NONLINEAR ANALYSIS OF CABLE STRUCTURES

By

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DECLARATION

I hereby, declare, that the work included in this thesis in part or whole, has not been submitted for any other academic qualification at any institution.

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ABSTRACT

Large structures are widely used in the modern construction industry for infra-structure facilities development. Among these, long span structures with cables are becoming increasingly popular. In this category of structures deformations are large and estimations based on small deformation theory in the normal analysis are inadequate.

The large deformation analysis results in nonlinear behavior where principle of superposition does not hold. In geometrical nonlinear analysis, the equations of equilibrium are based on the deformed geometry after the load application. The length of a curved deflected line is longer than the initial length and the basic assumptions used in linear analysis may cause inaccuracies when the deformations are very large. It is also essential that bending effects of cables are considered.

Here we deal with large deformations, but small strain problems with linear stress-strain relationships. Although there are many methods found in literature to analyze cables exhibiting large deformation nonlinear behavior, it is hard to find a universal approach to describe the exact behavior of a cable considering all geometrical nonlinearity issues.

The analysis described in this study recognizes all such influences contributing to geometrical non-linearity. The procedure developed is versatile and gives a state-of-the-art analytical tool. This work fills a void in the current practice recognizing large deformation issues without any knowledge of small or large strains as opposed to what is required in commercial software.

A numerical solution procedure has been evolved to solve the resulting nonlinear non-homogeneous integral differential equation. The procedure is converging and a computer program has been developed for practical use. The results are compared with those in literature to validate the findings and to ensure the accuracy of the new large deformation nonlinear analysis technique.

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H. C. T. Peiris

DEDICATION

To

my parents

and

all those who are interested and committed in advancement of science

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ABBRIVIATIONS

A	Cross sectional area
A_θ	Matrix contains the derivatives of the displacements
B	Strain - displacement gradient relationship matirix / Equilibrium matrix
C	Carry over factor
CC	Compatibility conditions
$[C_c]$	Compatibility matrix
D	Nodal coordinate vector
DDR	Deformation displacement relationship
$[D_f^1]$, $[D_f^2]$	differential operator matrix
E	Modulus of elasticity
e, ε	Material strain
e_1, e_2	errors in finite difference
E_{EQ}	Equivalent modulus of elasticity
f	external load vector
F	Independent member forces
F_r	Nodal forces due to temperature effects
GL	Green Lagrange
$[G_s]$	Flexibility matrix of the whole structure
$[G_t]$	geometry matrix
h	Interval of finite difference technique
I	Second Moment of area
i,j,k,it,iii,ii,r	integers
IE	Internal energy
IFM	Integrated Force Method
[J]	Jacobian transformation matrix

k	Uniformly distributed load along the member.(Catenary udl)
[K], [S]	Stiffness matrix
$[\mathbf{K}_G],[S_g]$	Geometrical Stiffness matrix
$[\mathbf{K}_M],[S_e]$	Elastic/ Material Stiffness matrix
l	Horizontal distance between the supports
L	Span
L_C	Curved length of the member
L_0	Original Undeformed length of the cable
L_U	Undeformed length of the cable (in an iteration)
M	Bending moment
n	Number of divisions of the member in finite difference
N	Axial force
N_i / H_i	Shape function
$\overline{p}$	surface forces per unit area of the deformed body $\overline{A}$,
P	Force / Point load
$p(x)$	Polynomial curve approximation for the profile
P_{eq}	equivalent nodal forces due to $\{q\}$ and $\{\overline{p}\}$.
q	body forces per unit mass
Q	Unbalance force/ Out of balance force
r	Radius of curvature
s	Length of the member measured along the profile
S	Rotational stiffness
s,c	Sine and cosines
$[\overline{S}_e]$	elastic stiffness matrix of the node
$[\overline{S}_g]$	geometric stiffness matrix of the node
$[\overline{S}_t]$	Tangential stiffness matrix of the node

Stdv	Standard deviation
T	Tensile force
[t]	Transformation matrix
T_1 T_2	Force components along the axis of the cable
TL	Total Lagrangian
u	Displacement within the element
V	Volume
w	Uniformly distributed load along a projected line of the member.
W	Total load along the cable
$\rightarrow X$	Coordinate axis for x measurements
x	Measurement along X/ measurement in x direction of the of other points in finite difference technique when considered point is measured as X
X	measurement in x direction of the considered point in finite difference technique
$\rightarrow Y$	Coordinate axis for y measurements
y	Measurement along Y/ measurement in y direction of the of other points in finite difference technique when considered point is measured as Y
Y	measurement in y direction of the considered point in finite difference technique
$\rightarrow Z$	Coordinate axis for z measurements
z	Measurement along Z
α	$\sqrt{T / EI}$
β	Constant in finite difference
γ	Constant in finite difference
Δ	Small increment
δ	Linear displacement of a point
ε*	effective strain
η	Inclination of direction of load w to the normal axis of the cable

θ	Angular deformation measurement
ϑ	generalized internal deformations of the element
λ	cosines of the angle between the member local axis and the global axes
μ	Non dimensional stress parameter
ξ	Isoparametric coordinate
π	Total potential energy functional
ρ	mass density and
σ	Axial stress
υ	Tolerance value
ϕ	Inclination of direction of load k to the normal axis of the cable
φ	Inclination of the cable to the horizontal
Φ	Scalar function
$[\Omega]$	Compliance matrix
ω	Angle of deviation at supports of a pinned supported member
$^{(0)}$	(superscript) Value at the start of the current cycle
$^{(1)}$	(superscript) Refers to the state at cycle end
$_{in}$, ^{0}	(subscript, superscript) Initial states of the problem
$_{e}$	(subscript) use to symbolize elemental properties
$_{L}$, $_{NL}$	(subscript) Linear, Nonlinear component
[]	Brackets indicate rectangular or square matrix.
{}	Curly brackets indicate a vector.
' , ''	First, Second derivative