STRUCTURAL CONNECTIVITY OF

TWO-DIMENSIONAL ASSEMBLIES

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The Research Thesis was submitted in partial fulfilment of the requirements for the Degree of Master of Science of Engineering

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June, 2020

DECLARATION

I declare that this is my own work and this thesis does not incorporate without acknowledgement any material previously submitted for a Degree or Diploma in any other University or institute of higher learning and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text.

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Prof. W. P. S. Dias

ABSTRACT

STRUCTURAL CONNECTIVITY OF TWO-DIMENSIONAL ASSEMBLIES

The characterization of structures based only on their geometrical configuration and independent of loading is a novel approach for evaluating and designing structures to be robust against accidental damage. The concept of 'structural connectivity' is introduced to assess the connectivity of the structure at multiple hierarchical levels. An adaptation of the Bristol approach is tentatively suggested as providing the most appropriate measure for structural connectivity. Three other measures, conventional connectivity indices in Graph theory, Newman's approach based on Network theory and Route structure analysis (originally developed to analyse road networks) are used to compare the results obtained from the Bristol approach. Three trusses of the same outer shape but differing in geometric configuration were analysed using all four methods to find the best connected truss. The configurations analysed are Fractal, Warren and Fan-type trusses. Axial rigidity of the chord members were increased to check its effects on structural connectivity. The different measures gave different results for the same structure, though there is some degree of consistency. Graph theory and Unweighted Newman's approach suggest that the Fractal truss has the most well-connected configuration, whereas the Bristol approach favours the Fan-Type type truss. Weighted Newman analysis and Route structure analysis indicate that Warren truss to be the most wellformed configuration. All three methods indicate that truss ends and central regions of chord members are the least connected areas. All weighted analysis methods show that increasing the chord member stiffness benefits structural connectivity of all truss forms. Separately, a frame (4 bays x 5 floors) with different column elements removed was also analysed, in order to determine the column removal that would result in the least degree of frame connectivity. Though different methods indicated different column removals to cause the highest loss in structural connectivity, all methods agree that the middle column removals causes higher loss in connectivity than side column removal in the corresponding floor. An idealised "A-Level" road network in Sri Lanka was analysed as proof that concept of structural connectivity can be applied to assemblies other than structures.

Key Words:

structural connectivity, hierarchical clustering, disproportional collapse, network connectivity, road network

DEDICATION

I would like to dedicate this thesis to my family, friends and my research supervisor, without whom this research would not be.

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I would like to pay gratitude to everyone who gave support during the research study as it would have never succeeded without them.

I would like to express my profound gratitude and warm regards to the supervisor of this research, Prof. W.P.S. Dias, Senior Professor, Department of Civil Engineering, University of Moratuwa, for proving me a research opportunity, guiding me with continuous supervision and sharing his broad knowledge and time for my research project. I'm indebted to him for the support he has given me throughout my extended research duration and in producing this thesis.

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LIST OF ABBREVIATIONS

Abbreviation	Description
GMR	Ground floor- Middle column removal
GSR	Ground floor- Side column removal
JS	Joint stiffness
MMR	Middle floor- Middle column removal
MSR	Middle floor- Side column removal
RSA	Route Structure Analysis
TMR	Top floor- Middle column removal
TSR	Top floor- Side column removal
WF	Wellformedness

1 INTRODUCTION

1.1 Introduction to research

The ultimate goal of structural engineering is to create or build structures that would serve and benefit society for a very long time. Safety of these engineering structures plays an important part in achieving this goal. The need for designing resilient and robust structures has become evident. The traditional design approaches such as limit state design and reliability theory serves well in designing structures to safety. However their implementation is hampered either by their own limitations or the complexity of calculations. It is evident that a fresh perspective in analysis of structures is required in addition to what is available now.

Only limited research has been carried out in this context. Alternate path analysis, to find the key element in a structure, is one such research topic. In this research, it is sought to analyse the form of the structure.

The theory of structural vulnerability (Lu, 1999), which also deals with the form of the structure, states that if any damage to the structure causes consequences that are disproportionate to the inflicted damage, then that structure is said to be vulnerable to that damage. This is reciprocal of the concept of robustness. Thus by making the structure less vulnerable, it can be made more robust.

Vulnerability of a structure is identified in three parts (Figure 1.1). (1) Internal vulnerability, is the vulnerability in the form of the structure which is based on its internal configuration. (2) Specific action related vulnerability, is the vulnerability of a structure/system for a specific damage/action. (3) Overall vulnerability, deals with actions occurring over a period of time. This is basically an accumulation of several action related vulnerabilities in addition to the internal vulnerability.

The factors relating to the action related and overall vulnerabilities are addressed in the traditional design approaches. The internal vulnerability can be reduced by changing the internal configuration of the structure. However the question remains as to how to quantify and identify a better configuration or form of the structure. It was decided to quantify how the structure is connected together, through measuring the connectivity of the structure. The concept of 'structural connectivity for structures' is introduced to measure the connectivity of the structure.

The concept of structural connectivity for structures can be seen as self-explanatory, although the literature has failed to give any definition, let alone a measurement index for it. This research seeks to understand 'structural connectivity for structures' and provide a definition for it. It is also attempted to propose a measurement index, that can be used for measuring 'structural connectivity' of any two dimensional system.

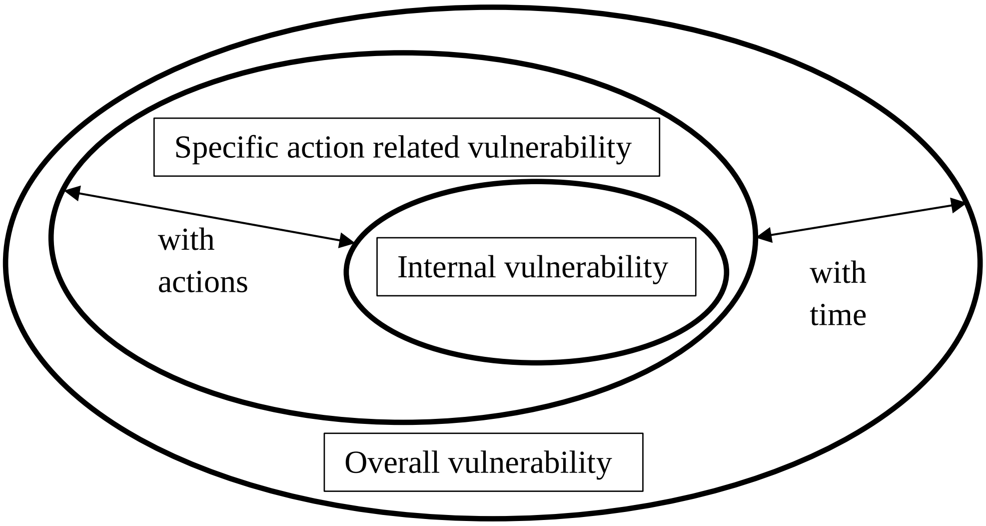


Figure 1.1 Relationship between types of vulnerability (Yu,1997)

1.2 Need for the study.

Whether a study is a philosophical one or practical one, the need for the study should be realised. The need to establish and measure the structural connectivity for structures is explained under this heading. The ability to measure the connectivity in the context of structural characteristics means that knowledge can be gained on how the members are connected together structurally as one unit and as multiple clusters locally. This will lead to discovery of weakly connected members - i.e. members which are most likely to be affected in case of any arbitrary loading.

This feeds into the concept of robustness, which can be defined as an ability of a structure not to show disproportionate consequence for an experienced damage. This can be understood as the structure being able to resist any arbitrary force. If a structure is to resist any arbitrary force, then it should be wellformed or well-connected enough to be a stable structure irrespective of any loading. Since the analytical approaches in this research don’t consider the loading on the structure, it is possible to identify the weak links that are inherent to the form of the structure, irrespective of the loading. A case is made that if a structure is wellformed irrespective of the loading then it should perform better under any arbitrary loading - i.e. members of a structure can be configured in a manner that would make them well-connected enough to resist any disproportionate damage. This can be achieved by measuring the structural connectivity of that structure.

1.3 Scope

This research seeks to understand and explain the concept structural connectivity with respect to the structures. The scope of this research is limited to analysis of two-dimensional systems. Two dimensional structures such as trusses and a two-dimensional frame are analysed using different methods to determine their structural connectivity. Three different configurations of trusses having the same outline are analysed to find the most wellformed configuration. The frame is analysed as intact and with loss of columns in different locations to determine which column loss will most affect the connectivity of the structure. The selected analytical methods will yield indices for the structural connectivity of structures. Adaptability of such indices for a road network is also evaluated.

Two limitations of the approach are acknowledged. The first is that these methods account only for the concept of connectivity. When particular structural forms are selected as the best on the basis of their connectivity, it may not mean that their structural performance such as load carrying capacity is the best too. The intention in this work is to highlight an aspect of structures that has been rather neglected, i.e. connectivity. But other aspects such as load capacity and constructability (or in the case of roads, motorability) will need to be taken into account too. The other limitation is that of validation. Although a particular index will suggest that one structure is "more connected" than another, how do we actually validate that? The approach used here is to use different methods and seek convergence in the selections made with respect to connectivity, with however varying degrees of success. It would also be difficult to validate our methods experimentally, since we define connectivity as a loading-independent property. The issue of validation clearly needs further study.

1.4 Objectives

In summary, the primary objectives for this study can be listed as;

- Presenting a definition for the concept of 'structural connectivity for structures'.
- Proposing measurement systems which can assess the structural connectivity of the structures.
- Applying the proposed measurement index to different types of assemblies, to assess the relevance of the said measurement systems.

In order to achieve these primary objectives, a set of secondary objectives were derived. Those can be listed as;

- Understanding the concept of connectivity and the structural connectivity, in their current application and context.

- Researching and analysing other concepts relating to structural connectivity.
- Selecting two-dimensional assemblies to be analysed.
- Finding the suitability of applying these concepts' measures for assessing the structural connectivity for structures.
- Finding networks similar to a two-dimensional structure and applying the proposed measurement to that system.

1.5 Structure of the thesis

'Connectivity' is one of the most widely used concepts that everybody interacts with at multiple levels on a daily basis. This is the main reason why the concept of structural connectivity feels intuitively familiar. The concept of (quantifiable) connectivity originates from the Graph theory; thus it is quite essential that the basic concepts in the Graph theory should be analysed in order to define the structural connectivity for structures. However, it should be noted that Graph theory is vast collection of concepts but our scope is limited to connectivity aspects of it. The connectivity with respect to the Graph theory is analysed in the Chapter 02. This will include analysis of the selected trusses using selected indices in Graph theory.

As measuring the connectivity is important in several sectors such as networking, different indices have been created to do so. Though these are offshoots of the Graph Theory, each has its own defining characteristic due to their application. Concepts and relating indices which resonate with our definition of structural connectivity for structures were sought and identified. It should be noted that the indices found were modified to suit this research. Chapter 03 to Chapter 05 cover these concepts and measurement indices.

The theory of structural vulnerability (Lu, 1999) is an important approach in analysing the form of the structure. This is the result of a series of research carried out at Bristol University, UK. This is an agglomerative approach which directly deals with the structural characteristics of the members as well as the joints. In Chapter 03, this approach is used to analyse the selected trusses.

One of the main applications of the Graph theory is network analysis; this ranges from analysing the behavioural patterns of monkeys in a zoo to analysis of World Wide Web. One of the notable approaches in network analysis was proposed by M.E.J. Newman (2004). This is a decompositive approach that takes a network and keeps removing the weak links to find strong communities within it. Analysis of the selected trusses using this theory is presented in Chapter 04.

In Chapter 05, the concept of Route Structure Analysis (RSA) is used to analysis the trusses. This was first introduced by Stephan Marshal (2003) to measure the structuredness of road networks. In this approach, overall connectivity as well as the relative connectivity of each element can be assessed.

It should be noted that through Chapter 03 to 05, only the trusses will be analysed. The analysis of the selected frame using all of the approaches is carried out in Chapter 06. An idealised Sri Lankan A-Level road network is analysed for structural connectivity in Chapter 07. Finally, Chapter 08 offers a summary of all discussions in the previous chapters. The conclusions of this research are given in Chapter 09.

The nature of this study is highly derivative, thus the traditional structure of reporting, which includes separate sections for Literature review, Methodology and Results, is dropped in favour of writing all the above components together for each concept in a separate chapters. For example, Chapter 02 includes the following in a single chapter.

- Introduction to Graph theory.
- General concepts in the Graph theory.
- General indices in the Graph theory.
- Results of indices for the selected structures.
- Brief discussion on the results.

Since most of the discussed concepts are not widely known to the engineering community, this format will enable the reader to absorb the concept with relative ease. All the chapters will have a sample calculation explaining the concept.

1.6 Introduction to structural connectivity

The term structural connectivity is an intuitive one. In a layman’s terms, it can be explained as ‘how the structure is connected together’. However, there isn’t a definition for structural connectivity in the context of structures. Currently the term structural connectivity is used to explain the connections in the brain’s structure for example.

In order to understand the term in context of structures, the term is divided and analysed separately (Figure 1.2). The first half of the term, structure, is self-explanatory; some structural configurations have to be selected for the analysis. This is further explained in Section 1.7. The analysis of the connectivity is the crucial one under investigation here. Since, a firm definition for the structural connectivity of structures is not available, it is acceptable to use measures of similar concepts to understand the said concept.

Analysing Graph theory will provide the basic understanding about connectivity as it would introduce the terms associated with connectivity as well as most of the concepts relating to it. General idea of connectivity is measured by assessing how many vertices or edges need to be removed to separate a graph. But some measures seek to signify how a graph is closely connected within itself. From this analysis, a definition for the structural connectivity can be formulated.

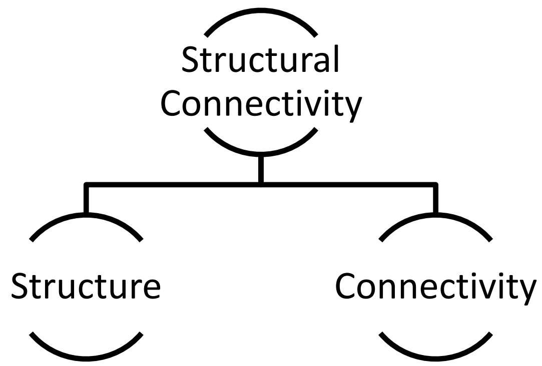


Figure 1.2 Decomposition of the term Structural Connectivity

Analysis of the form of the structure is the main aspect of the theory of structural vulnerability. In this approach structural members are to from clusters that will be most tightly connected among themselves than the members outside it. Then the clusters are expanded by adding more members to form bigger clusters that would result in the structure itself. This analysis takes the structural characteristics of the members as well as the joints into account in determining the 'wellformedness' of the structure. Due to this, a slightly modified version of this Bristol Approach is proposed as the index for the structural connectivity for structures. However, this can be validated by comparing the analysis results with some other methods. For this Newman's Method and Route Structure Analysis (RSA) are used.

Newman's Method is a network analysis tool that is used to find the strong communities in a given network. This analysis finds the most in-between link in the network by calculating the shortest paths in the network. A new quantification of the removal of the identified weak link is proposed in the research. This index is used to measure the form of the network.

The Route Structure Analysis, which was developed by Stephan Marshall, tries to find the structuredness of a road network. This approach has little to do with Graph theory, which makes it an important tool of comparison in this research.

1.7 The Structure

The scope of this study is limited to two dimensional systems. The main two dimensional structural systems considered in this research are;

- Trusses
- Frames

Three types of trusses are selected for comparative analysis. These have the same outline and only differs in the internal configuration of the members. The general arrangements of the structures is given in Figure 1.3.

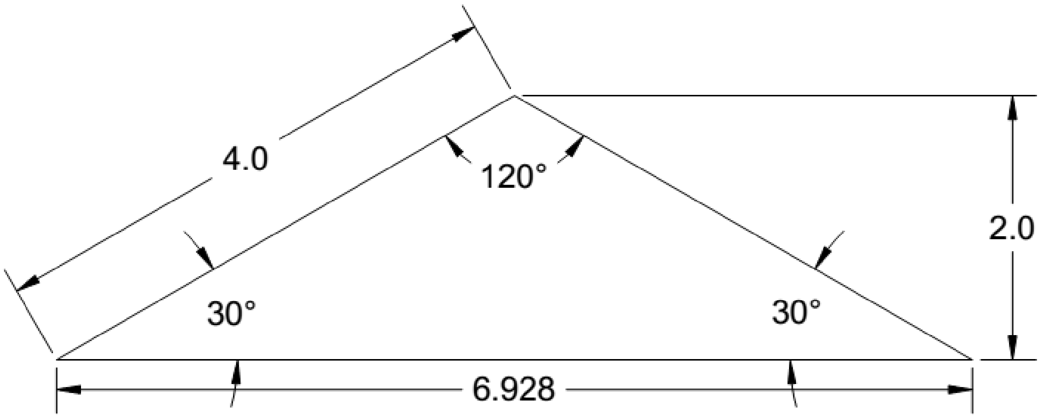


Figure 1.3 General Footprint of the truss

The different configuration of the trusses and their name IDs are given in Figures 1.4 to 1.6. The number of joints and number of members are kept same for all three configurations. Here all the members are assumed to have axial rigidity of unity. But, in real life, the chord members usually will have higher axial rigidity than the internal members. In order to account for that, a separate analysis is performed on similar trusses with chord members having double and four times the axial rigidity of internal members.

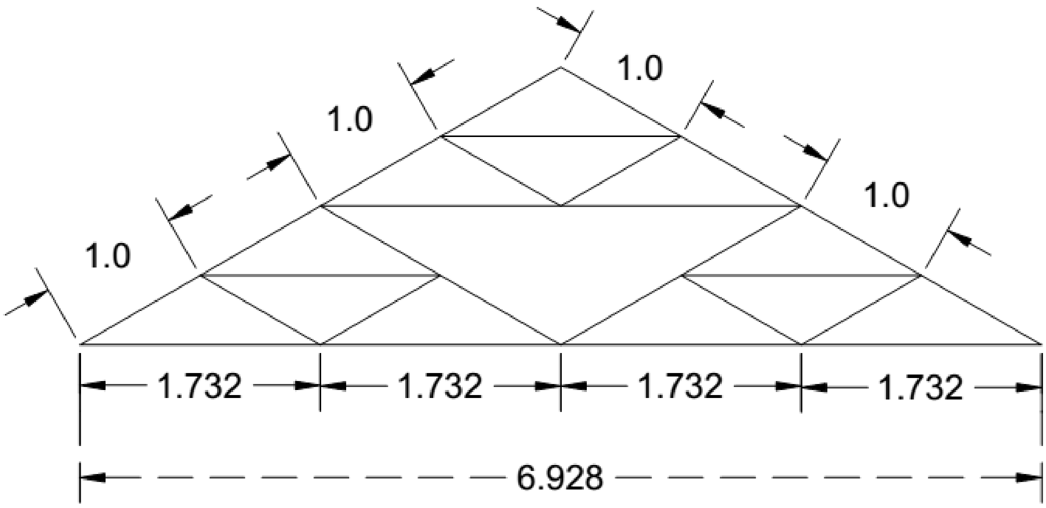


Figure 1.4 T1-Fractal Truss

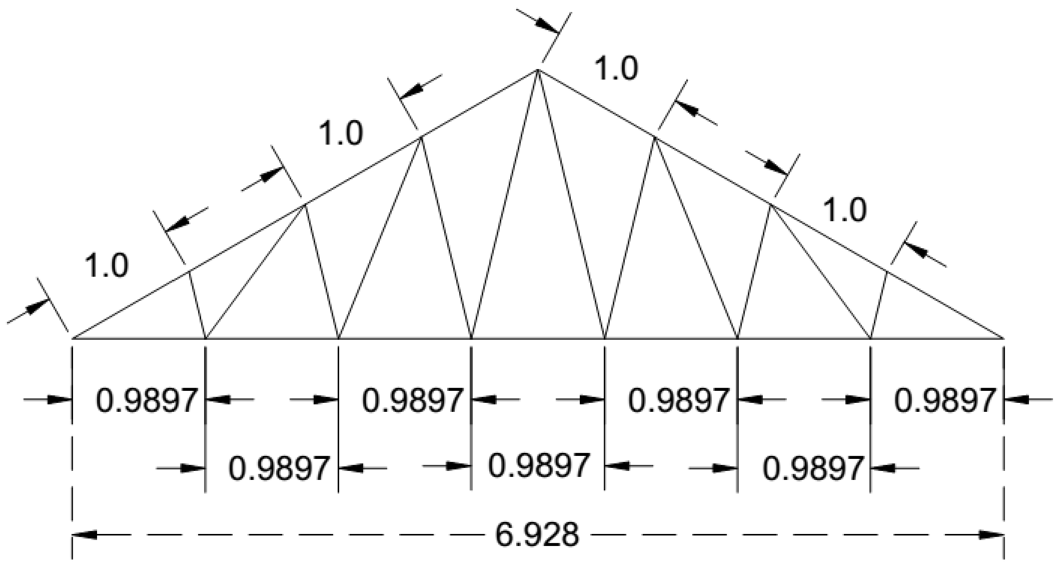


Figure 1.5 T2- Warren Truss

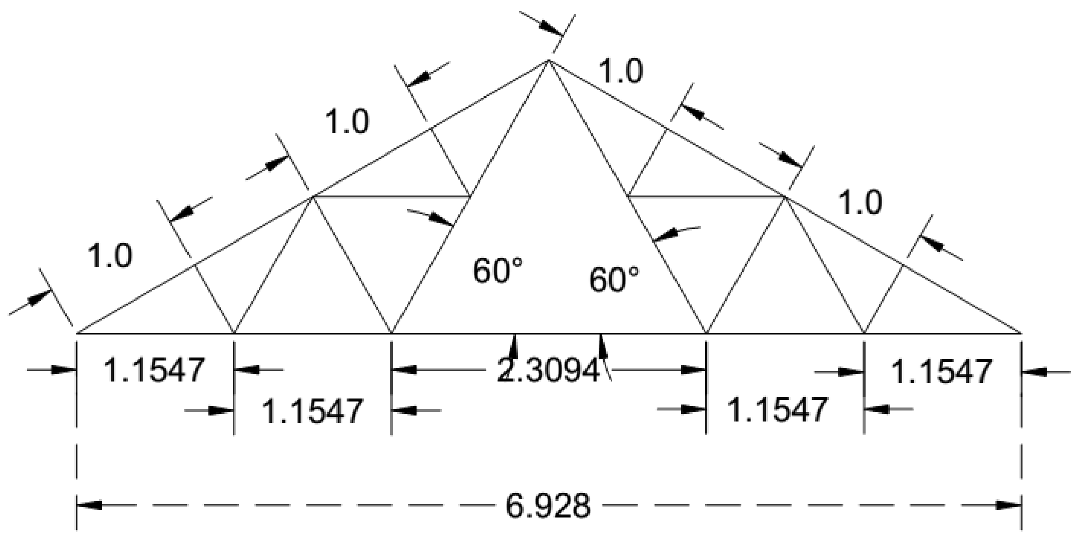


Figure 1.6 T3-Fan-Type Truss

The concept of a fractal can be explained as a repeating pattern that is self-similar across different scales. The truss T1 is created by using this concept. The other two trusses, Warren and Fan, are basic forms of most commonly used trusses.

The frame selected has four bays and five floors. The columns are of 400mm*400mm and beams are of 300mm*400mm (width x depth) and both are made of C30 concrete. The configuration of the structure is given in Figure 1.7. The member damage scenarios considered for the analysis are as follows;

- Ground floor- Side column removal (GSR)
- Ground floor- Middle column removal (GMR)
- Middle floor- Side column removal (MSR)

- Middle floor- Middle column removal (MMR)
- Top floor- Side column removal (TSR)
- Top floor- Middle column removal (TMR)

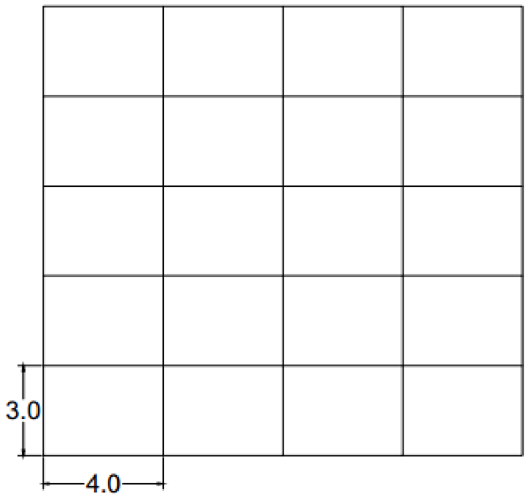


Figure 1.7 Frame object

Analysis of the selected structures along with the selected methods can be tabulated as shown in Table 1.1. The road network is described in Chapter 07.

Table 1.1 Summary of Analysis

Structure \ Method	Graph theory	Bristol Approach	Newman's Method		RSA (Unweighted)
			Unweighted	Weighted	
T1	✓	✓	✓	✓	✓
T2	✓	✓	✓	✓	✓
T3	✓	✓	✓	✓	✓
Frame	✕	✓	✓	✕	✓
Road	✕	✕	✓	✕	✓

2 Graph Theory

2.1 Introduction to Graph theory

In this chapter, the concepts of Graph theory relevant to this study are introduced and then the analysis for the structures is carried out. In order to analyse the structure as a graph, it has to be first converted in to a graph model. The process called 'Object oriented modelling' is used for this process. This is basically idealising the structure in to a skeletal form of itself. The joints and members of the structure will be converted in to vertices and edges respectively. Since this research focuses on two dimensional structures, this process is very straightforward. The structural information of the members are converted as the weight of the edge. The structural information of the joints are stored as the type of the vertices. Summary of the objected oriented modelling is given in Table 2.1. The terminology is explained in Section 2.2 & 2.3. The concept of weighted analysis is presented in Section 2.4. Results from Graph theory indices are given in Section 2.5 followed by the discussion in Section 2.6.

Table 2.1 Comparison of terminology

Structural terminology	Mathematical (Graph) terminology
Structural system	Graph model
Joint	Vertex
Member	Edge
Structural Cluster	Subgraph

2.2 Terms in Graph theory

The terms in the Graph theory can be found in many text books and online resources such as Handbook of Graph theory by Gross J.L (2003), Wikipedia-Graph theory, 2020 and many more.

A Graph, "*G=(V,E), consists of two sets, where, V is a set of vertices and E is a set of pairs of distinct vertices called edges. The graph is finite if sets V, E are both finite.*"

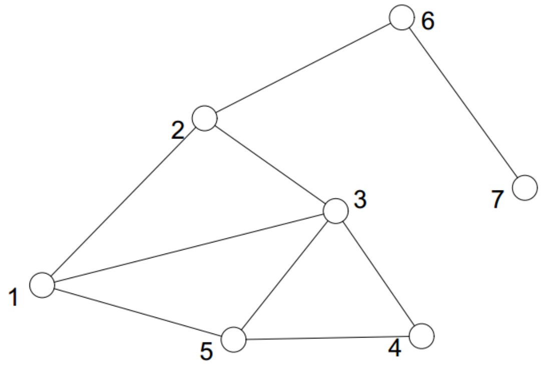
*"If all edges of the graph can be represented by ordered pairs of vertices then the graph can be classified as **a directed graph**",* otherwise the graph is identified as **an undirected graph**. An undirected graph can be treated as a two way directed graph.

In this research, all the graphs are finite and undirected.

Adjacency matrix: One of the major inputs of our analysis and the easiest representation of a graph is an adjacency matrix. For a graph having n number of vertices, the adjacency matrix, A, will be of nxn size and can be given as shown in equation 2.1.

$$Adjacency\ matrix, A = \left\{ \begin{matrix} aii = 0 & \\ aij = 1 & ; if\ i\ and\ j\ are\ connected \\ aij = 0 & ; other\ \ wise \end{matrix} \right\} \quad \textbf{equation 2.1}$$

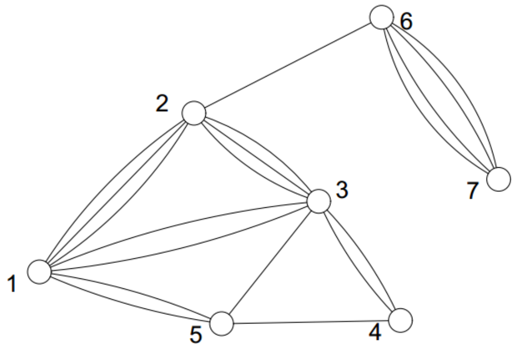
An example is given in Figure 2.1,



$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Figure 2.1 Example for simple adjacency matrix

If a_{ii}=1, there exists a self-loop in the joint i. In this study self-loops won't be analysed. If a_{ij}>1, there exists more than one edge parallel to each other connecting same pair of vertices i, j. This concept, which is called as multigraph, is used in analysing the weighted graph.



$$A = \begin{bmatrix} 0 & 3 & 2 & 0 & 2 & 0 & 0 \\ 3 & 0 & 3 & 0 & 0 & 1 & 0 \\ 2 & 3 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{bmatrix}$$

Figure 2.2 Example for multi-graph

Subgraph: *"A graph G'=(V',E') is a subgraph of G=(V,E) if V' and E' are respectively subsets of V,E such that an edge (v_i,v_j) is in E' only if v_i,v_j are in V'."*

A path in graph is defined as *"a finite alternating sequence of vertices and edges"*. This can be also represented by sequence of vertices as well.

"The first and last vertices will be named as the terminal vertices and the rest shall be identified as the interior vertices. All the vertices shall be distinct" (Wu, 1991).

Length of the path: *"Number of links in the path is the length of the path".*

Degree of a vertex: *"Degree of the vertex is the number of edges incident in the particular vertex".*

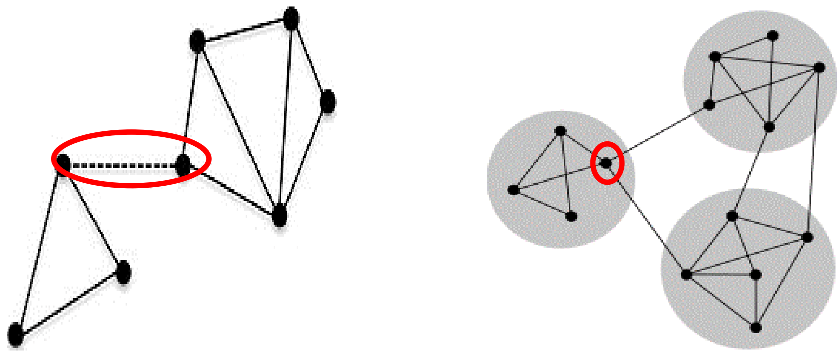
Degree matrix: *"Degree matrix is a diagonal matrix which stores the information of number of edges connected to each vertex (degree of vertex)."*

For a graph with n vertices, degree matrix would be nxn size can be given by equation 2.2.

$$Degree\ matrix, D = \begin{Bmatrix} Dii = degree\ of\ the\ vertex\ i \\ Dij = 0 \end{Bmatrix}$$ **equation 2.2**

The connectivity in Graph theory is defined as *"The minimum number of elements (vertices or edges) that need to be removed to disconnect the remaining nodes from each other".*

Basically this is a decompositive measure, we start to remove elements and check at what point the graph is broken into its subgraphs. The Figure 2.3 shows two graphs which ban be broken in to their subgraphs by removing the highlighted elements.



(a) Edge removal (b) Vertex removal

Figure 2.3 Graph connectivity through removed elements

Connected Graph: *"A graph G (V,E) is said to be connected if for every pair of vertices u,v (ϵ V), there exists a path from u to v."*

Connected Component: *"A connected component of an undirected graph is a sub graph in which any two vertices are connected to each other by paths, and which is connected to no additional vertices in the super graph"* (Figure 2.4).

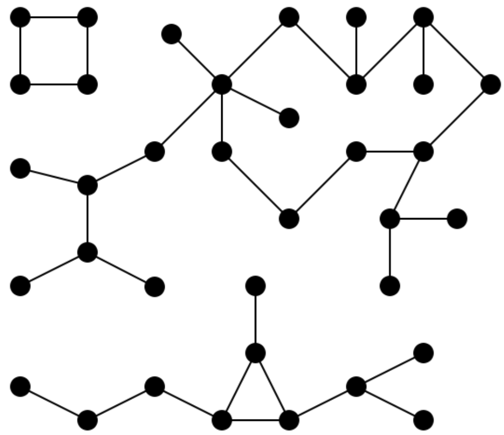


Figure 2.4 Graph of three connected components

One of the main theorems in the Graph theory with respect to the connectivity is the **Menger's theorem** (1927). The definition of Edge connectivity and Vertex connectivity are given as follows;

Edge Connectivity: *"Let G be a finite undirected graph and x and y two distinct vertices. Then the size of the minimum edge cut for x and y (the minimum number of edges whose removal disconnects x and y) is equal to the maximum number of pairwise edge-independent paths from x to y."*

Vertex Connectivity: *"Let G be a finite undirected graph and x and y two nonadjacent vertices. Then the size of the minimum vertex cut for x and y (the minimum number of vertices whose removal disconnects x and y) is equal to the maximum number of pairwise vertex-independent paths from x to y."*

Cluster analysis is defined as *"the methodology which optimizes intra-group homogeneity in a given population."*

In simple terms, the cluster analysis seeks to create groups within the given population, in such way that population contained in a group will have more in common within the group rather than the population of other groups. In other words, intra-group connections will be very high compared to inter-group connections.

Because of the nature of this analysis, it is used in various fields such as engineering networks, environmental analysis, big data analysis, artificial intelligence and many more. There are several techniques in the clustering analysis, such as;

- Hierarchical techniques
- Optimisation partitioning techniques
- Density or mode seeking techniques
- Clumping techniques

Though there are several techniques, the basic idea of clustering analysis is simple. It can be broken down like following;

"A set of N individuals, all having an attribute set, will be grouped into n classes depending on their attribute set"

The connections between the individuals is the main attribute of this analysis, thus Clustering analysis is one of the tools that will be employed in this study. Here we are mainly concerned with hierarchical clustering analysis. This can be broken in to two major types based on the clustering method. They are;

- **Agglomerative method**: Creates many tightly connected clusters and then adds them together to make bigger clusters.
- **Decompositive method**: Removes weakly connected members/ intra-group connections to create big clusters, then keeps breaking them down into small tightly connected clusters.

Both of these methods are used in this study.

Clusters: *"A sub-division in a population/ sub-graph in a graph which has more connections within the cluster than outside it."*

This can be regarded as the subgraphs within a structure that is represented as a graph.

2.3 Graph connectivity measures

Based on the connectivity concepts and theorems discussed above, several connectivity measures have been developed.

Strength of a Graph: *"The strength of an undirected graph corresponds to the minimum ratio edges removed/components created in a decomposition of the graph in question."*

An example of this is given in Figure 2.5,

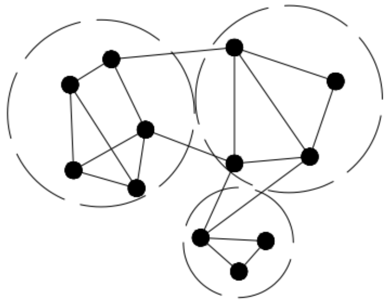


Figure 2.5 Example for Strength of Graph

Based on Figure 2.5,

Number of edges removed =4

Number of total components after removal =3

Components created =3-1

=2

Strength of the graph =4/2

=2

Graph Toughness: *"A graph G is said to be t-tough for a given real number t if, for every integer k > 1, G cannot be split into k different connected components by the removal of fewer than tk vertices."*

An example is given in Figure 2.6

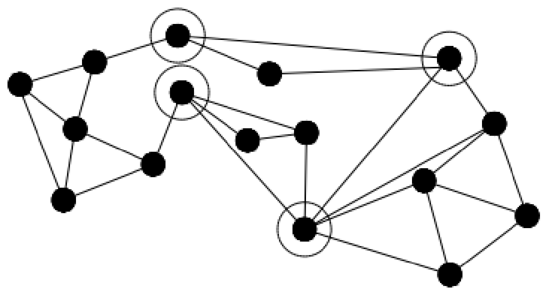


Figure 2.6 Example for Graph Toughness

From Figure 2.6, it can be noted that when the highlighted vertices are removed four different connected components are created.

Number of components, k =4

Number of removed vertices, tk =4

Graph toughness, t =1

Clustering coefficient:

Clustering coefficient is measured in two contexts, as global clustering coefficient (equation 2.3) and average local clustering coefficient (equation 2.5). The global clustering coefficient is based on triplets of nodes. A triplet is a structure of three connected nodes.

$$The\ global\ clustering\ coefficient, C = \frac{3*number\ of\ triangles}{number\ of\ connected\ triplets\ of\ vertices}$$

equation 2.3

A triangle would have three connected triplets. This measures checks the scenario of whether X and Z are connected given that both are connected to Y. The local clustering coefficient (equation 2.4) is measured for each vertex in the graph and then the average of those clustering coefficient can be used to measure the connectivity of the graph.

$$The\ local\ clustering\ coefficient, Ci = \frac{number\ of\ triangles\ connected\ to\ vertex\ i}{number\ of\ triplets\ connected\ to\ vertex\ i}$$

equation 2.4

$$The\ average\ Ci, C = \frac{1}{n} * \sum_{i=1}^{n} Ci$$ **equation 2.5**

where n is the number of vertices in the graph

Algebraic connectivity: This is the value of the second smallest eigen value of the Laplacian matrix of a graph, G. Laplacian matrix is the matrix representation of a graph created by equation 2.6.

$$L = D - A$$ **equation 2.6**

where, L =Laplacian matrix

D =Degree matrix

A = Adjacency matrix.

Geodesic path/ Shortest path: *"The path between two vertices which has the minimum length."*

As stated in Section 2.2, the distance is measured as the number of edges in that path.

Eccentricity of a vertex: *"The maximum geodesic distance between a given vertex and all other vertices in the graph."*

Diameter of a Graph: *"The maximum distance between any pair of vertices in the graph. This can be also given as the maximum eccentricity of any vertex."*

Radius of a Graph: *"The minimum eccentricity of any vertex in graph."*

Centrality measures: The set of analysis methods used to identify most important vertices in a graph. The type of centrality measures are;

- Closeness centrality
- Betweenness centrality
- Eigenvector centrality
- Degree centrality
- Katz centrality
- Harmonic centrality

The Closeness centrality: This measures how much the nodes are central in a connected graph. This is done by calculating average of the lengths of geodesic paths to all other vertices in the graph. Equation to calculate the closeness centrality of a vertex is given in equation 2.7. Average of every vertex's closeness centrality should give an idea of how much the graph is tightly connected (equation 2.8).

$$C(x) = N / \sum_y d(y,x)$$ **equation 2.7**

where, C(x) = closeness centrality of vertex x

N = number of vertex in a graph

d(y,x) =distance between the vertices x and y

$$Average\ C = {\sum_x C(x)}/{N}$$ **equation 2.8**

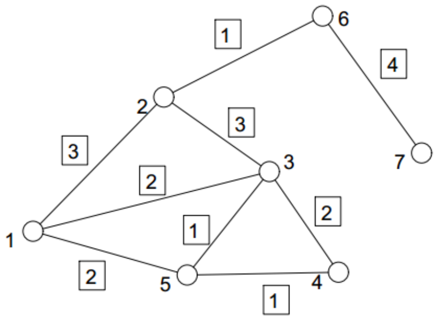
The Betweenness centrality: This measures how much a vertex is in between different groups of vertices. This is quantified by measuring how many shortest paths are passing through that specific vertex, when the shortest paths from all nodes to all nodes are calculated. This is further explained in Newman's Method in Chapter 4.

2.4 Weighted Analysis

The weight is a numerical value assigned as a label to a vertex or edge of a graph. This can be made to represent the structural characteristics of the member. Normally, the weighted analysis only considers the length of the edge. However, to present the measures in the context of structural terms, member's local stiffness can be used as the weight.

The weighted adjacency matrix is defined as shown in equation 2.9;

$$Weighted\ Adjacency\ matrix, A = \begin{Bmatrix} aii = 0 & \\ aij = w & ; if\ i\ and\ j\ are\ connected\ by\ edge\ of\ weight\ w \\ aij = 0 & ; other\ \ wise \end{Bmatrix}$$ **equation 2.9**



$$A = \begin{bmatrix} 0 & 3 & 2 & 0 & 2 & 0 & 0 \\ 3 & 0 & 3 & 0 & 0 & 1 & 0 \\ 2 & 3 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{bmatrix}$$

Figure 2.7 Example for weighted graph

It can be observed that multigraph from Figure 2.3 and weighted graph from Figure 2.7 has same adjacency matrix. This means that weight of an edge can be represented as multiple parallel edges connecting the same vertices. This is used in the analysis of networks in Newman's method.

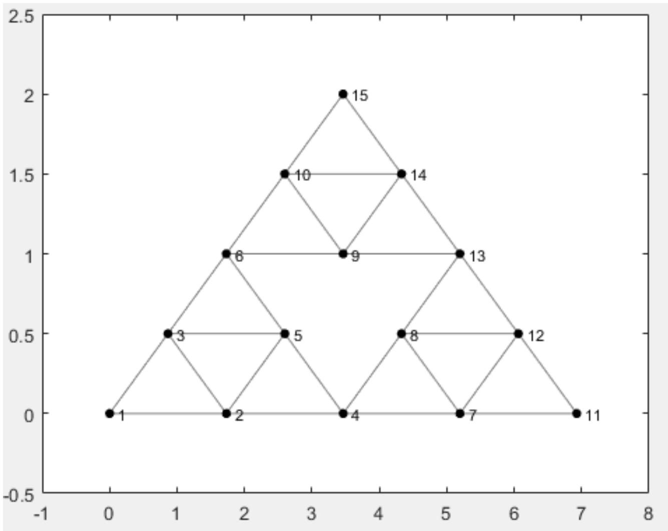
How the assigned weights affects the analysis and the results, depends on the analysis method. Weighted closeness centrality calculates the shortest paths considering the summation of weights as the length of each paths. However, The Newman method calculates the shortest paths as in unweighted method and uses the weights in the later part of the analysis (see Chapter 04). In the closeness centrality measure, the stiffness should be given a reciprocal relationship with the weight in order to make sure members of high stiffness (edges representing them) is given a higher priority in clustering.

2.5 Results

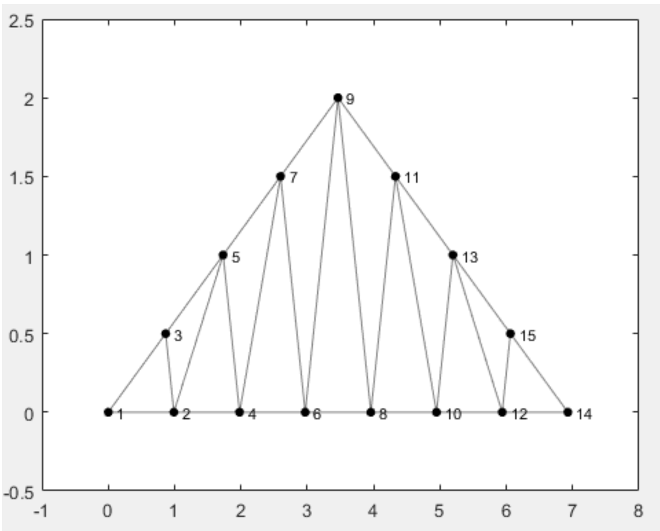
Section 2.5.1 shows the truss forms as represented in the software used for the analysis. The unweighted analysis results are given in Section 2.5.2 followed by the weighted analysis results in Section 2.5.3.

2.5.1 Truss forms

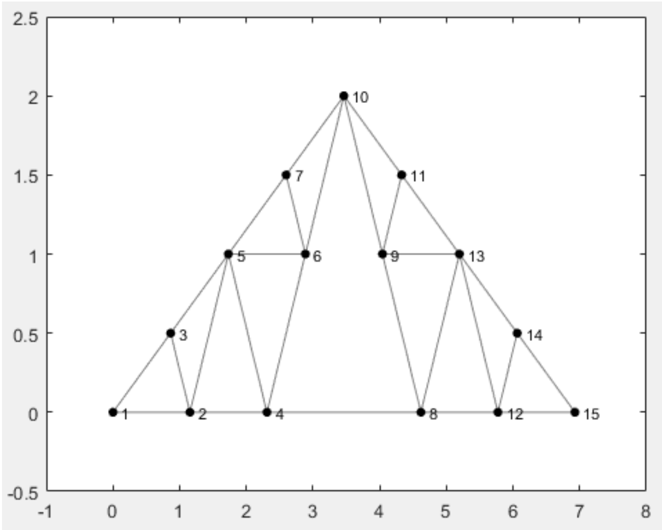
The analysis is carried out in the software package "MATLAB". The vertical scale of the results are modified to showcase the trusses as equilateral triangles for easier presentation. The truss forms given as the output of the programme are given in Figure 2.8.



(a) Fractal truss



(b) Warren truss

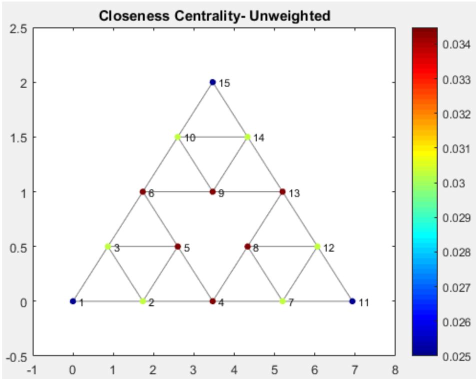


(c) Fan type truss

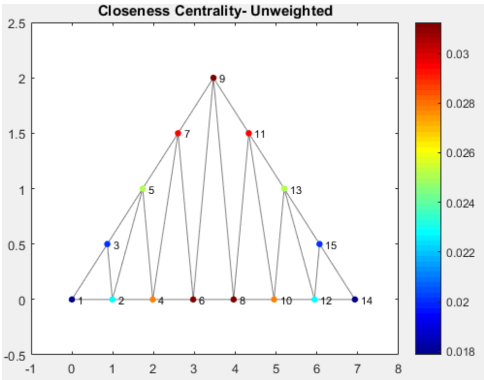
Figure 2.8 MATLAB representation of trusses

2.5.2 Unweighted Analysis

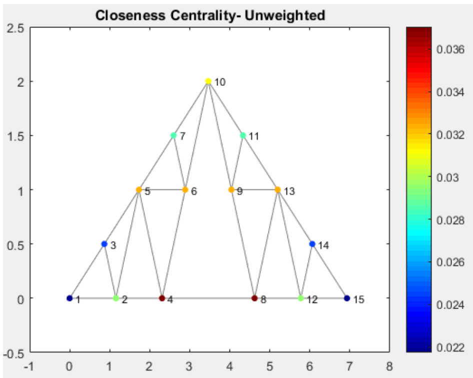
The closeness centrality results from the unweighted analysis are given in Figure 2.9 and the betweenness centrality results are presented in Figure 2.10.



(a) Fractal truss

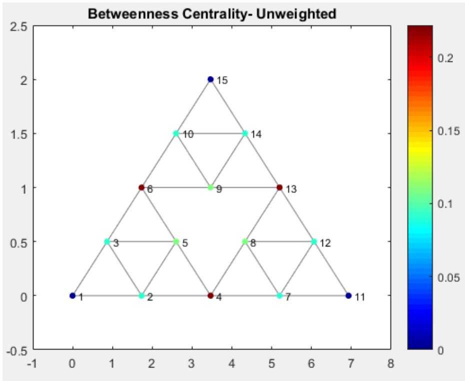


(b) Warren truss

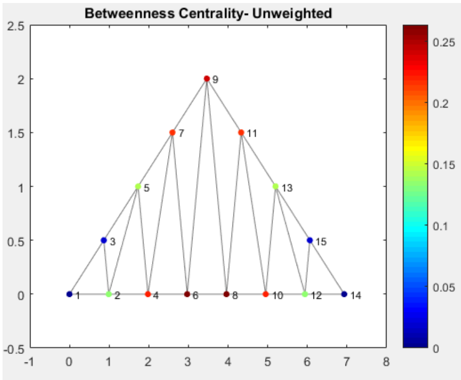


(c) Fan type truss

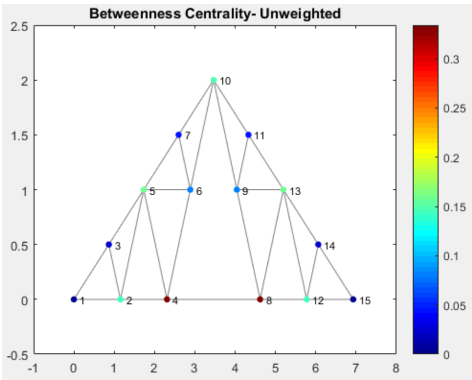
Figure 2.9 Results of unweighted closeness centrality analysis



(a) Fractal truss



(b) Warren truss

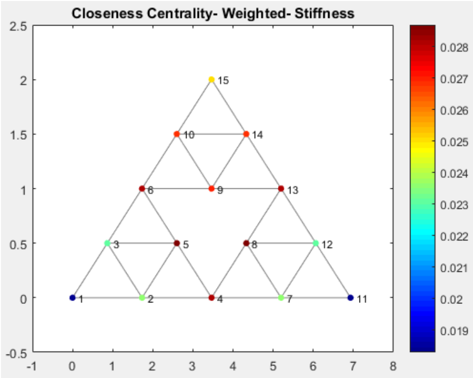


(c) Fan type truss

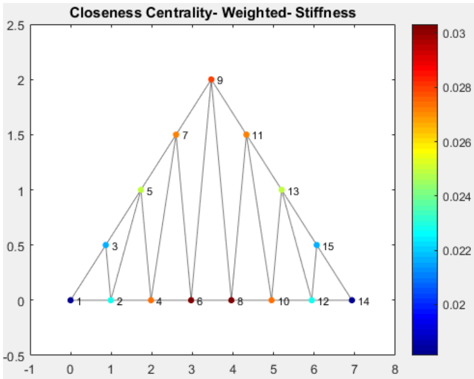
Figure 2.10 Results of unweighted betweenness centrality analysis

2.5.3 Weighted Analysis

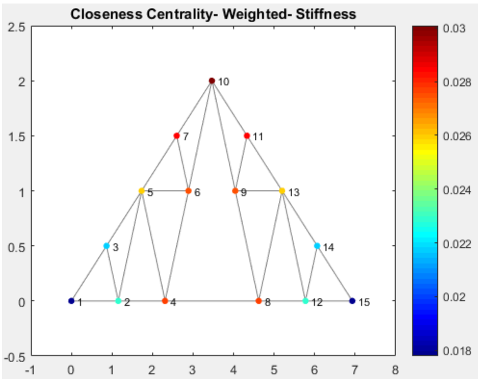
Results from the weighted closeness centrality measures are given in Figure 2.11 and Figure 2.12. Figure 2.11 shows the case where all member axial rigidity equals unity. Figure 2.12 shows the case where chord member axial rigidity is twice the web member axial rigidity.



(a) Fractal truss

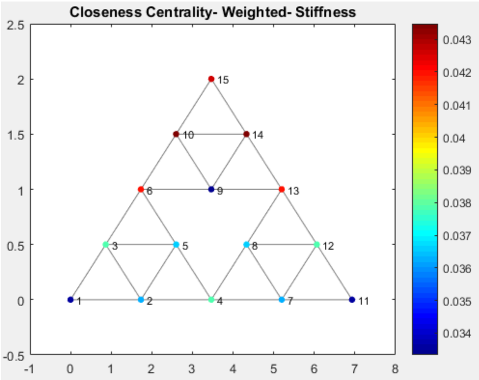


(b) Warren truss

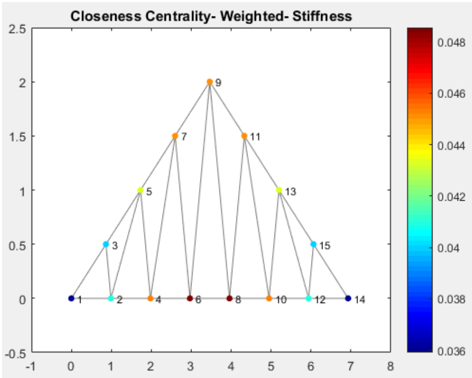


(c) Fan type truss

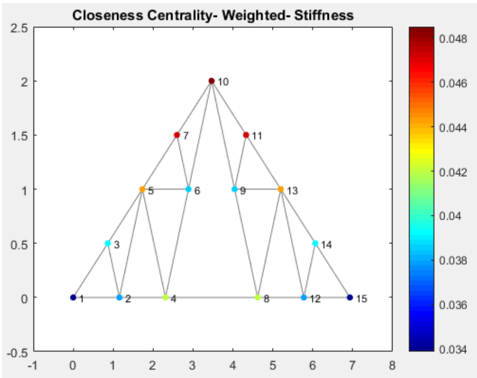
Figure 2.11 Results of weighted closeness centrality analysis for AE=1



(a) Fractal truss



(b) Warren truss



(c) Fan type truss

Figure 2.12 Results of weighted closeness centrality analysis for chord AE=2

A summary of all the results is given in Table 2.2.

Table 2.2 Summary of results

Indices	T1	T2	T3
Algebraic connectivity	0.5703	0.2158	0.3352
Diameter	4	7	5
Graph radius	3	4	3
Average shortest path	2.3429	2.9333	2.4857
Graph toughness	1	1	1
Strength of the graph	4	3	3
Clustering coefficient, C	0.2353	0.2708	0.2308
Average Ci, clustering coefficient	0.5667	0.5889	0.5644
Closeness centrality- unweighted	0.0309	0.0253	0.0295
Closeness centrality- weighted (AE)	0.0251	0.0248	0.0249
Closeness centrality- weighted (2AE)	0.0382	0.0429	0.0410

2.6 Discussion

In this chapter the selected truss forms were analysed using the indices for connectivity found in Graph theory. Though weighted analysis was carried out, the most important insight from this chapter should be the unweighted analysis. The unweighted analysis sees the structure just as a graph in its skeletal form. This gives us some idea what would be the best form of the truss without even considering the properties of the members, and the main purpose of the chapter is to introduce basic concepts of Graph theory, which are used in the following chapters. The betweenness centrality measure, which was introduced in this chapter serves as the introduction to Newman's Method, and discussed in the appropriate chapter.

The best configuration of the truss depending on the Graph theory indices is highlighted in the Table 2.3, which is a modified version of Table 2.2.

Table 2.3 Selecting truss configuration

Indices	T1	T2	T3	Selected truss
Algebraic connectivity	0.5703	0.2158	0.3352	T1
Diameter	4	7	5	T1
Graph radius	3	4	3	T1& T3
Average shortest path	2.3429	2.9333	2.4857	T1
Graph toughness	1	1	1	T1, T2 & T3
Strength of the graph	4	3	3	T1
Clustering coefficient, C	0.2353	0.2708	0.2308	T2
Average Ci, clustering coefficient	0.5667	0.5889	0.5644	T2
Closeness centrality- unweighted	0.0309	0.0253	0.0295	T1
Closeness centrality- weighted (AE)	0.0251	0.0248	0.0249	T1
Closeness centrality- weighted (2AE)	0.0382	0.0429	0.0410	T2

It can be said that for a closely knitted graph, the indices like diameter of graph, radius and the average shortest paths will be less, as they show how a graph is closely formed. From these indices, it can be seen that Truss T1 is closely formed. This is also reflected in the closeness centrality measure as shortest paths are used in calculation of the centrality measures. But the other trusses also have very similar values of closeness centrality. The higher Algebraic connectivity relates to higher connectivity within the truss and truss T1 is selected by this criterion.

Having a higher value for the strength of the graph means that higher number of edges needs to be removed to separate the graph; and truss T1 is selected in this category as well. But the truss T2 is selected in both clustering coefficient scenarios by having higher values for them. It can be clearly observed that, most of the selected Graph theory measures indicate that the fractal truss configuration (truss T1) has better connectivity. The results also show that the Warren truss (truss T2) is more triangulated than the other two trusses.

The change in weighted centrality measures as the chord member axial rigidity are doubled, should be given special consideration. The important thing is that all of the truss closeness centrality measures increases with the increased axial rigidity, indicating that weighted analysis can actually represent some structural sense. It can be deduced that T2 benefits most from this scenario. This will be further explored in the following chapters.

Analysing the theoretical background and the results of the Graph theory gives a clear understanding about structural connectivity. At its core, connectivity always relates to the member/vertex whose removal disconnects the graph. This can be translated directly in to the structural terms. However, it should also be understood that that there can be strong formations of members within the structure, usually connected by a member/vertex that would define the connectivity of the structure-i.e. weakly connected member. From this we can define structural connectivity in terms of structure as;

“In a field of a finite number of joints/nodes/points of interests connected by finite number of members/paths/connections, ‘structural connectivity’ seeks to not only assess how strongly the members are connected at a given joint but also how strongly the groups of members are connected to each other.”

3 Bristol Approach

3.1 Introduction

The Bristol approach was developed at Bristol University, UK. One of the major components of this approach is the Theory of structural vulnerability (Lu, 1999). This theory tries to quantify the vulnerability of a structural system only with respect to its structural form and connectivity using the characteristics of members and joints. Many new concepts were defined in the process of deriving of this theory. These concepts are also based on the Graph theory. Comparison between the terms of Graph theory and the Bristol Approach is shown in the Table 3.1.

Table 3.1 Comparison of Terms

Graph theory	Terms in Bristol Approach
A graph	A structure
Sub-graph	Sub-structures/ Clusters
Edges	Members
Vertices	Joints
Adjacency matrix	Association matrix
Paths	Structural paths
Cycles	Structural loops
Degree of vertices	Degree of joints

These terminology and the basic concepts of the Bristol approach are explained in the Section 3.2. The modifications made to the original approach to suit this research are also discussed after introducing the concepts. The clustering process is explained in Section 3.3. Section 3.4 & 3.5 details on measurement indices used in the Bristol approach. The results of the analysis is given in Section 3.6. Section 3.7 presents the discussion on results.

3.2 Basic Concepts

3.2.1 Introduction to the concepts

These basic concepts are taken from the researchers that defined the Bristol approach (Wu 1991, Yu 1997).

"A structural cluster is a subset of the graph model in which the objects are in some sense more tightly connected to each other than to other objects outside of the cluster."

"A structural ring is a sequence of a maximum of three connected clusters which can resist an arbitrary set of applied forces. It is the basic unit of a two-dimensional structure."

The concepts of structural cluster and structural ring are the most important part of this research, which reflects the concept of clusters in the Graph theory.

"A leaf cluster (or a primitive cluster) contains a single structural member and adjacent joints."

"A branch cluster is a cluster that contains more than one structural member. A structural ring is a branch cluster."

In this study the structural rings are taken as the leaf cluster and the combination of clusters are treated as the branch clusters. This is further explained in the Section 3.3 of this chapter.

"Reference cluster is normally the ground. It is the cluster from which the structure has to be separated for the total failure scenario to occur."

The Bristol Approach deals with the damage demand (i.e. effort needed to cause failure) of every failure scenario leading to the complete failure. However, this research is mainly concerned with the initial failure, thus this concept is not explored in this research.

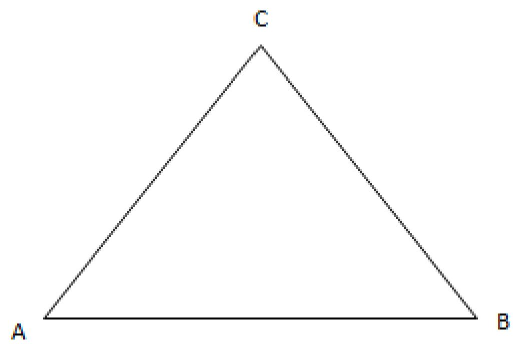
"Root cluster contains the entire structure including the ground."

In this research, the concept of root cluster is not explored in detail as reference clustering is not considered for the analysis.

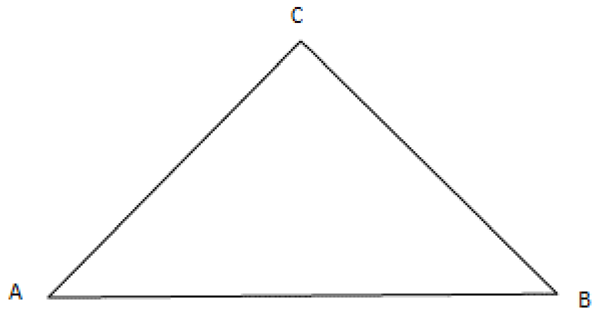
3.2.2 Wellformedness

"Wellformedness of a cluster Q is a measure of the form of a structure, which is closely related to the principal stiffness coefficients of the joints, the type of joint, the stiffness of the members and the configuration of the members in the structure" (Yu 1997)

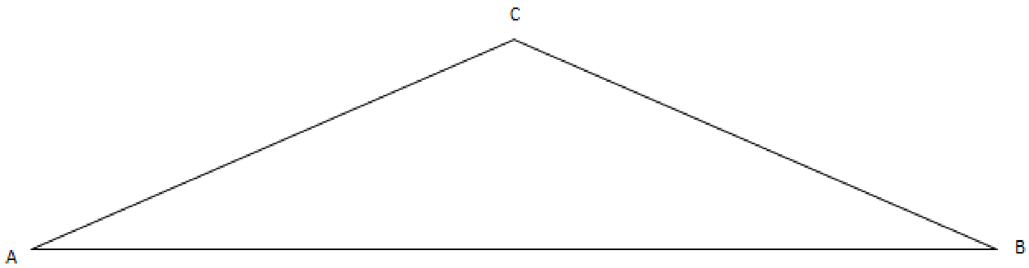
For example, let's look at the three different triangles having the same height as shown in Figure 3.1. From intuition, it can be said that the equilateral triangle is more wellformed than the other two isosceles triangles. The procedure given in the Bristol approach can be followed to quantify this intuition. Initially, the stiffness of the joints needs to be found.



(a) Equilateral triangle



(b) Isosceles triangle with side angle of 45^0



(c) Isosceles triangle with side angle of 30^0

Figure 3.1 Triangles differing in side angle with same height

3.2.3 Joint Stiffness (q_i)

In order to understand the joint stiffness, the stiffness matrix of a joint have to be derived with respect to a member. The derivation of this is explained in detail below. The coordinate system used for this derivation is given in Figure 3.2 and the displacement and force vectors along this coordinate system in Figure 3.4 and 3.5 respectively. The local coordinate system with respect to the member orientation is denoted by X', Y' and the global coordinate system is denoted by X, Y. The displacement vectors along the local coordinate system are given by u', v' whereas the displacement vectors along the global coordinate system are given by u, v. Similarly the force vectors along the local and global coordinate system are given by f'and f respectively.

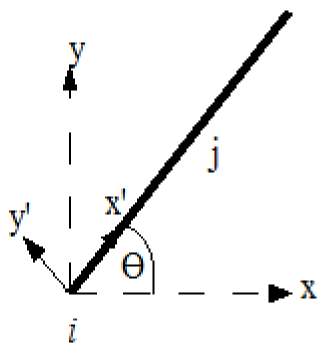


Figure 3.2 Coordinate system for the member

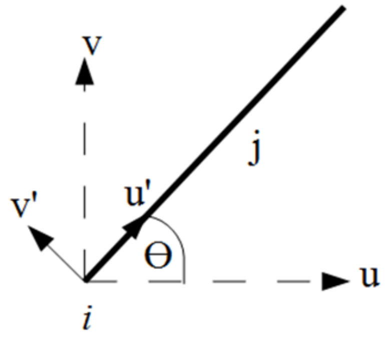


Figure 3.3 Displacement vectors for the joint i

The displacement along the local coordinates (x', y') is given by equation 3.1 and 3.2.

$$u' = u\,cos(\Theta) + v\,sin(\Theta)$$ **equation 3.1**

$$v' = -u\,sin(\Theta) + v\,cos(\Theta)$$ **equation 3.2**

The equations 3.1 & 3.2 can be combined in to the matrix form as shown in equation 3.3.

$$\begin{Bmatrix} u' \\ v' \end{Bmatrix} = \begin{bmatrix} \cos(\Theta) & \sin(\Theta) \\ -\sin(\Theta) & \cos(\Theta) \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix}$$ **equation 3.3**

But since the truss members have only axial forces, the v' component is always zero. Thus the matrix can be rewritten as shown in equation 3.4.

$$\{u'\} = [\cos(\Theta) \quad \sin(\Theta)] \begin{Bmatrix} u \\ v \end{Bmatrix}$$ **equation 3.4**

The deformation/elongation of the member due to the load is given by e' and e in the local and global coordinate systems respectively. The matrix e' and e are related by matrix [C] which is the transformation matrix.

$$\{e'\} = [C]\{e\}$$ **equation 3.5**

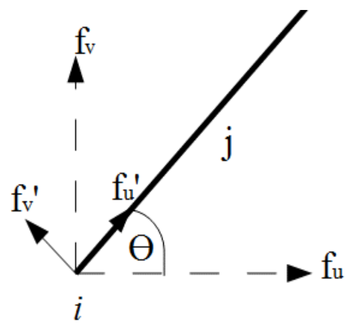


Figure 3.4 Force vectors for joint i

Similarly force in the member along its local axis can be written as shown in equation 3.6. Here the forces along global coordinates are denoted by fu & fv.

$$\{\boldsymbol{fu'}\} = [\mathbf{cos}(\Theta) \quad \mathbf{sin}(\Theta)]\begin{Bmatrix}\boldsymbol{fu}\\ \boldsymbol{fv}\end{Bmatrix}$$ **equation 3.6**

We can simplify the equation 3.6 into equation 3.7 by substituting the transformation matrix.

$$\{\boldsymbol{f'}\} = [\boldsymbol{C}]\{\boldsymbol{f}\}$$ **equation 3.7**

By Hook's law given in equation 3.8; [k] is the stiffness matrix relating the axial deformation of the truss member to force in the member. For the truss member, stiffness depends on the axial rigidity and the length of the member as shown in equation 3.9.

$$\{\boldsymbol{f}\} = [\boldsymbol{k}]\{\boldsymbol{e}\}$$ **equation 3.8**

In local coordinates, for a member j

$$\{\boldsymbol{f'}\} = \left[\frac{\boldsymbol{EA}}{\boldsymbol{L}}\right]\{\boldsymbol{e'}\}$$ **equation 3.9**

Substituting with equation 3.7 and 3.5, we can get equation 3.10.

$$[C]\{f\} = \left[\frac{EA}{L}\right]_j [C]\{e\}$$

equation 3.10

We can multiply both side of the equation 3.10 by $[C]^{-1}$ to arrive at the equation 3.11.

$$\{f\} = [C]^{-1}\left[\frac{EA}{L}\right]_j [C]\{e\}$$

equation 3.11

We can simplify this equation into equation 3.12 by substituting equation 3.13;where $[K]_{ij}$ is the component stiffness matrix of member j w.r.t the joint i.

$$\{f\} = [K]_{ij}\{e\}$$

equation 3.12

$$[K]_{ij} = [C]^{-1}\left[\frac{EA}{L}\right]_j [C]$$

equation 3.13

$$[K]_{ij} = \begin{bmatrix}\cos(\Theta)\\ \sin(\Theta)\end{bmatrix}\left[\frac{EA}{L}\right]_j [\cos(\Theta) \quad \sin(\Theta)]$$

$$[K]_{ij} = \left[\frac{EA}{L}\right]_j \begin{bmatrix}\cos(\Theta)\\ \sin(\Theta)\end{bmatrix}[\cos(\Theta) \quad \sin(\Theta)]$$

This can be solved as equation 3.14.

$$[K]_{ij} = \left[\frac{EA}{L}\right]_j \begin{bmatrix} \cos^2(\Theta) & \cos(\Theta)\sin(\Theta) \\ \cos(\Theta)\sin(\Theta) & \sin^2(\Theta) \end{bmatrix}$$ **equation 3.14**

After finding this component of the stiffness matrix, we can find the joint stiffness of joint i. The computation of the joint stiffness for joint 'i' is given in equation 3.15 where n is the number of joints connected at the joint i.

$$\boldsymbol{q_i} = \left|\sum_{j=1}^{j=n} [\boldsymbol{K}]_{ij}\right|$$ **equation 3.15**

Wellformedness of the structural ring/cluster is calculated by averaging the joint stiffness of the joints connected to that structural ring. The wellformedness of a structural ring/cluster can be calculated as shown in equation 3.15, where N is the number of members connected in the structural ring / cluster.

$$\boldsymbol{Q_i} = \frac{\sum_{i=1}^{i=N} q_i}{N}$$ **equation 3.16**

Let's take the examples that were presented above and calculate the wellformedness of each triangle. Here the axial rigidity (AE) of the members is taken as unity, and the equation for the joint stiffness can be written as

$$\boldsymbol{Qi} = \frac{\Sigma \sin^2(\Theta)}{L} * \frac{\Sigma \cos^2(\Theta)}{L} - \Sigma\left(\frac{\sin(\Theta) * \cos(\Theta)}{L}\right)^2$$ **equation 3.17**

The calculation of wellformedness of the selected triangles is shown in Tables 3.2 to 3.4.

Case (1): Equilateral triangle

Table 3.2 Wellformedness calculation for equilateral triangle

Joint	Member	Angle from global axis (Θ)	Length of member (L)	sin(Θ)^2	cos(Θ)^2	sin(Θ)* cos(Θ)	Joint Stiffness
A	AB	0	2.31	0.00	0.43	0.00	
	AC	60	2.31	0.32	0.11	0.19	0.14
B	BA	180	2.31	0.00	0.43	0.00	
	BC	120	2.31	0.32	0.11	-0.19	0.14
C	CA	240	2.31	0.32	0.11	0.19	
	CB	300	2.31	0.32	0.11	-0.19	0.14
Wellformedness							0.14

Case (2): Isosceles triangle (45' side angle)

Table 3.3 Wellformedness calculation for isosceles triangle (45^0 side angle)

Joint	Member	Angle from global axis (Ө)	Length of member (L)	sin(Ө)^2	cos(Ө)^2	sin(Ө)* cos(Ө)	Joint Stiffness
A	AB	0	4.00	0.00	0.25	0.00	
	AC	45	2.83	0.18	0.18	0.18	0.04
B	BA	180	4.00	0.00	0.25	0.00	
	BC	135	2.83	0.18	0.18	-0.18	0.04
C	CA	225	2.83	0.18	0.18	0.18	
	CB	315	2.83	0.18	0.18	-0.18	0.13
Wellformedness							0.07

Case (3): Isosceles triangle (30' side angle)

Table 3.4 Wellformedness calculation for isosceles triangle (30^0 side angle)

Joint	Member	Angle from global axis (Ө)	Length of member (L)	sin(Ө)^2	cos(Ө)^2	sin(Ө)* cos(Ө)	Joint Stiffness
A	AB	0	6.93	0.00	0.14	0.00	
	AC	30	4.00	0.06	0.19	0.11	0.01
B	BA	180	6.93	0.00	0.14	0.00	
	BC	150	4.00	0.06	0.19	-0.11	0.01
C	CA	210	4.00	0.06	0.19	0.11	
	CB	330	4.00	0.06	0.19	-0.11	0.05
Wellformedness							0.02

The wellformedness of the triangles from the above calculation are tabulated in Table 3.5.

Table 3.5 Wellformedness of different triangles with changing side angles

Side angle of the triangle in degrees	Wellformedness of the triangle
60	0.14
45	0.07
30	0.02

As judged intuitively, the equilateral triangle is shown to be the most wellformed of three triangles.

3.3 Clustering

3.3.1 Introduction

In a Truss, the triangle can be regarded as the basic form of the cluster (structural ring). In the original Bristol Approach, the clustering process is divided into three different stages. The first stage which is identified as 'structural clustering stage I (initial clustering stage)'is where we do the clustering in a manner that the wellformedness of the resultant cluster always increases without the addition of the reference cluster. In the 'structural clustering stage II (secondary clustering stage)', the wellformedness of the resultant cluster is allowed to decrease, but addition of reference cluster is not permitted at this stage. In the third and final stage, the reference cluster is added to the resultant cluster from stage II, and this is identified as the reference clustering stage.

In Wu's work (1991), the wellformedness of all the basic units (a triangle) are calculated prior to initiating the clustering process. Later researchers initiated the clustering from the most wellformed basic units and allowed addition of members to that cluster. However, this will showcase behaviour similar to prestige attachment centrality; for example people access established webpages more often and flights prefers to go through most renowned airports. In this study, it was decided to adopt Wu's rendition of the Bristol approach. Due to this concepts as leaf cluster and branch cluster are different from that presented in 'A Theory of structural vulnerability' (Lu,1999). In our research, a structural ring is considered as the leaf cluster as opposed to the members. Due to this, the 'structural ring' is redefined as a sequence of a maximum of three connected members which can resist an arbitrary set of applied forces.

Since we are concerned with the initial failure or the penultimate cluster in this process, we are not concerned with stage of reference stage clustering.

3.3.2 Clustering criteria

The original Bristol approach (Wu,1991) states that, the selection of a cluster as a leaf cluster (most tightly connected cluster) or selection of a cluster for addition to the existing cluster is governed by the following factors. For the selection, the maximum of these factors were be used in the following order of preference

- Wellformedness
- Minimum damage demand
- Nodal connectivity
- Distance from reference
- Free choice

We have not considered the concept of the damage demand in this research. The damage demand is used to measure the vulnerability of the structure in the original approach. In this research it was decided to use an index called “Relative Separateness” to measure the loss of wellformedness. Another modification was made to the original approach with regard to symmetrical structures.

In symmetrical structures there can be a situation where addition of two different rings/clusters to the main cluster will result in same wellformedness. Even in the first step of clustering, there can be two structural rings whose wellformedness are identical. In such situation the preference order would be used. As the last option, random choice is used in the original approach.

However if the respective clusters are not intersecting and they are in a symmetrical position to each other, both clusters can be chosen as initiation clusters (in initiation stage) or added together to the main cluster if their position allows it (in later stage of clustering). This is done considering the fact that both clusters will have same probability of existing.

As a result of this step, there would be simultaneous progression of symmetrical clusters acting in the same manner up to convergence of such clusters.

3.3.3 Clustering Example

In order to clarify the clustering process, clustering of truss T1 is shown as an example.

Step 01: Calculation of Joint Stiffness. The calculated joint stiffness (J.S) as per equation 3.15 are given in Figure 3.5.

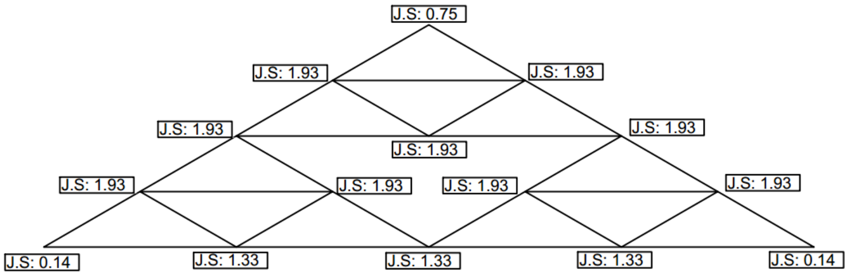


Figure 3.5 Joint Stiffness

Step 02: Calculation of Wellformedness of the Structural Ring. The calculated wellformedness (W.F) of the structural rings as per equation 3.16 are shown in Figure 3.6. Note that C.N refers to cluster number.

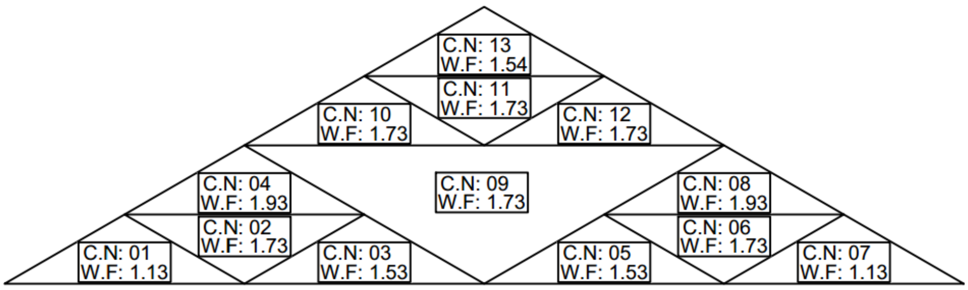


Figure 3.6 Wellformedness of the Structural Rings

From here we can see that cluster numbers 04 and 08 have the highest value of wellformedness and that they are in symmetrical position. Thus the clustering can be initiated using those clusters.

Step 03: Structural Clustering Stage I

The result of the first step of clustering is shown in Figure 3.7.

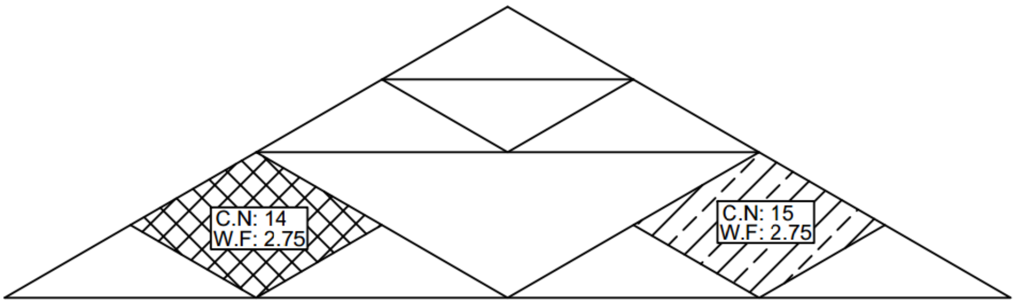


Figure 3.7 Structural Clustering Stage I- Step 1

Though clusters no 10 and 02 have same wellformedness (Figure 3.6) it would be geometrically impossible to cluster the C.N 10 with C.N 04, thus the clustering is progressed using C.N 02 as shown in Figure 3.8.

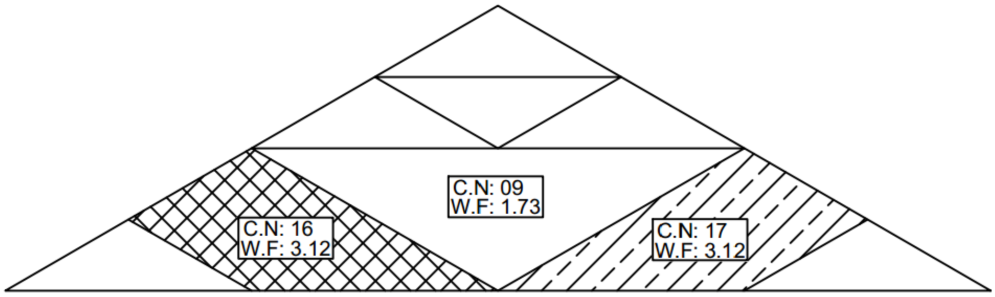


Figure 3.8 Structural Clustering Stage I- Step 2

As per our research, both C.N 16 and C.N 17 would merge with C.N 09 simultaneously. However, this step is broken into two parts to illustrate the change in wellformedness during this process as shown in Figures 3.9 and 3.10.

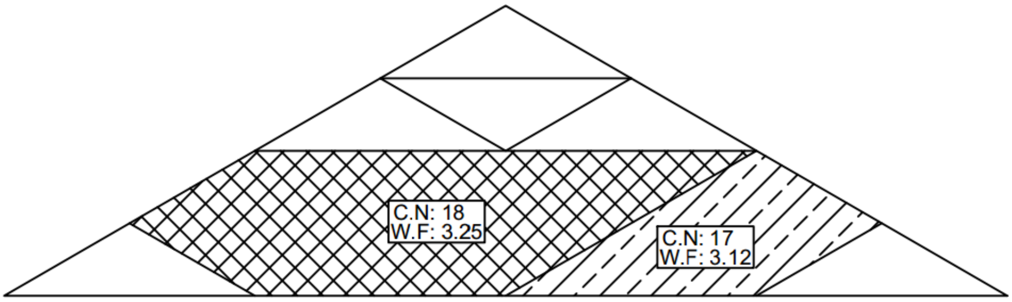


Figure 3.9 Structural Clustering Stage I- Step 3.1

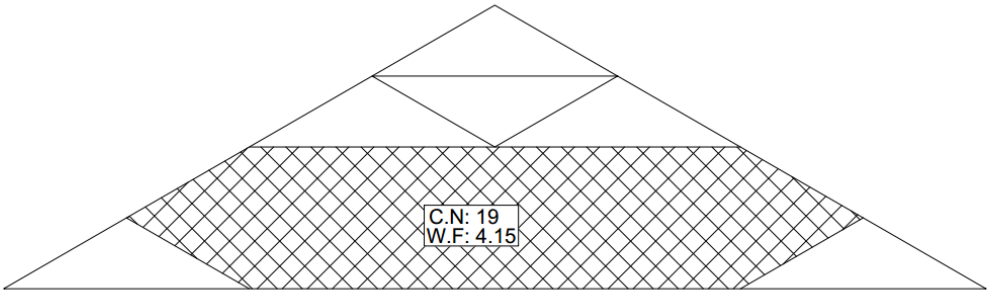


Figure 3.10 Structural Clustering Stage I- Step 3.2

At this stage, any more addition to the existing cluster will reduce its wellformedness. The new cluster is initialised using the same procedure explained above. Figure 3.11 shows this. The progress of the clustering is shown in Figures 3.12& 3.13.

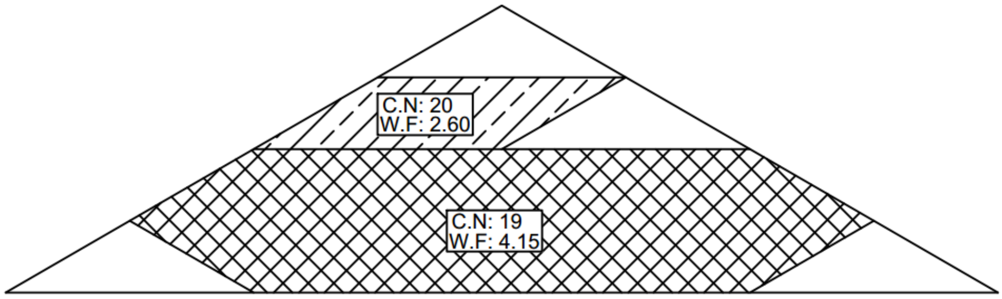


Figure 3.11 Structural Clustering Stage I- Step 4.1

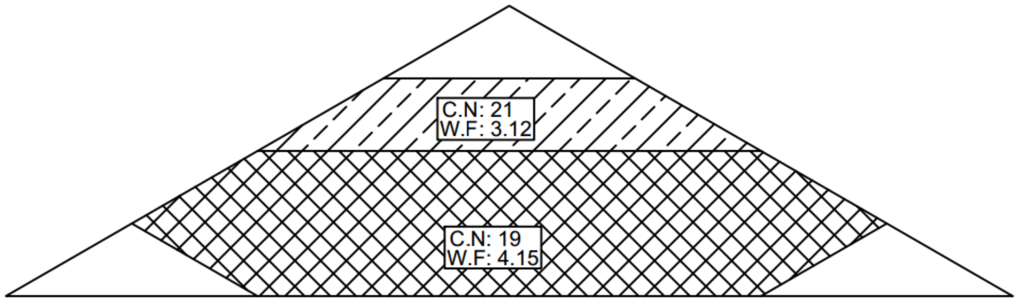


Figure 3.12 Structural Clustering Stage I- Step 4.2

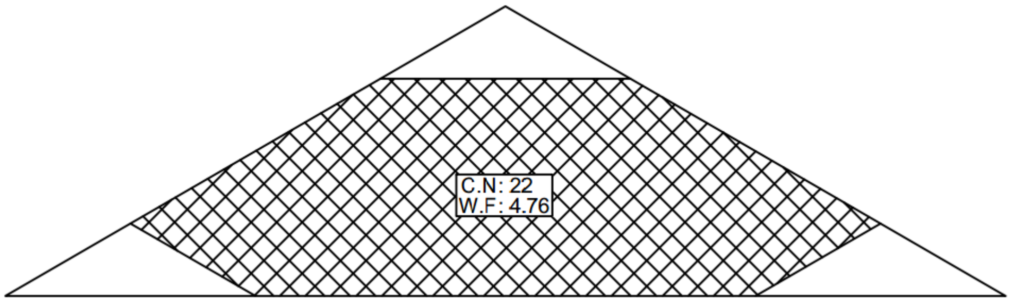


Figure 3.13 Structural Clustering Stage I- Step 5

Adding an already existing cluster to the emerging cluster is allowed in the 'Structural Clustering Stage I'. This marks the end of the first clustering stage as any more structural rings will reduce the wellformedness of the existing cluster.

Step 04: Structural Clustering Stage II

Figure 3.14 shows the first step of structural clustering stage II. This is also the penultimate cluster of this process. Figure 3.15 & 3.16 shows the final output comprising the entire structure. Wellformedness of this cluster is the wellformedness of the structure.

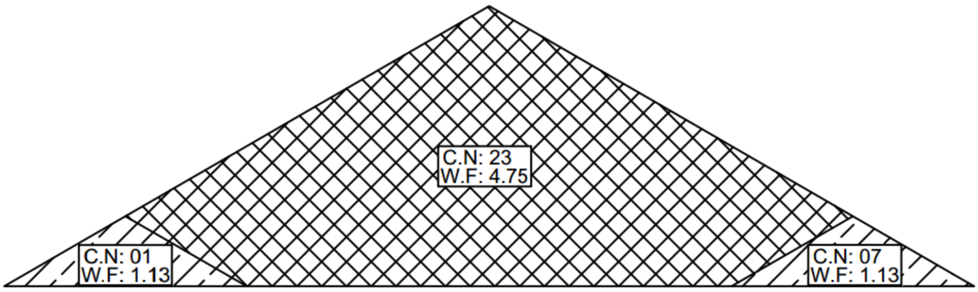


Figure 3.14 Structural Clustering Stage II- Step 1

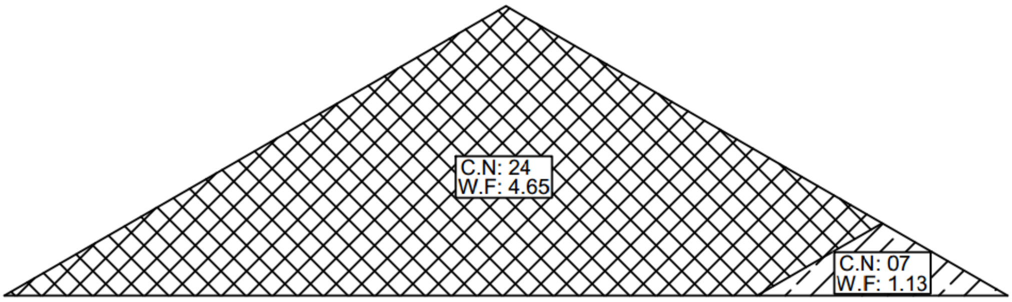


Figure 3.15 Structural Clustering Stage II- Step 2.1

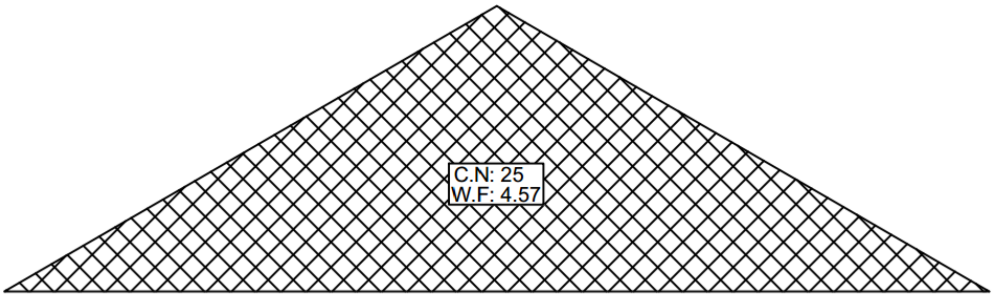


Figure 3.16 Structural Clustering Stage II- Step 2.2

3.4 Relative Separateness

Relative separateness is introduced to measure loss of wellformedness during the initial failure. This is defined as the ratio between the loss of wellformedness to the wellformedness of the intact structure. The wellformedness of the damaged structure is recalculated again from the start as any damage will change the clustering characteristics of the damaged structure. Wellformedness of the intact structure is taken as the summation of wellformedness of clusters existing at the end of Structural Clustering Stage I. The loss of wellformedness is the difference between the wellformedness of intact structure and the wellformedness of the final cluster of damaged structure.

$$\boldsymbol{Relative\ Separateness} = \frac{\boldsymbol{Loss\ of\ Wellformedness}}{\boldsymbol{Wellformedness\ of\ the\ intact\ structure}} \qquad \textbf{equation 3.18}$$

However, when a damage occurs in a truss it turns into a mechanism. Due to this, rather than using the relative separateness, wellformedness of the final cluster is used to determine the Structural connectivity of the trusses.

3.5 Material Cost of Increase Wellformedness

Usually, the chord members in trusses would have an increased axial stiffness compared to the web members. When chord member stiffness is increased, the overall wellformedness of the structure should increases. This was tested by increasing the axial stiffness by two times and four times. However, the material cost of this process also should be considered to keep the design economic. Here the increment in axial rigidity was assumed to be provided by increasing the cross-sectional area of the chord members.

3.6 Results

It was observed that in addition to the wellformedness of the final cluster, analysing the penultimate cluster reveals information about the location of the possible failure. The penultimate clusters of the all three trusses for cases of differing chord axial rigidity is given in Figures 3.17 to 3.25. The wellformedness of the final clusters are given in Table 3.6.

Case 1: All members' axial rigidity= 1

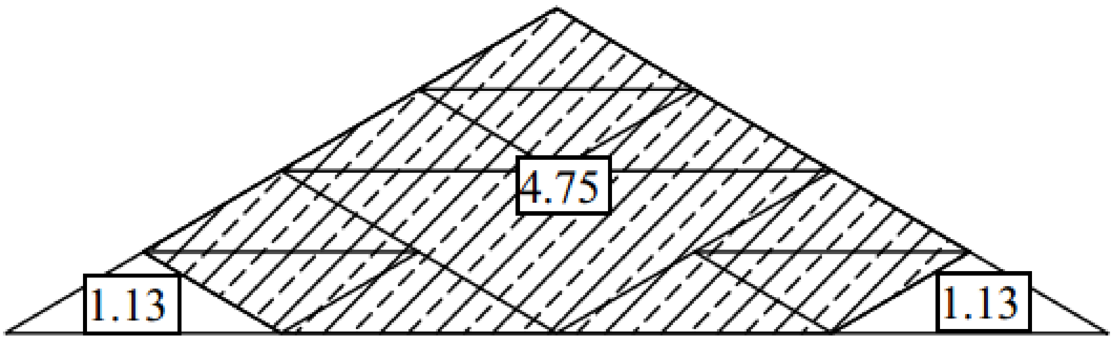


Figure 3.17 Penultimate Cluster of T1 for Case 1

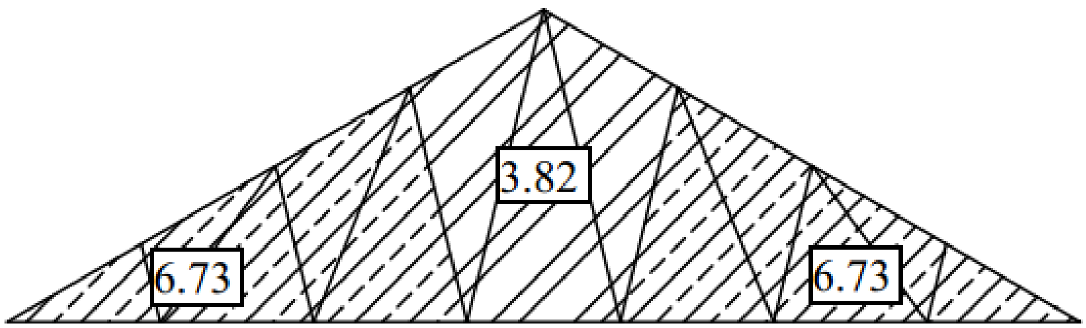


Figure 3.18 Penultimate Cluster of T2 for Case 1

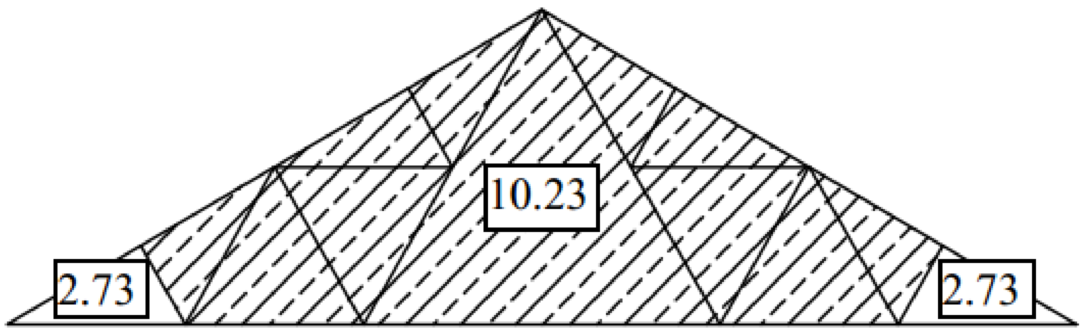


Figure 3.19 Penultimate Cluster of T3 for Case 1

Case 2: Chord members' axial rigidity= 2

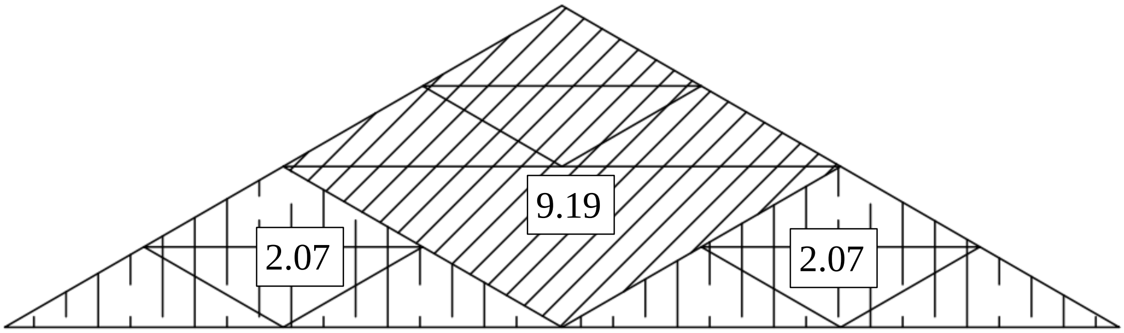


Figure 3.20 Penultimate Cluster of T1 for Case 2

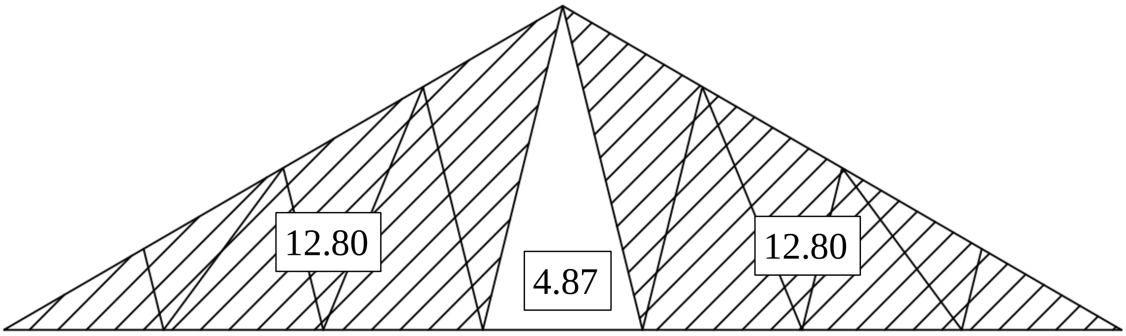


Figure 3.21 Penultimate Cluster of T2 for Case 2

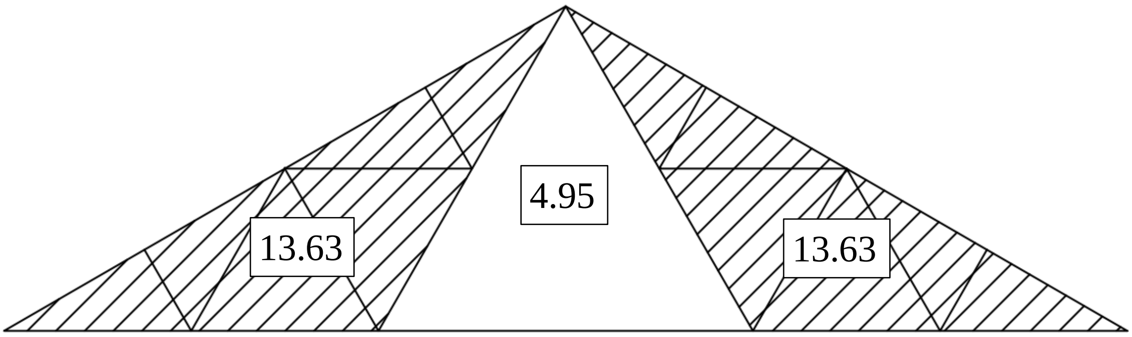


Figure 3.22 Penultimate Cluster of T3 for Case 2

Case 3: Chord members' axial rigidity= 4

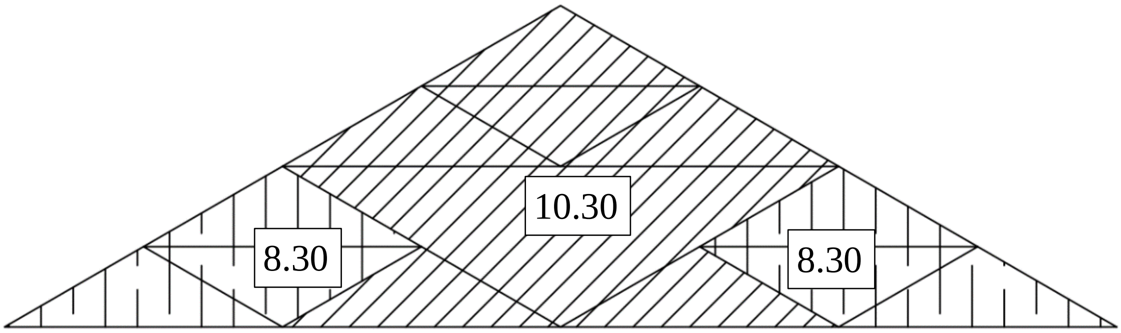


Figure 3.23 Penultimate Cluster of T1 for Case 3

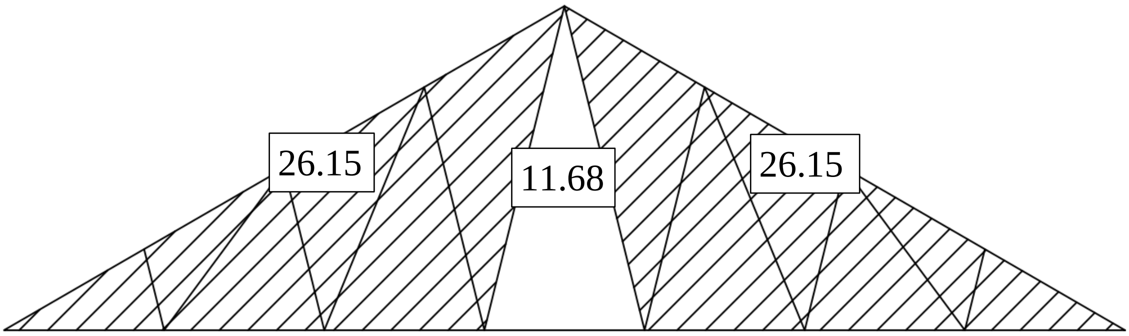


Figure 3.24 Penultimate Cluster of T2 for Case 3

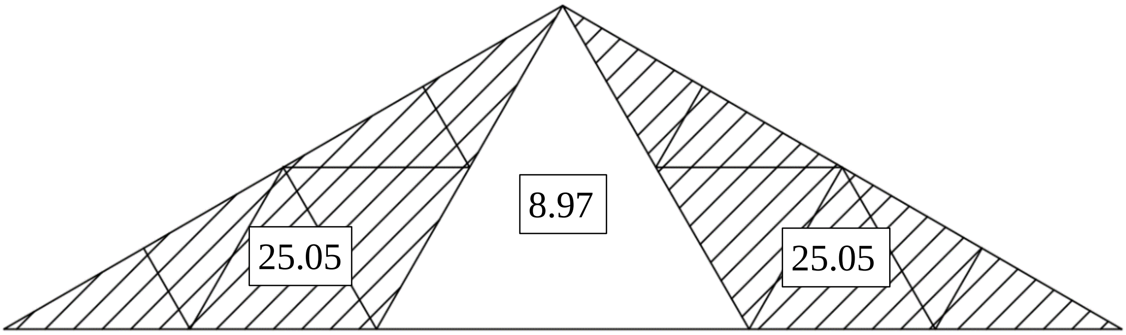


Figure 3.25 Penultimate Cluster of T3 for Case 3

Table 3.6 Summary of Results

	T1	T2	T3
AE	4.57	7.56	9.96
2AE	7.27	14.75	16.19
4AE	13.09	30.23	29.71

The material cost of the increased wellformedness is tabulated in Table 3.7. The change in wellformedness with the increased material cost is presented in graphical form in absolute values is presented in Figure 3.26 and relative percentages of increment taking Case 1 (all member AE=1) as the base model is presented in Figure 3.27. The graph shows results for four cases including chord member axial rigidity as one through four. The calculation for the case where chord member axial rigidity equals three is not presented here.

Table 3.7 Summary of Material cost

	T1	T2	T3
AE	33.59	30.97	28.78
2AE	48.52	45.89	43.71
4AE	78.37	75.75	73.57

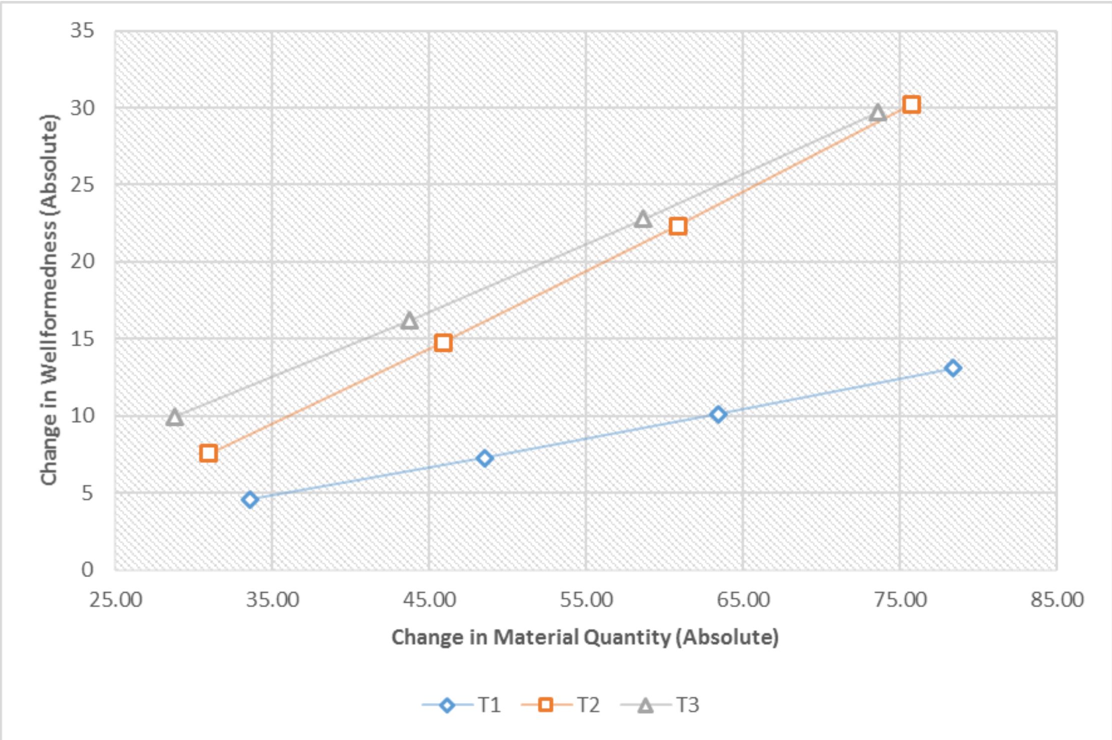


Figure 3.26 Change in Wellformedness vs Material Quantity (Absolute)

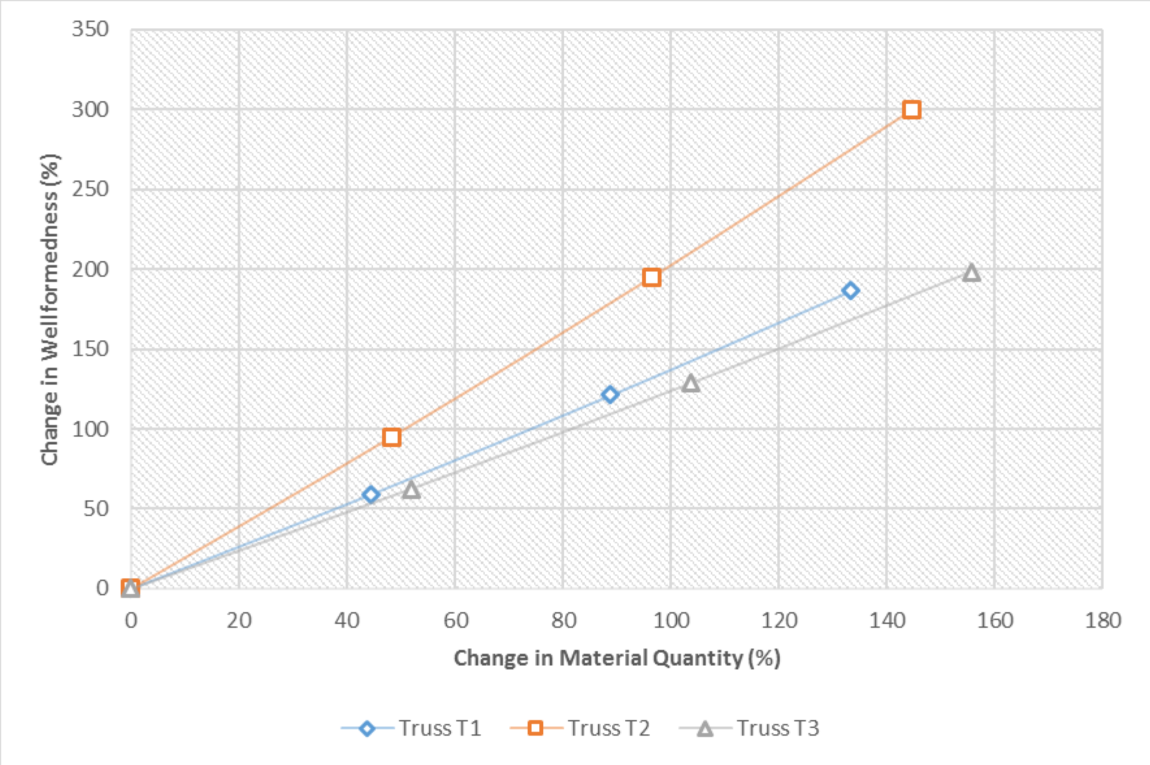


Figure 3.27 Change in Wellformedness vs Material Quantity (%)

3.7 Discussion

As observed from Table 3.6, truss T3 is selected as the most wellformed configuration as it has the highest wellformedness in Case 1 (where all members' axial rigidity equals one). Truss T3 is again identified as the most wellformed configuration when the chord member axial rigidity is doubled (case 2). It should be noted that wellformedness of all truss forms increased between case 1 and 2. This trend is observed again between case 2 and 3 as well. However, truss T3 is identified to have higher wellformedness in case 3, thus the most wellformed configuration. Another thing to note that Truss T2 and T3 display wellformedness values close to each other and there is a significant difference between them and truss T1 values in all three cases.

The penultimate clusters needs to be analysed to gain further understanding about the weakly connected members/ zones. In the case 1, the penultimate cluster of T1 shows that the support nodes would be the most weakly connected to the rest of the structure. Truss T3 also showcases a similar behaviour, but the wellformedness of this arrangement is high compared to the truss T1, meaning the possibility of loss of connectivity is lower than truss T1. Though the wellformedness of the truss T2 is higher than truss T1, the location of failure is located in the top chords, any failure in this region will completely damage the structure. This scenario can be said as a low risk-high damage scenario. Structural connectivity analysis can be used to avoid this kind of scenarios. It should be observed that all the failure is taking place in the chord of the trusses; thus it will be beneficial to increase the axial rigidity of the chord members and check the changes in the penultimate clusters.

In the case 2, where the axial rigidity of the chord members is doubled, the failure location is shifted away from the support nodes to middle of the chord members in truss T1. Though the location of the possible failure is unfavourable, the possibility of the failure will be low due to the increased wellformedness. This is similar to truss T3 in case 1. This is the first time, which the web members are involved in the failure scenario. Because of the simplicity of the selected configurations, a local web failure was not achieved using the Bristol Approach. A local web failure is more favourable than a chord member failure. Truss T2 and T3 show the failure has moved to the middle of the bottom chord. This is also more preferable to failure in the middle of the top chord as truss can avoid a complete collapse by using inherent arch action.

In case 3, when the axial rigidity of the chord members is doubled again, failure location of the truss T2 & T3 have not changed but the wellformedness has increased. This shows that more effort needs to be employed to break that connection between the case 2 and 3. Truss T1 shows that the failure now involves more web members, this is also more preferable.

As discussed above, the values of wellformedness for Truss T2 and T3 are close to each other. Material cost can be used to differentiate between these trusses. From the Figure 3.27, it is quite evident that truss T2 gives more returns for increment in the chord axial rigidity in terms of wellformedness.

4 Newman's Method

4.1 Introduction

In order to understand Newman's Method, understanding should be gained on the concept of Network Theory. Network Theory is the idealisation of real world networks into graphs of specific topology and the study of those graphs as the representation of relations/connections between discrete objects. This is presented in Section 4.2. Sections 4.3 & 4.5 discusses the unweighted and weighted Newman analysis respectively. An index for structural connectivity based on Newman's method is proposed in Section 4.4. Section 4.6 details the modifications made to original Newman's method in adapting it to this research. Results of the analysis are given in Section 4.7 followed by discussion on those results in Section 4.8.

4.2 Network Theory

4.2.1 Introduction to Network theory

The initial process of Network theory would be idealisation or representation of a network in a graph form. The structure of network can be found in many areas in the natural and artificial (man-made) world. For example, there exists networks like the metabolism network, air-traffic network, social network, internet and banking network. Most networks cannot be easily idealised into nodes and links. The concept of a network paradigm is used in this process.

4.2.2 The Network paradigm

The term paradigm refers to the principles and assumptions made to idealise a real world phenomenon (Network Theory Course - YouTube, 2016). Another important thing to note is that the topology of the idealised network doesn't necessarily follow the Euclidian geometry of the original network. The most important thing of the network representation is the connections in the network; this is expressed as the links in the network. The network paradigm also includes data on how the network emerges and the complexity and non-linearity of the network, but these concepts are out of scope of this study. In simple terms, the Network paradigm is a set of principles and assumptions used to idealise a real word network into a graph representing its topology and connectivity. Some of the common network topologies are given in Figure 4.1.

In this study, the joints are nodes and members are links. The topology doesn't need to be altered and the connections are given by the adjacency matrix of the graph.

4.2.3 The network connections

The amount of connections a network has alters the structure of the network and how it is interpreted. When there are only a few connections between the nodes, the characteristics of the network can be explained as the sum of the characteristics of the nodes-i.e. change in one node will change the network proportionally, since that change won't affect the other nodes in the network. When the number of connections increases between the nodes, a change in one node will affect other nodes in the network depending on the connections and the change in the network is not proportional to the change in the nodes. In essence, the characteristic of the network will be described by the connections between the nodes rather than the nodes themselves. It should be noted that due to this, how connected an individual node is becomes a key metric of its importance in the network.

Based on the network topology, the networks can be divided into three main types.

- Centralised network
- Decentralised network
- Distributed Network

Centralised networks are known to have an important hub node as the central node, most of the other nodes being connected to it (example: star network in Figure 4.1). This is very robust against random attacks but highly vulnerable to strategic attacks. In the decentralised network, we can observe local clusters emerging with their own hubs. This network topology represents the concept called "Small World Phenomena" - i.e. in a graph where most nodes are not neighbours but most nodes can be connected in few steps. Distributed network represents a network where all or most have same degree of connections (example: fully connected network in Figure 4.1). This would be the most robust network topology as it has no hierarchy of connections. But this is also a less efficient network topology. Our engineering structures can be represented by decentralised networks.

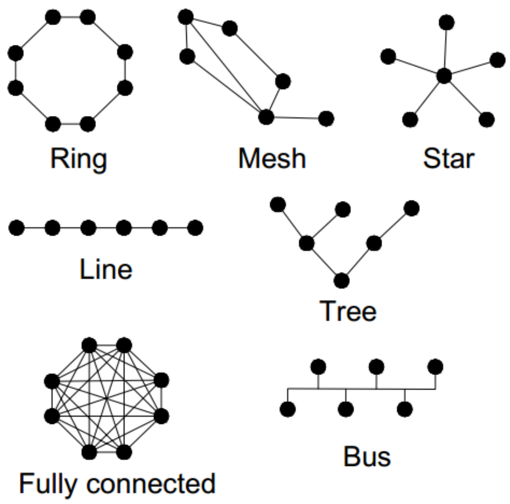


Figure 4.1 Typical network topologies

A node's importance in a network can be measured by its degree and its centrality. Degree of connectivity represents the immediate likelihood of catching whatever is passing through the network. Importance of node with respect to its location is measured by the centrality measures (also discussed in Chapter 02). Some of the centrality measures are;

- Degree centrality
- Closeness centrality
- Betweenness centrality
- Prestige attachment

Closeness centrality of the selected trusses are discussed in Chapter 02. This shows how likely a node can affect the rest of the nodes around it. Prestige attachment relates to the situation where nodes are get attached to identified important nodes to form clusters. This is true for citation networks or the World Wide Web. Betweenness centrality is the core of the Newman method.

4.2.4 Network Robustness and Resilience

As we discussed, there exist several types of networks at different levels. It is common that most of these networks are constantly attacked naturally and otherwise, but how the network responds to these damages depend on its characteristics. It should be noted that a network's robustness and connectivity are interrelated. Without the connectivity, the parts of the network will become disconnected and disintegrated. Thus the robustness of a network can be assessed by removing one/many components and seeing how affects the rest of the network. If this analysis was done with respect to the nodes, it can be explained by the concept of node percolation, and by edge percolation if it was done with respect to the links.

Edge betweenness is basically an index for identifying the in-between edges in a network. These links also act as the irreplaceable connections between one cluster and another. Whether the failure will have a cascading effect will determine the robustness of the network. For example, failure of a station in a power grid could have a cascading failure over the entire network. Another thing to consider is that whether the edge/ link removal is random or strategic. In this research, strategic removal of edges is considered for the analysis. Since the analysis is used to assess the inherent characteristic 'structural connectivity'; it is required to find the weakest link in the network and remove it to check its effects. This was earlier explored in the Chapter 02 in the description of Menger's theorem.

Menger's theorem is normally interpreted as the max-flow, min-cut theorem. This means that most effective way to disconnect to vertices it to remove the edges that have the maximum shortest paths between those vertices. This is the basic foundation of Newman's Method.

4.3 Newman's unweighted analysis

Newman's method involves three stages of analysis. These are;

- Calculating shortest paths between all vertex pairs.
- Assigning the betweenness values for the edges.
- Finding communities in the network.

The first stage of the analysis can be carried out by using any of the existing methods to find the shortest paths such as depth first algorithm or breadth first algorithm. The example used in Newman (2004) will be followed to explain this procedure. Figure 4.2 shows the selected network with its node numbers. The procedure to calculate the shortest paths is given below (breadth first algorithm) - (Newman 2004);

1. *"The initial vertex s is given distance $d_s = 0$ and a weight $w_s = 1$.*
2. *Every vertex i adjacent to s is given distance $d_i = d_s + 1 = 1$, and weight $w_i = w_s = 1$.*
3. *For each vertex j adjacent to one of those vertices i we do one of three things:*
   a. *If j has not yet been assigned a distance, it is assigned distance $d_j = d_i + 1$ and weight $w_j = w_i$.*
   b. *If j has already been assigned a distance and $d_j = d_i + 1$, then the vertex's weight is increased by w_i, that is $w_j \leftarrow w_j + w_i$.*
   c. *If j has already been assigned a distance and $d_j < d_i + 1$, we do nothing.*
4. *Repeat from step 3 until no vertices remain that have assigned distances but whose neighbours do not have assigned distances."*

In step 2, in order to check whether the node has been visited or not, a weight is assigned to the nodes that are visited in the algorithm. This weight is also used to record the total number of shortest paths crossing a specific node. It should be noted that this weight is not a structural characteristic but just an indicator. This has to be repeated for all vertex pairs.

"If two vertices i and j are connected, with j farther than i from the source s, then the fraction of a geodesic path from j through i to s is given by w_i/w_j "(Newman 2004).

This procedure was automated using the MATLAB software. The MATLAB code was referred from the "MATLAB Tools for Network Analysis" provided by MIT Strategic Engineering (MATLAB Tools for Network Analysis (2006-2011), 2011). This was used to verify the author's own MATLAB code. This MATLAB code is provided in the appendix. The results from the first stage of analysis is stored in a matrix form (n x n x n).

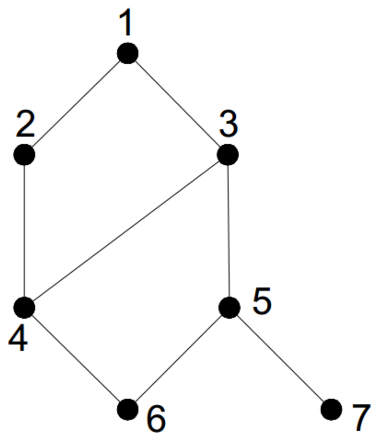


Figure 4.2 Selected Network

The shortest paths from node 1 to all other nodes are shown in the Figure 4.3. From the node 1, the distance of shortest path to other nodes and each node weights are given in Figure 4.4.Since this is an unweighted analysis, a link will be regarded as a step and the shortest paths are calculated by counting the number of steps taken to reach each node from node1.

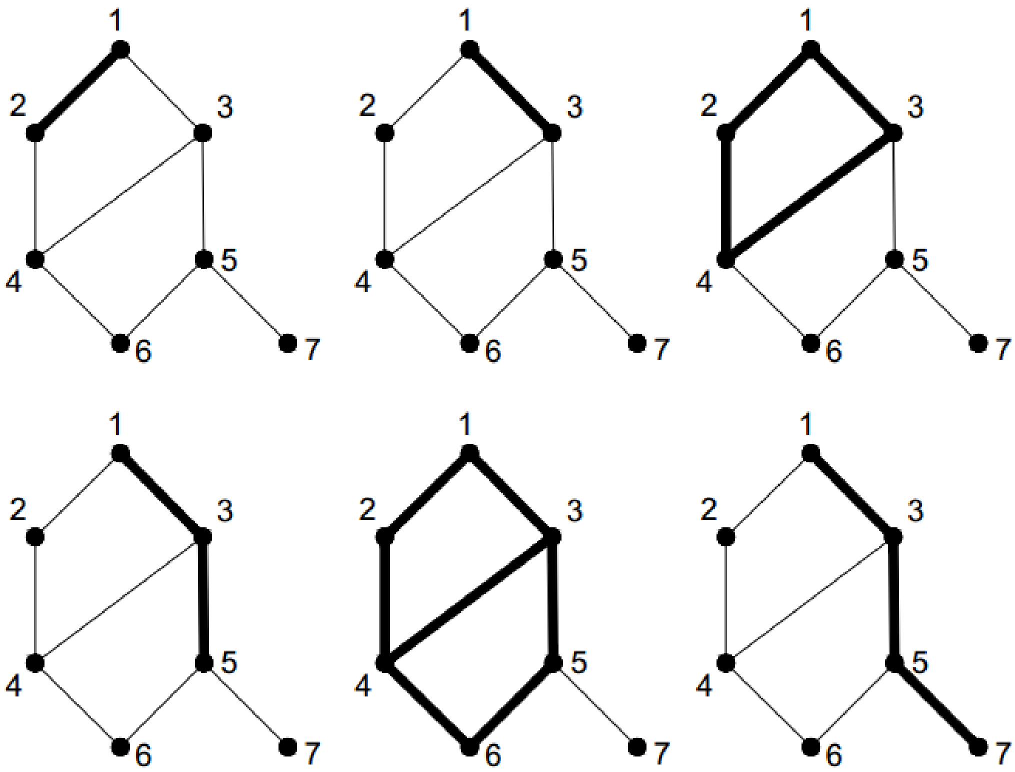


Figure 4.3 Shortest Paths from node 1

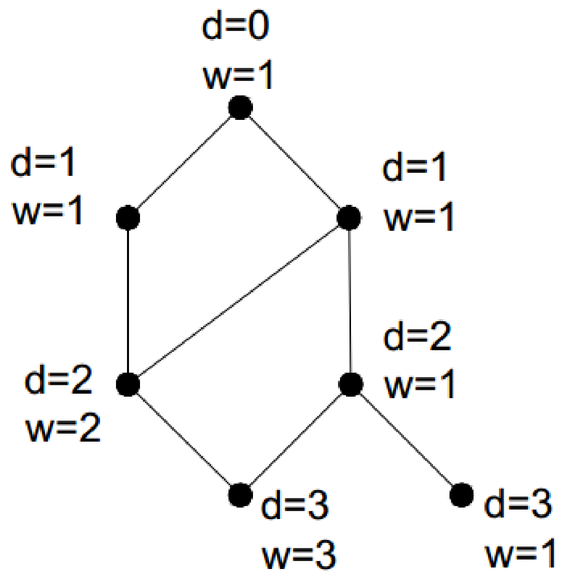


Figure 4.4 Shortest distance from node 1 to other nodes

The second stage of analysis was carried out as per the procedure given below (Newman-2004).

1. *"Find every "leaf" vertex t, i.e., a vertex such that no paths from s to other vertices go through t.*
2. *For each vertex i neighbouring t assign a score to the edge from t to i of w_i/w_t.*
3. *Now, starti with the edges that are farthest from the source vertex s and work up towards s. To the edge from vertex i to vertex j, with j being further from s than i, assign a score that is 1 plus the sum of the scores on the neighbouring edges immediately below it (i.e., those with which it shares a common vertex), all multiplied by w_i/w_j .*
4. *Repeat from step 3 until vertex s is reached.*
5. *Now repeating this process for all n source vertices s and summing the resulting scores on the edges gives us the total betweenness for all edges."*

For easier understanding, the procedure explained above is simplified in Figure 4.5 & 4.6. Figure 4.5 shows the edge betweenness scores for the shortest paths found in the Figure 4.3.

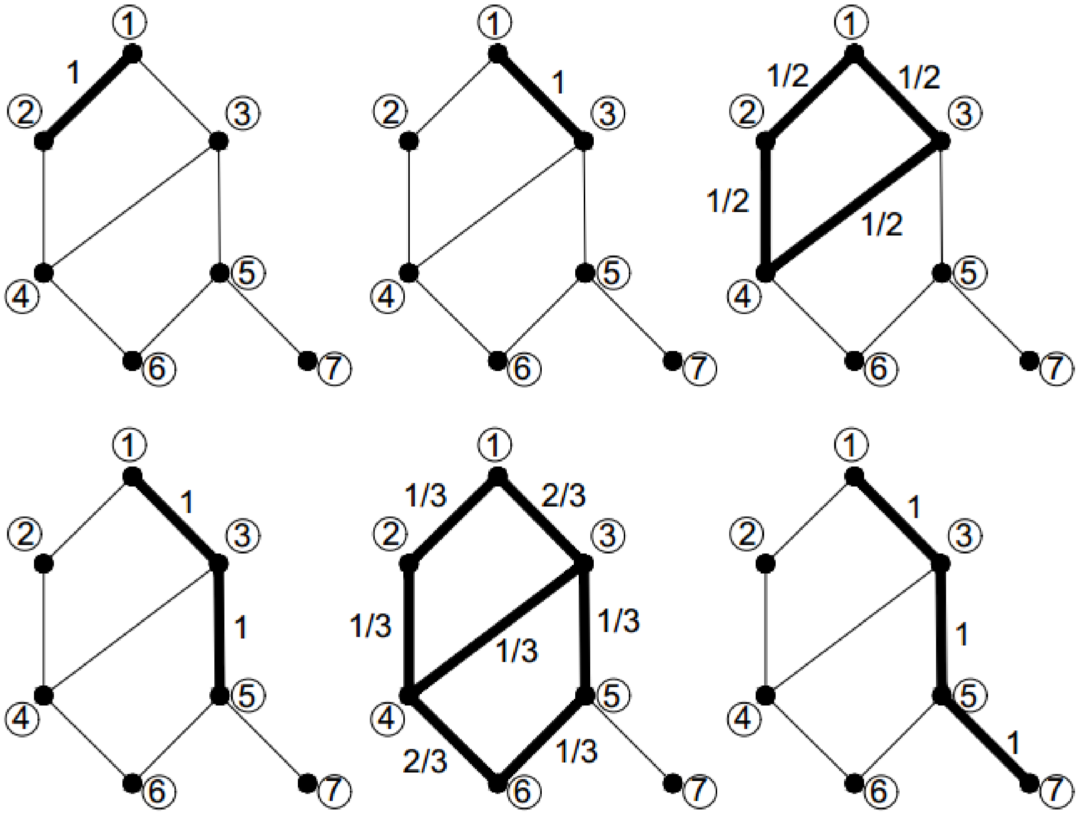


Figure 4.5 Betweenness score for edges taking node 1 as source

The edge betweenness scores are added for each edge and given in Figure 4.6.

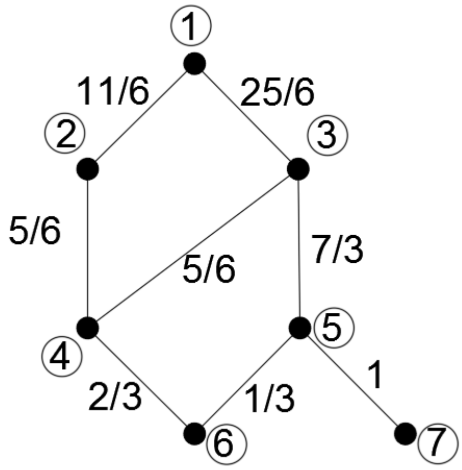


Figure 4.6 Resultant Edge betweenness taking node 1 as source

The edge betweenness after considering all the nodes as the source nodes is given in Figure 4.7. Even calculating edge betweenness of small networks such as the one shown here has too much computations to be done by hand. This is one of the reasons to develop the MATLAB code.

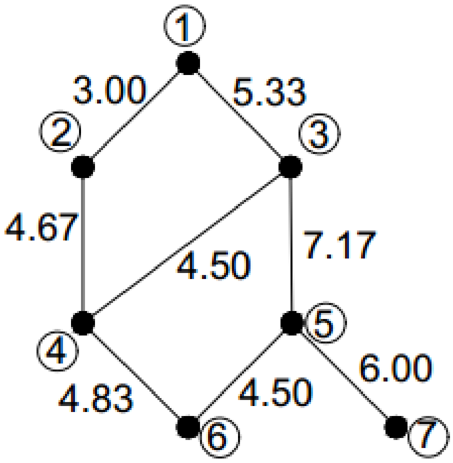


Figure 4.7 Final edge betweenness

As can be seen in the Figure 4.7, the edge betweenness for the edge 5-7 is 6.0. All the shortest paths from node 7 to all other six node needs to travel through the node 5, resulting in the edge betweenness of 6.0. This can be taken as the validation for our MATLAB code. It can be observed that the highest edge betweenness occurs in the edge 3-5. As per our theory, the edge with the highest edge betweenness is most likely to be inter-cluster link and will be susceptible to higher damage from a random attack as it would have a lesser connectivity with the rest of the structure.

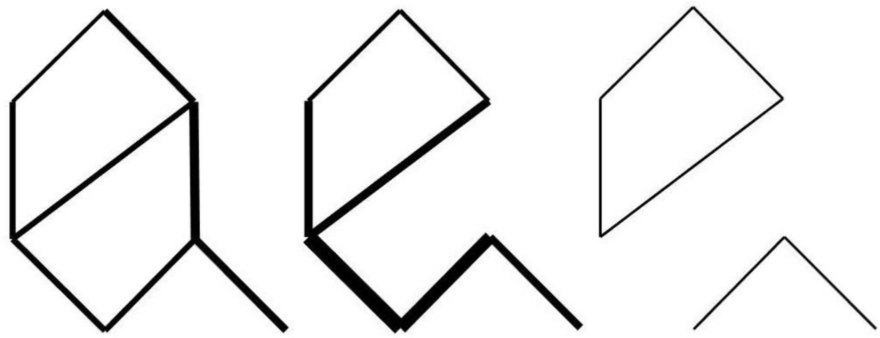
In order to find the actual clusters in the structure, the links that have the highest edge betweenness have to be removed in each step. In addition to checking the structure after removal of each link, an index is required to show the characteristic such as how in-between was the removed link.

4.4 Relative betweenness

To avoid confusion, each iteration of the algorithm will be called a generation from now on. Relative betweenness is the ratio of the edge betweenness of the edge removed (highest edge betweenness in generation no “r”) to the sum of edge betweenness of all remaining edges in the next generation (“r+1”).

$$\textit{Relative Betweenness} = \frac{\textit{Edge betweenness of the edge removed}}{\textit{Sum of edge betweenness of all edges in the next step}} \qquad \textbf{Equation 4.1}$$

If the value of the relative betweenness is low that means that there are more inter-cluster links than what was removed in that particular generation. If the value is high then the link removed was one of the few or only inter cluster link. Figure 4.8 shows the generations of the selected network according to the algorithm and Table 4.1 shows the highest edge betweenness and the sum of edge betweenness of all remaining edges in the next step and the relative betweenness of the removed edge in each generation. Figure 4.8 comprise outputs from the MATLAB code. It was coded to represent the edge betweenness of that generation as the thickness of the edge.



(a) Generation 00 (b) Generation 01 (c) Generation 02

Figure 4.8 Steps in the analysis

Table 4.1 Relative betweenness values

Generation No	Highest edge betweenness	Sum of edge betweenness in next generation	Relative betweenness
1	7.17	48.00	0.15
2	12.00	12.00	1.00
3	2.00	-	-

From Table 4.1, it can be observed that in generation 2 when the edge 4-6 is removed (Figure 4.7), the structure becomes separated and this clearly reflects in the relative betweenness of that edge. This can justify using relative betweenness to assess the connectivity of the network.

4.5 Newman's weighted analysis

As discussed in the Chapter 2.3, a weighted graph can be represented by a multi-path model. The edge betweenness represents how many shortest paths are going through an edge, i.e. a pair of nodes. If there exists more than one path between a pair of vertices, the total number of shortest paths going through them won't change. For example, if two paths exist between two cities with same travel time, though the drivers may choose either one of the paths, the total number of trips completed between those two cities will be same as the number of trips completed in the event of a single path existing between those two cities. Thus in weighted Newman analysis the initial step to compute the edge betweenness is carried out in the same way as for the unweighted case.

But when it comes to removing the edges, the weight of the edges should be taken into consideration as they represent a number of parallel edges between a pair of vertices. In the previous example, the number of trips through a path will be half of the total number of paths between the two cities. Thus the edge betweenness of the edge is half of the unweighted analysis results. In the second stage of the weighted analysis, the edge betweenness values taken from the unweighted analysis should be divided by the edge weights. Figure 4.9 shows a weighted network, this has the same structure as the network in Figure 4.2.

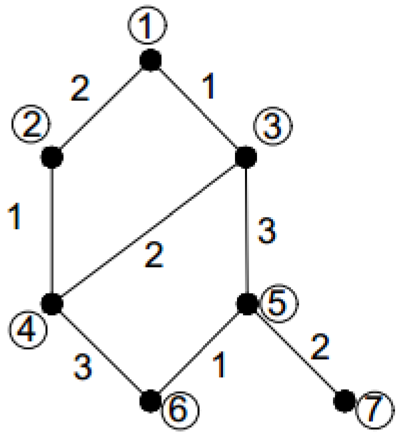


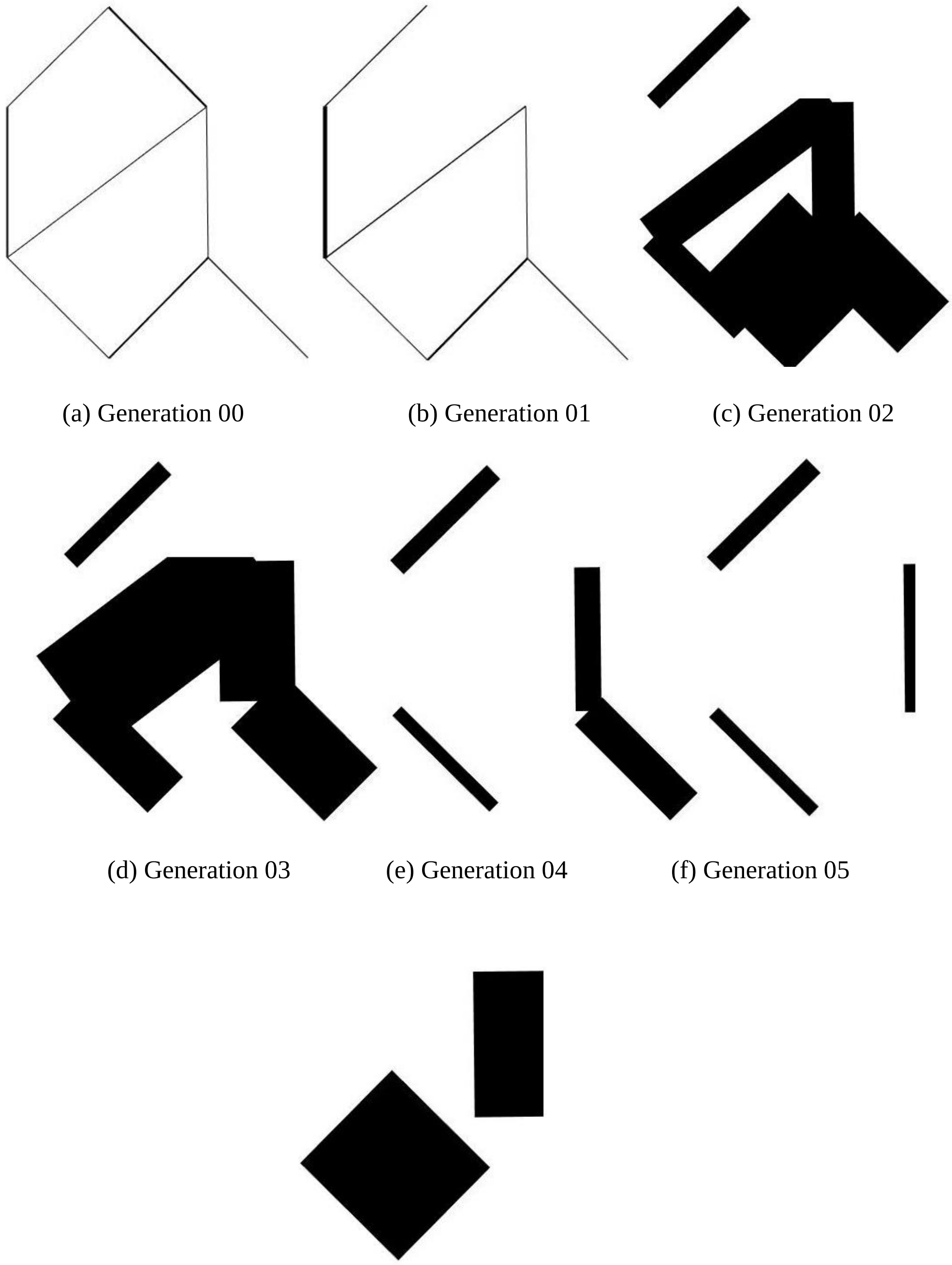
Figure 4.9 A weighted network

Table 4.2 shows the calculation of edge betweenness of the links of the selected network for the first generation of the analysis.

Table 4.2 Edge betweenness of weighted analysis

Edge ID	Edge betweenness from unweighted analysis	Edge weight	Edge betweenness of weighted analysis
1-2	3.00	2	1.50
1-3	5.33	1	5.33
2-4	4.67	1	4.67
3-4	4.50	2	2.25
3-5	7.17	3	2.39
4-6	4.83	3	1.61
5-6	4.50	1	4.50
5-7	6.00	2	3.00

It can be observed from Table 4.2 that the edge with highest edge betweenness has changed from edge 3-5 to edge 1-3. After removing this edge, the analysis will be carried out in similar manner until no edges remain. This weighted analysis results are shown in Figure 4.10 and computed relative betweenness in Table 4.3.



(a) Generation 00 (b) Generation 01 (c) Generation 02

(d) Generation 03 (e) Generation 04 (f) Generation 05

(g) Generation 06

Figure 4.10 Breakdown of network during the weighted analysis

Table 4.3 Relative betweenness of weighted analysis

Generation No	Highest edge betweenness	Sum of edge betweenness in next step	Relative betweenness
1	5.33	28.75	0.19
2	10.00	9.25	1.08
3	3.50	8.83	0.40
4	3.00	2.50	1.20
5	1.00	1.17	0.86
6	0.50	0.67	0.75
7	0.33	-	-

As expected from the findings in the unweighted analysis, the edge removals in generation no 2 and 4 (Figure 4.10 (c) & (e)) have the highest relative edge betweenness as their removal separates the structure. Thus the index "relative separateness" can be justified to be used as a measure of connectivity of the weighted networks as well.

4.6 Modifications to Newman's method

For the analysis of trusses it was decided to take member stiffness as the weight of the edges. This is done to maintain uniformity in analysis between different analytical methods considered and to account for the fact that a member with a higher stiffness will have a lower possibility of failure.

If we have a very large value as the weight, then its influence on the relative betweenness is more than what it should be. Same argument can be made for the number of shortest paths through an edge. A network having higher number of nodes has higher number of possible shortest paths through an edge, thus when comparing networks of different size, arrangements need to be made to offset this factor.

It was decided to divide the number of shortest paths through an edge by number of all possible paths in that network, i.e. n*(n-1) where n= number of nodes in a network. From this, edge betweenness is given as the fraction of possible paths in a network. This will automatically scale the edge betweenness to be compared with a network of any size. However, the same can be achieved in calculating relative betweenness. Thus this step can be omitted if we are comparing relative betweenness and not the edge betweenness of the networks.

To normalise the influence of weights in the calculation of relative betweenness it was decided to divide the weight of every edge by the average of all edge weights. From this the edge weights are given as the multiples of average edge weight.

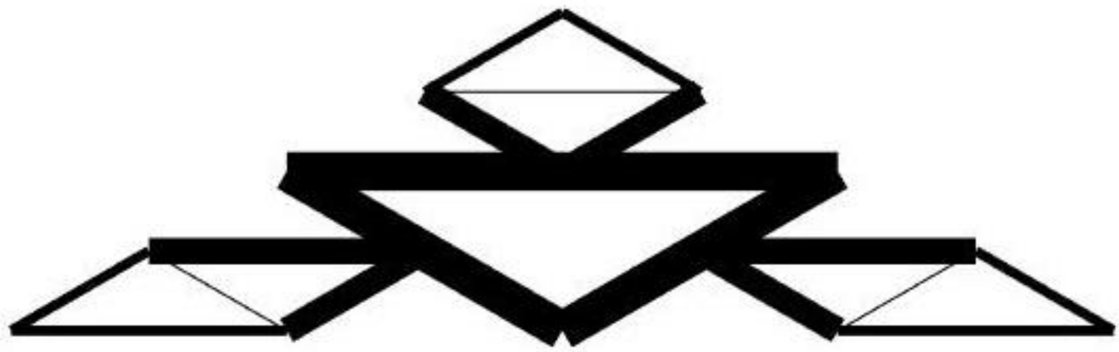
As discussed in the previous two chapters, the structural connectivity is characterised by the first edge to be removed. This is even truer for trusses as any loss of member will turn it into a mechanism. In this study we are comparing the relative betweenness of the first edge to be removed.

4.7 Results

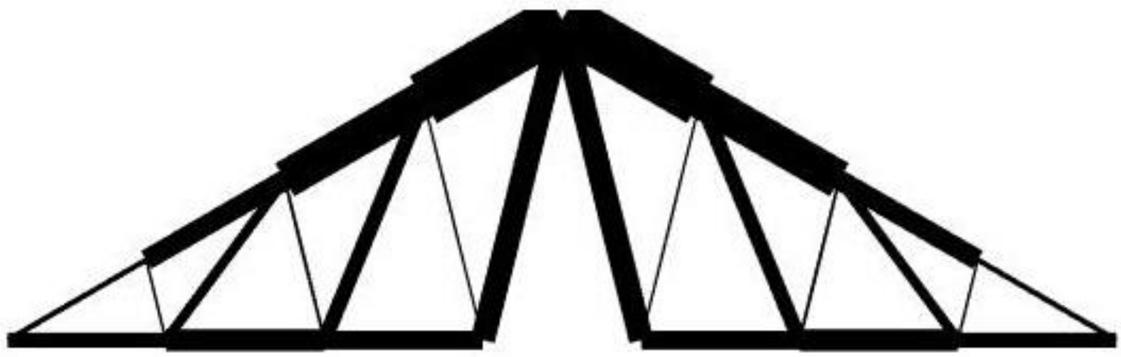
The results form Newman's method are given in this section. The unweighted analysis results are presented in Section 4.6.1 and the weighted analysis results in the Section 4.6.2. The summary of the relative edge betweenness for the considered analysis scenarios are given in Table 4.4.

4.7.1 Unweighted Analysis Results

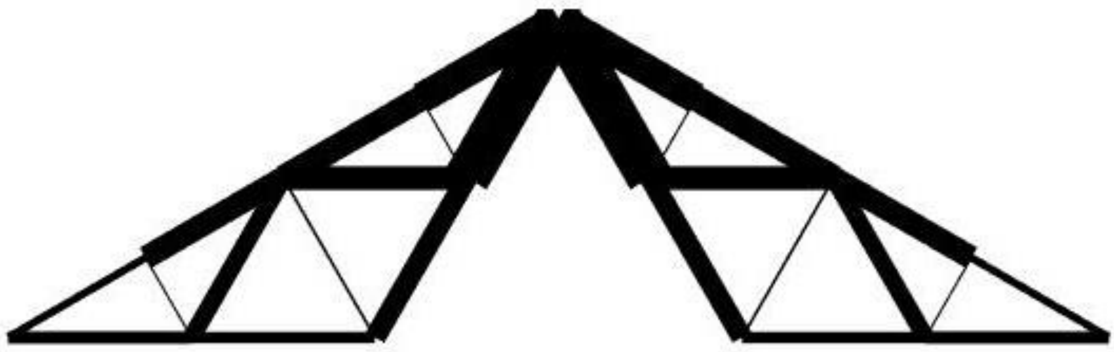
The results for the unweighted analysis are given in Figure 4.11. This Figure shows the trusses after the initial step of member removal which corresponds to the penultimate cluster in the Bristol method.



(a) Generation 01of Truss 01- Penultimate cluster in unweighted analysis



(b) Generation 01of Truss 02- Penultimate cluster in unweighted analysis

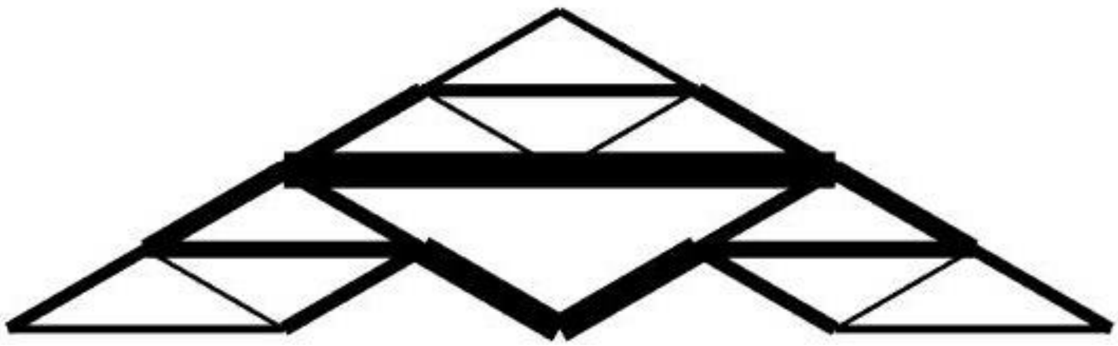


(c) Generation 01of Truss 03- Penultimate cluster in unweighted analysis

Figure 4.11 Results of the unweighted analysis

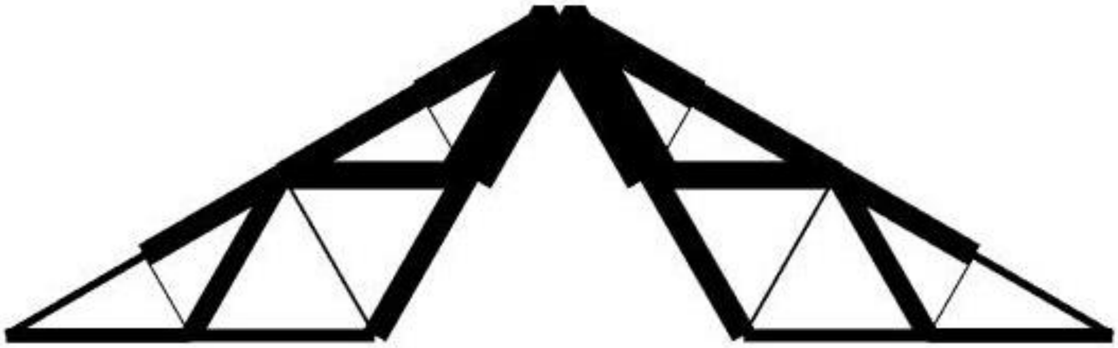
4.7.2 Weighted Analysis Results

This results of the weighted analysis in the Newman method are presented in this Section. Different axial rigidities for the chord members were analysed to check the effect of increased axial rigidity in the overall connectivity. The axial rigidities which were used in the Bristol approach were used here as well. The results for the chord axial rigidities AE=1, 2& 4 are given in the Figures 4.12, 4.13& 4.14 respectively; and a summary given in Table 4.4.



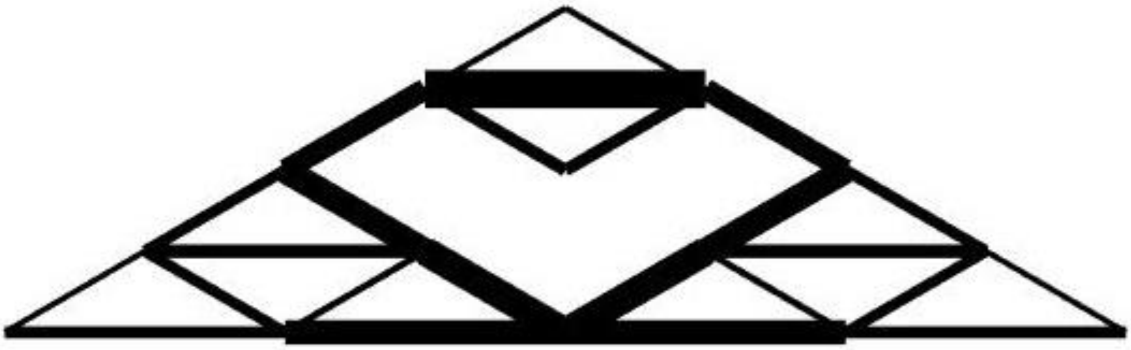
(a) Generation 01of Truss 01- Penultimate cluster in weighted analysis

(b) Generation 01of Truss 02- Penultimate cluster in weighted analysis



(c) Generation 01of Truss 03- Penultimate cluster in weighted analysis

Figure 4.12 Weighted analysis results- AE=1 (Case 1)



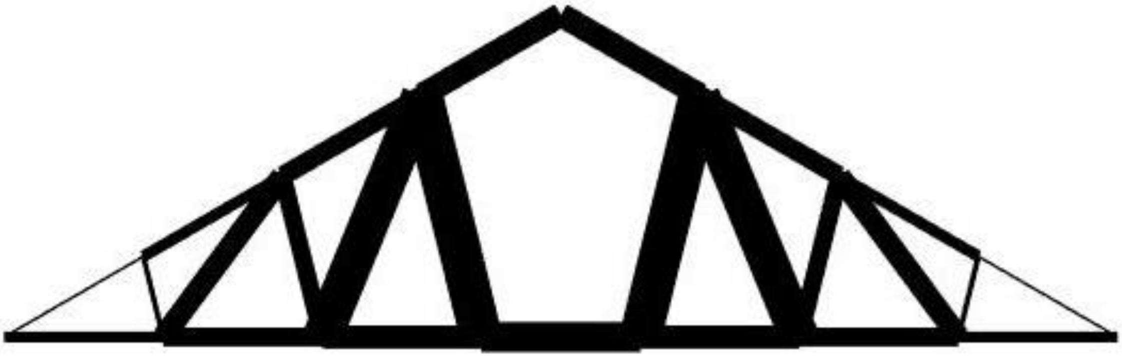
(a) Generation 01of Truss 01- Penultimate cluster in weighted analysis

(b) Generation 01of Truss 02- Penultimate cluster in weighted analysis

(c) Generation 01of Truss 03- Penultimate cluster in weighted analysis

Figure 4.13 Weighted analysis results- AE=2 (Case 2)

(a) Generation 01of Truss 01- Penultimate cluster in weighted analysis



(b) Generation 01of Truss 02- Penultimate cluster in weighted analysis

(c) Generation 01of Truss 03- Penultimate cluster in weighted analysis

Figure 4.14 Weighted analysis results- AE=4 (Case 3)

Table 4.4 Summary of relative edge betweenness of Generation 01

	Unweighted	AE	2AE	4AE
T1	4.52	7.80	10.28	12.93
T2	8.47	7.21	5.32	8.21
T3	11.49	23.55	15.57	9.16

4.8 Discussion

As it was done for the Bristol approach, results from the Newman method have to analysed referring both the penultimate cluster (i.e. generation 01 in Newman method) and the index for connectivity (relative betweenness). Initially the generation 01 of the structures are taken into consideration. Failure in a web member (internal member) is more preferable to failure in the chord member. One of the reasons to increase the chord axial rigidity is to check whether the failure can be moved from the chord to web member and if it happens to check at which ratio of chord to web member axial rigidity that will happen.

Since Newman's method deals with edges and removes the most inter-cluster edge, it can be used to check the failure location directly. As seen in the Figures 4.11 and 4.12, both unweighted and weighted analysis (case 1) indicate that the failure to happens in the chord member.

Unweighted analysis shows the analysis of the structure's form only without any consideration of the member characteristic. Even if the truss selected in the unweighted analysis is found to be the least connected one in the weighted analysis, we can say that by changing the member characteristics in a strategic manner it can be made to be the most connected one. This is due to the fact that by being the best in the unweighted analysis, the truss's form has more potential to be the most connected form in the weighted analysis. This is the advantage of unweighted Newman analysis and Graph theory analysis.

The relative edge betweenness of the truss T1 is the lowest in the unweighted analysis. Figure 4.11 shows that six edges are removed in this generation. It can be interpreted as these six edges having the same edge betweenness. The trusses T2 & T3 show that the bottom chord middle member is the most in-between member in those trusses. However, the value of relative edge betweenness of T2 is lower than that of T3, indicating that T2's form is more wellconnected than that of T3.

In the weighted analysis, when all members have same axial rigidity, the relative betweenness is the lowest for truss T2 and closely followed by truss T1. The relative betweenness of T3 is very high compared to trusses T1 and T2. In all three trusses, the bottom chord middle members are indicated as the weakly connected members.

The major change between in the unweighted analysis and case 1 of weighted analysis is simply the division of unweighted analysis results by the stiffness of the member. Since the axial rigidity equals unity, change in the results directly corresponds to the length (or multiple of average length) of the member. This is clearly seen in truss T1 results as the longer members out of six members identified in the unweighted analysis are again identified as the weakly connected members in the case 1 of the weighted analysis. This also justifies the sudden increase in edge betweenness of the truss T3 as the identified member is the longest in the truss.

When the chord axial rigidity is doubled, the failure location of the truss T1 is now moved to a web member. In addition to that both trusses T2 and T3 show decrease in the relative edge betweenness from case 1, though the same members are selected in this case as well. Value of the edge betweenness can be observed to have been increased from Case 1 to Case 2 for truss T1, but this cannot be directly compared to the Case 1 results as the failure has moved away from the original location.

When the axial rigidity of the chord members was doubled again, one can see that the failure in T2 also has moved away from the chord to the web member, and as observed with the truss T1 in case 2 the value for the relative edge betweenness is more than what it was in case 1. This can be interpreted as the web member having a higher possibility of failure than the chord member. This was also observed in truss T1 where the value of the relative edge betweenness was increased from case 2 to case 3 for the same members. Though the same member was identified as the most weakly connected member throughout all three cases in the weighted analysis for truss T3, the value for the relative edge betweenness kept reducing. Observed results indicate that increasing the axial rigidity of the chord members proves beneficial for the truss forms irrespective of the configuration.

5 Route Structure Analysis (RSA)

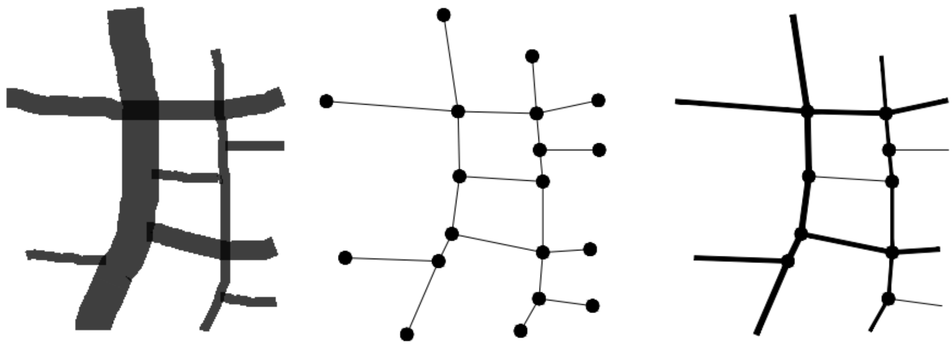
5.1 Introduction

Route structure analysis (RSA) was introduced by S. Marshall (2003) as a new method for characterising the network and route types. This was mainly developed for analysing the road network. This concept was later advanced into the line structure representation for road network analysis; however in this research the original route structure analysis was adopted. This approach sets out to model the structure of road network and analyse it irrespective of the network movement. Section 5.2 introduces the concept of routes and guidelines of route generation. The properties of routes are explained in Section 5.3. Results are given in Section 5.4 and followed by discussion on results in Section 5.5.

5.2 Road network representation- Routes & Joints

In the conventional road network representation, the road segments between junctions are represented by the links and junctions by the nodes. Figure 5.1 shows the difference between the conventional network representation and the RSA.

"The routes are defined as the linear aggregation of links and the joints are defined to have only one conjoined route through them" (Marshall, 2003).



(a) Layout plan (b) Conventional network (c) Route structure

Figure 5.1 Conventional and RSA representation of road network

Various possible route structure representations can be made based on how the links are aggregated into routes. It is important that a set of rules or guidelines are made to systemise this process of aggregating links into routes. The contextual guidance given by Marshall (2003) for route formation is given below,

"1. Where the designated route classification is known, this classification can be used to form routes. Hence, at any junction, a single route may be selected from two links with the same route designation (e.g. 'A' road class).

2. Where the route structure is not resolved by (1), then actual junction priority may be used, where known.

3. Where the route structure is not resolved by (1) or (2), then continuity of physical alignment may be used to select the through route.

4. Other possible means of determining the through route would be continuity of street name, or designation according to which route was constructed first, historically."

It can be observed that with detailed knowledge about the road network, better route structure can be formulated to represent the said road network. Since the routes can be used to represent a continuous member in a structure, this analysis provides a different perspective for structural connectivity.

5.3 Route structure properties

The route structure analysis is based on three main properties, which are measured for each route as well as for the overall network. The properties as defined by Marshall (2003) are presented here,

Continuity: *"The number of links that make up a specific route."*

This will represent how continuous a member is, in a structure. For example, when fabricating a truss joint where two members are crossing, a member is kept continuous and the other one is cut and welded/bolted on either side of the continuous member. This index identifies that main members in a structure.

Connectivity: "*The number of routes a given route connects or comes in contact with."*

This also represents the nodality of the joints a route is passing through. If two routes meet at more than one joint, the connectivity is calculated at each instance.

Depth: *"The property which measures how distant a route is from a particular datum, measured in number of steps of adjacency."*

This represents how many steps need to be taken to reach a specific route from a datum. Figure 5.2 shows the depth of the routes identified in Figure 5.1 as line thickness. It should be noted that the connectivity and other properties are determined for routes (representing members) rather than the joints. The naming convention used in Figure 5.2 follows the branching pattern of the routes. In this naming convention, the length of the name shows the depth of the route. The properties of all routes in Figure 5.2 are listed in Table 5.1.

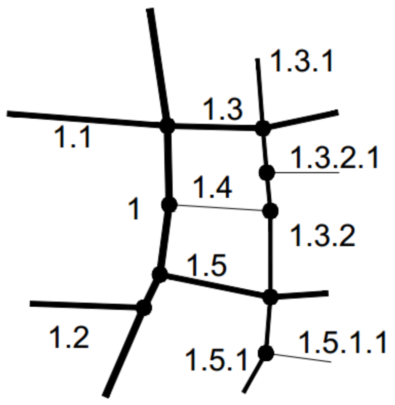


Figure 5.2 Depth of a route structure

Table 5.1 Route structure properties of Fgure 5.1

Route	Continuity	Connectivity	Depth	Route Type
1	5	5	1	a
1.1	1	2	2	b
1.2	1	1	2	c
1.3	2	3	2	d
1.4	1	2	2	e
1.5	2	2	2	f
1.3.1	1	2	3	g
1.3.2	3	6	3	h
1.5.1	2	3	3	i
1.3.2.1	1	1	4	j
1.5.1.1	1	1	4	j
Total	20	28	28	10 types

From Table 5.1 it can be observed that, route no 1 is the most continuous route and route 1.3.2 is the most connective one. Ten type of routes are identified in this network. In order to characterise the routes the following indices are introduced:

- Relative continuity (equation 5.2)
- Relative connectivity (equation 5.3)
- Relative depth (equation 5.3)

For a single route;

$s = continuity + connectivity + depth$ Equation 5.1

$relative\ continuity = continuity/s$ Equation 5.2

$relative\ connectivity = connectivity/s$ Equation 5.3

$relative\ depth = depth/s$ Equation 5.4

For a network;

$S = \sum continuity + \sum connectivity + \sum depth$ Equation 5.5

$relative\ continuity = \sum continuity\ /S$ Equation 5.6

$relative\ connectivity = \sum connectivity\ /S$ Equation 5.7

$relative\ depth = \sum depth\ /S$ Equation 5.8

The relative continuity, connectivity and depth for the route types stated in Table 5.1 is given in Table 5.2.

Table 5.2 Relative properties of routes and network

Route	Relative Continuity	Relative Connectivity	Relative Depth	Route Type
1	0.455	0.455	0.091	a
1.1	0.200	0.400	0.400	b
1.2	0.250	0.250	0.500	c
1.3	0.286	0.429	0.286	d
1.4	0.200	0.400	0.400	e
1.5	0.333	0.333	0.333	f
1.3.1	0.167	0.333	0.500	g
1.3.2	0.250	0.500	0.250	h
1.5.1	0.250	0.375	0.375	i
1.3.2.1	0.167	0.167	0.667	j
1.5.1.1	0.167	0.167	0.667	j
Network	0.263	0.368	0.368	

From Table 5.2, route id 1.3.2 can be identified as the most connected route. However, for the analysis, it was decided to check the overall network connectivity rather than individual route connectivity. Though our analysis is mainly concerned with the relative connectivity, relative depth also can be analysed in a structural context. If relative depth is low, most of the routes are accessible with fewer steps from the datum. This reduces likelihood of inter-cluster links and increases number of members available for load resisting.

The same procedure can be applied for the selected trusses. The results of RSA of the selected trusses are given in the Section 5.4.

Figure 5.3 shows the routes identified in truss T1. The members' continuity is considered in selecting the routes. For example, the chord members are taken as three separate routes. The bottom chord member is taken as the datum. This can be justified as bottom chord resembles a beam, which would be the most basic way to span between two points.

5.4 Results

The identified routes in the trusses T1, T2 & T3 are given in Figures 5.3, 5.4 & 5.5 respectively. The absolute and relative properties of the identified routes are given in Tables 5.3 through 5.8. Table 5.9 shows a summary of results.

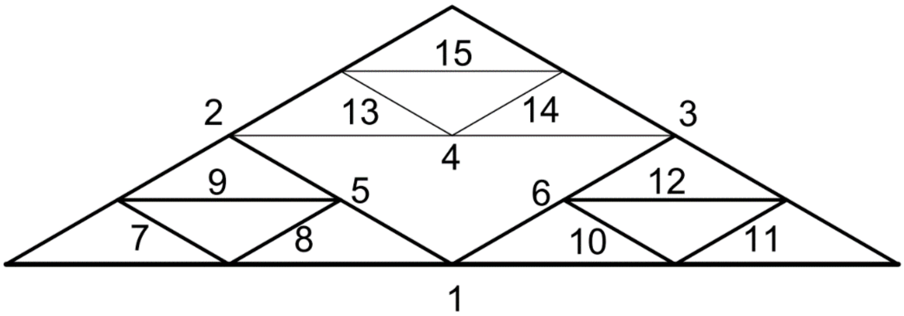


Figure 5.3 Routes of truss T1

Table 5.3 Properties of T1 truss

Route No	Route ID	Continuity	Connectivity	Depth
1	1	4	8	1
2	1.1	4	8	2
3	1.2	4	8	2
4	1.1.1	2	6	3
5	1.3	2	6	2
6	1.4	2	6	2
7	1.5	1	4	2
8	1.6	1	4	2
9	1.1.2	1	4	3
10	1.7	1	4	2
11	1.8	1	4	2
12	1.2.1	1	4	3
13	1.1.3	1	4	3
14	1.2.2	1	4	3
15	1.1.4	1	4	3
Total		27	78	35

Table 5.4 Relative properties of T1 routes

Route No	Relative Continuity	Relative Connectivity	Relative Depth	Route Type
1	0.308	0.615	0.077	a
2	0.286	0.571	0.143	b
3	0.286	0.571	0.143	b
4	0.182	0.545	0.273	c
5	0.200	0.600	0.200	d
6	0.200	0.600	0.200	d
7	0.143	0.571	0.286	e
8	0.143	0.571	0.286	e
9	0.125	0.500	0.375	f
10	0.143	0.571	0.286	e
11	0.143	0.571	0.286	e
12	0.125	0.500	0.375	f
13	0.125	0.500	0.375	f
14	0.125	0.500	0.375	f
15	0.125	0.500	0.375	f
Network	0.193	0.557	0.250	

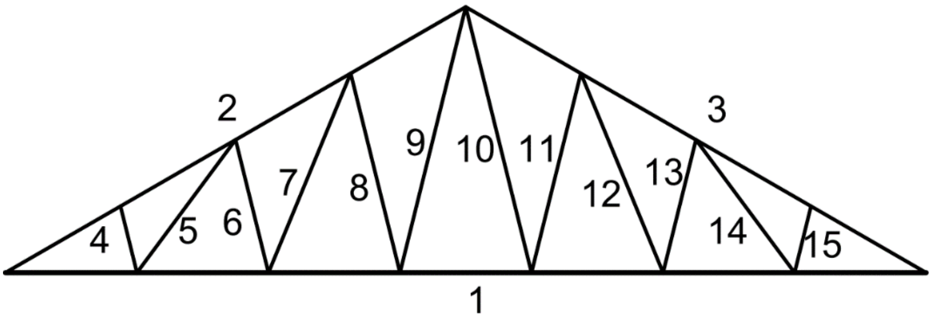


Figure 5.4 Routes of truss T2

Table 5.5 Properties of T2 truss

Route No	Route ID	Continuity	Connectivity	Depth
1	1	7	14	1
2	1.1	4	9	2
3	1.2	4	9	2
4	1.3	1	3	2
5	1.4	1	4	2
6	1.5	1	4	2
7	1.6	1	4	2
8	1.7	1	4	2
9	1.8	1	5	2
10	1.9	1	5	2
11	1.10	1	4	2
12	1.11	1	4	2
13	1.12	1	4	2
14	1.13	1	4	2
15	1.14	1	3	2
Total		27	80	29

Table 5.6 Relative properties of T2 routes

Route No	Relative Continuity	Relative Connectivity	Relative Depth	Route Type
1	0.318	0.636	0.045	a
2	0.267	0.600	0.133	b
3	0.267	0.600	0.133	b
4	0.167	0.500	0.333	c
5	0.143	0.571	0.286	d
6	0.143	0.571	0.286	d
7	0.143	0.571	0.286	d
8	0.143	0.571	0.286	d
9	0.125	0.625	0.250	e
10	0.125	0.625	0.250	e
11	0.143	0.571	0.286	d
12	0.143	0.571	0.286	d
13	0.143	0.571	0.286	d
14	0.143	0.571	0.286	d
15	0.167	0.500	0.333	c
Network	0.199	0.588	0.213	

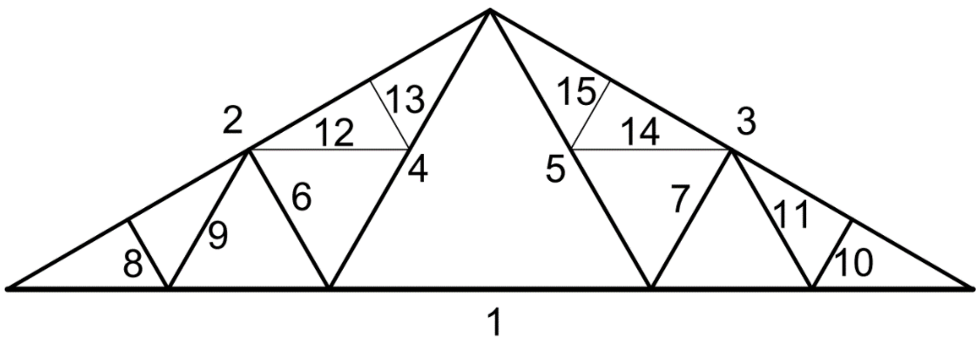


Figure 5.5 Routes of truss T3

Table 5.7 Properties of T3 truss

Route No	Route ID	Continuity	Connectivity	Depth
1	1	5	10	1
2	1.1	4	9	2
3	1.2	4	9	2
4	1.3	2	7	2
5	1.4	2	7	2
6	1.5	1	5	2
7	1.6	1	5	2
8	1.7	1	3	2
9	1.8	1	5	2
10	1.9	1	3	2
11	1.10	1	5	2
12	1.1.1	1	5	3
13	1.1.2	1	3	3
14	1.2.1	1	5	3
15	1.2.2	1	3	3
Total		27	84	33

Table 5.8 Relative properties of T3 routes

Route No	Relative Continuity	Relative Connectivity	Relative Depth	Route Type
1	0.313	0.625	0.063	a
2	0.267	0.600	0.133	b
3	0.267	0.600	0.133	b
4	0.182	0.636	0.182	c
5	0.182	0.636	0.182	c
6	0.125	0.625	0.250	d
7	0.125	0.625	0.250	d
8	0.167	0.500	0.333	e
9	0.125	0.625	0.250	d
10	0.167	0.500	0.333	e
11	0.125	0.625	0.250	d
12	0.111	0.556	0.333	f
13	0.143	0.429	0.429	g
14	0.111	0.556	0.333	f
15	0.143	0.429	0.429	g
Network	0.188	0.583	0.229	

Table 5.9 Summary of results

	Relative Connectivity	Relative Depth
T1	0.557	0.250
T2	0.588	0.213
T3	0.583	0.229

5.5 Discussion

The relative connectivity of the overall structure is used to analyse the connectivity of the structures. Since, the links are accumulated into routes, the failure location or weak joint cannot be identified using this approach. The structure is analysed without using any of the member or joint structural characteristics. This is similar to Graph theory or the unweighted Newman's method. In this approach, it is made possible to account for continuous members. The concept through-route reflects the real world practice in connection between the members.

The relative depth shows how close the members are to the datum. As discussed in Section 5.3, having a lower depth would imply the reduced possibility of inter-cluster links. Thus a structure having lower relative depth value is expected to have a good structural form.

The number of routes in all three trusses are the same and the total absolute continuity is equal between all three trusses as they have the same number of members in their configurations. Truss T1 is a fractal form, in which a form replicates itself in reducing scale. Due to this, the finer fractal forms (triangles, in this case) are not in connection with each other and only connected with the fractal of larger scale. This has reduced the connectivity of routes comprised of links making the finer fractals.

In truss T2, all web members are connected to the chord members directly, increasing the connectivity count. Truss T3 also shows similar behaviour. It can be observed from Table 5.9, that the value for relative connectivity is high for truss T2, followed closely by truss T3. This is also reflected in the values for the relative depth i.e. the relative depth is seen to be inversely proportional to relative connectivity. From this it can be concluded that as per route structure analysis, the truss T2 is chosen as the most connected truss form and truss T3 is considered a close second. It should be noted however, that the above indices are rather weak in discriminatory power, since the values for the three trusses (both relative connectivity and relative depth) are fairly close to each other.

6 Analysis of Frame

6.1 Introduction

In this chapter, the selected frame (Figure 6.1) is analysed in order to measure the structural connectivity using the methods introduced in chapters 03 to 05, namely Bristol approach, Newman method and RSA. The methodology of adapting the said methods for application to frames is discussed in Section 6.2. In addition to analysing the intact frame, possible failure states of the said frame were also analysed using the same methods to determine the loss in structural connectivity in those failure states. These failure states relate to specific column removals as shown in Figure 6.2. The results of the analysis are given in Section 6.3. The Section 6.4 gives the discussion of the results.

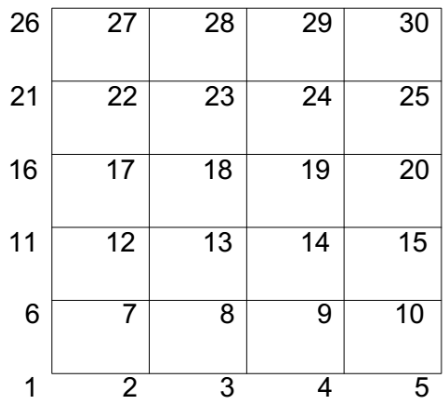
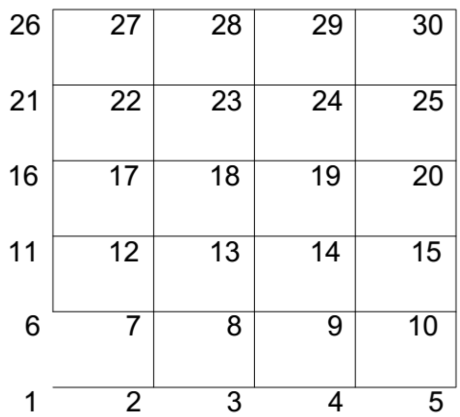
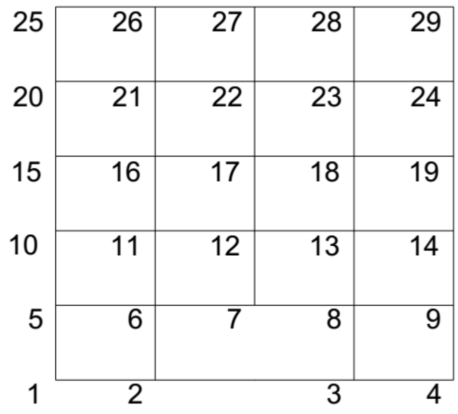


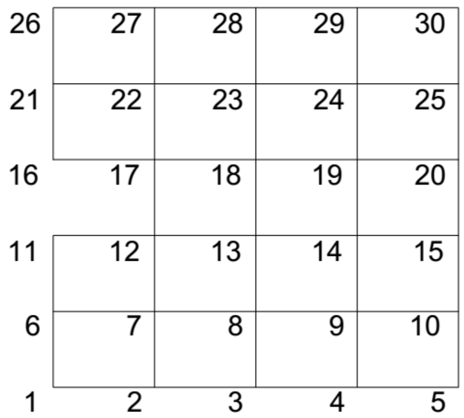
Figure 6.1 Selected frame



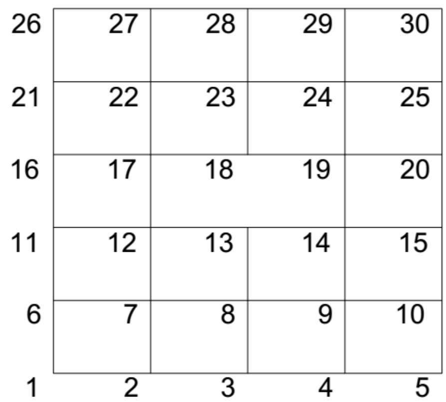
(a) Ground floor side column removal (GSR)



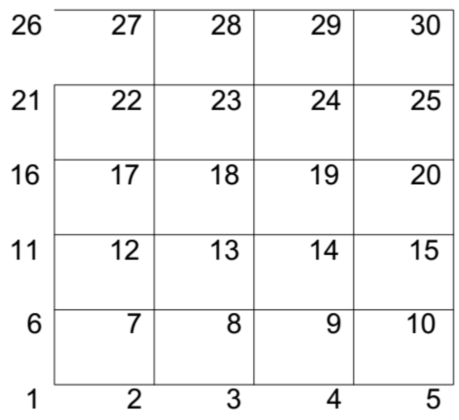
(b) Ground floor middle column removal (GMR)



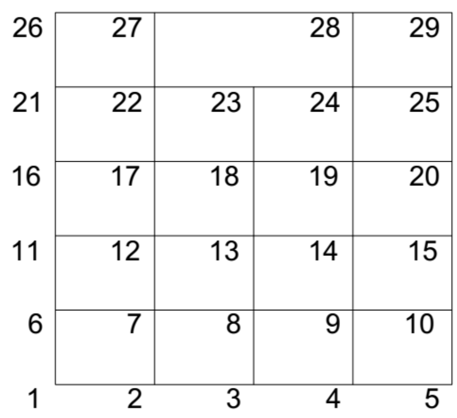
(c) Middle floor side column removal (MSR)



(d) Middle floor middle column removal (MMR)



(e) Top floor side column removal (TSR)



(f) Middle floor middle column removal (TMR)

Figure 6.2 Failure scenarios considered in frame

6.2 Methodology

The methodology of applying the Bristol approach for frames is given in Section 6.2.1 and Section 6.2.2 explains the steps applied in adopting the Newman method. The Section 6.2.3 defines the same for RSA.

6.2.1 Bristol approach

Following the same procedure given in Section 3.2.1, the joint stiffness for a frame joint can be derived. Here the each node will have three degrees of freedom since a frame element undergoes axial deformation as well as bending. Each degree of freedom will correspond to axial force, shear force and moment. It should be noted that the forces and displacements in the local axial direction are independent of the other two degrees of freedom. Stiffness matrix (in local coordinates) for a frame element is given equation 6.1. The ends of the elements are taken as fixed. The unit displacements and forces along the local axis are marked with "u_i" and "q_i" respectively.

$$\begin{Bmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \end{Bmatrix} = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{Bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & 0 & 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & 0 \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} * \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix}$$ **equation 6.1**

Transformation matrix [T] (this is the inverse of matrix [C] discussed in Chapter 03) can be formulated from equations 6.2, 6.3 & 6.4. This transformation matrix states the geometric relationship between the local displacements u_i and global displacements v_i, as shown in equation 6.5.

$u_1 = v_1 \cos\theta + v_2 \sin\theta$ **equation 6.2**

$u_2 = -v_1 \sin\theta + v_2 \cos\theta$ **equation 6.3**

$u_3 = v_3$ **equation 6.4**

$$T = \begin{bmatrix} \cos\theta & \sin\theta & 0 & & & \\ -\sin\theta & \cos\theta & 0 & & 0 & \\ 0 & 0 & 1 & & & \\ & & & \cos\theta & \sin\theta & 0 \\ & 0 & & -\sin\theta & \cos\theta & 0 \\ & & & 0 & 0 & 1 \end{bmatrix}$$ **equation 6.5**

The equations 6.6, 6.7 & 6.8 show the relationship between the local forces q_i and global forces f_i. From the equations it can be shown that the same transformation matrix is used to translate the local displacements to global displacements and local forces to global forces, as shown by equations 6.9 and 6.10.

$q_1 = f_1 \cos\theta + f_2 \sin\theta$ **equation 6.6**

$q_2 = -f_1 \sin\theta + f_2 \cos\theta$ **equation 6.7**

$q_3 = f_3$ **equation 6.8**

$u = T * v$ **equation 6.9**

$q = T * f$ **equation 6.10**

As per Hook's law, the relation between the forces and displacements are given in equations 6.11(same as equation 6.1) and 6.12 in terms of local and global coordinates respectively.

$q = k * u$ **equation 6.11**

$f = K * v$ **equation 6.12**

Combining the equations 6.9, 6.10 & 6.11, the equation 6.13 can be derived.

$f = T^T * k * T * v$ **equation 6.13**

Combining the equations 6.12 and 6.13, the equation 6.14 can be derived. This equation states the relationship between the local and global stiffness matrixes. The global stiffness matrix can be derived as shown in equation 6.15.

$K = T^T * k * T$ **equation 6.14**

$$K = \begin{bmatrix} \frac{AE}{L}c^2 + \frac{12EI}{L^3}s^2 & c.s(\frac{AE}{L} - \frac{12EI}{L^3}) & -\frac{6EI}{L^2}s & -\frac{AE}{L}c^2 - \frac{12EI}{L^3}s^2 & -c.s(\frac{AE}{L} - \frac{12EI}{L^3}) & -\frac{6EI}{L^2}s \\ c.s(\frac{AE}{L} - \frac{12EI}{L^3}) & \frac{AE}{L}s^2 + \frac{12EI}{L^3}c^2 & \frac{6EI}{L^2}c & -c.s(\frac{AE}{L} - \frac{12EI}{L^3}) & -\frac{AE}{L}s^2 - \frac{12EI}{L^3}c^2 & \frac{6EI}{L^2}c \\ -\frac{6EI}{L^2}s & \frac{6EI}{L^2}c & \frac{4EI}{L} & \frac{6EI}{L^2}s & -\frac{6EI}{L^2}c & \frac{2EI}{L} \\ -\frac{AE}{L}c^2 - \frac{12EI}{L^3}s^2 & -c.s(\frac{AE}{L} - \frac{12EI}{L^3}) & \frac{6EI}{L^2}s & \frac{AE}{L}c^2 + \frac{12EI}{L^3}s^2 & c.s(\frac{AE}{L} - \frac{12EI}{L^3}) & \frac{6EI}{L^2}s \\ -c.s(\frac{AE}{L} - \frac{12EI}{L^3}) & -\frac{AE}{L}s^2 - \frac{12EI}{L^3}c^2 & -\frac{6EI}{L^2}c & c.s(\frac{AE}{L} - \frac{12EI}{L^3}) & \frac{AE}{L}s^2 + \frac{12EI}{L^3}c^2 & -\frac{6EI}{L^2}c \\ -\frac{6EI}{L^2}s & \frac{6EI}{L^2}c & \frac{2EI}{L} & \frac{6EI}{L^2}s & -\frac{6EI}{L^2}c & \frac{4EI}{L} \end{bmatrix}$$

equation 6.15

For the structure shown in Figure 6.3, the stiffness matrix for the members 1-2 (a) and 1-3 (b) can be simplified as shown in equations 6.16 and 6.17 respectively.

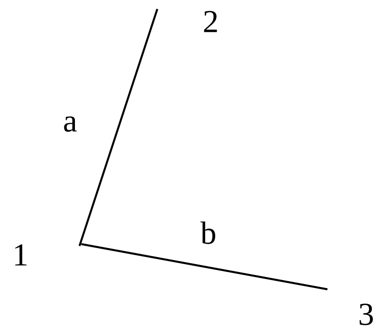


Figure 6.3 Simple frame joint

$K_a = \begin{bmatrix} k_{11}^a & k_{12}^a \\ k_{21}^a & k_{22}^a \end{bmatrix}$ **equation 6.16**

$K_b = \begin{bmatrix} k_{11}^b & k_{13}^b \\ k_{31}^b & k_{33}^b \end{bmatrix}$ **equation 6.17**

At the joint 1, the addition of the matrices is carried out as shown in equation 6.18 to derive the joint stiffness matrix.

$$K_1 = \begin{bmatrix} k_{11}^a + k_{11}^b & k_{12}^a & k_{13}^b \\ k_{21}^a & k_{22}^a & 0 \\ k_{31}^b & 0 & k_{33}^b \end{bmatrix}$$ **equation 6.18**

The basic unit of clustering, the structural ring, is chosen as a loop containing four frame joints; this follows the approach given by Wu (1991). Following this step, the rest of the procedure is identical to the one used for the trusses.

The clustering of the intact frame is shown in Figure 6.4 to Figure 6.17 in order to explain the steps of clustering. Figure 6.4 shows the formation of structural rings. The selection of leaf clusters and initiation of structural clustering stage I is shown in Figure 6.5. Here cluster numbers 6 and 10 are combined to make cluster number (C.N) 21. Though there is a possibility to join C.N 6 and C.N 7, this was omitted to account for the symmetry of the structure. Clustering stage 1 is continued up to Figure 6.15; following that structural clustering stage II is initiated and the clustering is completed in Figure 6.17. In Figures 6.9 and 6.13, new clusters are initiated as any addition to the existing clusters will reduce its wellformedness.

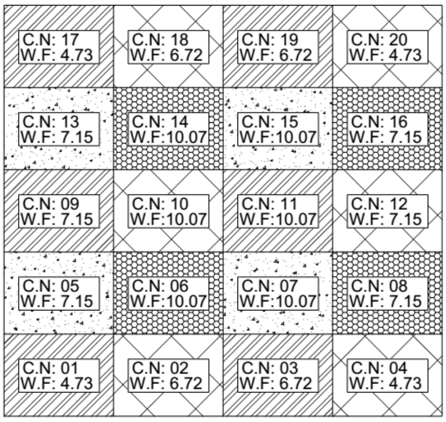


Figure 6.4 Formation of structural rings

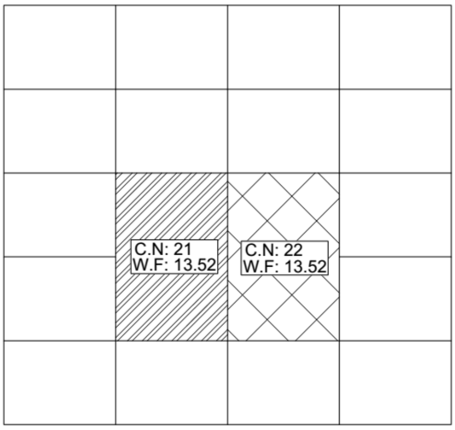


Figure 6.5 Structural clustering stage 1- Step 1

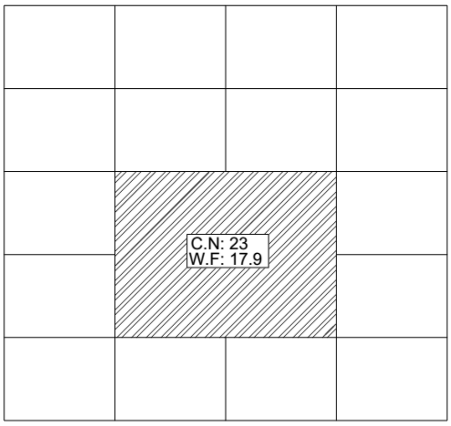


Figure 6.6 Structural clustering stage 1- Step 2

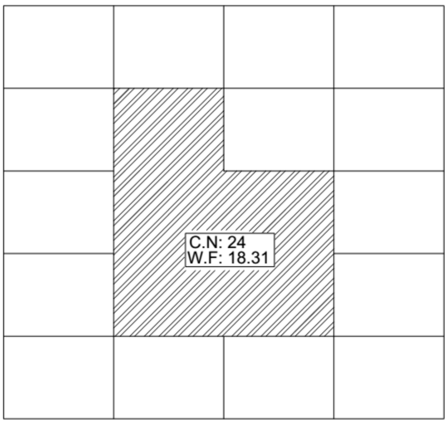


Figure 6.7Structural clustering stage 1- Step 3a

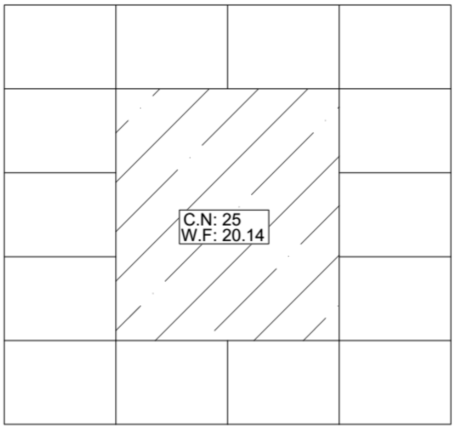


Figure 6.8 Structural clustering stage 1- Step 3b

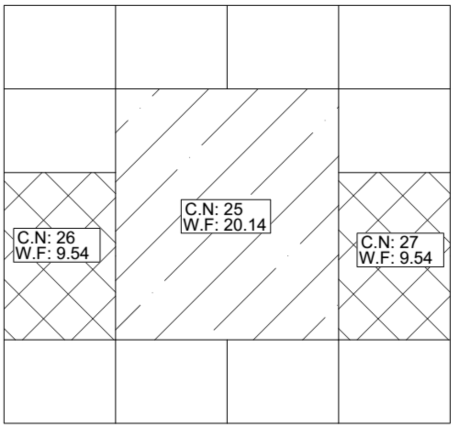


Figure 6.9 Structural clustering stage 1- Step 4

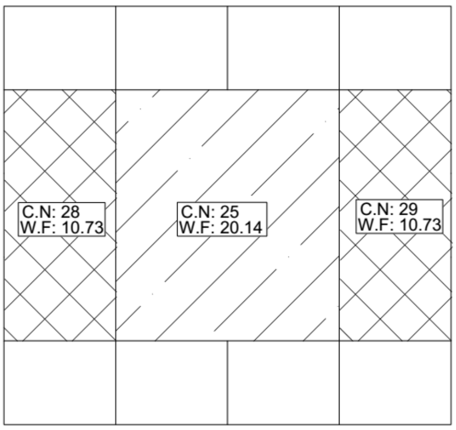


Figure 6.10 Structural clustering stage 1- Step 5

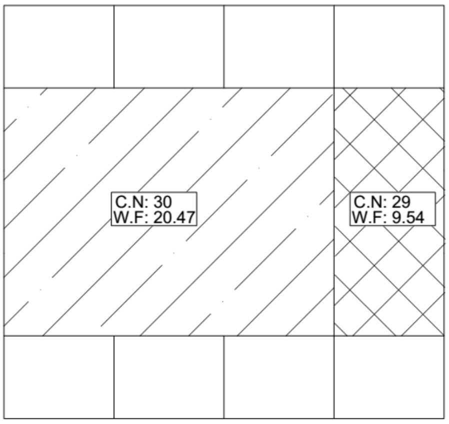


Figure 6.11 Structural clustering stage 1- Step 6a

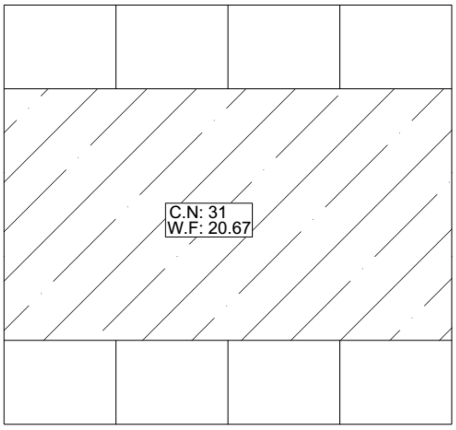


Figure 6.12 Structural clustering stage 1- Step 6b

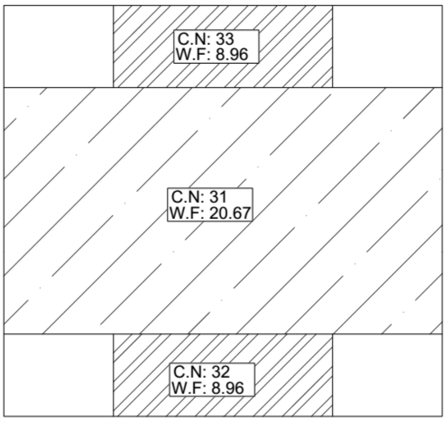


Figure 6.13 Structural clustering stage 1- Step 7

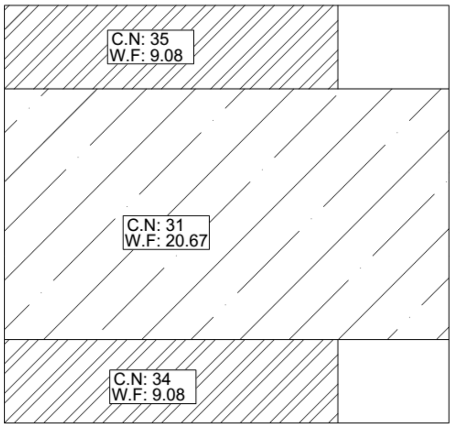


Figure 6.14 Structural clustering stage 1- Step 8

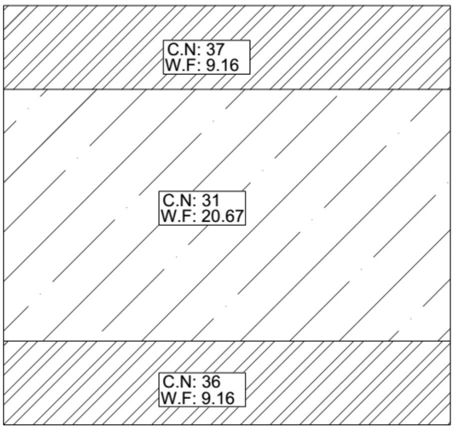


Figure 6.15 Structural clustering stage 1- Step 9

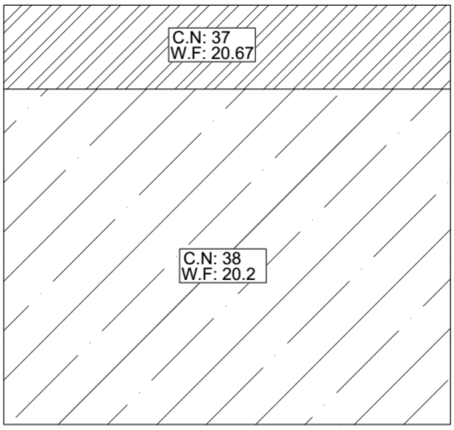


Figure 6.16 Structural clustering stage 2- Step 10

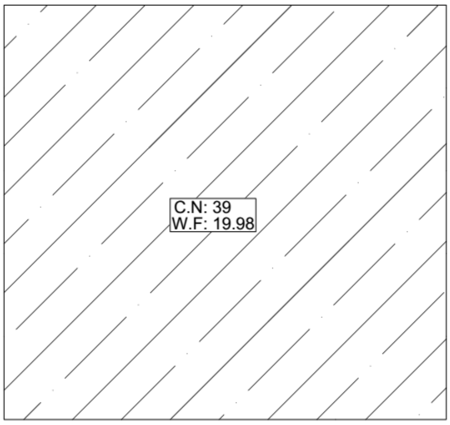


Figure 6.17 Structural clustering stage 2- Step 11

The change in wellformedness due to certain member removals are calculated using the index relative separateness as shown in equation 3.18.

6.2.2 Newman's method

No additional steps needs to be adopted to implement the Newman method to the frames for unweighted analysis. But for the weighted analysis, the edge weights are in the form of a matrix, whose determinants are zero. Due to this, the edge weights cannot be directly applied to the analysis as it was done for the trusses. To solve this, an approach was adopted to create equivalent edge weights based on the node weights.

The strength of a node in a weighted network is calculated as shown in equation 6.19. This shows that the strength of the node is an accumulation of all edge weights of the edges connecting to the node.

$s_i = \sum_{j=1}^{N} a_{ij} w_j$ **equation 6.19**

a_{ij}= component of the adjacency matrix connecting node *i* and node *j*

w_j= edge weight of the link connecting node *i* and node *j*

s_i = strength of the node

N = number of links connected to node *i*

This process can be reversed to find the equivalent edge weights of edges based on nodal weights. In order to determine the equivalent edge weights for the network shown in Figure 6.18, the nodal weights of nodes 1 to 4 should be determined first. The values of nodal weights will either be available or can be derived from true edge weights. The joint stiffness that was determined in the Bristol approach is an example of a derived nodal weight. Then equivalent edge weight for member "a" with respect to node 4 can be determined as shown in equation 6.20. This shows that the contribution to equivalent edge weight of a member from a node is equal to weight of that node multiplied by a fraction of weight of the opposite node of the considered link to sum of nodal weights of all nodes connected the node considered.

This process is applied to all the nodes. The equivalent edge weight is calculated by adding the contribution by both nodes of a member, as shown in equation 6.21.

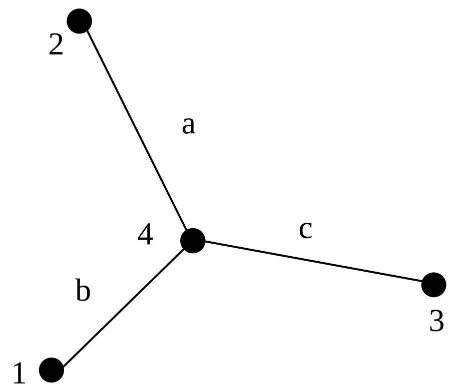


Figure 6.18 Sample network

$$w_{ij-i} = \frac{s_i \times s_j}{\sum_{k=1}^{N} a_{ik} s_k}$$ **equation 6.20**

w_{ij-i}= The contribution form node *i* to the equivalent edge weight of member *i-j*

$$w_{ij} = w_{ji} = w_{ij-i} + w_{ji-j}$$ **equation 6.21**

w_{ji-j}= The contribution form node *j* to the equivalent edge weight of member *i-j*

w_{ij}= The equivalent edge weight of member *i-j*

After calculating the equivalent edge weights, the process of analysis is the same as that used for weighted Newman's method for trusses.

6.2.3 Route structure analysis

The application of RSA to the frames is very similar to the application to the trusses. The determination of through routes needs special consideration. Normally the columns can be determined as more through than the beams, as they carry load from beams of all floors. But at the ground level, in the case of a raft foundation, the raft slab will be more continuous than the columns above it. In the meantime, in the case of pile foundations, the columns can be taken to be more continuous than the tie-beams at ground level.

When a floor column is removed, the columns directly above them will become same level of thoroughness or less than the beam connecting them to the adjacent column, as they now have to transfer their loads to that beam. For corner column removals, the columns above the removed column and the beam connecting them to the adjacent column can be modelled as on route. For the middle column removals, the beams above the removed column is considered to make a route and to have higher thoroughness than the columns above them.

6.3 Results

The results for the analysis carried out using the methodologies discussed in the Section 6.2, is given here. The results of the Bristol approach are presented in Section 6.3.1, followed by results of Newman's method in 6.3.2. The results of RSA are given in Section 6.3.3.

6.3.1 Results-Bristol approach

The penultimate clusters of the intact frame and the considered column removal cases are given in Figures 6.19 to 6.25. Table 6.1 shows the summary of wellformedness and the relative separateness for the cases considered.

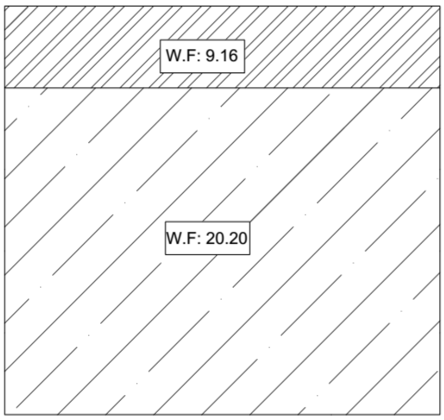


Figure 6.19 Penultimate cluster of intact frame

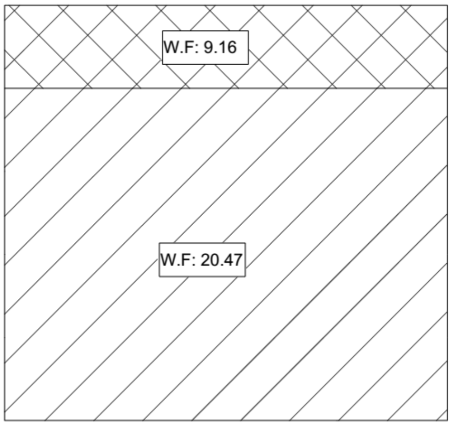


Figure 6.20 Penultimate cluster of case GSR

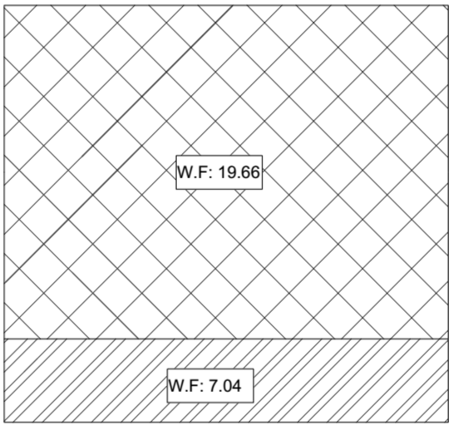


Figure 6.21 Penultimate cluster of case GMR

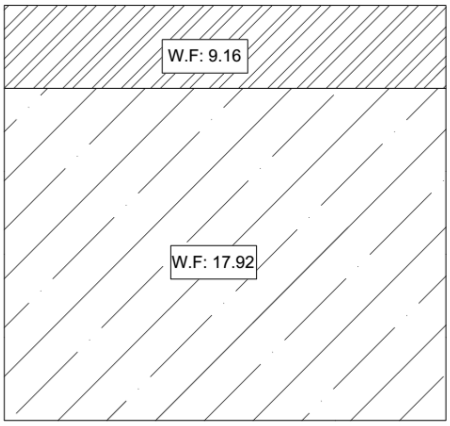


Figure 6.22 Penultimate cluster of case MSR

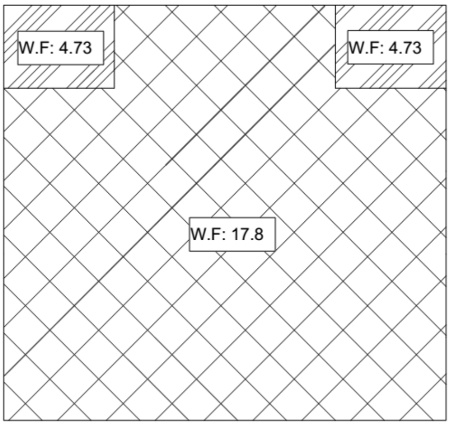


Figure 6.23 Penultimate cluster of case MMR

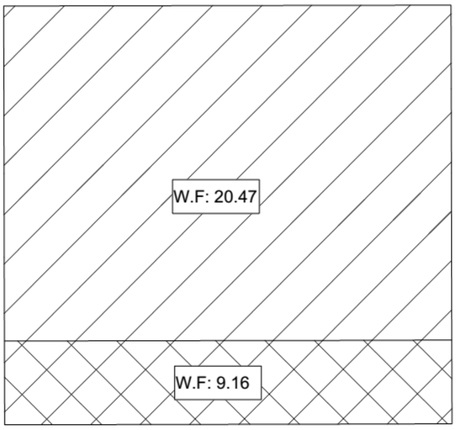


Figure 6.24 Penultimate cluster of case TSR

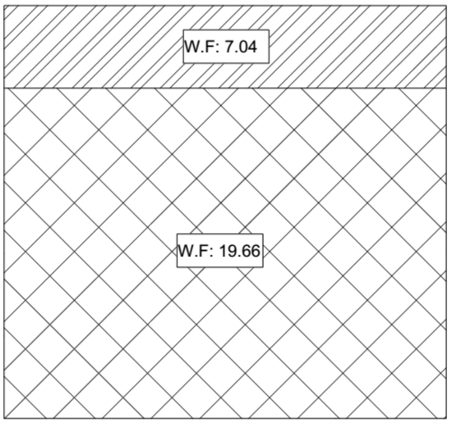


Figure 6.25 Penultimate cluster of case TMR

Table 6.1 Summary of Wellformedness and separateness

Frame condition	Wellformedness at the end of clustering stage-1.	Final Wellformedness	Relative Separateness
Intact	3.90E+27	1.99E+27	
GSR		2.03E+27	48.00
GMR		1.91E+27	50.92
MSR		1.95E+27	50.02
MMR		1.79E+27	54.15
TSR		2.03E+27	48.00
TMR		1.91E+27	50.94

6.3.2 Results of Newman's method

The generation 01 of the unweighted analysis are given in Figures 6.26 to 6.32. Table 6.2 gives a summary of relative edge betweenness for the unweighted Newman's method. Similarly the weighted analysis results are given in Figures 6.33 to 6.39. The summary of the weighted analysis is given in Table 6.3.

Results of Unweighted analysis

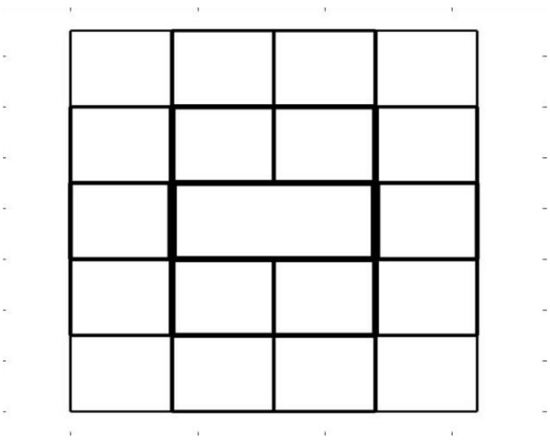


Figure 6.26 Generation 01-Intact frame

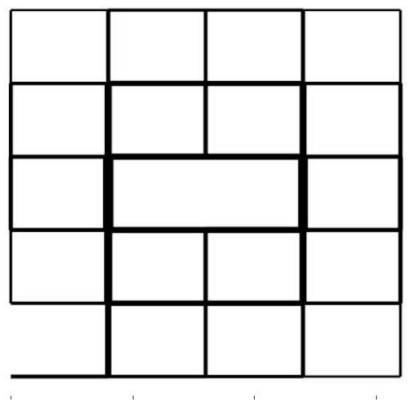


Figure 6.27 Generation 01-Case GSR

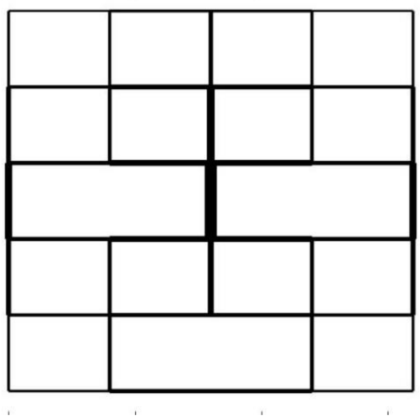


Figure 6.28 Generation 01-Case GMR

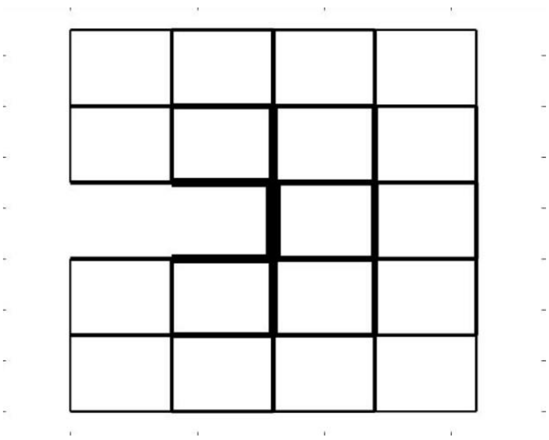


Figure 6.29 Generation 01-Case MSR

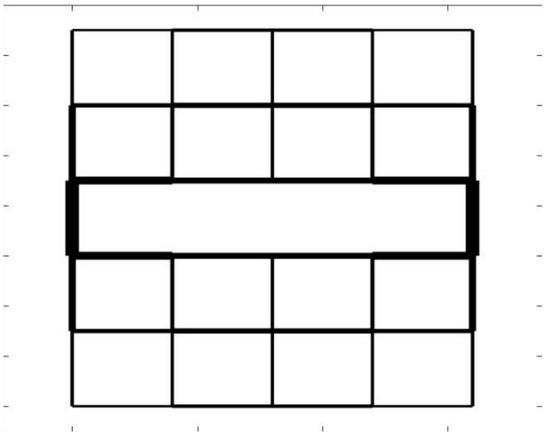


Figure 6.30 Generation 01-Case MMR

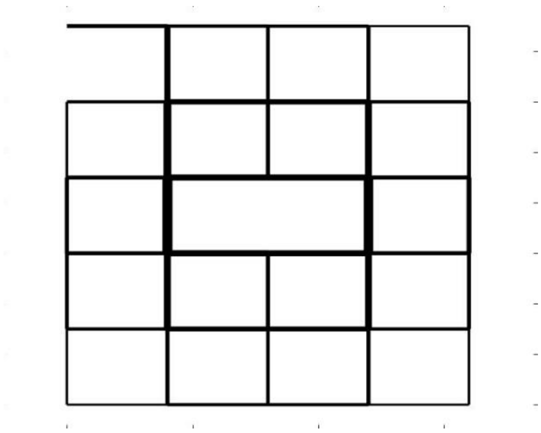


Figure 6.31 Generation 01-Case TSR

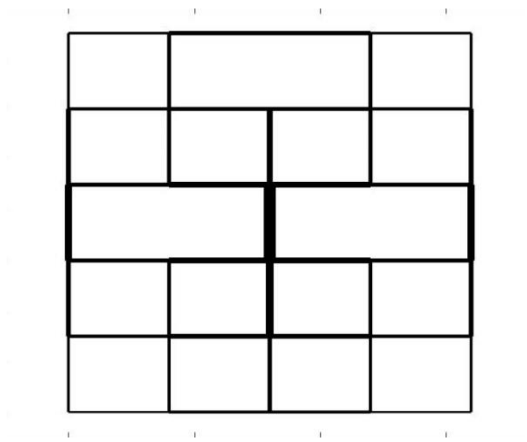


Figure 6.32 Generation 01-Case TMR

Table 6.2 Summary of unweighted analysis results

Frame Condition	Relative edge betweenness (x100)
Intact	3.367
GSR	3.385
GMR	3.366
MSR	4.075
MMR	4.071
TSR	3.385
TMR	3.366

<u>Results of Weighted analysis</u>

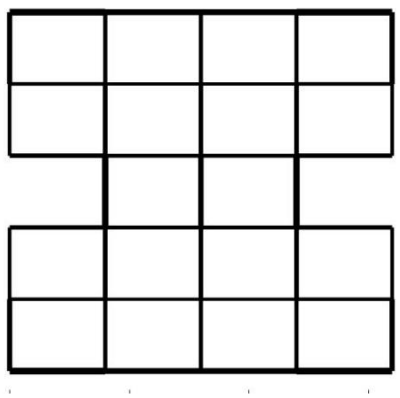


Figure 6.33 Generation 01-Intact frame

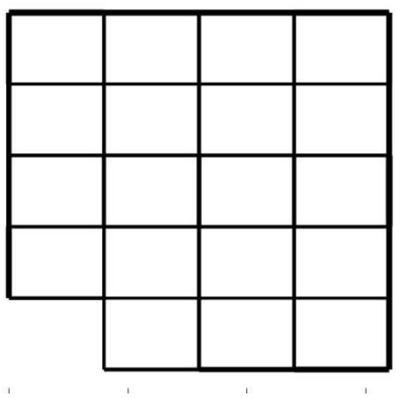


Figure 6.34 Generation 01-Case GSR

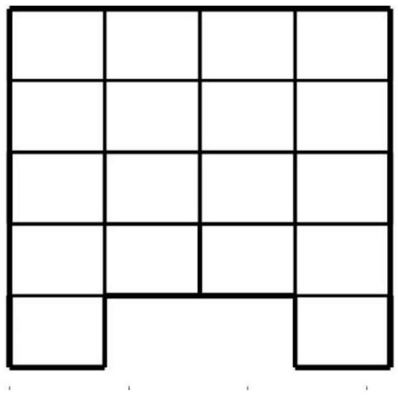


Figure 6.35 Generation 01-Case GMR

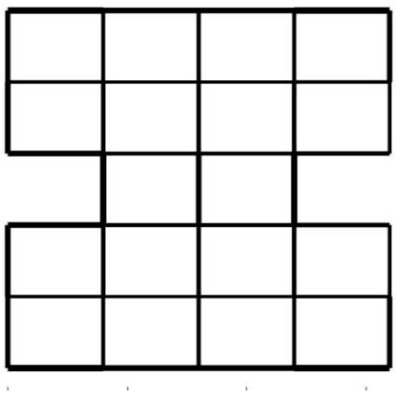


Figure 6.36 Generation 01-Case MSR

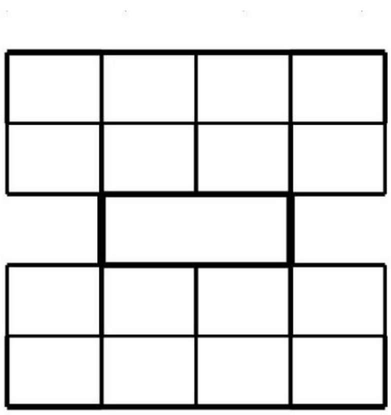


Figure 6.37 Generation 01-Case MMR

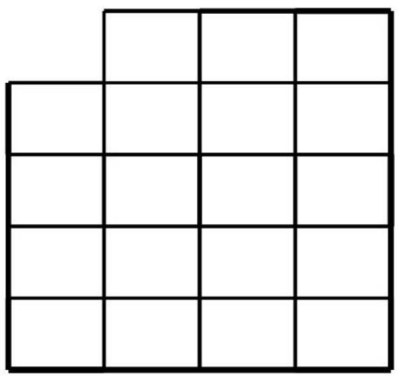


Figure 6.38 Generation 01-Case TSR

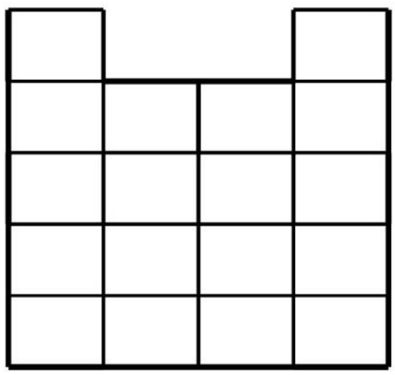


Figure 6.39 Generation 01-Case TMR

Table 6.3 Summary of weighted analysis results

Frame Condition	Relative edge betweenness (x100)
Intact	3.045
GSR	7.619
GMR	8.354
MSR	2.994
MMR	3.265
TSR	7.619
TMR	8.354

6.3.3 Results of RSA

The Figures 6.40 to 6.46 show the identified routes for the intact frame as well as the column removal scenarios. The depth of the routes are shown as route thickness. Tables 6.4 to 6.11 show the properties of route types as well as the relative properties of network. The summary of relative connectivity of all the frame cases is given in Table 6.12.

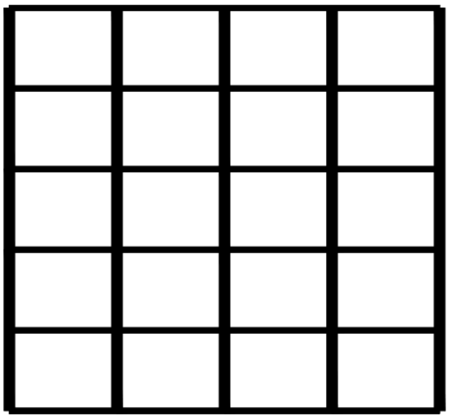


Figure 6.40 Identified routes in intact frame

Table 6.3 Route properties- Frame intact

Type	Continuity	Connectivity	Depth	Nos
1	5	6	1	2
2	5	12	1	3
3	1	3	2	12
4	1	4	2	12
Total	49	132	53	
Relative	0.209	0.564	0.226	

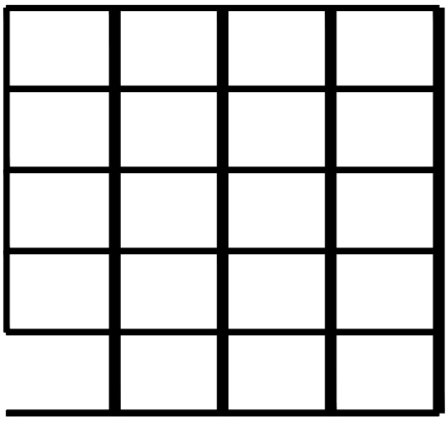


Figure 6.41 Identified routes in case GSR

Table 6.4 Route properties- Case GSR

Type	Continuity	Connectivity	Depth	Nos
1	5	11	1	1
2	5	12	1	2
3	5	6	1	1
4	1	3	2	11
5	5	6	2	1
6	1	4	2	11
7	1	2	2	1
Total	48	126	52	
Relative	0.211	0.561	0.228	

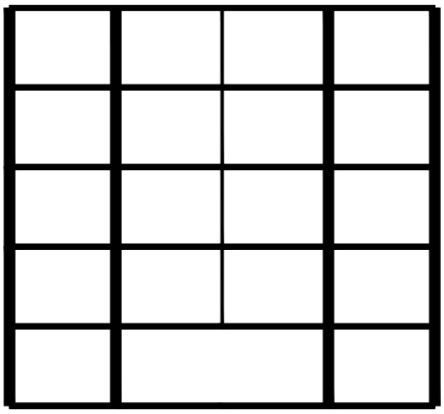


Figure 6.42 Identified routes in case GMR

Table 6.5 Route properties- Case GMR

Type	Continuity	Connectivity	Depth	Nos
1	5	6	1	2
2	5	12	1	2
3	4	9	3	1
4	1	3	2	12
5	1	4	2	8
6	2	5	2	1
7	2	4	2	1
Total	48	124	51	
Relative	0.217	0.552	0.231	

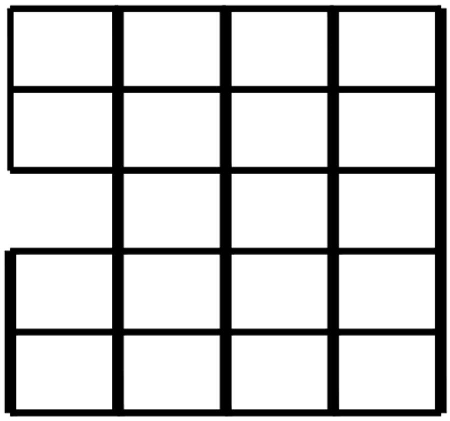


Figure 6.43 Identified routes in case MSR

Table 6.6 Route properties- Case MSR

Type	Continuity	Connectivity	Depth	Nos
1	1	2	3	1
2	5	12	1	3
3	5	6	1	1
4	3	4	2	1
5	1	3	2	11
6	1	4	2	12
Total	48	130	53	
Relative	0.208	0.563	0.229	

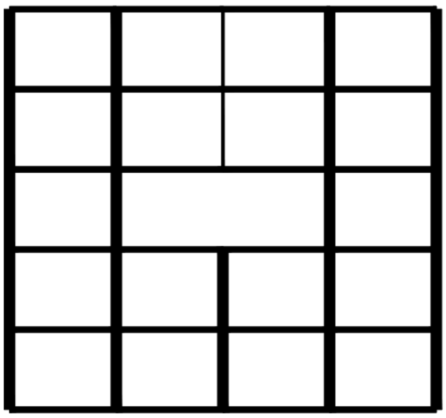


Figure 6.44 Identified routes in case MMR

Table 6.7 Route properties- Case MMR

Type	Continuity	Connectivity	Depth	Nos
1	5	6	1	2
2	5	12	1	2
3	2	5	3	1
4	2	6	1	1
5	1	3	2	12
6	1	4	2	10
7	2	5	2	1
Total	48	128	54	
Relative	0.209	0.557	0.235	

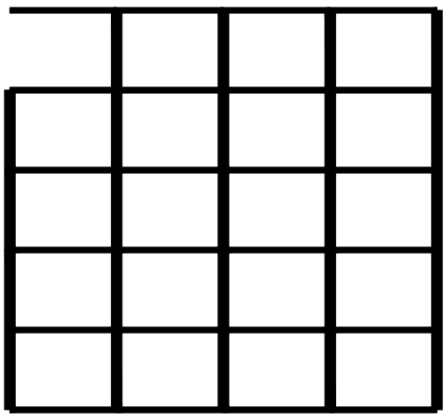


Figure 6.45 Identified routes in case TSR

Table 6.8 Route properties- Case TSR

Type	Continuity	Connectivity	Depth	Nos
1	5	6	1	2
2	5	12	1	3
3	4	5	1	1
4	1	3	2	11
5	1	4	2	12
6	1	2	2	1
Total	48	130	53	
Relative	0.208	0.563	0.229	

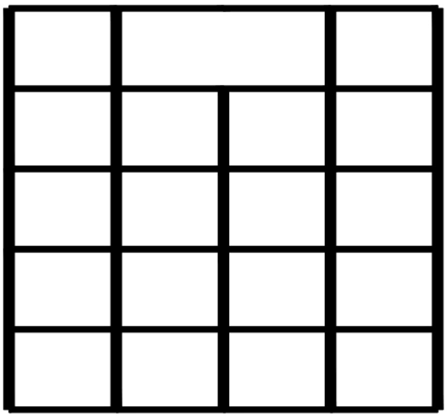


Figure 6.46 Identified routes in case TMR

Table 6.9 Route properties- Case TMR

Type	Continuity	Connectivity	Depth	Nos
1	4	10	1	1
2	2	4	2	1
3	5	12	1	2
4	5	6	1	2
5	1	3	2	12
6	1	4	2	10
Total	48	126	51	
Relative	0.213	0.56	0.227	

Table 6.10 Summary of Relative connectivity and relative depth

Frame Id	Relative Connectivity	Relative Depth	% reduction in relative connectivity	% increase in relative depth
Intact	0.5641	0.2265		
GSR	0.5614	0.2281	0.48	0.70
GMR	0.5520	0.2308	2.14	1.89
MSR	0.5628	0.2294	0.24	1.30
MMR	0.5565	0.2348	1.34	3.66
TSR	0.5268	0.2294	0.24	1.30
TMR	0.5600	0.2267	0.73	0.08

6.4 Discussion

It can be observed that in the Bristol approach, the top or bottom part of the frame is the last cluster to be clustered into the main cluster. This implies that middle portion of the frame is more tightly connected than the top or bottom. Even though the loss of middle floor column causes a change in the way of clustering, the top portion still remains the last to be clustered. This can be due to the fact that most of the joints with higher joint stiffness remain in the middle portion of the frame.

Table 6.1 shows that the middle floor middle column removal causes the highest relative separateness-i.e. highest loss in structural connectivity. It can also be observed that the middle column removals cause higher loss in structural connectivity than the side column removal in any floor. The wellformedness results also reflect the symmetry in column removal. The wellformedness of case TSR and GSR are same even though their penultimate clusters differ. The same can be said for cases TMR and GMR.

In the Newman unweighted analysis, the columns in middle floors are identified having the highest relative edge betweenness in all the scenarios considered. Since all the shortest paths from the top and bottom of the frame need to pass directly through the middle portion, the middle floor columns are identified as the inter-cluster links. From Table 6.2, the MSR case is identified as to have the highest possibility to cause the next failure. In this analysis it was found that removing a side column in a floor will have higher possibility of next failure following it, rather that removing a middle column. Removal of a middle column will increase the number of shortest paths carried by the adjacent columns by no more than half of what it used to carry. But removal of a side column will increase the number of shortest paths carried by the adjacent column by the number of paths it used to carry. This could be the reasoning behind this observation.

In the Newman weighted analysis, the intact frame shows that middle floor middle column is the most likely to be removed. The cases GSR, GMR, TSR &TMR result in failure of beams immediately adjacent to the columns removed. The TSR and TMR show that the beams above the removed columns are failing. This is easier to interpret in a real world context. However, the cases GMR & GSR are showing that the beams below the removed columns are failing; and this is hard to interpret in a real world context. Table 6.3 shows that the relative edge betweenness for GMR and TMR are the highest and GSR and TSR are also equal. This echoes the symmetry observed in the Bristol approach results. This trend can be observed in the unweighted Newman analysis as well. It would be beneficial to incorporate the depth from the datum into the index to differentiate the top and bottom column removal. The cases MSR and MMR result in removal of middle floor columns, which is similar to the unweighted analysis. From Table 6.3, it can be determined that the middle column removal will have higher possibility of propagating failure rather than the side column removal in any floor.

The RSA was carried out in a manner to differentiate the ground floor column removal and top floor column removal. Figure 6.41 shows that after removal of ground floor side column, the columns above it and the beam immediately above the removed column are considered as the one through route terminating at the adjacent column. Whereas as shown in Figure 6.45, top floor column removal wouldn't have similar effect as there is no column above it. This is done to incorporate the concept of possible load paths after a failure, to a small extent. In Figure 6.44 (MMR), it can be observed that beams immediately below the removed column are considered as two different routes, as they are still transferring load from that floor to the adjacent columns, same as before the failure. However the beams immediately above the removed column are considered as single route, as they have to transfer the load from the columns above to the adjacent columns. A similar philosophy was used all scenarios.

Table 6.10 shows that the relative connectivity of the damaged frames will always be lower than of the intact frame. This can be seen as justifying the implementation of RSA to the frames. Table 6.10 also shows that the relative depth of the intact frame is always lower than its counterparts. It can be derived that relative depth can be used to measure the degree of damage of the structure. Similar to the Bristol approach, the middle column removals cause severe loss in the connectivity, compared to the respective side column removal. It can be noted that the least loss of connectivity due to a middle column removal (TMR) is greater than the largest loss in connectivity due to a side column removal (GSR). The loss in connectivity due to column removals reduces as it moves away from the ground - i.e. loss in connectivity can be repeated as GMR>MMR>TMR>GSR>MSR>TSR. The ground floor middle column causes the highest loss in connectivity. The middle floor middle column causes the largest increase in relative depth.

7 Analysis of Road Network

7.1 Introduction

In the previous chapters, the structures (truss and frame) were analysed in an attempt to quantify the structural connectivity. Three different methods were employed for this purpose. Based on the knowledge gained, it is attempted to measure the structural connectivity of a two dimensional network, different to the structures. The road network of Sri Lanka is chosen as the network to be analysed. Section 7.2 introduces that network and its idealisation. The analysis method and results are given in Sections 7.3 and 7.4 respectively. Section 7.5 presents the discussion of results.

7.2 The network idealisation

Like many other networks, a road network is complex in nature. Figure 7.1 shows the national road network of Sri Lanka and its complexity. The road network of Sri Lanka is reported to have density of 1.76km/sq.km. This can be simplified by choosing the level of resolution of network to be analysed. The major roads in road network are classified in to the following six different categories (Sri Lanka Tagging Guidelines - OpenStreetMap Wiki, 2017).

- Expressway (E)
- Main Road (A-Level)
- Main Road (B-Level)
- Secondary/Minor Road (C&D)
- Jeep/Car road
- Footpath

By selecting only the expressways and the main A-level roads, the resolution of the network is reduced, i.e. reduced resolution offers less details of network. Figure 7.2 shows only the A-Level roads and the expressways extracted from Figure 7.1. As can be observed, the complexity of the network is greatly reduced. This leads to easier computation with respect to identification of shortest paths. With increased resolution, it would be possible to get more accurate results regarding the network; however the computation effort also increases. In this research, it was decided to limit the resolution to expressways and main A-level roads.

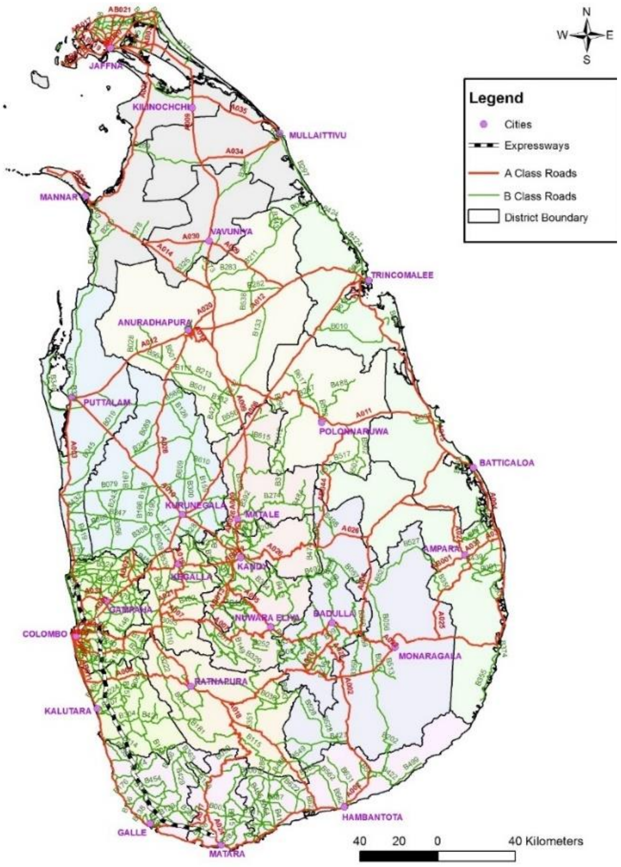


Figure 7.1 National Road Network (RDA-2017)

Figure 7.2 A-Level main roads and expressways

When selecting the roads for the analysis, in addition to the A-level roads and expressways, the roads connecting the expressways to the A-level roads are included. It was noted that parallel roads of the same level don't commonly exist in the network at this resolution; thus it was decided not to include the number of parallel roads as the edge weight for the analysis in this research. This can be explored in the analysis of network of larger resolution, as it would have a higher number of parallel roads.

The road junctions are modelled as joints. The road is modelled as a link connecting these two joints, while the orientation of the road is dictated by the joint location. The length of the link is kept as the actual physical length of the road segment between the said joints. The idealised road network is given in Figure 7.3. This network has 70 nodes and 105 links. The physical road lengths were rounded off to nearest 2 km.

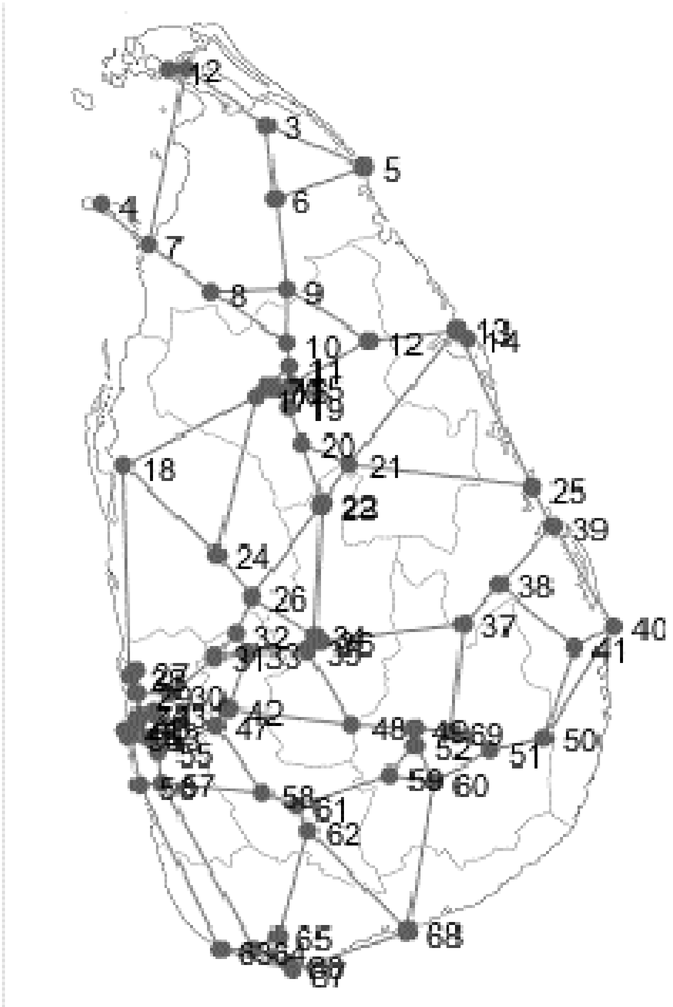


Figure 7.3 Idealised network

7.3 The analysis method

7.3.1 Introduction

The Bristol approach was identified as the most suitable index to measure the structural connectivity of the structures. To adopt the Bristol approach to the road network, the traffic modulus of every link should be determined. Traffic modulus is the equivalent of the elastic modulus for structures and deals with traffic flow (analogous to stress) and traffic strain (analogous to mechanical strain) (Liu, 2016). Computation of traffic modulus requires data on the traffic capacity and many other aspects which would not be available for everyone, or cannot be assumed as fixed over a period of time.

It was decided to adopt a combination of Newman's method and RSA (which is developed specifically for roads) to quantify the structural connectivity of the road network. The Newman method was used to disintegrate the network by removing the most in-between road segments and the RSA was used to identify the overall connectivity of the network in each generation of the Newman method.

7.3.2 Weighted geodesic paths

As explained in Chapter 5, the Newman method uses shortest paths to calculate the edge betweenness. This is carried out in an unweighted manner, i.e. distance between the nodes are given by the number of steps taken to travel between them. The weight of the link is not taken into consideration when calculating the shortest path, it is calculated as one step. The edge weight is later used to divide the edge betweenness from the unweighted analysis to produce the edge betweenness for the weighted analysis, thus influencing the analysis. This approach is appropriate for most scenarios.

But in the case of road network, the calculation of shortest paths cannot be carried out in the unweighted manner. Selection of travel path between two cities will largely depend on the physical distance of the paths between the said cities. The formula of the Newman method was modified to calculate the weighted shortest paths. It should be noted that only the calculation of the shortest paths are carried out in the weighted manner; but the calculation of the edge betweenness and the rest of the analyses are carried out in the unweighted manner.

The algorithm was developed based on the concept that if a weighted shortest path to node 'x' from node 'y' is passing through two adjacent nodes 'i' and 'j', then the distance between the node 'i' and 'j' should be equal to the difference between the shortest distances from 'i' and 'j' to node 'y'. Since all graphs/ network are undirected and connected, shortest path from node 'x' to node 'y' is equivalent to shortest path from node 'y' to node 'x'.

The algorithm is done in two stages. The initial stage is to calculate the weighted shortest distances to all the nodes from a specific node. The algorithm is explained using network shown in Figure 7.4 as the example. This is a modified version of network shown in Figure 4.2. The weights (distances) of the links are shown as circled values in Figure 7.4.

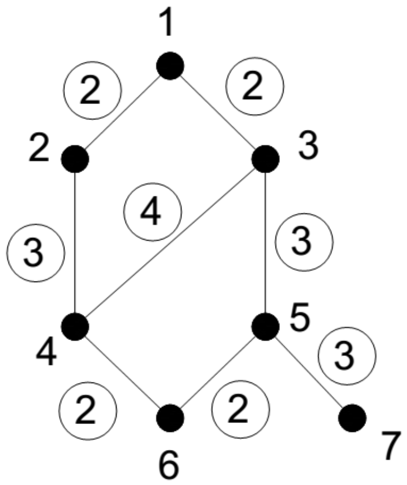


Figure 7.4 Selected weighted network

Step 01: Select a node, ‘s’ as the source node.

Note 01: node 1 is selected as the source node in this example.

Step 02: Initial node‘s’ is given, distance d_s=0.

Step 03: Every node ‘i’ adjacent to node ‘s’ is given distance

$d_i=d_s+D_{si}$; where D_{si}= distance between node ‘s’ and ‘i’.

Note: nodes 2 and 3 are given distance of 2.

Step 04: For every node ‘j’ adjacent to node ‘i’, do one of the following;

- If node ‘j’ has not been assigned a distance, then it is assigned distance

  $d_j=d_i+D_{ij}$; where D_{ij}= distance between node ‘i’ and ‘j’.

- If node ‘j’ has already been assigned a distance and $d_j>d_i+D_{ij}$ then it is replaced with

  $d_j=d_i+D_{ij}$; where D_{ij}= distance between node ‘i’ and ‘j’.

- If node ‘j’ has already been assigned a distance and $d_j<d_i+D_{ij}$ then we do nothing

Note 02: when the node 2 is taken as node ‘i’, node 4 is the only adjacent node (node ‘j’) and given the distance of 5.

Note 03: Following that node 3 is taken as ‘i’, it has two adjacent nodes (node 4 &5 as node ‘j’). The shortest distance from node 1 to node 4 through node 3 is equals to distance 6. However this is more than the distance assigned to node 4 in the Note 03, thus this step is omitted. Node 5 is assigned distance of 5.

Step 04: Repeat the step 03, taking the node 'j' as node 'i', until all the nodes are assigned a distance.

Note 04: The process is propagated away from the source node towards the farthest node.

Step 05: Repeat steps 03 and 04, counting each iterations as a generation, until the shortest distance from node 's' to all nodes remains the same in two consecutive generations.

Note 05: In large networks, the shortest path to a node that is away from the source node may be calculated incorrectly in the first trial. Step 05 is introduced to correct this.

This concludes the initial stage of the algorithm. Now, the weighted shortest distance to all nodes from node 's' is available for the second stage of the analysis. Distances from 's' can now be seen as an attribute of all the nodes. Figure 7.5 shows the shortest distances from node 1 to all other nodes.

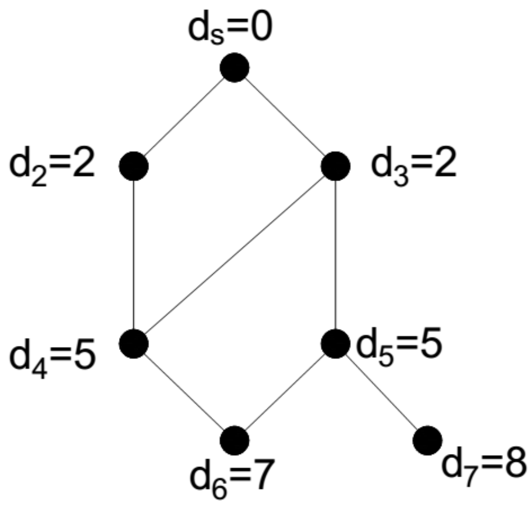


Figure 7.5 Shortest distance to all nodes from the source node

Step 06: For all nodes 'i' other than the source node 's', find the adjacent nodes 'j' and group them into two groups, up_neighbour and down_neighbour with respect to the distances to node 's', using the conditions given here;

- up_neighbour will contain all the nodes 'j' through which the shortest paths from source node 's' to a specific node 'i' will run through. For such nodes 'j',

  d_i= shortest distance to node 'i' from node 's'.

  d_j= shortest distance to node 'j' from node 's'.

D_{ij}= distance between node 'i' and 'j'.

If $d_j<d_i$ AND $d_j=d_i-D_{ij}$, then node 'j' is assigned to up_neighbours (i.e. node 'j' closer to 's' than 'i')

Note 06: For example, the node 4 has three adjacent nodes in Figure 7.4. Out of this three nodes, node 2 and node 3 have distances lower than that of node 4. However, only node 2 is assigned to up_neighbour according to the algorithm.

- Similarly, down_neighbours will contain all the nodes 'j' whose shortest paths from source node 's' will run through specific node 'i'.

If $d_j>d_i$ AND $d_j=d_i+D_{ij}$, then node 'j' is assigned to down_neighbours (i.e. node 'i' closer to 's' than 'j').

Note 07: node 7 is assigned as down_neighbour of node 4.

- If $d_j=d_i$, then do nothing.

Step 06-1: Arrange the nodes according to the shortest distance from the source node in ascending order, staring with the source node 's'.

Note 08: This order for Figure 7.5 would be nodes 1, 2, 3, 4, 5, 6 and 7.

Step 06-2: Assign a weight flag to source node as w_s=1. For all other nodes weight flag shall be equal to summation of weight flags of nodes in the up_neighbour group.

Note 09: Figure 7.6 shows the weight flag assigned for nodes of Figure 7.4.

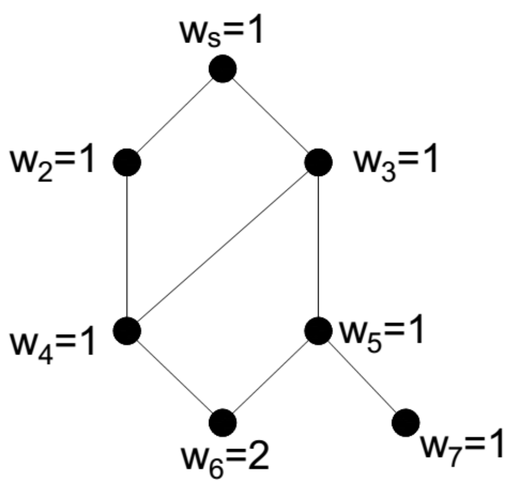


Figure 7.6 Weight flags of the nodes

Step 07-1: Arrange the nodes according to the shortest distance from the source node in descending order, finishing with the source node 's'.

Note 10: This order for Figure 7.5 would be nodes 7, 6, 5, 4, 3, 2 and 1.

Step 07-2: For each node 'i', in descending order, find the nodes in the down_neighbours 'j' and add the edge betweenness of the connecting edges (i-j). This would be zero for the first node in the order (farthest form the source node).

Step 07-3: Then find the nodes j in the up_neighbour and assign the edge betweenness as given in equation 7.1.

Edge betweenness of i-j= (w_j/w_i)*(1+ summed edge betweenness) **equation 7.1**

> Note 11: The edge connecting the farthest node (node 7) to its upneighbour (node 5) is edge 5-7. Edge betweenness for this edge can be calculated as 1.
>
> Note 12: Edge betweenness of edges 4-6 and edge 5-6 (the edges connecting node 6's to its upneighbours) is calculated as 0.5 each.
>
> Note 13: Edge betweenness of edge 3-5 is calculated as 2.5. This is the summation of edge betweennesses of edge 5-6 and edge 5-7 added with value one then multiplied by 1.
>
> Figure 7.7 shows the edge betweenness calculated for all edges considering node 1 as the source node.

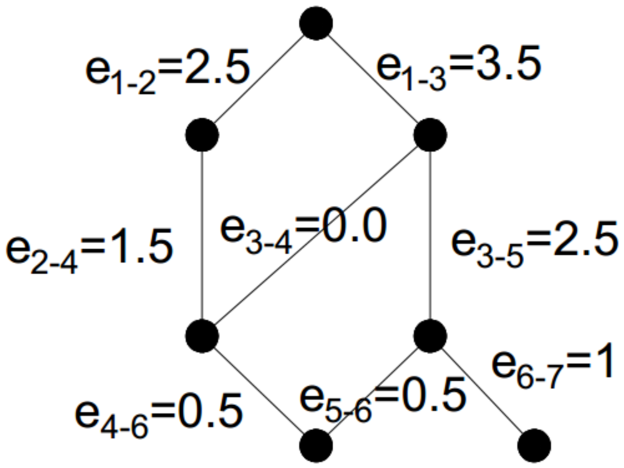


Figure 7.7 Edge betweenness considering node 1 as the source node

After this, all seven steps need to be carried out for all nodes in the network and the edge betweenness of an edge is the summation of all edge betweennesses of that specific edge throughout the analysis, i.e. considering all nodes as source edges. The removal of edges follow the normal unweighted Newman analysis method. It should be noted that weight flag, w, is used to account for multiple paths but doesn't constitute weighted analysis.

7.3.3 The route structure

Even though only the expressways and level-A main roads are considered for the analysis, some priority basis needs to be established for route formation. Expressways are considered as more through than the level-A roads. Between the level-A roads, the naming was used to determine the thoroughness of the route. Lower the naming number, the road was considered as more through e.g. the road A1 is more through than A2 or A3. The Newman method is used to remove the most in-between edge and the RSA is used to calculate the overall connectivity of the network. As the Newman method progresses to remove edges incrementally, the overall connectivity is recalculated by re-identifying routes. When the network is broken into two or more groups, the process is kept the same while the relative connectivity and relative depth are now calculated separately for each cluster as well as the overall network. Calculation of connectivity of overall network even after it is disconnected has been explored by previous researchers (Cartledge, 2011). For example, Table 7.1 shows the route properties identified for the intact network. All the routes are considered to have the same depth.

Table 7.1 Route properties for intact network

Stage 0	Continuity	Connectivity	Depth
E03	3	4	1
E02	1	3	1
E01	3	7	1
A00	1	2	1
A01	7	10	1
A02	6	9	1
A03-1	2	4	1
A03-2	1	4	1
A04	10	15	1
A05	6	8	1
A06	7	10	1
A07	2	3	1
A08-1	1	3	1
A08-2	1	3	1
A09-1	1	2	1
A09-2	10	12	1
A10-1	2	5	1
A10-2	2	5	1
A11-1	1	3	1
A11-2	1	3	1
A12-2	4	7	1
A12-1	2	5	1
A13	1	2	1
A14	3	3	1

A15	2	5	1
A16	2	3	1
A17-1	2	3	1
A17-2	1	4	1
A18	2	3	1
A19	1	3	1
A20	1	2	1
A21	1	3	1
A22	1	2	1
A23	1	3	1
A24	2	3	1
A25	1	3	1
A26	1	3	1
A27	1	3	1
A28	1	2	1
A29	1	3	1
A30	1	3	1
A31	1	3	1
A32	1	2	1
A33	1	2	1
A34	1	2	1
A35	1	2	1

7.4 Results

Figures 7.8 to 7.11 show the first three generations of the modified Newman's algorithm for the road network. Figure 7.12 shows the 19^{th} generation of the algorithm. The edge betweenness of each edge is represented as the thickness of the edges in the given Figures. Table 3.2 shows the relative connectivity and relative depth of the network for first ten generations of the algorithm. This data is also represented in the Figure 7.9 as a graph. When the network is broken into different clusters, the parameters for the new clusters are also tracked.

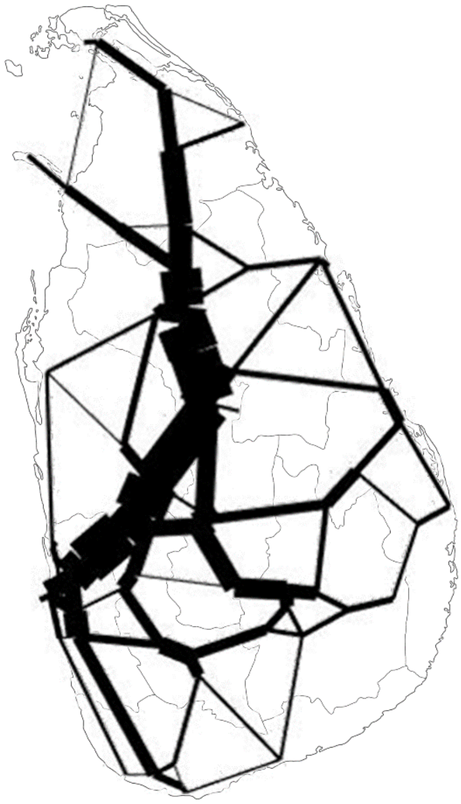


Figure 7.8 Generation 00 (Intact network)

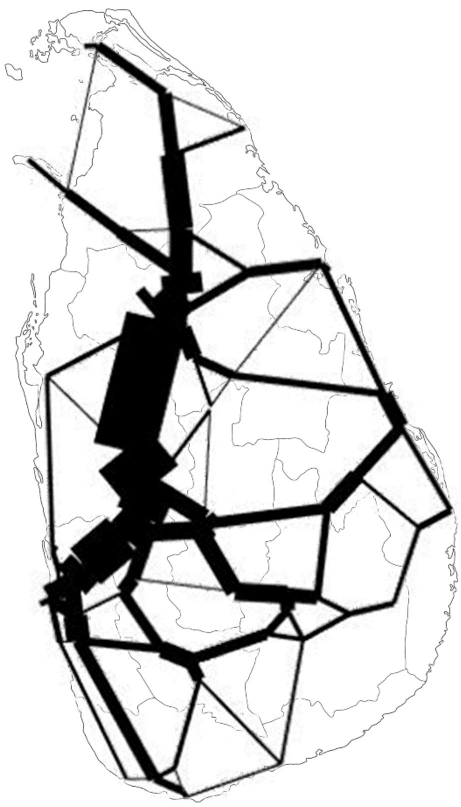


Figure 7.9 Generation 01 (Penultimate cluster)

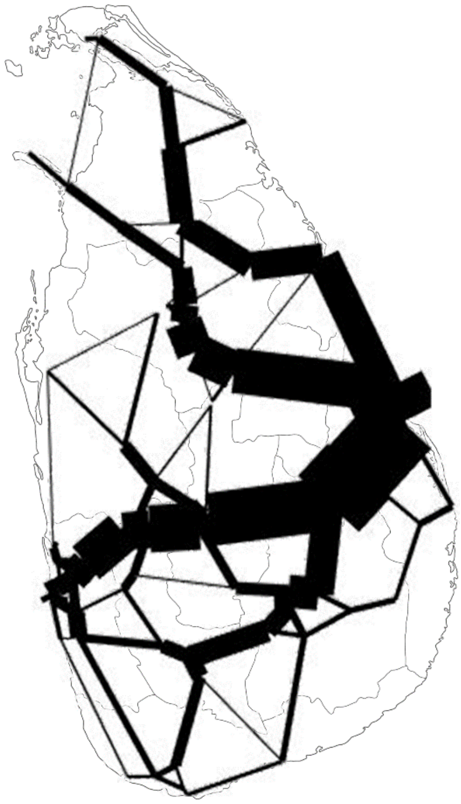


Figure 7.10 Generation 02

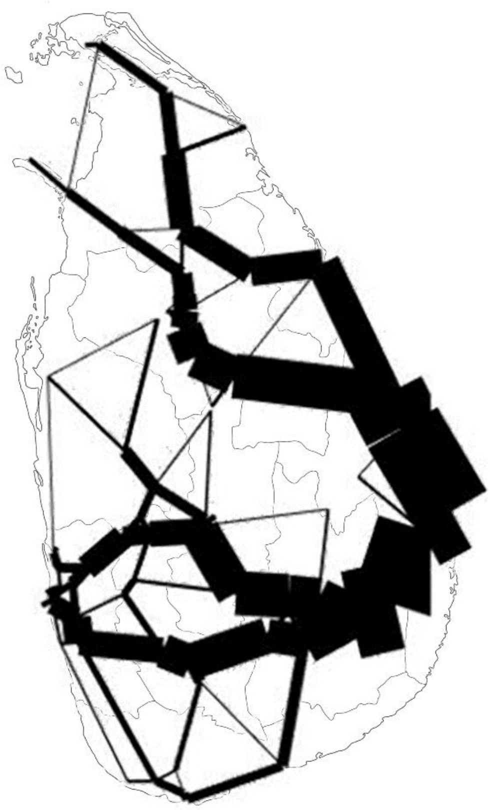


Figure 7.11 Generation 03

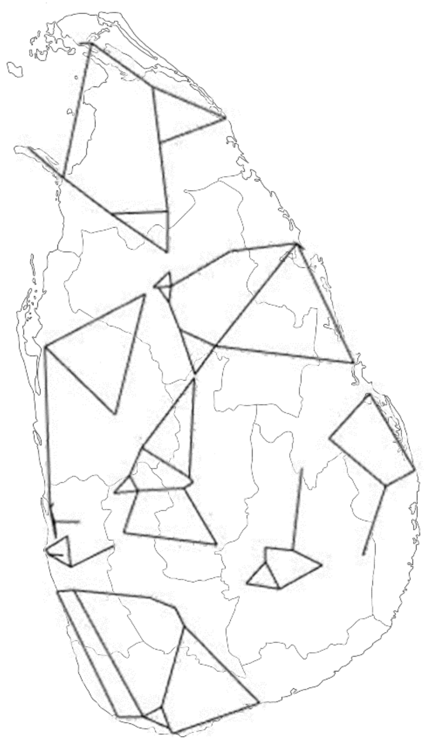


Figure 7.12 Generation 19

Table 7.2 Relative connectivity and relative depth of road network

Step	Relative Connectivity-Overall	Relative Connectivity-Cluster 1	Relative Connectivity-Cluster 2	Relative Connectivity-Cluster 1.1	Relative Connectivity-Cluster 1.2	Relative Depth-Overall	Relative Depth-Cluster 1	Relative Depth-Cluster 2
0	0.56232					0.13333		
1	0.56232					0.13623		
2	0.56105					0.13953		
3	0.56105					0.14244		
4	0.55977	0.56279	0.55469			0.14577	0.13953	0.15625
5	0.55977	0.56279	0.55469			0.14869	0.14419	0.15625
6	0.55882	0.56132	0.55469			0.15000	0.14623	0.15625
7	0.55882	0.56132	0.55469			0.15294	0.15094	0.15625
8	0.55786	0.55981	0.55469			0.15430	0.15311	0.15625
9	0.55689	0.55825	0.55469			0.15569	0.15534	0.15625
10	0.55319	0.55224	0.55469	0.55422	0.55085	0.15805	0.15920	0.15625

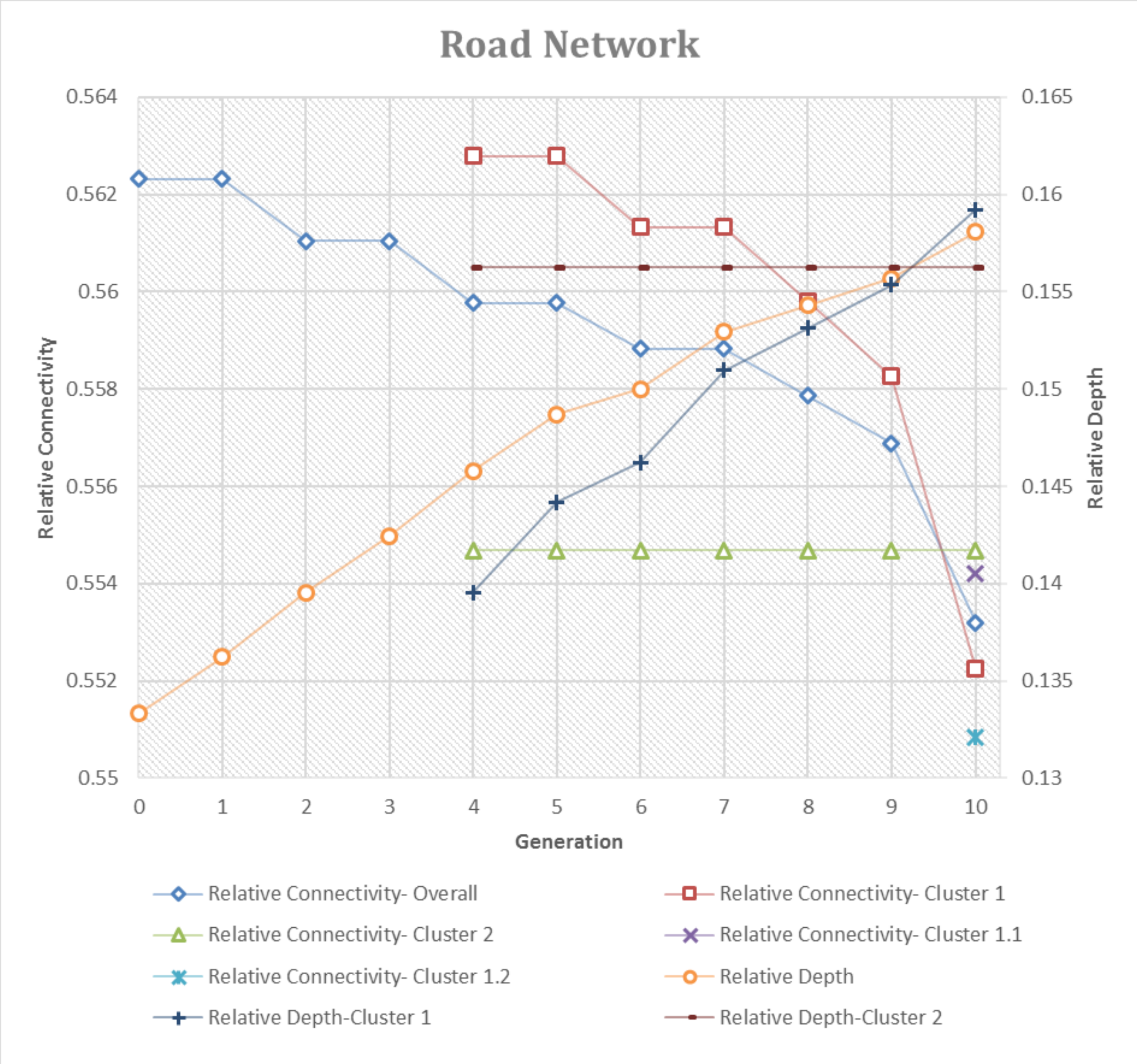


Figure 7.13 Relative connectivity and relative depth of road network

7.5 Discussion

In this chapter, the road network was analysed as an attempt to determine the applicability of structural connectivity to networks other than structures. It was also attempted to check adoptability of the indices considered for structural connectivity of structures to the road network. Though the preferred index of structural connectivity of structures (Bristol approach) was not applicable to the road network in its current form, mainly due to its requirement of data that may not be available to public, a suitable replacement index was formulated by combining Newman's method and route structure analysis.

Analysis using weighted multiple shortest paths provides a realistic approach towards the road network analysis, as travel time is the major component influencing the route selection in travelling. However, this analysis is carried out on an idealised network of low resolution, limiting the possible alternate shortest paths that can be formulated within the network. It is also assumed that all roads have similar traffic capacity, traffic flow and same comfort level of travel. Since all the roads are of A-level or above, this is a reasonable assumption. The expressways are deemed to have higher throughness due to lesser congestion and higher comfort level. The influence of the traffic capacity can be taken as an edge weight for a weighted analysis along with the parallel roads between two junctions. Since this is an unweighted analysis, it was assumed that all nodes have same traffic generating potential, this can be modelled as nodal weights for weighted analysis of road network. Due nature of the analysis (unweighted) and scope of this research, the said assumptions are deemed to suffice.

It can be observed from Figure 7.13 and Table 7.2 that the relative connectivity of the overall network always reduces or stays constant in consecutive generations. It is also observed that the modified Newman's algorithm seeks out the cluster with the highest relative connectivity, after the network is broken into clusters (at generation 4). Though the relative connectivity remains constant between some generations, the overall relative depth always increases. This trend is observed in the broken clusters as well. The relative depth of the cluster 1 always increases as the cluster continues to lose edges. This shows that the relative depth can be used as an index for level of damage in a network (overall as well as individual clusters).

The Figures 7.8 to 7.11 show the generations of modified Newman's algorithm. The most in-between edge detected in the 1st generation of the algorithm is the road segment shared by A9 (Kandy- Jaffna) and A6 (Ambepussa- Trincomalee) roads near Dambulla. This can be regarded as one of the most strategic road segments as most of the west-east/north, south-north & central-north/east trips will occur through this road segment. This is consistent with everyday experience and gives credibility to the applied algorithm and the analysis method. In addition, by generation 19, the road network is broken in to clusters that almost resemble the established provincial divisions as shown in Figure 7.12. This shows that the analysis method captures the spatial connectivity of the road networks

8 Summarizing Discussion

8.1 Introduction

This chapter acts as a summary and combination of the discussion sections in the previous chapters. Initially the discussion on the selected analysis methods and selected networks is presented in Section 8.2. The Section 8.3 discusses the trusses and offers comparison between results from the different analysis methods. The frame results are discussed in the Section 8.4 followed by discussion on road network results in Section 8.5. Section 8.6 highlights the indices that are most helpful for analysing the connectivity of two dimensional assemblies whether intact or damaged.

8.2 Analysis methods and selected structures

The aim of this research was to give definition for structural connectivity in the context of structures and to propose an index for it. Three kinds of two-dimensional assemblies were analysed in this research and each type was used for different kind of purpose. Three different trusses were analysed to check which form has the better structural connectivity. Effect of increasing the chord member stiffness was checked as well. The frame was analysed to check which column loss would cause the highest loss in structural connectivity. The road network was checked to confirm the applicability of structural connectivity to networks other than structures. The change in structural connectivity during the network decomposition was measured for the road network. Figures 1.4 to 1.6 shows the selected truss forms and Figure 1.7 the selected frame configurations. The idealised road network is given in Figure 7.3.

The analysis methods can be grouped into the following two categories;

- Unweighted analysis methods
- Weighted analysis methods

Graph theory, unweighted Newman’s analysis and route structure analysis can be called unweighted analysis methods. These analysis methods don’t consider the structural characteristics of the links or nodes in the idealised network. Since they don’t consider the length of the links in the analysis, the spatial location of the nodes don’t influence the results. The main factor in deciding the structural connectivity is the connections between the nodes. But the route structure analysis for the road network (not the trusses) takes the spatial positioning of the links and nodes into consideration.

The weighted Newman’s method and the Bristol approach are the weighted analysis methods used in this research. These methods take the actual form and structural characteristics of the structure into account during the analysis. These characteristics can be the joint type and member stiffness, among others. All the unweighted methods offer an insight into the form of the structure/network without any structural characteristics, while the weighted analysis methods offer information on the structural feature of the structure as well. If a structure has high structural connectivity in the unweighted analysis, then it has the potential to be the form with the highest structural connectivity in the weighted analysis.

Out of the selected four analysis methods, only the Bristol approach and Newman's method have the ability to predict the possible failure location. However there exists a fundamental difference between Newman's method and the Bristol approach. The Bristol approach, being an agglomerative method, creates clusters in the first step of the analysis. The last cluster to be added to the main cluster is identified as the least connected one. This last cluster can either be a structural ring or a cluster that has grown to some extent; this means that it is not always possible to identify a single member as a weak link but rather a weakly connected region. Newman's method being a decompositive method seeks out the least connected link in the first step itself.

8.3 Trusses

In Table 2.3, it was shown that indices such as closeness centrality, diameter of graph and graph radius select truss T1 to be most closely knitted as one. These indices indicate how closely the nodes in a graph are connected together. The strength of graph, which is similar to Newman's method, shows that truss T1 is the strongest. It should be noted that the unweighted Newman analysis also selects truss T1 to be the most wellformed. Truss T1 is also selected to have the highest connectivity when it comes to algebraic connectivity index. Truss T2 is identified to be the most triangulated one as it has the highest values for both global and average local clustering coefficients.

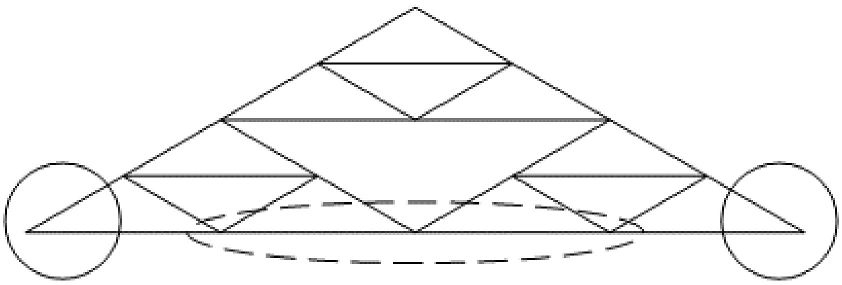
It was observed that the unweighted analysis methods such as Graph theory and the unweighted Newman analysis indicate that truss T1 to be the most connected one. However the Route Structure Analysis (RSA) results don't reflect this. In the RSA, truss T1 is shown to be the least connected one. It should be noted that though RSA is identified as an unweighted analysis, the spatial relationship of the network is taken into consideration during this analysis. This is somewhat similar to weighted analysis methods and truss T1 is not favoured in any of the weighted analyses. From Table 5.9, it can be seen that truss T2 has the highest relative connectivity in RSA. It should be noted that in the Graph theory indices select the truss T2 to be second most wellconnected truss form.

Newman's method is the only unweighted analysis that can show the possible failure location or the weak link. It was noted that all failure locations are located on the chords of the trusses (Figure 4.11). This is also observed in the weighted analysis.

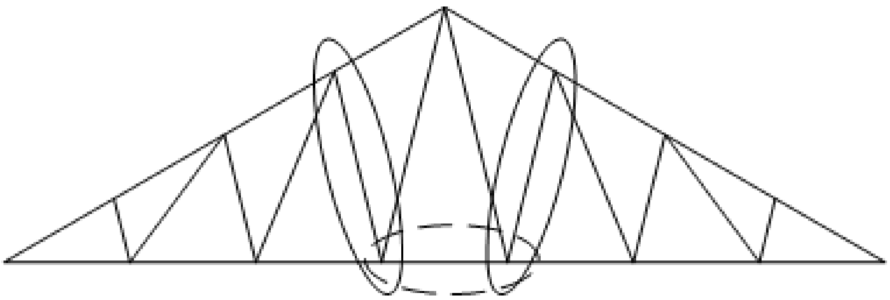
In the weighted analysis, three cases were considered for the analysis, varying the axial rigidity of the chord members while keeping the axial rigidity of the web members as unity. These cases were;

- Case 1 : Axial rigidity of all the members are unity
- Case 2 : Axial rigidity of the chord members are double that of the web members
- Case 3 : Axial rigidity of the chord members are four times that of the web members

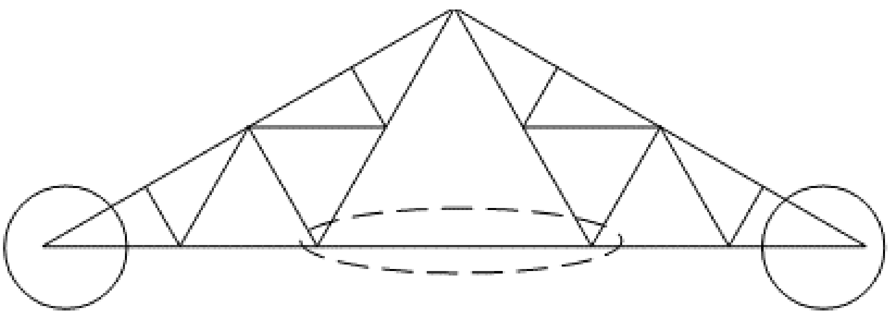
Understanding the penultimate cluster of the Bristol approach (or generation 01 of Newman's method) is important to identify the possible failure location in a structure. Figures 4.1 to 4.3 show the possible failure locations identified in the three cases considered. Solid loops are used to indicate the failure location identified by the Bristol approach while the dashed loops to indicate weakest link identified by Newman's method. Table 8.1 gives a summary of analysis results for trusses.



(a) Fractal truss

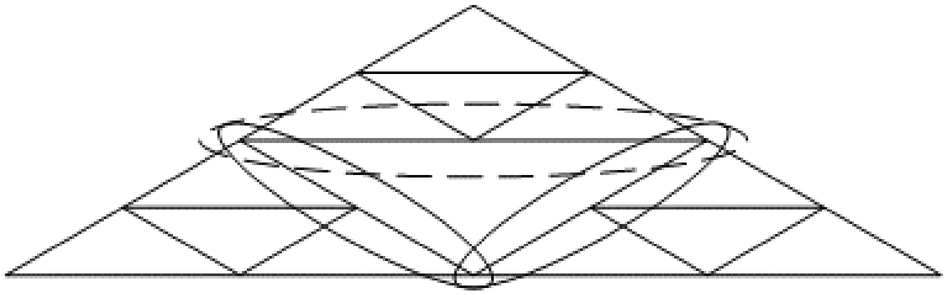


(b) Warren truss

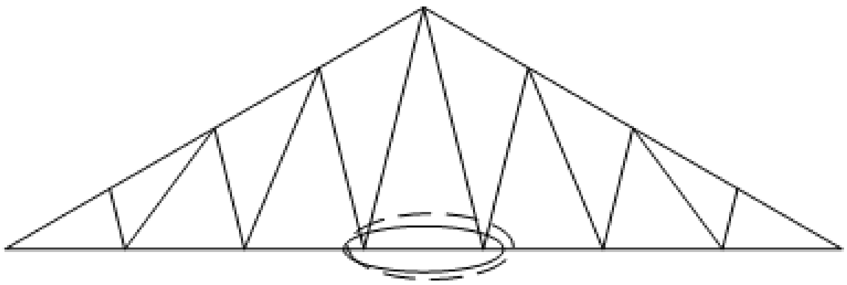


(c) Fan type truss

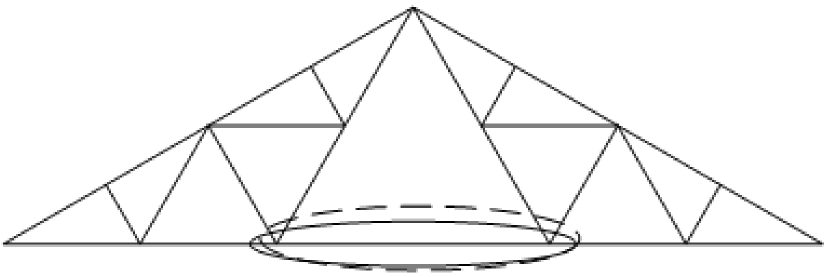
Figure 8.1 Results of Case 1



(a) Fractal truss

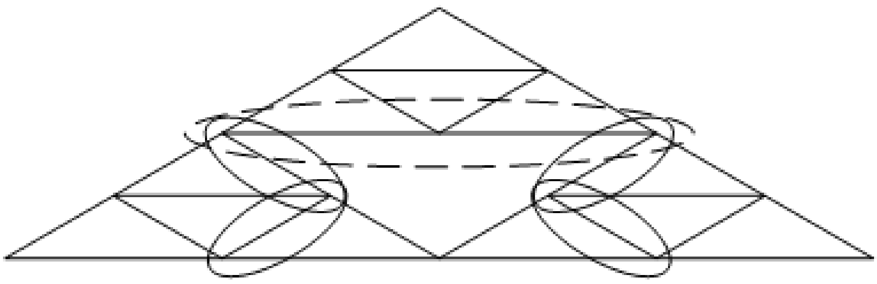


(b) Warren truss

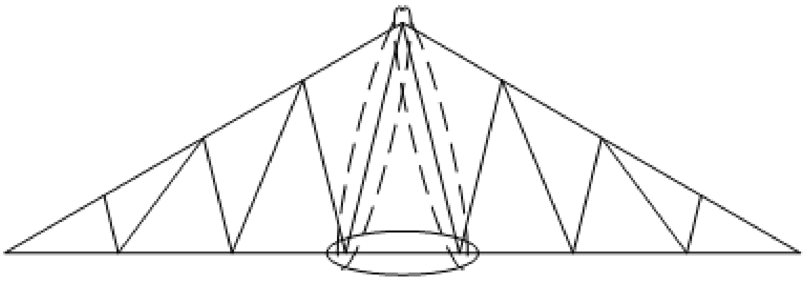


(c) Fan type truss

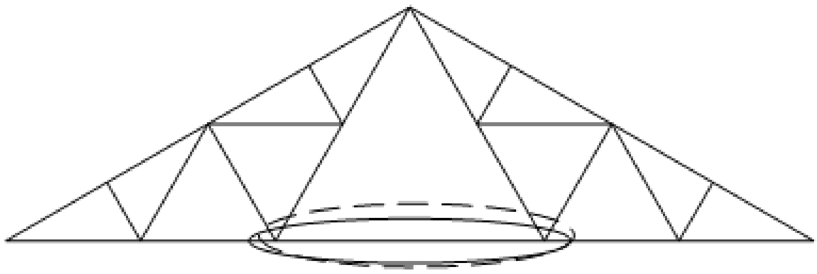
Figure 8.2 Results of Case 2



(a) Fractal truss



(b) Warren truss



(c) Fan type truss

Figure 8.3 Results of Case 3

Figures 8.1 to 8.3 shows that the failure locations predicted by the different methods don’t always agree with each other. It is generally regarded that failure of a web member is preferable to failure of a chord member. Due to simplicity of the chosen truss forms and the nature of agglomerative clustering, the Bristol approach is unable to point to a failure that is purely a web failure. Newman’s method is able achieve this.

Figure 8.1 shows that the failure is located in the chords of the trusses for both methods. Newman’s methods identifies the middle of the bottom chord as the possible failure location for all three trusses. Though failure in the chord is not desirable, the inherent arching action of trusses may negate complete failure even after failure of middle bottom chord. For truss T1 and T3, the Bristol approach shows that the failure might occur in the corner nodes of the trusses. These are usually the support nodes. Truss T2’s failure is expected to happen in the middle of the top chords. This kind of failure can lead to complete failure and should be avoided. In Figure 8.1, none of the failure locations predicted by two methods match each other.

Figure 8.2 shows that truss T2 and T3 indicate the same failure locations for both analysis methods. Middle of the bottom chord is identified as the failure location for both of these trusses. The failure location identified by both methods lie in the middle fractal of truss T1. The failure of truss T1 now affects the top chord which is not preferable but the overall wellformedness has increased indicating the effort to cause the failure will be higher than in case 1. As per Newman’s method, the failure has shifted from the chord to a web member, and this is much preferable.

In Figure 8.3, only truss T3 shows the same failure location for both Bristol approach and Newman's method which remains at the middle of bottom chord. For truss T1, Newman's method indicates the same location of failure while the region of failure region indicated by the Bristol approach moves away from the support nodes. Newman's analysis for truss T2 shows that the failure has moved from chord to web completely. However the Bristol approach continues to indicate that middle of bottom chord is susceptible to failure even after quadrupling the axial rigidity from case 1.

Table 8.1 Summary of truss analysis results (shading indicates better connectivity)

	Truss T1			Truss T2			Truss T3		
	Case 1	Case 2	Case 3	Case 1	Case 2	Case 3	Case 1	Case 2	Case 3
Wellformedness	4.57	7.27	13.09	7.56	14.75	30.23	9.96	16.19	29.71
Relative betweenness	7.80	10.28	12.93	7.21	5.32	8.21	23.55	15.57	9.16
Relative connectivity	0.557			0.588			0.583		
Relative Depth	0.250			0.213			0.229		

In Bristol approach, the structural connectivity is measured by the wellformedness (for trusses and intact frames) and by the relative separateness (for damaged frames) irrespective of the possible failure location. Table 8.1 shows that increasing the chord member stiffness benefits all the truss forms. This is easily observable for the Bristol approach - increasing the chord axial rigidity always results in increased wellformedness for all three trusses. The relative betweenness of the Newman method is highly dependent on the removed edge; this was the reason behind the sudden increase in relative betweenness between case 1 and case 2 for truss T1and between case 2 and case 3 for truss T2. Figures 8.1 and 8.2 shows that the failure has moved from the chord to web member of truss T1 and Figures 8.2 and 8.3 show the same for truss T2. In case 3 for truss T1, the relative betweenness again increases indicating that the web member has become more in-between which in turn indicates that the possibility of a chord failure has reduced even further. The failure location of truss T3 remains same in Newman analysis for all three cases. Table 8.1 shows that the relative betweenness of truss T3 reduces as the axial rigidity is increased. Since the indicated failure location is in the chord of truss T3, this indicates that identified member is becoming less in-between. This is favourable.

As per the Bristol approach, truss T3 is selected to be the most wellformed in case 1 and 2. But in the case 3, truss T2 is chosen as the most wellformed truss. In Newman's method, truss T2 is selected to be most wellconnected throughout all three cases. The RSA also selects truss T2 to have highest connectivity. The material cost for increasing the wellformedness was compared in Figure 3.27. Truss T2 is shown to be most effective in this aspect.

8.4 Frame

Unlike for the trusses, detailed comparison of results from different methods is already presented in Section 6.4. A brief summary of the mentioned discussion is presented in this section. Table 8.2 shows a summary of results from different analysis methods for different column loss conditions considered.

Table 8.2 Summary of frame analysis results (shading indicates poorer connectivity)

Frame condition	GSR	GMR	MSR	MMR	TSR	TMR
Relative separateness	48.00	50.94	50.02	54.15	48.00	50.94
Relative betweenness (unweighted)	3.385	3.366	4.075	4.071	3.385	3.366
Relative betweenness (weighted)	7.619	8.354	2.994	3.265	7.619	8.354
% Reduction in relative connectivity	0.48	2.14	0.24	1.34	0.24	0.73
% increase in relative depth	0.70	1.89	1.30	3.66	1.30	0.08

Among the selected analysis methods for frames, only Newman's method offers a purely unweighted analysis. It is shown in Table 8.2 that middle floor side column removal causes the highest possibility of propagating the failure, closely followed by middle floor middle column removal scenario. It can be concluded that as per unweighted analysis middle floor column removals cause higher loss in connectivity than top or ground floor column removals. This is understandable as all shortest paths from top and bottom of the structure have to travel through the middle floor columns. This result is also reflected in the Bristol approach to some degree. It is also noted in the unweighted Newman analysis, the side column removals has a greater effect than middle column removals.

Table 8.2 shows that the middle floor middle column removal has the highest relative separateness. The middle portion of the structure has the joints with highest joint stiffness causing the clustering to be initiated from there. Loss of a member in the middle portion of structure causes the highest loss in structural connectivity. The middle floor middle column removal causes the largest increase in the relative depth in RSA, which is an index used to check the damage to the structure. It should be noted that in the Bristol approach middle column removal at any floor has a greater effect that the respective side column removals. This is reflected in both weighted Newman analysis and RSA.

The ground floor middle column removal and top floor middle column removals are chosen as the mostly likely scenarios to propagate failure in the weighted Newman analysis. In the RSA results it can be observed that percentage loss in relative connectivity for each scenario can be arranged in the ascending order such that the least loss in relative connectivity by a middle column removal is greater than the greatest loss by a side column removal. It was also observed that ground floor middle column removal will have the greatest effect on the relative connectivity followed by middle floor middle column removal. Loss in connectivity of selected scenarios can be arranged as GMR>MMR>TMR>GSR>MSR>TSR.

It can be observed from the Table 8.2, that both Newman's method and Bristol approach results are symmetric in nature-i.e. the result of GMR is equal to TMR and GSR is equal to TSR. This behaviour is not reflected in the RSA results. Actual spatial relationship of the member is taken into consideration in the RSA, enabling the analysis method to assess the top column and ground column removals separately. This highlight the importance of having an index that can account the spatial arrangement of the structure. Bristol approach employs a parameter called 'distance from datum' to account this.

8.5 Road Network

The analysis of the road network was carried out to confirm the adaptability of the concept of structural connectivity to other two dimensional networks. The response of a network to strategic link removal was also explored during this. Due to the difficulty of adopting Bristol approach to road network, only a combination of Newman's method and RSA was used analyse the road network.

In order to find the weak links in the road network, Newman's community finding algorithm was modified to use weighted multiple shortest paths. This modified algorithm could prove to be useful in many future network analyses. The RSA was used to find the relative connectivity of the overall network and clusters that formed by removing the in-between edges. The relative depth proved to be a good index to analyse the damage level of the network.

8.6 Key Indices

Key indices used in this research are tabulated against the assemblies in Table 8.3. While the Bristol method accounts for many of the structural features in a structure, its implementation is not so straightforward for transferring to other assemblies such as roads. The time tested Newman methods are probably the most versatile, based as they are on even more established graph theory principles. The route structure analysis, relatively unknown to the structural engineering community, also appears to be promising.

Table 8.3 Key indices in this research

Analysis Method	Index	Trusses	Damaged Frame	Roads
Bristol approach	Wellformedness	✓	✓	
	Relative separateness		✓	
Newman's method	Relative betweenness	✓	✓	(✓)
RSA	Relative connectivity	✓	✓	✓
	Relative depth	✓	✓	✓

9 Conclusions

The following are the conclusions from this research.

- The concept of structural connectivity for assemblies is defined as follows; "In a field of a finite number of joints/nodes/points of interests connected by a finite number of members/paths/connections, the concept of 'structural connectivity' seeks to not only assess how strongly the members are connected at a given joint but also how strongly the groups of members are connected to each other."
- All selected analysis methods are identified to have their own merits. However, out of the selected analysis methods, Bristol approach (wellformedness) is tentatively recommended as the index for measuring the structural connectivity of structures. This is because it uses direct structural mechanics properties in the analysis.
- The importance of the unweighted analysis is realised in this research as it reveals the form that can have the highest structural connectivity in the weighted analysis. The Route Structure Analysis (RSA) is recommended as the preferred unweighted analysis method as it takes the spatial relationship of the structure into account in the analysis.
- Newman's method proved to be the most flexible one, given its adaptability for several uses. Being able to determining appropriate weights for each analysis is the strongest characteristic of this analysis method. Though Bristol approach method is recommended as the measure for structural connectivity, Newman's method should be appreciated for being able to determine the weakest link in the structure at the first step of the analysis with acceptable accuracy.
- In addition to the measurement indices, the penultimate cluster of Bristol approach and generation 01 of Newman's method gives important information regarding structural connectivity and weakly connected zones/members in a structure.
- Though some of results from the different analysis methods are similar to each other, similarity is not consistent throughout the results. This is particularly noted in the determination of weak links. However, the results did lead to a convergence in the identification of the ends and mid-span of trusses as the general location of weak links. It was also identified that chord members are the weak links in all scenarios.
- Bristol approach shows the truss T3, Fan type truss, to have the highest structural connectivity followed by Warren type truss (truss T2). Both weighted Newman analysis and RSA show truss T2 to have the highest structural connectivity. Considering all the results, truss T2 is chosen to have a good structural form.

- It was observed that increasing the axial rigidity of the chord member benefits all the truss forms. This was observed in both weighted analysis methods. Increasing the axial rigidity of the chord members either increases the structural connectivity of the trusses and/or moves the weak link/ possible failure location from chord member to web member. Truss T2 gives highest percentage increase in wellformedness compared to the material cost incurred to achieve it.
- For the framed structure, analysis methods agree that middle column removal causes higher loss in connectivity than side column removal in the corresponding floor. The different methods indicate different middle column removals to have the highest effect on the structural connectivity.
- The Bristol approach and weighted Newman analysis show symmetric results- i.e. loss in structural connectivity for GMR & GSR is equal to TMR & TSR respectively. This might not be reflected in a real world context. The RSA shows that the loss in connectivity can be arranged as GMR> MMR> TMR> GSR> MSR> TSR.
- Based on this, it is recommended to incorporate spatial relationships of the structure in calculation of structural connectivity to a greater degree in future research.
- The concept of structural connectivity for structures is proved to be applicable to two dimensional networks other than structures. The A-level road network of Sri Lanka was successfully analysed using the concept of structural connectivity.
- The relative depth from RSA can be used as an index to measure the damage to two-dimensional assemblies.

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APPENDIX A: MATLAB Code

The MATLAB code to calculate betweenness stated in Newman's method is presented here. The code is broken into different executable M-files. Appendix A-1 gives the code for the normal network and Appendix A-2 the code for road networks. Though this might not be the most efficient coding, it was fit for its purpose.

Appendix A-1: MATLAB code for analysing normal network

M-file: Combined_input_code.m

```
% Input Area

M=xlsread('FPRTM21.xls');
C=xlsread('FC.xls');

% Creating basic adjacency matrix
adj=sparse(M(:,1),M(:,2),1);
adj=full(adj);

%assigning cordinates
nm=length(M(:,1));
nj=length(C(:,1));

n1=M(:,1);
n2=M(:,2);

y2 = zeros(nm,1); y1 = y2; x2 = y1; x1 = x2;
for i=1:1:nm
    x1(i,1)=C(n1(i,1),1);
    y1(i,1)=C(n1(i,1),2);
    x2(i,1)=C(n2(i,1),1);
    y2(i,1)=C(n2(i,1),2);
end

% Assigning other charecteristics

A=M(:,5);
I=M(:,6);
E=M(:,7);
R=M(:,8);

% Length computation
l=zeros(nm,1);
for i=1:nm
l(i)=hypot((x2(i)-x1(i)),(y2(i)-y1(i)));
end

% creating full adjacency matrix
n=length(adj);
[a,b]=size(adj);
adj1=zeros(max(a,b));
adj1(1:a,1:b)=adj;
adj=adj1;
% Check 1= Graph
```

```
G=graph(adj,'upper');

plot(G,'XData',C(:,1),'YData',C(:,2),'EdgeColor','black','NodeColor'
,'black');
 % Full matrix
 AdjM2=adj+adj';

dof= zeros(6,1,nm);
doft=zeros(4,1,nm);
T = zeros(6,6,nm);
K_e_local= zeros(6,6,nm);
K_e_global= zeros(6,6,nm);
K_t_local=zeros(2,2,nm);
K_t_global=zeros(4,4,nm);
K_assembled = zeros(3*nj,3*nj);
K_tassembled = zeros(2*nj,2*nj);
Tt = zeros(2,4,nm);

K_f=zeros(6*nm,6);
K_f_W1=zeros(nm,2);
K_f_W=zeros(nm,1);
for i=1:nm
    T(:,:,i)=matrix_tr(l(i),x1(i),y1(i),x2(i),y2(i)); % creating
transformation matrix for frame
    K_e_local(:,:,i) = s_matrix(l(i),A(i),I(i),E(i));% creating
stiffness matrix
    al1=l(i);aA1=A(i);aI1=I(i);aE1=E(i);
    b1=det(K_e_local(:,:,i));b2=4*aE1*aI1/(al1^2);
    K_f_W1(i,:)=[b1,b2];
    K_f_W(i,1)=K_f_W1(i,R(i,1)); % assigning newman weight based on
column/ beam
    K_e_global(:,:,i) = T(:,:,i)'*K_e_local(:,:,i)*T(:,:,i); %
global stiffness matrix
     csvwrite('tryf.csv',K_e_global(:,:,i));
    K_f(6*i-5:6*i,1:6)=csvread('tryf.csv'); % global storage
differently

    dof(:,:,i) = [3*n1(i)-2; 3*n1(i)-1; 3*n1(i); 3*n2(i)-2; 3*n2(i)-
1; 3*n2(i)];
    K_assembled(dof(:,:,i),dof(:,:,i)) =
K_assembled(dof(:,:,i),dof(:,:,i)) + K_e_global(:,:,i); % assembled
frame matrix
end
K_t=zeros(4*nm,4);
K_t_W=zeros(nm,1);
for i=1:1:nm
    Tt(:,:,i)=matrix_ttr(l(i),x1(i),y1(i),x2(i),y2(i));
    K_t_local(:,:,i) =  E(i)*A(i)/l(i)*[1 -1;-1 1];
    K_t_W(i,1)=E(i)*A(i)/l(i);
    K_t_global(:,:,i) = Tt(:,:,i)'*K_t_local(:,:,i)*Tt(:,:,i);
    csvwrite('tryi.csv',K_t_global(:,:,i));
    K_t(4*i-3:4*i,1:4)=csvread('tryi.csv');
    doft(:,:,i) = [ 2*n1(i)-1; 2*n1(i);2*n2(i)-1; 2*n2(i)];
    K_tassembled(doft(:,:,i),doft(:,:,i)) =
K_tassembled(doft(:,:,i),doft(:,:,i)) + K_t_global(:,:,i);
end

save('K_t.mat','K_t') % global stiffness matrix for truss separetely
save('K_f.mat','K_f') % global stiffness matrix for frame separetely
save('K_tas.mat','K_tassembled')
save('K_fas.mat','K_assembled')
```

```
save('t_W.mat','K_t_W')
save('f_W.mat','K_f_W')
save('AdjM2.mat','AdjM2') %full adjacency matrix
save('Location.mat','C')
save('Prop.mat','M')
save('length.mat','l')
```

M_file: matrix_tr.m

```
function [T] = matrix_tr(l,x1,y1,x2,y2)
cost = (x2-x1)/l;
sint = (y2-y1)/l;

k = [cost sint 0;-sint cost 0;0 0 1];
y = zeros(3);

T = [k y;y k];

end
```

M_file: matrix_ttr.m

```
function [T] = matrix_tr(l,x1,y1,x2,y2)
cost = (x2-x1)/l;
sint = (y2-y1)/l;


T = [cost sint 0 0;0 0 cost sint];

end
```

M_file: s_matrix.m

```
function [K_local] = s_matrix(l,A,I,E)

a = E*A/l;
b = 12*E*I/(l^3);
c = 6*E*I/(l^2);
d = 4*E*I/(l);
e = 2*E*I/(l);

K_local = [a 0 0 -a 0 0;0 b c 0 -b c;0 c d 0 -c e;-a 0 0 a 0 0;0 -b
-c 0 b -c;0 c e 0 -c d];

end
```

M_file: s_tmatrix.m

```
function [K_local] = s_tmatrix(l,A,E)

a = E*A/l;

K_local = [a -a;-a a];
```

```
end
```

M_file: Jointsiffness.m

```
clc
clear all
load('K_fas.mat')
af=length(K_assembled)/3;
Kf=zeros(3,3,af);
Cf=zeros(af,1);
for i=1:1:af
    Kf(:,:,i)=K_assembled(3*i-2:3*i,3*i-2:3*i);
    Cf(i,1)=det(Kf(:,:,i));
end
load('K_tas.mat')
at=length(K_tassembled)/2;
Kt=zeros(2,2,at);
Ct=zeros(at,1);
for i=1:1:at
    Kt(:,:,i)=K_tassembled(2*i-1:2*i,2*i-1:2*i);
    Ct(i,1)=det(Kt(:,:,i));
end

save('C_t.mat','Ct')
save('C_f.mat','Cf')
```

M_file: MultiSPaths.m

```
function [P D TW]= MultiSPaths(AdjR)
n=length(AdjR);
D=ones(n)*10000;
P=cell(n);
for i=1:1:n
    AdjMod=AdjR;
    D(i,i)=0;
    qu=i;
    Vi=zeros(1,n);
    while(~isempty(qu))
        v=qu(1);
        for j=1:1:n
            if (AdjMod(v,j)==1)
                if (Vi(v)==0)
                    qu=cat(2,qu,j);
                    if(D(i,j)==10000)
                        D(i,j)=D(i,v)+1;
                    end
                    if (D(i,j)==D(i,v)+1)
                        P{i,j}=cat(2,P{i,j},v);
                    end
                end
            end
        end
        qu(1)=[];
        Vi(v)=1;
    end
end
Pa=cell(n);
```

```
for i=1:1:n;
    for j=1:1:n;
        Pa{i,j}=zeros(n);
    end
end
for i=1:1:n;
   for j=1:1:n
       Tr=j;
       PaMat=zeros(n);
       while(~isempty(Tr))
               PaMat(P{i,Tr(1)},Tr(1))=1;
               Tr=cat(2,Tr,P{i,Tr(1)});
               Tr(1)=[];
       end
       Pa{i,j}=PaMat;
   end
end
W=cell(n);
for i=1:1:n
    for j=1:1:n
        W{i,j}=zeros(n);
    end
end
for i=1:1:n
    for j=1:1:n
        WMat=zeros(n);
        PaMat=Pa{i,j};
        MBr=max(sum(PaMat,2));
        We=1;
        WStc=[j];
        while(~isempty(WStc))
            Po=WStc(end);
            WStc(end)=[];
            WStc=cat(2,WStc,P{i,Po});
            WMat(P{i,Po},Po)=WMat(P{i,Po},Po)+We;
        end
        NPa=sum(WMat(i,P{j,i}));
        if(NPa~=0)
            W{i,j}=WMat/NPa;
        end
    end
end
TrW=cell(n);
for i=1:1:n
    for j=1:1:n
        TrW{i,j}=zeros(n);
    end
end
for i=1:1:n
    for j=1:1:n
        TrWMat1=W{i,j};
        TrWMat2=W{j,i};
        Tmp=zeros(n);
        for k=1:1:n
            for m=1:1:n
                Tmp(k,m)=max(TrWMat1(k,m),TrWMat2(m,k));
            end
        end
        TrW{i,j}=Tmp;
        TrW{j,i}=Tmp';
    end
```

```
end
TW=zeros(n);
for i=1:1:n
    for j=1:1:n
        TW=TW+TrW{i,j};
%           TW=TW+W{i,j};
    end
end
for i=1:1:n
    for j=1:1:n
        TW(i,j)=TW(i,j)/(n*(n-1));
    end
end
```

M_file: EndPlot.m

```
function EndPlot(AdjR,C,TW)
% St=['ABCDEFGHIJKLMNO'];
n=length(AdjR);
St=['ABCDEFGHIJKLMNOPQRSTUVWXYZabcd'];
F=figure;
global ima;
for i=1:1:n
    for j=1:1:n
        x=[C(i,1) C(j,1)];
        y=[C(i,2) C(j,2)];
        if(AdjR(i,j)==1)
            plot(x,y,'Color',[0,0,0]','LineWidth',TW(i,j)*0.7);
            %axis([-1 8 -1 3]);
            axis([-1 17 -1 17]);
            axis equal;
            hold on;
        end
    end
end
S=[num2str(ima) '.jpg'];
saveas(F,S);
```

M_file: DGen.m

```
function [TWG]= DGen(t,Ge,WF,Li) % t= frame/ truss, Ge= Number of
generations, WF= Weight factors
close all
clc
load AdjM2;
load Location;
load length;
n=length(AdjM2);
TWN=cell(1,Ge);
AdjR=AdjM2;

load t_W;
%load f_W;
load C_f;
load Prop;

Adjtemp=AdjM2;
for i=1:1:length(AdjM2)
```

```
    for j=1:1:length(AdjM2)
        if(AdjM2(i,j)==1)
            Adjtemp(i,j)=Cf(j);
        end
    end
end

Adjsum=zeros(length(AdjM2),1);
for i=1:1:length(AdjM2)
Adjsum(i,1)=sum(Adjtemp(i,:));
end

Adjtemp2=zeros(length(AdjM2),length(AdjM2));
for i=1:1:length(AdjM2)
    for j=1:1:length(AdjM2)
        Adjtemp2(i,j)=Cf(i)*Adjtemp(i,j)/Adjsum(i);
    end
end

K_f_W= zeros(length(AdjM2),length(AdjM2));
for i=1:1:length(AdjM2)
    for j=1:1:length(AdjM2)
        if (j>i)
            K_f_W(i,j)=Adjtemp(i,j)+Adjtemp(j,i);
        end
    end
end

n1=M(:,1);
n2=M(:,2);
e1=M(:,5);

StiffnessMat=ones(sum(sum(AdjM2))/2,1);
StiffnessMat(:,1)=K_t_W;

WeightAdj=zeros(n,n);

for i=1:1:length(StiffnessMat)
    WeightAdj(n1(i),n2(i))=StiffnessMat(i,1);
end

if (t)
    WeightAdj=K_f_W;
end

if (Li)
    for i=1:1:length(StiffnessMat)
        WeightAdj(n1(i),n2(i))=e1(i,1);
    end
end

tot=sum(sum(WeightAdj))/((sum(sum(AdjM2))/2));

for i=1:n
    for j=1:n
        if WeightAdj(i,j)>0
            WeightAdj(i,j)=round(WeightAdj(i,j)/tot,2);
        end
    end
```

```
end

WeightAdj=WeightAdj+WeightAdj';
global ima;
ima=0;
z1=zeros(Ge,2);
for i=1:1:Ge
    [P D TW]= MultiSPaths(AdjR);
    if WF>0
        for a=1:1:n
            for b=1:1:n
                if WeightAdj(a,b)== 0
                    TW (a,b)=0;
                else
                    TW (a,b)=TW (a,b)/WeightAdj(a,b);
                end
            end
        end
    end
    TWG{i,i}=TW;
    aa=sum(sum(TW))/2;
    if aa<10
        power=(ceil(log10(aa)-1))-1;
        TW=TW/10^power;
        TW=round(TW*1000)/1000;
        TW=TW*10;
        ima=i;
        EndPlot(AdjR,C,TW);title(strcat('Generation :',num2str(i)));
        TW=TW/10;
        TW=TW*10^power;
    else
        TW=round(TW*100)/100;
        ima=i;
        EndPlot(AdjR,C,TW);title(strcat('Generation :',num2str(i)));
    end
    maxW=max(max(TW));
    z1(i,1)=maxW;
    TootW=sum(sum(TW))/2;
    z1(i,2)=TootW;
    [B]=find(TW==maxW);
    AdjR(B)=0;
end
xlswrite('aaaa.xls',z1);
```

Analysis procedure:

1. Run the “Combined_input_code.m” after adjusting the input values.
2. Run the “Jointsiffness.m” to determine the joint stiffnesses for Bristol approach.
3. Run the “DGen.m” inputting the values to determine the nature of the analysis such as truss/frame and weighted/ unweighted.

   t=1 => frame

   Ge => number of generations

   WF=1 => weighted analysis

Appendix A-2: MATLAB code for analysing road network

M_file: Combined_input_code.m

```
% Input Area

M=xlsread('RL6.xls');
C=xlsread('RC6.xls');

% Creating basic adjacency matrix
adj=sparse(M(:,1),M(:,2),1);
adj=full(adj);

%assigning cordinates
nm=length(M(:,1));
nj=length(C(:,1));

n1=M(:,1);
n2=M(:,2);

y2 = zeros(nm,1); y1 = y2; x2 = y1; x1 = x2;
for i=1:1:nm
    x1(i,1)=C(n1(i,1),1);
    y1(i,1)=C(n1(i,1),2);
    x2(i,1)=C(n2(i,1),1);
    y2(i,1)=C(n2(i,1),2);
end

% Assigning other charecteristics

l=M(:,4);

% creating full adjacency matrix
n=length(adj);
[a,b]=size(adj);
adj1=zeros(max(a,b));
adj1(1:a,1:b)=adj;
adj=adj1;
% Check 1= Graph
G=graph(adj,'upper');

% G1=graph(n1,n2,l);
x=C(:,1);
y=C(:,2);
D=distances(G);
p=plot(G,'XData',x,'YData',y,'EdgeColor',[0 0 0]);
axis equal;
axis([0 320 0 470]);

%
plot(G,'XData',C(:,1),'YData',C(:,2),'EdgeColor','black','NodeColor'
,'black');
%  % Full matrix
 AdjM2=adj+adj';


save('AdjM2.mat','AdjM2') %full adjacency matrix
save('Location.mat','C')
```

```
save('Prop.mat','M')
save('length.mat','l')
save('D.mat','D')
clear all
```

M_file: kneighbors.m

```
function kneigh = kneighbors(adj,ind,k)

adjk = adj;
for i=1:k-1; adjk = adjk*adj; end;

kneigh = find(adjk(ind,:)>0);


end;
```

M_file: numedges.m

```
function m = numedges(adj)

sl=selfloops(adj); % counting the number of self-loops

if issymmetric(adj) & sl==0    % undirected simple graph
    m=sum(sum(adj))/2;
    return
elseif issymmetric(adj) & sl>0
    sl=selfloops(adj);
    m=(sum(sum(adj))-sl)/2+sl; % counting the self-loops only once
    return
elseif not(issymmetric(adj))   % directed graph (not necessarily
simple)
    m=sum(sum(adj));
    return
end
```

M_file: numnodes.m

```
function n = numnodes(L)

n = length(L);
end;
```

M_file: edge_betweenness.m

```
function ew = edge_betweenness(adj)

el=adj2edgeL(adj);  % the corresponding edgelist
n = size(adj,1);  % number of nodes
m = numedges(adj);  % number of edges

ew = zeros(size(el,1),3); % edge betweenness - output
```

```
for s=1:n % across all (source) nodes

    % compute the distances and weights starting at source node i
    d=inf(n,1); w=inf(n,1);
    d(s)=0; w(s)=1; % source node distance and weight
    queue=[s];      % add to queue
    visited=[];

    while not(isempty(queue))
        j=queue(1); % pop first member
        visited=[visited j];
        neigh=kneighbors(adj,j,1); % find all adjacent nodes, 1 step
away

        for x=1:length(neigh)  % add to queue if unvisited
            nei=neigh(x);

            if isempty(find(visited==nei)) &
isempty(find(queue==nei)); queue=[queue nei]; end

        end
        for x=1:length(neigh)

            nei=neigh(x);
            if d(nei)==inf   % not assigned yet
                d(nei)=1+d(j);
                w(nei)=w(j);
            elseif d(nei)<inf & d(nei)==d(j)+1  % assigned already,
add the new path
                w(nei)=w(nei)+w(j);
            elseif d(nei)<inf & d(nei)<d(j)+1
                'do nothing';
            end
        end
        queue=queue(2:length(queue));  % remove the first element
    end

    eww = zeros(size(el,1),3);   % edge betweenness for every source
node (iteration)

    % find every leaf - no path from "s" to other vertices goes
through the leaf
    leaves = find(d==max(d)); % farthest away from source
    for l=1:length(leaves)
        leaf=leaves(l);
        neigh=kneighbors(adj,leaf,1);
        nei2rem=[];
        for x=1:length(neigh)

            if isempty(find(leaves==neigh(x))); nei2rem=[nei2rem
neigh(x)]; end

        end
        neigh=nei2rem;  % remove other leaves among the neighbors
        for x=1:length(neigh)
            indi=find(el(:,1)==neigh(x));
            indj=find(el(:,2)==leaf);
            indij=intersect(indi,indj);   % should be only one
element at the intersection
```

```
            eww(indij,3)=w(neigh(x))/w(leaf);
        end
    end

    dsort=unique(d);
    dsort=-sort(-dsort);  % reverse sort of unique distance values

    for x=1:length(dsort)
        leaves=find(d==dsort(x));
        for l=1:length(leaves)
            leaf=leaves(l);
            neigh=kneighbors(adj,leaf,1);
            up_neigh=[]; down_neigh=[];
            for x=1:length(neigh)
                if d(neigh(x))<d(leaf)
                    up_neigh=[up_neigh neigh(x)];
                elseif d(neigh(x))>d(leaf)
                    down_neigh=[down_neigh neigh(x)];
                end
            end
            sum_down_edges=0;
            for x=1:length(down_neigh)
                indi=find(el(:,1)==leaf);
                indj=find(el(:,2)==down_neigh(x));
                indij=intersect(indi,indj);
                sum_down_edges=sum_down_edges+eww(indij,3);
            end
            for x=1:length(up_neigh)
                indi=find(el(:,1)==up_neigh(x));
                indj=find(el(:,2)==leaf);
                indij=intersect(indi,indj);

eww(indij,3)=w(up_neigh(x))/w(leaf)*(1+sum_down_edges);
            end
        end
    end

    for e=1:size(ew,1); ew(e,3)=ew(e,3)+eww(e,3); end

end

for e=1:size(ew,1)
    ew(e,1)=el(e,1);
    ew(e,2)=el(e,2);
    %ew(e,3)=ew(e,3)/n/(n-1);   % normalize by the total number of
paths
end
```

M_file: edge_betweennessrr.m

```
function ew= edge_betweennessrr(adj,adjW)

el=adj2edgeL(adj);
n=numnodes(adj);
m=numedges(adj);

ew=zeros(size(el,1),3);
ew(:,1)=el(:,1);
```

```
ew(:,2)=el(:,2);
resultmat=zeros(n,2,n);

for s=1:n
    d=inf(n,1);
    d(s)=0;
    for trial=1:10
        queue=[s];
        visited=[];

        while not(isempty(queue))
           j=queue(1);
           visited=[visited j];
           neigh=kneighbors(adj,j,1);

           for x=1:length(neigh)
               nei=neigh(x);
               if isempty(find(visited==nei)) &
isempty(find(queue==nei))
                   queue=[queue nei];
               end
           end

           for x=1:length(neigh)
               nei=neigh(x);
               if d(nei)==inf
                   d(nei)=adjW(j,nei)+d(j);
               elseif d(nei)<inf & round(d(nei)-
(d(j)+adjW(j,nei)))>0
                   d(nei)=d(j)+adjW(j,nei);
               elseif d(nei)<inf & floor(10*abs(d(nei)-
(d(j)+adjW(j,nei))))==0
                   continue
               elseif d(nei)<inf & round(d(nei)-
(d(j)+adjW(j,nei)))<0
                   continue
               end
           end

           queue=queue(2:length(queue));
        end
    end


    Dcell=cell(1,length(d));
    Ucell=cell(1,length(d));

    for i=1:length(d)
        if i==s
            continue
        else
            up_neigh=[];
            down_neigh=[];
            neigh=kneighbors(adj,i,1);
            for y=1:length(neigh)
                bark=neigh(y);
                A1=d(i)-d(bark)-adjW(bark,i);
                A2=d(bark)-d(i)-adjW(i,bark);
                A3=d(i)-d(bark);
                if (d(bark)<d(i)) & (round(A1)==0)
```

```
                    up_neigh=[up_neigh bark];
                elseif (d(bark)>d(i)) & (round(A2)==0)
                    down_neigh=[down_neigh bark];
                elseif round(A3)==0
                    continue
                end
            end
            Ucell{i}= up_neigh;
            Dcell{i}= down_neigh;
        end
    end

    asort=unique(d);
    asort= sort(asort);
    w=zeros(n,1);
    w(s)=1;
    visit=[];
    for x=1:length(asort)
        roots=find(d==asort(x));
        for r=1:length(roots);
            root=roots(r);
            A=Ucell{root};
            for y=1:length(A)
                w(root)=w(root)+w(A(y));
            end
            visit=[visit root];
        end
    end



    eww=zeros(size(ew,1),1);
    dsort=unique(d);
    dsort=-sort(-dsort);
    for x=1:length(dsort)
        if dsort(x)==inf
            continue
        else
            leaves=find(d==dsort(x));
            for l=1:length(leaves)
                leaf=leaves(l);
                A=Ucell{leaf};
                B=Dcell{leaf};
                sum_down_edges=0;

                if ~isempty(B)
                    for y=1:length(B)
                        indi=find(ew(:,1)==leaf);
                        indj=find(ew(:,2)==B(y));
                        indij=intersect(indi,indj);
                        sum_down_edges=sum_down_edges+eww(indij,1);
                    end
                end
                for y=1:length(A)
                    indi=find(ew(:,1)==A(y));
                    indj=find(ew(:,2)==leaf);
                    indij=intersect(indi,indj);

eww(indij,1)=(w(A(y))/w(leaf))*(1+sum_down_edges);
                end
            end
```

```
        end
    end

    for e=1:size(ew,1);
        ew(e,3)=ew(e,3)+eww(e,1);
    end
    resultmat(:,1,s)=d;
    resultmat(:,2,s)=w;
end
    for e=1:size(ew,1);
        ew(e,3)=ew(e,3)/n/(n-1);
    end

save('resultmat.mat','resultmat')
```

M_file: RGen.m

```
function [TWN]= RGen(Ge,WF) % Ge= Number of generations, WF= Weight
factors
close all
clc
load AdjM2;
load Location;
load length;
n=length(AdjM2);
TWN=cell(1,Ge);
AdjR=AdjM2;


load Prop;

n1=M(:,1);
n2=M(:,2);
prt=M(:,4);

WeightAdj=zeros(n,n);

  for i=1:1:length(n1)
        WeightAdj(n1(i),n2(i))=l(i,1);
  end

WeightAdj=WeightAdj+WeightAdj';


global ima;
ima=0;
z1=zeros(Ge,2);

for i=1:1:Ge
    TW= zeros(n,n);
    ew= edge_betweenness(AdjR);
    if WF>0
        ew=edge_betweenessrr(AdjR,WeightAdj);
    end
    load resultmat;
    for j=1:size(ew,1)
        TW(ew(j,1),ew(j,2))= ew(j,3);
    end
```

```
    for a=1:n
        for b=1:n
            TW(a,b)=round(TW(a,b)*10000)/10000;
        end
    end

    Lower=zeros(n,n);
    Upper=Lower;
    Check=Lower;
    for a=1:n
        for b=1:n
            if a==b
                continue
            elseif a>b
                Lower(a,b)=TW(a,b);
            elseif a<b
                Upper(a,b)=TW(a,b);
            end
        end
    end
    Check=Upper-Lower';
    if sum(sum(Check))>0
        continue
    end
    TWN{1,i}=TW;
    aa=sum(sum(TW))/2;

        ima=i;
        EndPlot(AdjR,C,TW);title(strcat('Generation :',num2str(i)));

        maxW=max(max(TW));
    z1(i,1)=maxW;
    TootW=sum(sum(TW))/2;
    z1(i,2)=TootW;
    [B]=find(TW==maxW);
    AdjR(B)=0;
end
xlswrite('aaaa.xls',z1);
```

Analysis procedure:

1. Run the “Combined_input_code.m” after adjusting the input values.
2. Run the “RGen.m” inputting the values to determine the nature of the analysis.

   Ge => number of generations

   WF should be given as 1 to enable weighted shortest paths analysis.