2);
            end
            if vmax(i,3)>250
            vmax(i,9)=vmax(i,3);
        end
        if vmax(i,4)>300
            vmax(i,10)=vmax(i,4);
        end
        if vmax(i,5)>300
            vmax(i,11)=vmax(i,5);
        end
        if vmax(i,6)>300
            vmax(i,12)=vmax(i,6);
        end
end
```


## Sub functions 1

```
function [Orientation position] =
forwardkinamatic(theta_1,theta_2,theta_3,theta_4,theta_5,theta_6,T)
format short g
al_0=0;al_1=90,al_2=0;al_3=90;al_4=-90;al_5=90;
a_\overline{0}=0;a_1=0;a_2=210;a_3=\overline{7}5;a_4=0;a_5=0;
d_1=0;d_2=0;d_3=0;d_4=210;d_5=0;d_6=70;
t_1=theta_1;
t_2=90+theta_2;
t_3=90+theta_3;
t_4=theta_4;
t-5=theta-5;
t_6=theta_6;
T01 = [cosd(t_1), -sind(t_1), 0, a_0
sind(t_1)*\operatorname{cosd}(al_0), cosd(t_1)*cosd(al_0)
sind(al_0)*d_1;
sind(t 1)*sind(al 0), cosd(t 1)*sind(al 0), cosd(al 0),
cosd(a\overline{l_0)*d_1 ;}
0, - - 0,
; ]
T12 = [ cosd(t_2), -sind(t_2), 0, a_1
;
sind(t2)*cosd(al_1), cosd(t_2)*cosd(al_1),
sind(al_1)*d_2;
sind(t_\overline{2})*si\overline{n}d(al_1), cosd(t_2)*sind(al_1), cosd(al_1),
cosd(a\overline{l_1)*d_2 ;}
0, 0,
; ]
T23 = [ cosd(t_3), -sind(t_3), 0,
a_2 ;
sind(t_3)*cosd(al_2), cosd(t_3)*cosd(al_2), -sind(al_2), -
sind(al2)*d_3;
sind(t_3)*sind(al_2), cosd(t_3)*sind(al_2), cosd(al_2),
cosd(al_2)*d_3 ;
0, 0, 0, 1
; ]
T34 = [ cosd(t_4), -sind(t_4), 0,
a_3 ;
sind(t_4)*cosd(al_3), cosd(t_4)*cosd(al_3), -sind(al_3), -
sind(al3)*d_4;
sind(t_4)*sind(al_3), cosd(t_4)*sind(al_3), cosd(al_3),
cosd(al_3)*d_4 ;
0, 0, 0, 1
; ]
T45 = [ cosd(t_5), -sind(t_5),
a_4 ;
```

```
sind(t_5)*cosd(al4), cosd(t_5)*cosd(al_4),
sind(al_4)*d_5;
sind(t_\overline{5)*sind(al_4), cosd(t_5)*sind(al_4), cosd(al_4),}
cosd(al_4)*d_5 ;
0, 0
1 ; ]
T56 = [ cosd(t_6), -sind(t_6),
a_5 ;
sind(t_6)*cosd(al_5), cosd(t_6)*cosd(al_5), -sind(al_5), -
sind(a\_5)*d_6;
sind(t_\overline{6})*sind(al_5), cosd(t_6)*sind(al_5), cosd(al_5),
cosd(al_5)*d_6 ;
        0, 0,
1 ; ]
T_06 = T01 * T12 * T23 * T34 * T45 * T56 ;
Orientation = T_06(1:3,1:3)
position = T_06(1:3,4)
end
```


## Sub function 2

function [theta4 theta5 theta6 R6_3, T4_0] = theta456calc (theta_1, theta_2,theta_3,T)
\%DH parameters
al_0 $00 ; \mathrm{al} \_1=90$, al_2=0; al_3=90;al_4=-90;al_5=90;
$a_{-} 0=0 ; a_{-} 1=0 ; a_{2} 2=2 \overline{1} 0 ; a_{-} 3=\overline{7} 5 ; a_{-} 4=0 ; a_{-} 5=0 ;$
$d^{-} 1=0 ; d_{-}^{-} 2=0 ; d_{-}^{-} 3=0 ; d_{-} 4=210 ; d_{-}^{5}=0 ; d_{-}^{6}=70$;
t_1=theta_1;t_2 $=90+$ theta_2;t_3=90+theta_3;

```
T1_}0=[\operatorname{cosd(t_1),}\quad-\operatorname{sind(t_1),}0
a_0 ;
```

        sind (t 1)*cosd(al 0), cosd(t 1)*cosd(al 0)
    $-\operatorname{sind}\left(a l \_0\right) * d-\overline{1}$;
sind(t_1)*sind(al_0), cosd(t_1)*sind(al_0),
cosd(al_0)*d_1 ;
0 , 0,
; ]

$\begin{array}{ll}\text { T3 } \\ \text { a } 2 & {\left[\operatorname{cosd}\left(t_{-}^{2}\right),\right.}\end{array} \quad-\operatorname{sind}\left(t_{-} 3\right)$,
-sind(al_2)*d_3;
sind(t_3)*sind(al_2), cosd(t_3)*sind(al 2)
$\operatorname{cosd}(a l 2) * d \_3$;
0 , 0 ,
; ]
\%Calculating theta 4
T3_0 = T1_0*T2_1*T3_2;
R03 $=$ [T3_0(1, 1:3);
T3 0 (2, 1:3) ;
T3_0 (3, 1:3) ; ] ;
R03T $=$ transpose (R03);
R06 = [T(1,1:3);
T (2, 1:3) ;
T(3,1:3); ;
R6 $3=\operatorname{R03T} * R 06$;

```
theta4 = atan2d(R6_3(3,3),R6_3(1,3));
if theta4 == 180
    theta4 = 0;
end
%
%Calculating theta 5
T 4 = theta4;
T\overline{4}_3=[ cosd(t_4), -sind(t_4), 0,
a_3 ;
    sind(t_4)*cosd(al_3), cosd(t_4)*cosd(al_3), -sind(al_3),
-sind(al_3)*d_4;
    sind(t_4)*sind(al_3), cosd(t_4)*sind(al_3), cosd(al_3),
cosd(al_3)*d_4 ;
        0, 0,
                            ,
                            0,
                                    1
; ]
T4_0 = T3_0 * T4_3;
T4_0T = transpose(T4_0);
T46 = T4_0T * T;
theta5 =- asind(T46(1,3));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Calculating theta 6
t 4 = theta4;
t-5 = theta5;
T4
Sind(t_4
cosd(t_4)*cosd(al_3), -sind(al 3)
cosd(al_3)*d_4-4;
    0, 0
; ]
T5_4 [ cosd(t_5), -sind(t_5)
    sind(t_5)*cosd(al_4), cosd(t_5)*cosd(al_4),
-sind(al_4)*d_5;
    sind(t_5)*sind(al_4), cosd(t_5)*sind(al_4),
    0,
    0,
        -sind(al 4),
        cosd(al_4),
cosd(al_4)*d_5 ;
                            0,
                                    0,
                                    1
; ]
T5_0 = T3_0*T4_3*T5_4;
```

```
R05 = [T5_0(1,1:3);
    T5_0(2,1:3);
    T5_0(3,1:3);];
R5_0T = transpose(R05);
R6_0 = [T(1,1:3);
        T(2,1:3);
    T(3,1:3);];
R6_5 = R5_0T*R6_0;
%Calculating theta6
theta6 = asind(-R6_5(1,2));
end
```


## Sub function 3

```
function time = timecalc(line,joint)
global tt;
syms a1 a2 a3 a4 a5 v T dj ji
eqn = a1+(2*a2*T)+(3*a3*(T^2))+(4*a4*(T^3))+(5*a5* (T^4))-v == 0;
solt = solve(eqn,T);
initial_time=0;
final_time=2;
timestep=.05;
x=initial_time:timestep:final_time;%time 0 to 20 seconds
t = size(x,2);
if joint == 1
    v_max = 250;
else
        if joint == 2
            v_max = 187;
        else
            if joint == 3
                        v_max = 250;
            else
                        if joint == 4
                    v_max = 300;
                        else
                                    if joint == 5
                                    v_max = 300;
                                    else
                                    if joint == 6
                                    v_max = 300;
                            end
                                    end
                        end
            end
        end
end
if line == 1
    jointi=[51.84,12.298,-119.428,-103.03,-53.81,111.40];%home
jointf=[t_t(line,1),t_t(line,2),t_t(line,3),t_t(line,4),t_t(line,5),
t_t(line,6)];
    else
    jointi=[t_t(line-1,1),t_t(line-1,2),t_t(line-1,3),t_t(line-
1,4),t_t(line-1,5),t_t(line-1,6)];
```

```
jointf=[t_t(line,1),t_t(line, 2),t_t(line, 3),t_t(line,4),t_t(line,5),
t_t(line,}\overline{6})]
end
ji = jointi(joint);
jf = jointf(joint);
dj = jf - ji;
j_vi=0; % Initial Angular velocity is assumed as zero
j_vf=0; % Final Angular velocity is assumed as zero
j_ai=0;% Initial acceleration is assumed as zero
j_af=0; % Final acceleration is assumed as zero
%Coefficients calculation
a0=ji;
a1=j_vi;
a2=j_ai/2;
a3=[20*(dj)-(8*j_vf+12*j_vi)*T-(3*j_af-j_ai)*T.^2]/(2*T.^3);
a4=[30* (-dj) +(14*j_vf+16*j_vi)*T+(3*j_af-2*j_ai)*T.^2]/(2*T.^4);
a5=[12*(dj)-6*(j_vf}+j,vi)*\overline{T}-(j_af-j_a\overline{i})*T.^2]/(2*T.^5)
Coefficients_J1=[a0 a1 a2 a3 a4 a5]
vmaximum = (15*dj)/(8*T);
t_max = (15*dj)/(v_max*8);
time = abs(t_max);
end
```

```
global tt;
initial_time=0;
final_time=time;
timestep=.05;
x=initial_time:timestep:final_time;%time 0 to 20 seconds
t = size(x,2);
jtable(t,7)=0;%joint table
vtable(t,7)=0;%velociy table
atable(t,7)=0;%angular table
qtable(t,7)=0;%jerk table
jtable(:,1)=x;%table with time
vtable(:,1)=x;%table with time
atable(:,1)=x;%table with time
qtable(:,1)=x;%table with time
c = jtable(1:t,1);%time from 0 to 10seconds
t1 = size(c,1);
%joint parameters
    if line == 1
    %jointi=[0,-5.43,-22.0404,0,27.4438,0];%home position
[50,0,500,0,0,0]
    jointi=[51.84,12.298,-119.428,-103.03,-53.81,111.40];%home
position [230,140,215,0,-90,180]
jointf=[t_t(line,1),t_t(line, 2),t_t(line, 3),t_t(line,4),t_t(line,5),
t_t(line,}\overline{6})]
    else
    jointi=[t_t(line-1,1),t_t(line-1, 2),t_t(line-1,3),t_t(line-
1,4),t_t(line-1,5), t_t(line-1, 6)];
jointf=[t_t(line,1),t_t(line, 2),t_t(line, 3),t_t(line,4),t_t(line,5),
t_t(line,\overline{\sigma})];
        end
%Joint boundary conditions
for k=1:6
ji=jointi(k); %Initial position
jf=jointf(k); % Final position
j_vi=0; % Initial Angular velocity is assumed as zero
j_vf=0; % Final Angular velocity is assumed as zero
j_ai=0;% Initial acceleration is assumed as zero
j_af=0; % Final acceleration is assumed as zero
% Assume arm comes to the end position position with time
```

```
t_i=0;%initial time
t_f=x(1,t);%final time
T=t_f-t_i;%time difference
%Coefficients calculation
for i = 1:t
a0=ji;
a1=j_vi;
a2=j_ai/2;
a3=[20*(jf-ji)-(8*j_vf+12*j_vi)*T-(3*j_af-j_ai)*T.^2]/(2*T.^3);
a4=[30*(ji-jf)+(14*\overline{j}_vf+16*\overline{j}_vi)*T+(3*\overline{j}_af-\overline{2*j_ai)*T.^2]/(2*T.^4);}
a5=[12*(jf-ji)-6*(j_vf+j_vi)*T-(j_af-j_ai)*T.^\2]/(2*T.^5);
Coefficients_J1=[a0 a1 a2 a3 a4 a5]
jtable(i,k+1)=
a0+(a1*x(1,i))+(a2*(x(1,i))^2) +(a3*(x(1,i))^3)+(a4*(x(1,i))^4)+(a5*(
x(1,i))^5);
vtable(i,k+1)=
a1+(2*a2*x(1,i))+(3*a3*(x(1,i)^2))+(4*a4*(x(1,i)^3))+(5*a5*(x(1,i)^4
)) ;
atable(i,k+1)=
(2*a2) +(6*a3*x(1,i))+(12*a4* (x(1,i)^2)) +(20*a5* (x (1,i)^3));
qtable(i,k+1) = 6*a3+(24*a4*x(1,i))+(60*a5*(x(1,i)^2))
end
end
```


## Appendix B

Dimensions and the configuration of the Denso VP 6242
Machine head cover dimensions
Robotic cell layout

Dimensions and the configuration of the Denso VP 6242


## MACHINE HEAD COVER



ALL DIMENSIONS ARE IN MILLIMETERS

## ROBOTIC CELL LAYOUT



## Appendix C

Scenario 1 joint angles calculation results

| Line No: | Joint |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | J1 | J2 | J3 | J4 | J5 | J6 |
| 1 | -54.462 | -14.039 | 123.366 | 103.300 | 56.737 | 66.684 |
| 2 | -54.429 | -14.039 | 121.631 | 102.197 | 56.323 | 68.703 |
| 3 | -54.453 | -13.914 | 119.798 | 101.065 | 56.002 | 70.724 |
| 4 | -54.470 | -13.745 | 117.926 | 99.923 | 55.707 | 72.751 |
| 5 | -54.407 | -13.613 | 116.081 | 98.784 | 55.368 | 74.789 |
| 6 | -54.190 | -13.603 | 114.346 | 97.660 | 54.911 | 76.832 |
| 7 | -53.764 | -13.775 | 112.812 | 96.567 | 54.283 | 78.845 |
| 8 | -53.134 | -14.117 | 111.504 | 95.507 | 53.489 | 80.795 |
| 9 | -52.304 | -14.614 | 110.438 | 94.484 | 52.533 | 82.655 |
| 10 | -51.275 | -15.257 | 109.635 | 93.502 | 51.409 | 84.396 |
| 11 | -50.038 | -16.041 | 109.120 | 92.577 | 50.108 | 85.986 |
| 12 | -48.589 | -16.964 | 108.933 | 91.736 | 48.619 | 87.375 |
| 13 | -46.931 | -18.016 | 109.105 | 91.018 | 46.940 | 88.510 |
| 14 | -45.093 | -19.149 | 109.564 | 90.413 | 45.094 | 89.414 |
| 15 | -43.105 | -20.308 | 110.193 | 89.878 | 43.105 | 90.168 |
| 16 | -40.972 | -21.454 | 110.890 | 89.351 | 40.975 | 90.860 |
| 17 | -38.691 | -22.560 | 111.579 | 88.775 | 38.702 | 91.570 |
| 18 | -36.255 | -23.612 | 112.226 | 88.110 | 36.278 | 92.344 |
| 19 | -33.657 | -24.596 | 112.800 | 87.305 | 33.700 | 93.238 |
| 20 | -30.893 | -25.493 | 113.259 | 86.273 | 30.966 | 94.345 |
| 21 | -27.979 | -26.248 | 113.465 | 84.778 | 28.106 | 95.916 |
| 22 | -25.007 | -26.782 | 113.242 | 82.459 | 25.240 | 98.326 |
| 23 | -22.142 | -27.031 | 112.488 | 78.986 | 22.580 | 101.903 |
| 24 | -19.588 | -26.979 | 111.218 | 74.246 | 20.385 | 106.749 |
| 25 | -17.526 | -26.639 | 109.486 | 68.480 | 18.887 | 112.624 |
| 26 | -15.656 | -26.161 | 107.570 | 61.941 | 17.806 | 119.242 |
| 27 | -13.467 | -25.696 | 105.746 | 54.188 | 16.689 | 126.990 |
| 28 | -10.733 | -25.320 | 104.201 | 44.505 | 15.408 | 136.543 |
| 29 | -7.465 | -25.062 | 103.059 | 32.213 | 14.106 | 148.572 |
| 30 | -3.822 | -24.912 | 102.360 | 17.088 | 13.112 | 163.333 |
| 31 | 0.016 | -24.867 | 102.136 | -0.073 | 12.731 | 180.071 |
| 32 | 3.851 | -24.919 | 102.379 | -17.226 | 13.109 | 196.803 |
| 33 | 7.499 | -25.064 | 103.068 | -32.348 | 14.118 | 211.559 |
| 34 | 10.786 | -25.313 | 104.189 | -44.640 | 15.447 | 223.586 |
| 35 | 13.488 | -25.696 | 105.752 | -54.246 | 16.702 | 233.067 |
| 36 | 15.532 | -26.204 | 107.656 | -61.860 | 17.678 | 240.693 |
| 37 | 17.442 | -26.672 | 109.553 | -68.471 | 18.797 | 247.378 |
| 38 | 19.650 | -26.963 | 111.196 | -74.283 | 20.447 | 253.284 |
| 39 | 22.213 | -27.010 | 112.458 | -79.003 | 22.651 | 258.109 |


| 40 | 25.062 | -26.767 | 113.229 | -82.482 | 25.295 | 261.696 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 41 | 28.012 | -26.257 | 113.515 | -84.862 | 28.135 | 264.178 |
| 42 | 30.914 | -25.515 | 113.350 | -86.390 | 30.983 | 265.791 |
| 43 | 33.676 | -24.606 | 112.851 | -87.369 | 33.717 | 266.838 |
| 44 | 36.272 | -23.601 | 112.206 | -88.101 | 36.296 | 267.644 |
| 45 | 38.707 | -22.545 | 111.549 | -88.757 | 38.718 | 268.407 |
| 46 | 40.987 | -21.444 | 110.877 | -89.348 | 40.990 | 269.136 |
| 47 | 43.119 | -20.294 | 110.170 | -89.867 | 43.119 | 269.817 |
| 48 | 45.108 | -19.125 | 109.505 | -90.379 | 45.110 | 270.536 |
| 49 | 46.944 | -17.993 | 109.045 | -90.983 | 46.953 | 271.440 |
| 50 | 48.590 | -16.962 | 108.926 | -91.732 | 48.620 | 272.619 |
| 51 | 50.031 | -16.051 | 109.141 | -92.587 | 50.101 | 274.028 |
| 52 | 51.270 | -15.264 | 109.650 | -93.510 | 51.404 | 275.615 |
| 53 | 52.306 | -14.614 | 110.441 | -94.486 | 52.534 | 277.349 |
| 54 | 53.135 | -14.116 | 111.505 | -95.509 | 53.491 | 279.208 |
| 55 | 53.763 | -13.777 | 112.817 | -96.569 | 54.283 | 281.159 |
| 56 | 54.191 | -13.602 | 114.349 | -97.662 | 54.913 | 283.172 |
| 57 | 54.417 | -13.602 | 116.073 | -98.783 | 55.378 | 285.214 |
| 58 | 54.474 | -13.740 | 117.923 | -99.923 | 55.712 | 287.251 |
| 59 | 54.447 | -13.922 | 119.805 | -101.067 | 55.995 | 289.276 |
| 60 | 54.415 | -14.055 | 121.645 | -102.202 | 56.310 | 291.297 |
| 61 | 54.462 | -14.039 | 123.366 | -103.300 | 56.737 | 293.316 |

## Appendix D

Scenario 3 joint angles calculation results

| Line No: | Joint |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | J1 | J2 | J3 | J4 | J5 | J6 |
| 1 | -54.462 | -14.039 | 123.366 | 103.300 | 56.737 | 66.684 |
| 2 | -54.462 | -13.962 | 120.765 | 101.667 | 56.191 | 69.640 |
| 3 | -54.462 | -13.766 | 118.067 | 100.006 | 55.721 | 72.607 |
| 4 | -54.352 | -13.581 | 115.373 | 98.339 | 55.215 | 75.591 |
| 5 | -53.836 | -13.742 | 113.023 | 96.723 | 54.382 | 78.557 |
| 6 | -52.891 | -14.259 | 111.134 | 95.175 | 53.202 | 81.402 |
| 7 | -51.510 | -15.112 | 109.794 | 93.714 | 51.662 | 84.026 |
| 8 | -49.688 | -16.271 | 109.066 | 92.369 | 49.746 | 86.336 |
| 9 | -47.431 | -17.691 | 108.984 | 91.188 | 47.445 | 88.244 |
| 10 | -44.772 | -19.312 | 109.552 | 90.242 | 44.772 | 89.659 |
| 11 | -41.773 | -21.034 | 110.626 | 89.543 | 41.775 | 90.612 |
| 12 | -38.464 | -22.665 | 111.647 | 88.719 | 38.475 | 91.636 |
| 13 | -34.820 | -24.170 | 112.549 | 87.671 | 34.853 | 92.837 |
| 14 | -30.824 | -25.532 | 113.329 | 86.315 | 30.895 | 94.293 |
| 15 | -28.701 | -26.103 | 113.519 | 85.293 | 28.807 | 95.368 |
| 16 | -26.541 | -26.546 | 113.443 | 83.815 | 26.709 | 96.917 |
| 17 | -24.396 | -26.849 | 113.100 | 81.797 | 24.665 | 99.013 |
| 18 | -22.323 | -27.005 | 112.498 | 79.166 | 22.751 | 101.724 |
| 19 | -20.382 | -27.015 | 111.650 | 75.873 | 21.047 | 105.092 |
| 20 | -18.630 | -26.882 | 110.572 | 71.941 | 19.634 | 109.096 |
| 21 | -17.116 | -26.616 | 109.288 | 67.500 | 18.576 | 113.605 |
| 22 | -15.867 | -26.233 | 107.829 | 62.789 | 17.904 | 118.384 |
| 23 | -14.418 | -25.864 | 106.427 | 57.472 | 17.176 | 123.725 |
| 24 | -12.628 | -25.553 | 105.178 | 51.208 | 16.290 | 129.943 |
| 25 | -10.523 | -25.302 | 104.116 | 43.756 | 15.311 | 137.278 |
| 26 | -8.149 | -25.108 | 103.260 | 34.895 | 14.347 | 145.952 |
| 27 | -5.560 | -24.971 | 102.633 | 24.493 | 13.515 | 156.108 |
| 28 | -2.819 | -24.889 | 102.251 | 12.682 | 12.944 | 167.630 |
| 29 | 0.000 | -24.862 | 102.123 | 0.000 | 12.739 | 180.000 |
| 30 | 2.818 | -24.889 | 102.251 | -12.681 | 12.944 | 192.369 |


| 31 | 5.560 | -24.971 | 102.633 | -24.492 | 13.514 | 203.891 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 32 | 8.149 | -25.108 | 103.260 | -34.895 | 14.347 | 214.048 |
| 33 | 10.522 | -25.302 | 104.115 | -43.755 | 15.311 | 222.722 |
| 34 | 12.627 | -25.553 | 105.179 | -51.207 | 16.289 | 230.056 |
| 35 | 14.418 | -25.864 | 106.427 | -57.471 | 17.177 | 236.275 |
| 36 | 15.867 | -26.233 | 107.829 | -62.789 | 17.904 | 241.616 |
| 37 | 17.118 | -26.616 | 109.287 | -67.501 | 18.578 | 246.397 |
| 38 | 18.630 | -26.882 | 110.572 | -71.941 | 19.634 | 250.905 |
| 39 | 20.382 | -27.015 | 111.650 | -75.874 | 21.047 | 254.909 |
| 40 | 22.324 | -27.005 | 112.498 | -79.166 | 22.751 | 258.276 |
| 41 | 24.396 | -26.849 | 113.100 | -81.798 | 24.665 | 260.987 |
| 42 | 26.541 | -26.546 | 113.443 | -83.815 | 26.709 | 263.083 |
| 43 | 28.701 | -26.103 | 113.520 | -85.293 | 28.807 | 264.632 |
| 44 | 30.824 | -25.532 | 113.329 | -86.315 | 30.895 | 265.707 |
| 45 | 34.821 | -24.170 | 112.549 | -87.671 | 34.853 | 267.163 |
| 46 | 38.464 | -22.665 | 111.647 | -88.719 | 38.475 | 268.365 |
| 47 | 41.773 | -21.034 | 110.626 | -89.543 | 41.775 | 269.388 |
| 48 | 44.772 | -19.312 | 109.552 | -90.242 | 44.773 | 270.341 |
| 49 | 47.432 | -17.690 | 108.984 | -91.188 | 47.445 | 271.756 |
| 50 | 49.688 | -16.271 | 109.066 | -92.369 | 49.746 | 273.664 |
| 51 | 51.510 | -15.112 | 109.795 | -93.714 | 51.662 | 275.974 |
| 52 | 52.891 | -14.259 | 111.135 | -95.175 | 53.202 | 278.598 |
| 53 | 53.836 | -13.742 | 113.023 | -96.723 | 54.382 | 281.443 |
| 54 | 54.352 | -13.581 | 115.373 | -98.339 | 55.215 | 284.409 |
| 55 | 54.462 | -13.766 | 118.067 | -100.006 | 55.721 | 287.394 |
| 56 | 54.462 | -13.962 | 120.765 | -101.667 | 56.191 | 290.360 |
| 57 | 54.462 | -14.039 | 123.366 | -103.300 | 56.737 | 293.316 |

## Appendix E

Scenario 4 joint angles calculation results

| Line No: | Joint |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | J1 | J2 | J3 | J4 | J5 | J6 |
| 1 | -54.462 | -14.039 | 123.366 | 103.300 | 56.737 | 66.684 |
| 2 | -54.462 | -13.962 | 120.765 | 101.667 | 56.191 | 69.640 |
| 3 | -54.462 | -13.766 | 118.067 | 100.006 | 55.721 | 72.606 |
| 4 | -54.352 | -13.581 | 115.373 | 98.339 | 55.215 | 75.591 |
| 5 | -53.836 | -13.742 | 113.023 | 96.723 | 54.382 | 78.557 |
| 6 | -52.891 | -14.259 | 111.134 | 95.175 | 53.202 | 81.402 |
| 7 | -51.510 | -15.112 | 109.795 | 93.714 | 51.662 | 84.026 |
| 8 | -49.688 | -16.271 | 109.066 | 92.369 | 49.746 | 86.336 |
| 9 | -47.431 | -17.691 | 108.984 | 91.188 | 47.444 | 88.244 |
| 10 | -44.772 | -19.312 | 109.552 | 90.242 | 44.773 | 89.659 |
| 11 | -41.774 | -21.034 | 110.626 | 89.543 | 41.775 | 90.612 |
| 12 | -38.464 | -22.665 | 111.647 | 88.720 | 38.475 | 91.635 |
| 13 | -34.821 | -24.170 | 112.549 | 87.671 | 34.854 | 92.837 |
| 14 | -31.197 | -25.419 | 113.275 | 86.464 | 31.264 | 94.134 |
| 15 | -27.754 | -26.315 | 113.521 | 84.707 | 27.884 | 95.983 |
| 16 | -25.135 | -26.760 | 113.249 | 82.564 | 25.363 | 98.219 |
| 17 | -22.947 | -26.973 | 112.709 | 80.041 | 23.319 | 100.825 |
| 18 | -20.991 | -27.028 | 111.950 | 77.010 | 21.570 | 103.931 |
| 19 | -19.202 | -26.943 | 110.962 | 73.344 | 20.078 | 107.669 |
| 20 | -17.623 | -26.727 | 109.770 | 69.127 | 18.906 | 111.952 |
| 21 | -16.326 | -26.382 | 108.382 | 64.585 | 18.133 | 116.564 |
| 22 | -15.025 | -26.001 | 106.958 | 59.649 | 17.483 | 121.546 |
| 23 | -13.397 | -25.672 | 105.666 | 53.889 | 16.667 | 127.289 |
| 24 | -11.451 | -25.401 | 104.542 | 47.078 | 15.731 | 134.014 |
| 25 | -9.226 | -25.186 | 103.610 | 38.989 | 14.764 | 141.948 |
| 26 | -6.769 | -25.027 | 102.891 | 29.447 | 13.871 | 151.273 |
| 27 | -4.135 | -24.921 | 102.401 | 18.442 | 13.174 | 162.011 |
| 28 | -1.391 | -24.869 | 102.154 | 6.295 | 12.789 | 173.860 |
| 29 | 1.390 | -24.869 | 102.154 | -6.293 | 12.789 | 186.138 |
| 30 | 4.134 | -24.921 | 102.401 | -18.441 | 13.174 | 197.987 |


| 31 | 6.768 | -25.027 | 102.891 | -29.446 | 13.871 | 208.725 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 32 | 9.226 | -25.186 | 103.610 | -38.988 | 14.763 | 218.051 |
| 33 | 11.451 | -25.401 | 104.543 | -47.077 | 15.731 | 225.985 |
| 34 | 13.397 | -25.672 | 105.666 | -53.889 | 16.667 | 232.711 |
| 35 | 15.024 | -26.001 | 106.959 | -59.647 | 17.482 | 238.452 |
| 36 | 16.330 | -26.382 | 108.380 | -64.588 | 18.136 | 243.438 |
| 37 | 17.623 | -26.727 | 109.770 | -69.127 | 18.906 | 248.048 |
| 38 | 19.202 | -26.943 | 110.962 | -73.344 | 20.078 | 252.331 |
| 39 | 20.991 | -27.028 | 111.950 | -77.010 | 21.570 | 256.069 |
| 40 | 22.947 | -26.973 | 112.709 | -80.041 | 23.319 | 259.175 |
| 41 | 25.133 | -26.760 | 113.249 | -82.562 | 25.362 | 261.779 |
| 42 | 27.752 | -26.315 | 113.520 | -84.705 | 27.882 | 264.014 |
| 43 | 31.197 | -25.418 | 113.272 | -86.460 | 31.263 | 265.861 |
| 44 | 34.821 | -24.170 | 112.549 | -87.671 | 34.854 | 267.163 |
| 45 | 38.464 | -22.665 | 111.647 | -88.720 | 38.475 | 268.365 |
| 46 | 41.774 | -21.034 | 110.626 | -89.543 | 41.775 | 269.388 |
| 47 | 44.772 | -19.312 | 109.552 | -90.242 | 44.773 | 270.341 |
| 48 | 47.432 | -17.690 | 108.984 | -91.188 | 47.445 | 271.756 |
| 49 | 49.688 | -16.271 | 109.066 | -92.370 | 49.746 | 273.664 |
| 50 | 51.510 | -15.112 | 109.795 | -93.714 | 51.662 | 275.974 |
| 51 | 52.891 | -14.259 | 111.135 | -95.175 | 53.202 | 278.598 |
| 52 | 53.836 | -13.742 | 113.023 | -96.723 | 54.382 | 281.443 |
| 53 | 54.352 | -13.581 | 115.373 | -98.339 | 55.215 | 284.409 |
| 54 | 54.462 | -13.766 | 118.067 | -100.006 | 55.721 | 287.394 |
| 55 | 54.462 | -13.962 | 120.765 | -101.667 | 56.191 | 290.360 |
| 56 | 54.462 | -14.039 | 123.366 | -103.300 | 56.737 | 293.316 |
|  |  |  |  |  |  |  |

