

NONLINEAR ANALYSIS OF CABLE STRUCTURES

By



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DECLARATION

I hereby, declare, that the work included in this thesis in part or whole, has not been submitted for any other academic qualification at any institution.

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ABSTRACT

Large structures are widely used in the modern construction industry for infra-structure facilities development. Among these, long span structures with cables are becoming increasingly popular. In this category of structures deformations are large and estimations based on small deformation theory in the normal analysis are inadequate.

The large deformation analysis results in nonlinear behavior where principle of superposition does not hold. In geometrical nonlinear analysis, the equations of equilibrium are based on the deformed geometry after the load application. The length of a curved deflected line is longer than the initial length and the basic assumptions used in linear analysis may cause inaccuracies when the deformations are very large. It is also essential that bending effects of cables are considered.

Here we deal with large deformations, but small strain problems with linear stress-strain relationships. Although there are many methods found in literature to analyze cables exhibiting large deformation nonlinear behavior, it is hard to find a universal approach to describe the exact behavior of a cable considering all geometrical nonlinearity issues.

The analysis described in this study recognizes all such influences contributing to geometrical non-linearity. The procedure developed is versatile and gives a state-of-the-art analytical tool. This work fills a void in the current practice recognizing large deformation issues without any knowledge of small or large strains as opposed to what is required in commercial software.

A numerical solution procedure has been evolved to solve the resulting nonlinear non-homogeneous integral differential equation. The procedure is converging and a computer program has been developed for practical use. The results are compared with those in literature to validate the findings and to ensure the accuracy of the new large deformation nonlinear analysis technique.

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H. C. T. Peiris

DEDICATION

To

my parents

and

all those who are interested and committed in advancement of science



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CONTENTS

Declaration	ii
Abstract	iii
Acknowledgement	iv
Dedication	v
List of Figures	x
List of Tables	xiv
Abbreviations	xv
1 Introduction	1
1.1 Background	1
1.2 Non-Linear Load – Deflection Response.	2
1.2.1 Material Non-Linearity.	3
1.2.2 Geometric Non-Linearity	3
1.3 Linear vs. Non-Linear Analysis.	3
1.4 Effect of Axial Load on the Stiffness.	6
1.5 Use of Cables in Structures	7
1.6 Objectives of the Research	11
1.7 Scope	11
1.8 Organisation of the Thesis	11
2 Literature Review	13
2.1 Introduction	13
2.2 Historical Aspects of the Large Deformation Structures	14
2.3 Analytical Methods	16
2.3.1 Analysis of a Suspended Cables under Point Loads	16
2.3.2 Analysis of a Suspended Cable Subjected to Uniformly Distributed Loading	18
2.3.2.1 Elastic Catenary	19
2.3.2.2 Elastic Parabola	22
2.3.2.3 Summary	23
2.4 P-Delta Analysis for Large Deformation Structures	24

2.4.1 Effect on Stiffness of a Flexural member Subjected to a Tensile Load.	24
2.4.2 Effect on Stiffness of a Flexural member Subjected to Compression Loading	26
2.4.3 Summary on P-delta Analysis	28
2.5 Finite Element Methods	29
2.5.1 Tangent Stiffness Matrix of a member of plane or space truss.	29
2.5.1.1 Solution Procedure by Using Newton-Raphson's Technique.	34
2.5.1.2 Calculation in One Iteration Cycle	35
2.5.1.3 Convergence Criteria	37
2.5.1.4 Summary	37
2.5.2 Tangent Stiffness Matrix of a Member of Plane Frame.	38
2.5.2.1. Solution Procedure for Plane Frame Elements by Using Newton-Raphson's Technique.	42
2.5.3 The Total Lagrangian Bar Element	46
2.5.3.1 Element Kinematics	47
2.5.3.2 Strain Measure	49
2.5.3.3 Stress Measure	50
2.5.3.4 The Tangent Stiffness Matrix	51
2.5.3.5 Summary	53
2.5.4 Catenary Cable Element	53
2.5.4.1 Summary	55
2.5.5 Three-Node Isoparametric Cable Finite Element	55
2.5.5.1 Solution Procedure	61
2.5.5.2 Convergence Criteria	62
2.5.5.3 Summary	62
2.6 Nonlinear Analysis Using Integrated Force Method	63
2.6.1 Introduction	63
2.6.2 Basic theory of integrated force method	64
2.6.3 Nonlinear analysis by integrated force method	66
2.6.4 Development of elemental matrices	70
2.6.5 Summary	71
2.7 The Theory of Elastica	72
2.7.1 The Large Deformation of a Cantilever Beam with concentrated Load.	72

2.7.2 Flexible Cantilever Beams Subjected to Distributed and Combined Loadings.	76
2.7.3 Uniform Simply Supported Beam Subjected to a Concentrated Load	79
2.7.4 Simply Supported Beams Loaded with a Uniformly Distributed Loading	82
2.7.5 Statically Indeterminate Flexible Bars Subjected to a Concentrated Load.	84
2.7.6 Statically Indeterminate Single Span Beams Subjected to Distributed Loadings	87
2.7.7 Summary	89
2.8 Non-Linear Analysis in SAP 2000 Non-Linear	90
2.8.1 Nonlinear analyze cases	90
2.8.2 P-Delta forces in the Frame Element.	91
2.8.3 Initial P-Delta Analysis	92
2.8.4 Large Displacement Analysis	94
2.8.5 Initial Large –Displacement Analysis	94
2.9 Modified Elastic Stiffness	95
2.9.1 Other Formulae for Equivalent Modulus of Elasticity	97
2.9.2 Summary	98
2.10 Limitations in the Available Methods	99
2.11 Summary	101
3 Methodology	102
3.1 Introduction	102
3.2 Static Behaviour of a Cable	103
3.3 Finite Difference Solution for Differential Equations	108
3.3.1 Introduction	108
3.3.2 Representation of derivatives by finite differences.	109
3.3.3 Errors in finite-difference equations	110
3.4 Solution of Equations Using Gauss-Seidel Method	112
3.4.1 Introduction	112
3.4.2 Gauss-Seidel method	112
3.5 Polynomial Curve Fitting Near the Boundaries	113
3.6 Identification of the Problem	115

3.6.1 Cable Properties, Coordinate system, Sign Convention	115
3.6.2 Loading	115
3.6.3 Boundary Conditions	117
3.7 Solution Procedure	117
3.8 Flow Chart for the Computer Programme	134
3.9 Summary	137
4 Results and Analysis	138
4.1 Sensitivity of the Stiffness of Cables against Different Problem parameters	139
4.2 Validity of Superposition	147
4.3 Large Deformation vs Small Deformation.	152
4.4 Bending of Cables	156
4.5 Effect of Sag on Axial Stiffness	161
4.6 Comparison with the Catenary and Parabolic Approximations.	164
4.7 Comparison with the Finite Element Modeling	168
4.8 Summary	172
5 Conclusions and Recommendations	173
5.1 Introduction	173
5.2 Conclusions	173
5.3 Recommendations for Further Research	177
 References	 178
 Appendices	
A. Estimation of End Forces Caused by End Displacement in P-Delta Analysis	182
B. Computer Program for Large Deformation Geometrically Nonlinear Analysis of Cable Structures.	183

LIST OF FIGURES

Figure 1.1	Load deflection response diagram	2
Figure 1.2	General Deformation of a member	5
Figure 1.3	Deformation of a Cantilever Subjected to an Axial Force and a Point Load	6
Figure 1.4	General representation of a Structure Response Curve	7
Figure 1.5	a) Normandy Bridge b) Ikuchi Bridge,	8
Figure 1.6	a) Akashi-kaikyo bridge b) Humber Bridge	9
Figure 1.7	a) Millennium Dome b) Georgia Dome	9
Figure 1.8	a) Suspending cables in power transmission b) Guyed Towers	10
Figure 1.9	a) Ribbon stressed bridge, b) Lateral stiffening by using cables, c) Support structure for a pipe	10
Figure 2.1	Reference configuration	16
Figure 2.2	Deformed configuration	16
Figure 2.3	Suspended cable subjected to self weight and axial tension	19
Figure 2.4	Elastic Catenary / Parabola	21
Figure 2.5	Effects of tensile forces on bending deformations	24
Figure 2.6	Effects of compressive axial forces on bending deformations	26
Figure 2.7	Variation of S vs μ	27
Figure 2.8	Variation of C vs μ	28
Figure 2.9	Derivation of the geometric stiffness matrices for a truss	30
Figure 2.10	Derivation of tangent stiffness matrix for a plane frame member	40
Figure 2.11	Member end-forces.	43
Figure 2.12	A plane truss structure undergoing large displacements	46
Figure 2.13	The Geometrically nonlinear, two-node, two- dimensional bar element in total Lagrangian description.	47
Figure 2.14	The definition of displacement fields	48
Figure 2.15	Geometric Interpretations of the Quantities used in Element Kinematics	49
Figure 2.16	Forces on a catenary cable finite element	53
Figure 2.17	Geometry of the proposed Cable Finite Element	55

Figure 2.18 (a) General displacement of the element (b) Displacement of a arbitrary differential element	56
Figure 2.19 2D element for nonlinear analysis	70
Figure 2.20 (a) Large deformation of a cantilever beam of uniform cross section; (b) Free body diagram of a beam element.	74
Figure 2.21 (a) Uniform cantilever beam loaded with a distributed loading k ; (b) uniform cantilever beam; padded with a distributed load k and a concentrated load P at the free end.	76
Figure 2.22 Simply supported beam loaded with a concentrated load P at any point along its length.	80
Figure 2.23 Straight and deflected configuration of a simply supported beam loaded with a uniformly distributed load.	83
Figure 2.24 Statically indeterminate beam loaded with a concentrated load P at any point along its length.	85
Figure 2.25 Straight and deflected configuration of a statically indeterminate beam loaded with a uniformly distributed load.	87
Figure 2.26 (a) Parabolic profile of the cable (b) Typical element from the cable taken from the node 2	95
Figure 3.1 Cables subjected to different point loads	106
Figure 3.2 Different distributed loads on the cables	106
Figure 3.3 (a) deformed configuration for end rotations (b) Deformed configuration for combined end translation and rotations	107
Figure 3.4 Graph of function $y=f(x)$	109
Figure 3.5 Loading, Deformed configuration and other problem parameters	116
Figure 3.6 Sub-divisions of the Cable for apply the Finite Difference Schemes	124
Figure 3.7 Applying corrections to the Existing End Moments	126
Figure 3.8 Calculating Axial Force along the Tangential direction	131
Figure 3.9 Flow Chart for Arrangement of the Computer Programme	134
Figure 3.10 Flowchart for Subroutine 01	135
Figure 3.11 Flowchart for Subroutine 02	136

Figure 4.1	Comparison of the variation of S vs μ for different number of nodes in finite difference scheme	140
Figure 4.2	Comparison of the variation of C vs μ for different number of nodes in finite difference scheme	140
Figure 4.3	Comparison of the variation of S vs μ for members having different lengths	142
Figure 4.4	Comparison of the variation of C vs μ for members having different lengths	142
Figure 4.5	Comparison of the variation of S vs μ for members subjected to different end rotations	143
Figure 4.6	Comparison of the variation of C vs μ for members subjected to different end rotations	143
Figure 4.7	Variation of S vs μ for members subjected to different uniformly distributed loads along the projected length	144
Figure 4.8	Variation of C vs μ for members subjected to different uniformly distributed loads along the projected length	145
Figure 4.9	Variation of S vs μ for members subjected to different uniformly distributed loads along the deformed length (catenary loading)	145
Figure 4.10	Variation of C vs μ for members subjected to different uniformly distributed loads along the deformed length (catenary loading)	146
Figure 4.11	Variation of Fixed End Moment vs μ for different uniformly distributed loads along a projected length of the cable.	148
Figure 4.12	Variation of Fixed End Moment vs μ for different uniformly distributed loads along the Deformed length of the cable.	149
Figure 4.13	Comparison between the superimposed values and directly analyzed values.	149
Figure 4.14	Comparison of Fixed End Moment at node 01 vs μ , between the directly analyzed values and the superposed values	150
Figure 4.15	Comparison of Fixed End Moment at node 02 vs μ between the directly analyzed values and the superposed values for a W=50 kN/m UDL and a Unit rotation at node 02	151

Figure 4.16 Undeformed lengths vs μ of the member subjected to $W=50$ kN/m UDL, Unit rotation at node 02, and the combination of loading	151
Figure 4.17 Variation of the support moments vs μ for small and large displacement analysis	153
Figure 4.18 Variation of the undeformed length vs μ for small and large displacement analysis	154
Figure 4.19 Deformed configuration obtained from large deformation analysis and small deformation analysis	155
Figure 4.20 Variation of the Fixed End Moments vs μ for different small and large displacement analysis	156
Figure 4.21 a) deformed configurations for live load and dead loads, b)Variation of bending moments along the length of a cable and a beam.	158
Figure 4.22 a) deformed configurations for dead load and lateral deflection of the support b)Variation of bending moments along the length of a cable and a beam.	160
Figure 4.23 Applying incremental axial deformations to initially loaded cables	161
Figure 4.24 Variation of axial forces and sag vs axial deformation	162
Figure 4.25 Variation of axial forces and sag vs axial deformation	162
Figure 4.26 Comparison of the variation of axial forces vs axial deformation	164
Figure 4.27 a) Definition of the compared parameters. b) Deformed shape of a stay cable of Normandy Bridge from the output of the program.	165
Figure 4.28 Definition of the problem parameters for comparison of finite element analysis verses the developed analytical technique.	168
Figure 4.29 Sensitivity of the sag for the variation of the undeformed length	171
Figure 4.30 Sensitivity of the axial force	171

LIST OF TABLES

Table 2.1	Maximum Sags calculated using catenary and parabolic methods.	18
Table 4.1	Bending Moments and Displacements Due to Live loads of cable stressed structures	159
Table 4.2	Bending moments at supports due to unit vertical displacement at the support 1	161
Table 4.3	Comparison of the Deformational Characteristics of actual Cables vs the Results obtained by the new technique	166
Table 4.4	representation of the results of deformed cables in (Table) assuming a UDL along a projected length of the cable by using both analytical techniques and the new technique.	167
Table 4.5	Comparison between finite element method and the new analysis technique in analysis of the cables	169



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ABBREVIATIONS

A	Cross sectional area
A_θ	Matrix contains the derivatives of the displacements
B	Strain - displacement gradient relationship matrix / Equilibrium matrix
C	Carry over factor
CC	Compatibility conditions
$[C_c]$	Compatibility matrix
D	Nodal coordinate vector
DDR	Deformation displacement relationship
$[D_f^1], [D_f^2]$	differential operator matrix
E	Modulus of elasticity
e, ε	Material strain
e_1, e_2	errors in finite difference
E_{EQ}	Equivalent modulus of elasticity
f	external load vector
F	Independent member forces
F_r	Nodal forces due to temperature effects
GL	Green Lagrange
$[G_s]$	Flexibility matrix of the whole structure
$[G_t]$	geometry matrix
h	Interval of finite difference technique
I	Second Moment of area
i, j, k, l, m, n, r	integers
IE	Internal energy
IFM	Integrated Force Method
$[J]$	Jacobian transformation matrix

k	Uniformly distributed load along the member.(Catenary udl)
$[\mathbf{K}], [\mathbf{S}]$	Stiffness matrix
$[\mathbf{K}_G], [S_g]$	Geometrical Stiffness matrix
$[\mathbf{K}_M], [S_e]$	Elastic/ Material Stiffness matrix
l	Horizontal distance between the supports
L	Span
L_C	Curved length of the member
L_0	Original Undeformed length of the cable
L_U	Undeformed length of the cable (in an iteration)
M	Bending moment
n	Number of divisions of the member in finite difference
N	Axial force
N_i / H_i	Shape function
\bar{p}	surface forces per unit area of the deformed body A ,
P	Force / Point load
$p(x)$	Polynomial curve approximation for the profile
P_{eq}	equivalent nodal forces due to $\{q\}$ and $\{\bar{p}\}$.
q	body forces per unit mass
Q	Unbalance force/ Out of balance force
r	Radius of curvature
s	Length of the member measured along the profile
S	Rotational stiffness
s, c	Sine and cosines
$[\bar{S}_e]$	elastic stiffness matrix of the node
$[\bar{S}_g]$	geometric stiffness matrix of the node
$[\bar{S}_t]$	Tangential stiffness matrix of the node

Stdv	Standard deviation
T	Tensile force
$[t]$	Transformation matrix
$T_1 T_2$	Force components along the axis of the cable
TL	Total Lagrangian
u	Displacement within the element
V	Volume
w	Uniformly distributed load along a projected line of the member.
W	Total load along the cable
$\rightarrow X$	Coordinate axis for x measurements
x	Measurement along X/ measurement in x direction of the of other points in finite difference technique when considered point is measured as X
X	measurement in x direction of the considered point in finite difference technique
$\rightarrow Y$	Coordinate axis for y measurements
y	Measurement along Y/ measurement in y direction of the of other points in finite difference technique when considered point is measured as Y
Y	measurement in y direction of the considered point in finite difference technique
$\rightarrow Z$	Coordinate axis for z measurements
z	Measurement along Z
α	$\sqrt{T / EI}$
β	Constant in finite difference
γ	Constant in finite difference
Δ	Small increment
δ	Linear displacement of a point
ε^*	effective strain
η	Inclination of direction of load w to the normal axis of the cable

θ	Angular deformation measurement
\mathcal{G}	generalized internal deformations of the element
λ	cosines of the angle between the member local axis and the global axes
μ	Non dimensional stress parameter
ξ	Isoparametric coordinate
π	Total potential energy functional
ρ	mass density and
σ	Axial stress
ν	Tolerance value
ϕ	Inclination of direction of load k to the normal axis of the cable
φ	Inclination of the cable to the horizontal
Φ	Scalar function
$[\Omega]$	Compliance matrix
ω	Angle of deviation at supports of a pinned supported member
$(^0)$	(superscript) Value at the start of the current cycle
$(^1)$	(superscript) Refers to the state at cycle end
$in, ^0$	(subscript, superscript) Initial states of the problem
e	(subscript) use to symbolize elemental properties
L, NL	(subscript) Linear, Nonlinear component
$[\]$	Brackets indicate rectangular or square matrix.
$\{ \}$	Curly brackets indicate a vector.
$' , ''$	First, Second derivative