DEVELOPMENT OF A MULTIPURPOSE ROPE CLIMBING ROBOT

A dissertation submitted to the
Department of Electrical Engineering, University of Moratuwa
in partial fulfillment of the requirements for the
degree of Master of Science

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2010

Abstract

Wheeled vehicles or robots can achieve high speed locomotion with a relatively lower control complexity compared to other forms such as legged, hopping, or slithering robots. However, wheeled vehicles have several limitations on rough and uneven surfaces. For instance, wheel robots are much speedier than legged counterpart, but wheeled rope climbing robots are supposed to have very smooth ropes. Moreover, wheels come into contact with all details of the terrain causing higher energy losses in friction. More specifically, this causes higher losses on soft contact surfaces like a slacked rope. "Development of a multipurpose rope climbing robot" research is mainly focusing on developing a rope climbing robot that can be applied in various practical situations. Objective of this research is to design a four legged rope climbing robot that smoothly and steadily moves on a rope. Different mechanisms such as wheeled robot, brachalion robot, crawling type robots and different kind of legged robots have been developed to achieve this target and they have their own advantages and disadvantages. The proposed robot is a four legged robot and each leg has two degrees of freedom. One degree of freedom to rotate the leg forward and backward and second degree of freedom is for grip the rope by each leg. This robot is always grip the rope very steadily and moves on rope very smoothly. These two characteristics are important to carry a weight with a robot and easily can maintain an overall stability of the robot. The robot is planned to work on various situations and the nature of the rope or characteristics such as size of the rope, whether the rope vertical or horizontal directional and whether the rope is having some obstacles like knots or bend at some points on the rope. The robot is planned to have, the design is simple as possible and overall cost is minimized for use the robot in practical situations. For rescue operations, military operations, scientific researches operate on danger areas for humans and specific rope climbing operations can be achieved by the proposed robot. Results show the effectiveness of the proposed methodology and the practical implementation.

Keywords: Rope climbing robot, Legged robot, Brachiation robot, Mobile robotic systems, Micro-robotics, Robot controllers, Crawling type robots

DECLARATION

The work submitted in this dissertation is the result of my own investigation, except where otherwise stated.

It has not already been accepted for any degree, and is also not being concurrently submitted for any other degree.

UOM Verified Signature

N. D. Hewapathirana

We/I endorse the declaration by the candidate.

UOM Verified Signature

Prof. Lanka Udawatta

Contents

Acknowledgment

Thanks are due first to my project supervisor, Professor Lanka Udawatta and co-project supervisor Dr. Thrishantha Nanayakkara, for their great insights, perspectives, guidance and sense of humor. I would like to specially thank head of the department Dr. J. P. Karunadasa and coordinator Dr. Chandima Pathirana. Professor Sisil Kumarawadu, Dr. Nalin Wickramarachchi and Dr. Chandima Pathirana provided me very valuable comments and their feedback at each progress review meetings lead me through new paths to further develop my research. My sincere thanks go to the officers in Post Graduate Office, Faculty of Engineering, University of Moratuwa, Sri Lanka for helping in various ways to clarify the things related to my academic works in time with excellent cooperation and guidance. Sincere gratitude is also extended to the people who serve in the Department of Electrical Engineering office.

Lastly, I should thank many individuals, friends and colleagues who have not been mentioned here personally in making this educational process a success. May be I could not have made it without your supports.

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Chapter 1

1. Introduction

This research, titled "Development of a multipurpose rope climbing robot" is intended to design and constructing a rope climbing robot for general purposes. The proposed robot will be constructed to climb vertical or horizontal directional ropes. The robot is planed to work on wide-range of environments where the characteristics, physical limitations and obstacles on the rope are not much limiting factors to the movement of the robot. The grippers of the robot are designed to grip different kind of ropes. The robot can work with various types of ropes that have different diameters values and roughness values. The proposed robot is planned to have four legs for better steady gripping while the robot is moving and carrying a weight on it.

Wheeled vehicles or robots can achieve high speed locomotion with a relatively lower control complexity compared to other forms such as legged, hopping, or slithering robots. However, wheeled vehicles have several limitations on rough and uneven surfaces. For instance, wheel robots are much speedier than legged counterpart, but wheeled rope climbing robots are supposed to have very smooth ropes. If the rope is having obstacles wheel robots are not performed well whereas legged robots are performed with much higher efficiency. Moreover, wheels come into contact with all details of the terrain causing higher energy losses in friction. More specifically, this causes higher losses on soft contact surfaces like a slacked rope. The proposed four legged robot is designed to climb vertical or horizontal directional ropes with higher efficiency. The design of the robot is specially emphasized so that the robot should be able to grip different kinds of ropes with different diameters and roughness values.

Presently, there are only a handful of researches have been carried out for serve the above purpose. Few of the wheel robots have been constructed by different groups for rope climbs which they have their own advantages and disadvantages. Wheel robots are much more speedier than legged robot, but wheeled rope climbing robots are expect very smooth ropes for the propagation. If the rope is having some obstacles wheel robots do not perform well whereas legged robots are perform much higher degree than wheeled robots. The proposed legged robot will have greater number of degrees of freedom than wheeled robots. Brachalion robots and crawling type robots are also try to conquer the rope climbing world and they succeeded to a certain degree.

The proposed robot is expected to have following characteristics. The overall complexity of the robot construction should be minimal to reduce the total cost for practical usage of the robot. Smooth operation is important to carry a weight very smoothly. For example we can use this type of rope climbing robot for rescue missions and if the robot is carrying a wounded person it should move very smoothly and steadily. Rope's physical characteristics such as size, roughness etc should not be much of the limited factors for the robot movement.

1.1. Social and Scientific Value of the Research

1. The proposed robot is designed for multipurpose rope climbing applications. This robot can be used for rescue operations, military operations and any place that is risky for humans to climb by the rope etc. Most of the developed countries are using robots for such operations as possible. But developing countries are still reluctant to use robots for their activities. This is due to various reasons such as financial difficulties, lack of technical knowledge and poor trust on the robot machines etc. This research is planning to construct low cost rope climbing robot for general usage.
2. Most of the time when a fire taken place in a building, rescue person is using manual methods to save the victims of the incident. They use ropes from top of the building and climbing on them for save life of wounded persons. But that maneuver is a huge risk for the rescue personal as well as the wounded victim. The proposed type robot can be used for high risk operations and minimize the risk on human officers.
3. In Sri Lanka, there are no fully automated, semi-automated or manual controlled rope climbing robots are being used for practical operations. There are no frequent large scale natural disasters occurring in Sri Lanka. Still there are man-made disasters situations such as fire situations happening in buildings and terrorist attacks or high risk military operations etc. By using

automated robots for similar environments we can reduce the overall risk on the society.

4. Design and construct more generic rope climbing robot that is not depend on set of rope types. The media that can be found in the operating environment should be able to use as a rope or any existing rope like media. For example sometime after emergency situation happened we can not place special ropes for robot movements. If there are any ropes or bar already exist on that environment the robot should be able to operate on that media and work according to the situation.

1.2. Analyzed Robot Developments and Researches

Few of the robot developments done by various groups were analyzed to get better understanding about the current development in the rope climbing robot technology. The proposed robot is unique from some of the aspects of the existing robots. Following paragraphs will describe some of the analyzed robot developments in details.

1. SLOTH - Rope Climbing Robot [1]

SLOTH is a robot that is used for rope climbing. Studying the anatomy and movement of a living sloth has inspired SLOTH's mechanical design and it's climbing gaits. SLOTH robot is constructed by three small servos and is controlled through an SSC II serial servo controller. SLOTH robot can be used as a telepresence robotic system by carrying a small video camera to offer "visual access" in places where access by human presence is difficult and dangerous (like earthquake affected buildings, poisonous and toxic gas trapped rooms etc).

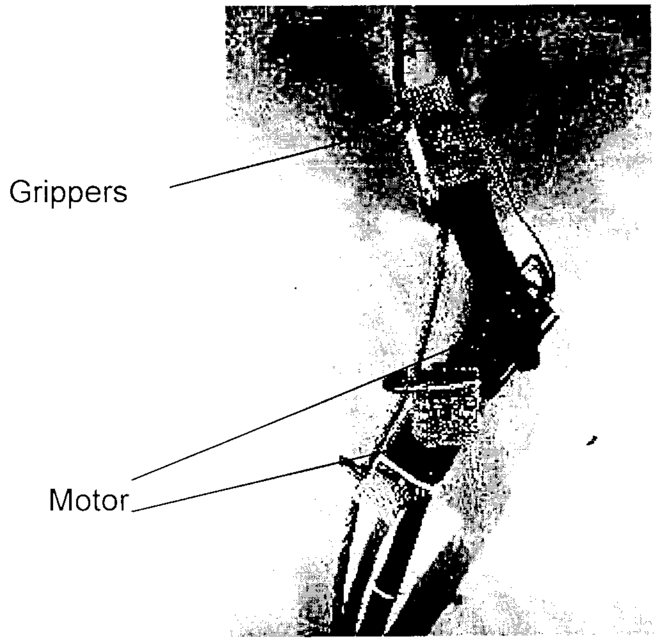

Figure 1.1: SLOTH - Rope Climbing Robot

2. Brachiating Robot by Professor : Toshio Fukuda's group

Professor Toshio Fukuda's group have been developed few brachiating robots for very similar operations like rope climbing. Two-link Brachiating Robot [2], 13-link Brachiation Robot [3] and Stair-Climbing of Multi-Locomotion Robot were analyzed prior to this proposal preparation. Two-link Brachiating Robot explore how dynamically dexterous tasks can be achieved using physical insight into the designated task and intrinsic dynamics of the system. Hierarchical behavior-based controller for dynamical motion, motion adaptation for the change of intervals between branches by off-line learning using steepest descent method and on-line adjustment using real-time video tracking system are major features of the 13-link Brachiation Robot. Stair-Climbing of Multi-Locomotion Robot is developed to realize the stair-climbing of the Multi-Locomotion Robot and establish a stable climbing method as the robot walks up the stair with railing.

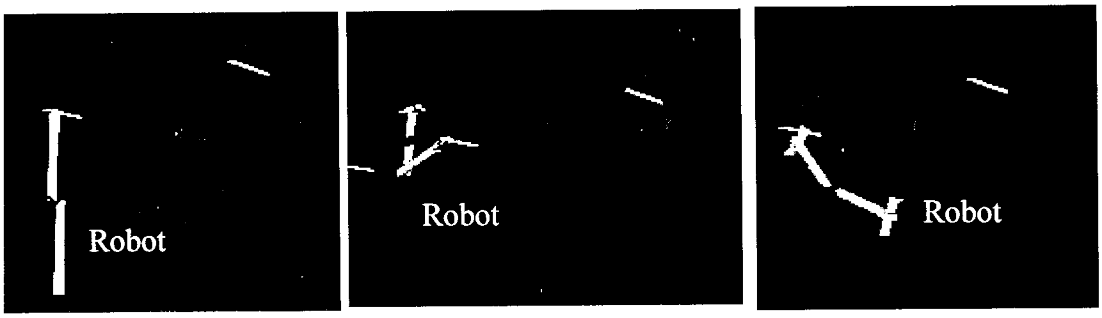

Figure 1.2: Two-link Brachiating Robot

Figure 1.2 shows how two-link brachiating robot is moving on the ropes. This figure illustrates three snapshots of the movement. This robot basically jumps from first point to second point and grab that point. If does not have very smooth operation, but as far as ropes are steady it can move steadily on the ropes. Figure 1.3 illustrates the advance version of two-link brachiating robot 13-link brachiating robot. This robot operates on little smooth track than previous one since it has move degrees of freedom. Figure 1.4 is about Stair-Climbing Multi-Locomotion Robot that is also share same design as previous with advance controller mechanisms.

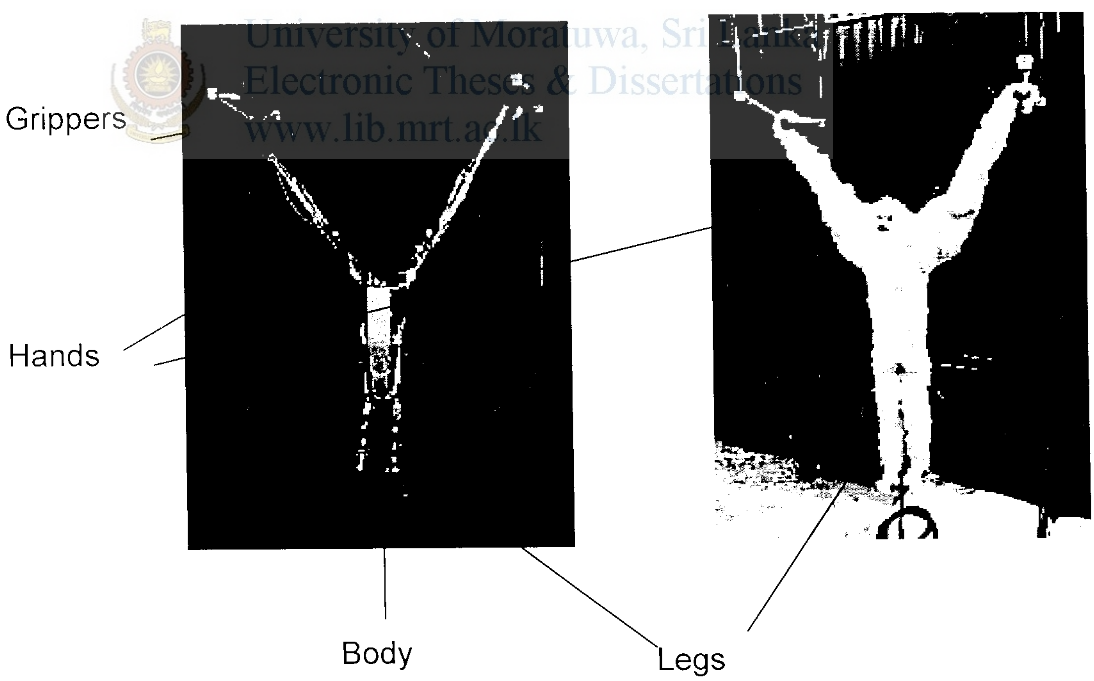

Figure 1.3: 13-link Brachiation Robot

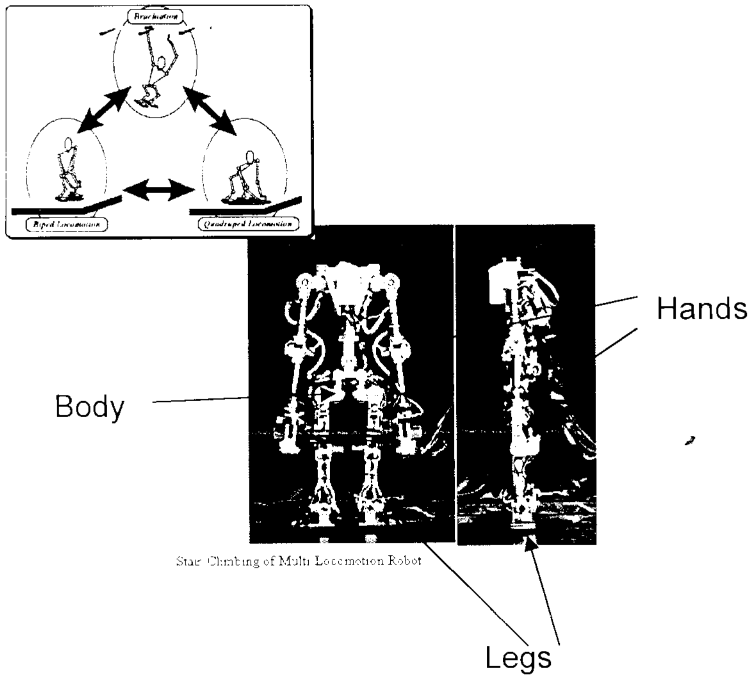

Figure 1.4: Stair-Climbing of Multi-Locomotion Robot

3. Rope Climbing Robot Inspects Wind Turbines

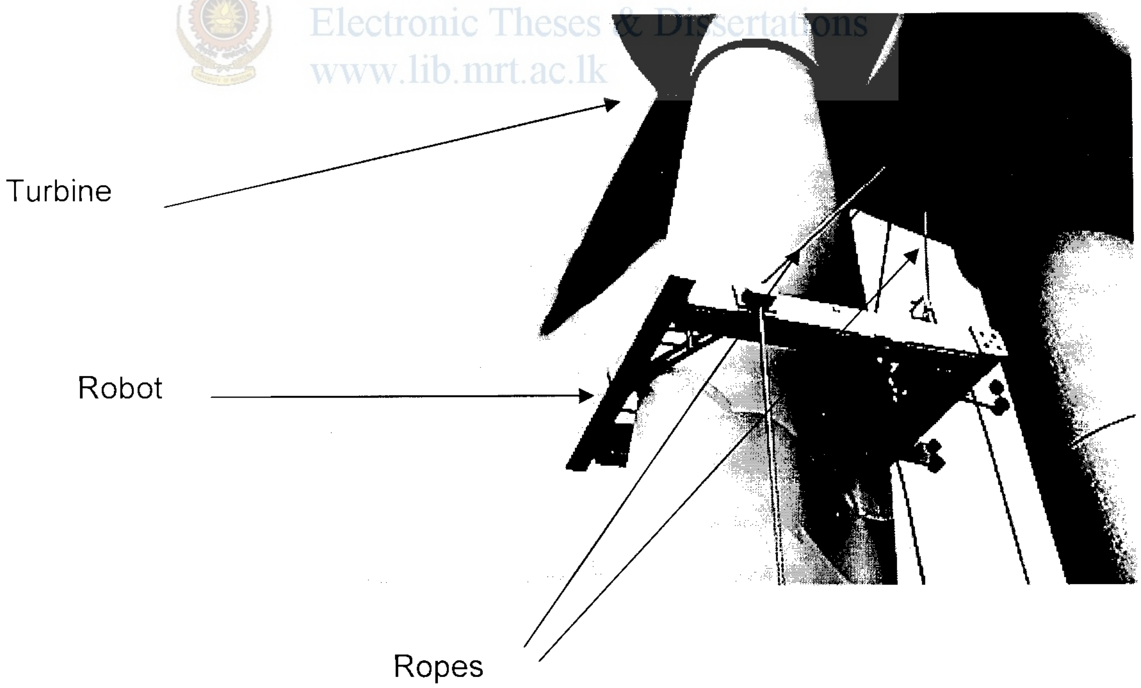

Figure 1.5: Rope Climbing Robot Inspects Wind Turbines

This rope climbing robot is used to inspect rotor blades of wind turbines. The inspection system consists of three elements: an infrared radiator conducts heat to the surface of the rotor blades and a high-resolution thermal camera records the temperature pattern and thus registers flaws in the material. In addition, an ultrasonic system and a high resolution camera are also on board, thus enabling the robot to also detect damage that would remain hidden to the human eye.

This design is based on wheeled mechanism and other mentioned robots are categorized as legged robots. Using set of wheels and motors the robot body moves upward and downward on the rope. This robot is having four ropes for higher stability and proper positioning of robot body on the wind turbine propeller wings.

4. Other robot developments

A wheeled robot that can run on a web by pinching the strings is presented in [5]. They proposed applying wheeled robots in assembling large structures in constructing Furoshiki satellite. In first step Furoshiki satellite is used to lay large web of strings that is used to crawls antenna elements on it using these wheeled robots. Next, all of the antenna elements crawl on the web with the help of the robots toward pre-allocated locations in order to realize a complete antenna. The biomimetic design and assembly of a 3g self-contained crawling robot fabricated through the integrated use of various micro-robot technologies is another successful attempt on the crawling type robots [6]. The hexapod structure is designed to move in an alternating tripod gait driven by two piezoelectric actuators connected by sliding plates to two sets of three legs. This research has successfully integrated various micro-robot technologies that have been developed including actuator design and fabrication, power and control electronics design, programming via a finite state machine, and the development of bio-inspired fiber arrays.

Chapter 2

2. Problem Statement

Rope climbing is a sport in which competitors, usually men, attempt to climb up a suspended vertical rope using only their hands. Not only as a sport, but also rope climbing is done for various other activities as well. Based on the experience human have gained, they have moved into construction of rope climbing machines and rope climbing robots. Different between ropes climbing machines and rope climbing robots are, machines are used to help people to climb ropes and robot are able to climb a rope their own. Climbing machines should control by a human and they have to use them to simplify their climbing work. Further moving to robot developments people have tried to construct robot that can climb automatically or semi-automatically.

Different types of rope climbing robots have being constructed and they are having their own advantages and disadvantages. Wheeled robots, legged robots, crawling type robots, brachalion type robots and hybrid robots with the combinations of above techniques are widely used in robotics developments. In rope climbing robots context different kind of wheeled robots are heavily used. Legged robots and brachalion robots are also play major role in rope climbing robot developments. With compare to the wheeled robots, legged robots are little slower. But wheeled robots always expected to have smooth ropes and they can be applied to predefined set of rope types only. Legged robots can be designed to have more generic than wheeled robots and they have more degrees of freedom. Also legged robots can be moved on the rope little steadily than wheeled robots and they can carry a weight on it smoothly.

In rope climbing robot context there are few limitations that designers can not totally eliminate. Basically any rope climbing robot design highly depends on the nature of the rope. The size of the rope, stiffness, roughness, whether the rope is shaking or steady, rope orientation and many other factors are there to consider when design a rope climbing robot. Different controller mechanisms might be needed when robot moving upward, downward and carrying a weight with it. Also rope climbing robot design should address how it works when some obstacles present on the rope.

2.1. Research Objectives

Design a Low Cost Multipurpose Rope Climbing Robot

This research is intended to design and constructing a rope climbing robot for general purposes. The robot is planned to have four legs and four grippers. Each leg and grip pair is having two degrees of freedom. The entire robot will have eight degrees of freedom. All joints are rotary joints and robot will move forward and backward using leg rotary joints. Grippers are not designed for specific type of ropes and generally it will try to grip any rope that can wrap by the gripper. At any given time at least two legs are steadily grip the rope and each leg functions individually. This method will make sure to have the robot move on a rope very steadily and smoothly. The robot design is expected to have low complex as possible to minimize the overall construction cost and maintenance cost. The low cost robots designs lead them to be used in various practical applications, without restricting robot researches into laboratories.

Design Kinematics and Dynamic model for rope climbing robot

Before construct robot prototype, kinematics and dynamic model are planned to prepare for further analyze research robot design. By doing a kinematics analysis tries to decide the minimum degrees of freedom needed to climb up a rope with a sag.

Simulate rope climbing robot and analyze the stability of the design

Based on the dynamic model, simulation is planned to carry for further analysis of the stability of the system. In this simulation, study how Gravitational force is apportioned to each motor torque for static stability. Also plane to analyze any friction coefficient for viscous friction.

Construct a rope climbing robot prototype

Upon successful completion of dynamic simulation of the robot design, it is planned to construct a proposed rope climbing robot prototype. This prototype will construct using 8 servo motors and micro-controller based control system. Each leg can operate separately for individual leg movements and to move robot forward and backward all four legs can operate simultaneously.

Further analyze of practical usages of the robot in Sri Lankan and International context

Also the research is aimed at analyzing further possible usages of the proposed robot in Sri Lankan and International context. Already it was identified that the proposed robot can be used for rescue operations, military operations since the robot climbs rope very steadily and smoothly. Also it is possible to apply proposed rope climbing robot in various other scenarios. Final objective of the research is to identify possible applications and if there any alterations required plan for them.

2.2. Facilities

Required knowledge is gained by referring related books and the Internet to move forward on the research topic. Internet has been heavily used to seek information on previously published articles on the topic. Required hardware components are planned to find from the local retail hardware shops and on-line e-commerce centers. Additional hardware materials for robot construction such as skeleton of robot body is planned to construct manually by using aluminum and wood materials. For the dynamic simulation of the robot design, it is planned to prepare mathematical model of the system based on the lecture notes of control system subjects of the degree program.

2.3. Feasibility

Required knowledge is gained by using various resources as discussed in previous topic. Prior to the physical construction of robot, dynamic model is prepared and do the simulation to check the stability of the proposed robot system. Based on the results of the simulation, it is planned to fine tune the design and proceed to physical construction of the robot. Servo motors, sensor devices and control circuit components are needed to purchase. Required software components are planned to be developed using free and open source software packages.

The proposed robot will be a four legs multi-locomotion rope climbing robot. The design is specially emphasis on reduce the overall implementation cost of the robot for higher usage of the practical situations. The prototype will be constructed to demonstrate the idea and completeness of the project.

Chapter 3

3. Research Methodology

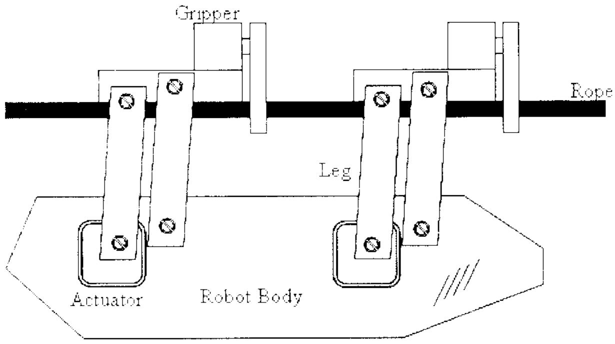

Figure 3.1: Proposed robot design

The proposed robot is a four legs rope climbing robot. This robot is contained four legs and four grippers for steady and smooth operation. To control these legs and grippers it is planned to use eight servo motors. Each leg is controlled by one servo motor and each gripper is controlled by another servo motor. Any leg can independently rotate forward and backward for individual leg positioning and all for legs operate simultaneously for entire robot's forward and backward movements. Each leg is having two rotary joints to control, but by using simple lever system it is possible to control both joint using single actuator. Robot foot of each leg is aligned parallel to the rope at any given time while it's moving due to the above lever system. Since foot is always parallel with the rope maximum surface of the foot touches with the rope and this lead to steady gripping facility for the robot movement.

This robot design uses same controller mechanism for leg and gripper motors for forward and backward movement of the robot. Since the robot is having four legs it has higher stability even one leg does not grip the rope properly. The design of the robot highly emphasis on more general type grippers that do not much depends on the characteristics of the rope. This will lead to function smoothly on unexpected situations such as rope is having some obstacles. This design does not directly operate properly when ropes cross each other.

3.1. Robot Leg Configuration

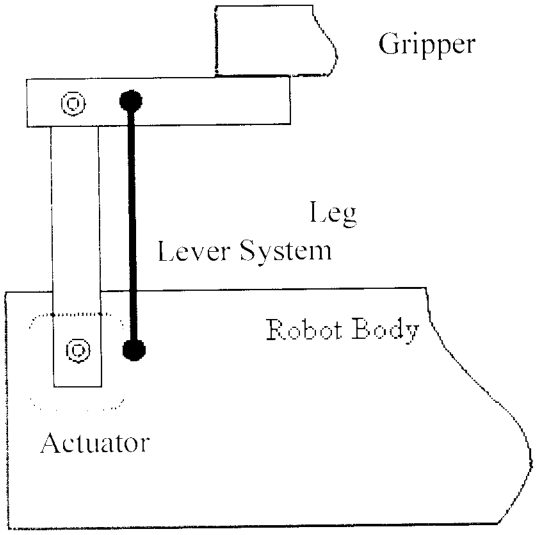

Figure 3.2: Robot leg configuration

The robot leg is fixed with robot body using a rotary joint. Single servo motor is used to control the individual leg. By rotating servo motor robot can move the leg forward and backward. By controlling individual servo motor at leg's joint robot can move individual leg and by controlling all four servo motors together robot can move its body forward and backward. Simple lever system is used to reduce the number of actuators required to construct the robot leg. Also foot movement on rope is smooth and aligns with the robot leg movement with this lever system and it reduced the controller complexity of the robot design. This is one of the mechanisms that used to reduce the complexity of the robot design.

3.2. Robot Gripper Configuration

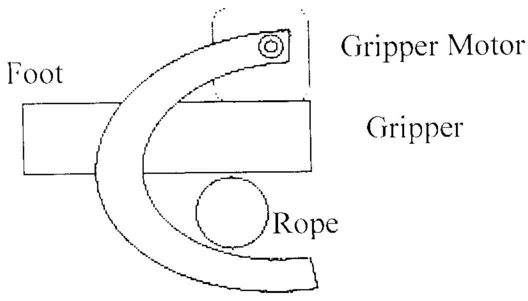

Figure 3.3: Robot gripper configuration

Robot gripper is a tool that is used to grip the rope for steady movement on the rope. This tool is not depended on the rope type or size. Each gripper is constructed using a single servo motor. Gripper is fixed to moving end of the leg. Gripper presses and releases the rope againts the robot foot.

3.3. Robot Movement on the Rope

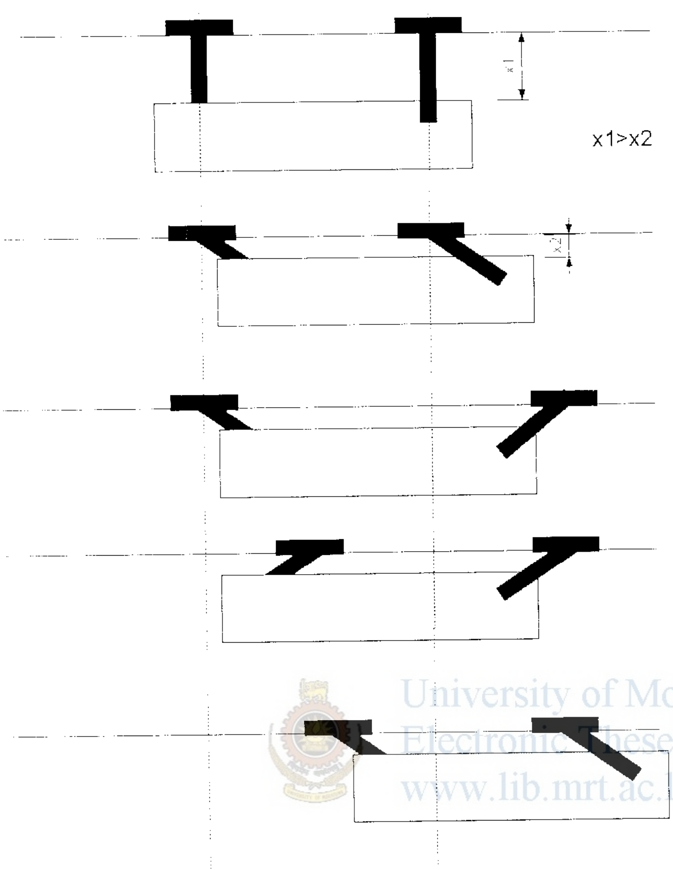

Figure 3.4: Robot movement on the rope

Figure 3.4 is illustrated how the robot is moving forward on the rope. This figure shows only one back leg and one front leg pair only. Same pattern is applied for other two legs in next leg movement cycle. While two legs are grip the rope steadily other two legs (either side) are moved forward then by rotating all four legs the robot is moved to forward. By repeating same cycle backward robot can move backward. Robot can stay on the rope in two configurations. Either with zero torque on leg actuators or it can stay any position with the continuous holding torque of the leg actuators. First step of the Figure 3.4 shows the zero holding torque position and last step shows the continuous holding torque position.

3.4. Robot Control System

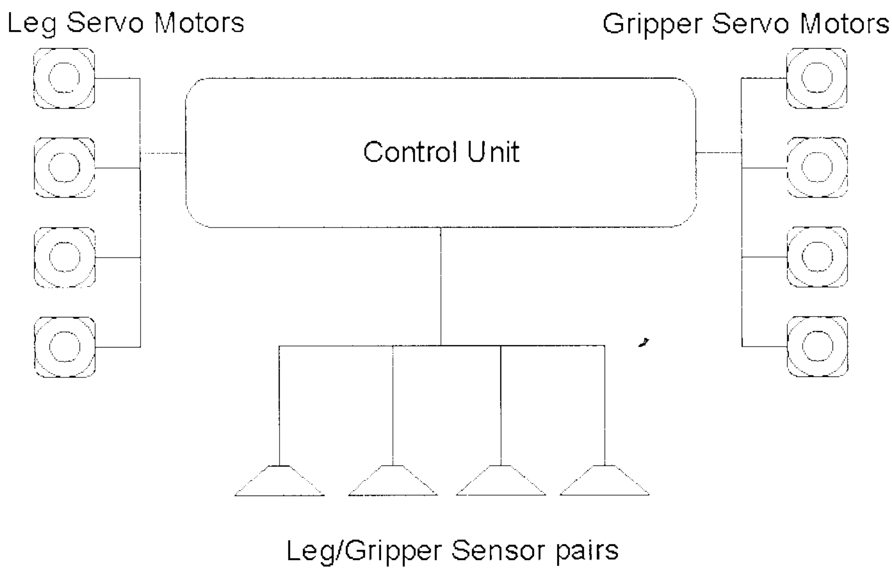

Figure 3.5: Robot Control System

Micro-controller based robot control system is used to control entire robot. Input parameters for the control system are come from leg/gripper sensor pairs. Leg servo motors and Gripper servo motors are controlled by the micro-controller, based on the input parameters and function that is needed to perform. Complete control unit consists with two major components. First component is responsible to take necessary decision or select proper high level control signal based on the signal parameters and function that robot needs to perform. These control signals are very high level signals and it does not operate any hardware components. Second component is responsible to decode above high level signals and perform underline operations. This component is intelligent enough to work with hardware components and control them properly.

This control system decomposition is very important when you develop complete controller program for the robot. Software programs for each component can be individually develop and test. Later by integrating individual components it is possible to construct complete robot control unit.

Chapter 4

4. Theoretical Development

4.1. The Robot Design

The robot design (Figure. 3.1) is a four legged multi-locomotion rope climbing variety that is having four legs and four grippers for steady and smooth operation. Each leg is controlled by a single servo motor and each gripper is controlled by another servo motor. Each leg and gripper pair is controlled by a single controller board.

The robot leg is fixed with its robot body using a rotary joint. Each leg is controlled by a dedicated motor. By controlling individual motor at leg's joint, robot can move individual leg independently and by synchronizing the movements of all four motors together can move its body forward and backward. The gripper is a tool that is used to grip the rope for steady movement on the rope. Each gripper is constructed using a single motor and is fixed to the moving extremity of the leg.

Micro-controller based control system (Figure. 3.5) is used to control the entire robot. Input parameters for the control system are retrieved from leg/gripper sensor pairs. Leg servo motors and gripper servo motors are controlled by the control unit based on the input feedback signals and required functions to be performed.

4.2. The Kinematics Analysis

Accurate calculation of the foot position is necessary when precise foot placement is critical, such as when robot is navigating on rough or otherwise uneven terrain. If the robot is provided with the ability to select its foot placements it must be able to move the leg to the appropriate positions. The position of the end effectors in space can easily be calculated if the joint angles are measured as shown in Figure 4.1.

Frame 0, denoted , and is considered to be an initial frame of reference for this calculation. This frame is attached to the robot body and will thus move with the robot. For present purpose, it can be considered to be an inertial frame. Frame 1 is attained via a rotation about the Z0 axis through angle. Frame 2 is attained via a translation of L1 along the X1 axis followed by a rotation about Z1 of –(180-).

The actual position of the foot can be calculated by describing the vector (4.1) to the end point in the inertial frame of the leg.

Note that symbols have their usual notations.

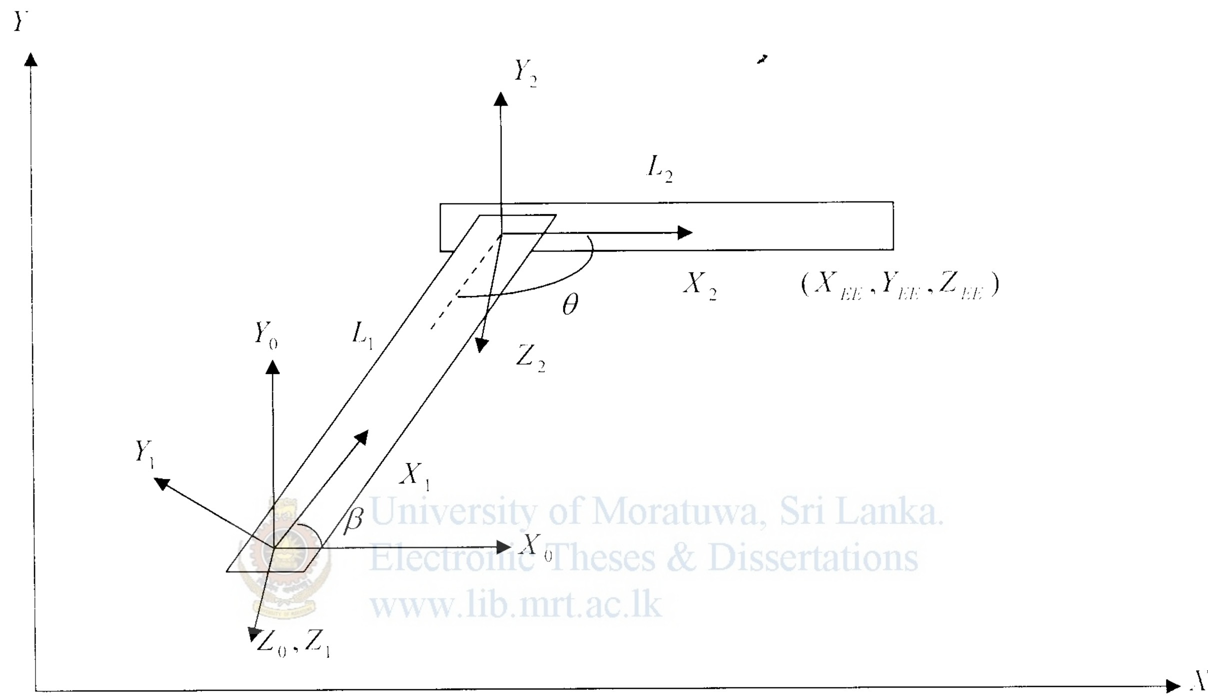

Figure 4.1: Robot leg end-effectors coordinate

L_1 – Length of link 1

L_2 – Length of link 2

β – Angle of link 1

θ – Angle of link 2

X_{EE}, Y_{EE}, Z_{EE} – End effector coordinates

X_0, Y_0, Z_0 – Axis of frame 0

X_1, Y_1, Z_1 – Axis of frame 1

X_2, Y_2, Z_2 – Axis of frame 2

By defining end-effector vector $\underline{r_{ee}}$ it is possible to derive end-effector coordinates that describe the position of the foot in space.

$$
\begin{aligned}
\underline{r_{EE}} &= \underline{r_1} + \underline{r_2} \qquad (4.1)\\
&= \Im_1^T \begin{bmatrix} L_1 \\ 0 \\ 0 \end{bmatrix} + \Im_2^T \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix}\\
&= \Im_0^T \left[C_{01}\underline{r_1} + C_{01}C_{12}\underline{r_2} \right]\\
&= \Im_0^T \left[C_{01}\left(\underline{r_1} + C_{12}\underline{r_2} \right) \right]\\
&= \Im_0^T \left\{ \begin{bmatrix} C_\beta & -S_\beta & 0 \\ S_\beta & C_\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} L_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} C_{180-\theta} & -S_{180-\theta} & 0 \\ S_{180-\theta} & C_{180-\theta} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix} \right) \right\}\\
&= \Im_0^T \left\{ \begin{bmatrix} C_\beta & -S_\beta & 0 \\ S_\beta & C_\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} L_1 \\ 0 \\ 0 \end{bmatrix} + \begin{pmatrix} L_2 C_{180-\theta} \\ L_2 S_{180-\theta} \\ 0 \end{pmatrix} \right) \right\}\\
&= \Im_0^T \left\{ \begin{bmatrix} C_\beta & -S_\beta & 0 \\ S_\beta & C_\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} L_1 + L_2 C_{180-\theta} \\ L_2 S_{180-\theta} \\ 0 \end{pmatrix} \right\}\\
&= \Im_0^T \left\{ \begin{bmatrix} C_\beta & -S_\beta & 0 \\ S_\beta & C_\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} L_1 + L_2 C_{180-\theta} \\ L_2 S_{180-\theta} \\ 0 \end{pmatrix} \right\}\\
&= \Im_0^T \begin{bmatrix} C_\beta (L_1 - L_2 C_\theta) - L_2 S_\theta S_\beta \\ S_\beta (L_1 - L_2 C_\theta) + L_2 S_\theta C_\beta \\ 0 \end{bmatrix}
\end{aligned}
$$

The position of the foot in space is given by

$$
\begin{aligned}
X_{EE} &= C_\beta (L_1 - L_2 C_\theta) - L_2 S_\theta S_\beta = L_1 C_\beta - L_2 (C_\beta C_\theta + S_\theta S_\beta)\\
Y_{EE} &= S_\beta (L_1 - L_2 C_\theta) + L_2 S_\theta C_\beta = S_\beta L_1 - L_2 (S_\beta C_\theta - S_\theta C_\beta)\\
Z_{EE} &= 0
\end{aligned}
$$

Based on the kinematics equations of the foot, it is possible to generate the Jacobian matrix (4.3) of the leg. By considering the partial derivative of the positions with respect to each of the joint angels, the Jacobian matrix (4.4) can be derived. This matrix can be used to build a correlate between Cartesian velocities of the foot to the joint velocities of the leg (4.2).

$X_{EE} = f(\theta)$: θ = joint angles

$$V_{EE} = \frac{dX_{EE}}{dt}$$

$$= \frac{\delta f}{\delta \theta}\frac{d\theta}{dt} + \frac{\delta f}{\delta t} \qquad (4.2)$$

$$= J\frac{d\theta}{dt}$$

$$\dot{\theta} = J(\theta)^{-1} V_{EE}$$

Derivation of transformation from Cartesian velocities to joint velocities.

$$J = \begin{bmatrix} \frac{\delta x}{\delta \beta} & \frac{\delta x}{\delta \theta} & 0 \\ \frac{\delta y}{\delta \beta} & \frac{\delta y}{\delta \theta} & 0 \\ \frac{\delta z}{\delta \beta} & \frac{\delta z}{\delta \theta} & 0 \end{bmatrix} \qquad (4.3)$$

with

$$\frac{\delta x}{\delta \beta} = -L_1 S_\beta + L_2(S_\beta C_\theta - C_\beta S_\theta)$$

$$\frac{\delta x}{\delta \theta} = L_2(C_\beta S_\theta - S_\beta C_\theta)$$

$$\frac{\delta y}{\delta \beta} = L_1 C_\beta - L_2(C_\beta C_\theta + S_\beta S\theta)$$

$$\frac{\delta y}{\delta \theta} = L_2(S_\beta S_\theta + C_\beta C_\theta)$$

$$\frac{\delta z}{\delta \beta} = 0$$

$$\frac{\delta z}{\delta \theta} = 0$$

Jacobian matrix relating Cartesian positions with angles.

$$J^{-1} = \begin{bmatrix} \frac{-1}{L_1 S_\beta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (4.4)$$

Finally, relationship between Cartesian velocities and joint velocities depend on S_β. That mean by controlling only angle β, we can control the robot leg. This concludes that each leg can be controlled by a single motor.

4.3. The Dynamic Analysis

The dynamic modeling of the robot is based on the Newton-Euler method with Luh-Walker-Paul's algorithm (Recur vise Newton-Euler). The diagram in Figure 4.2 is used to derive the dynamic model of the robot. Assume links to be uniform slender rods (homogeneous rods) with masses m1 and m2. Robot body weight is represented using w.

Forward equations of the Luh-Walker Paul's algorithm are derived to compute angular velocities, angular acceleration, and linear acceleration for individual revolute joint and based on them compute the linear acceleration of the center of mass for link 1, link 2 and robot body.

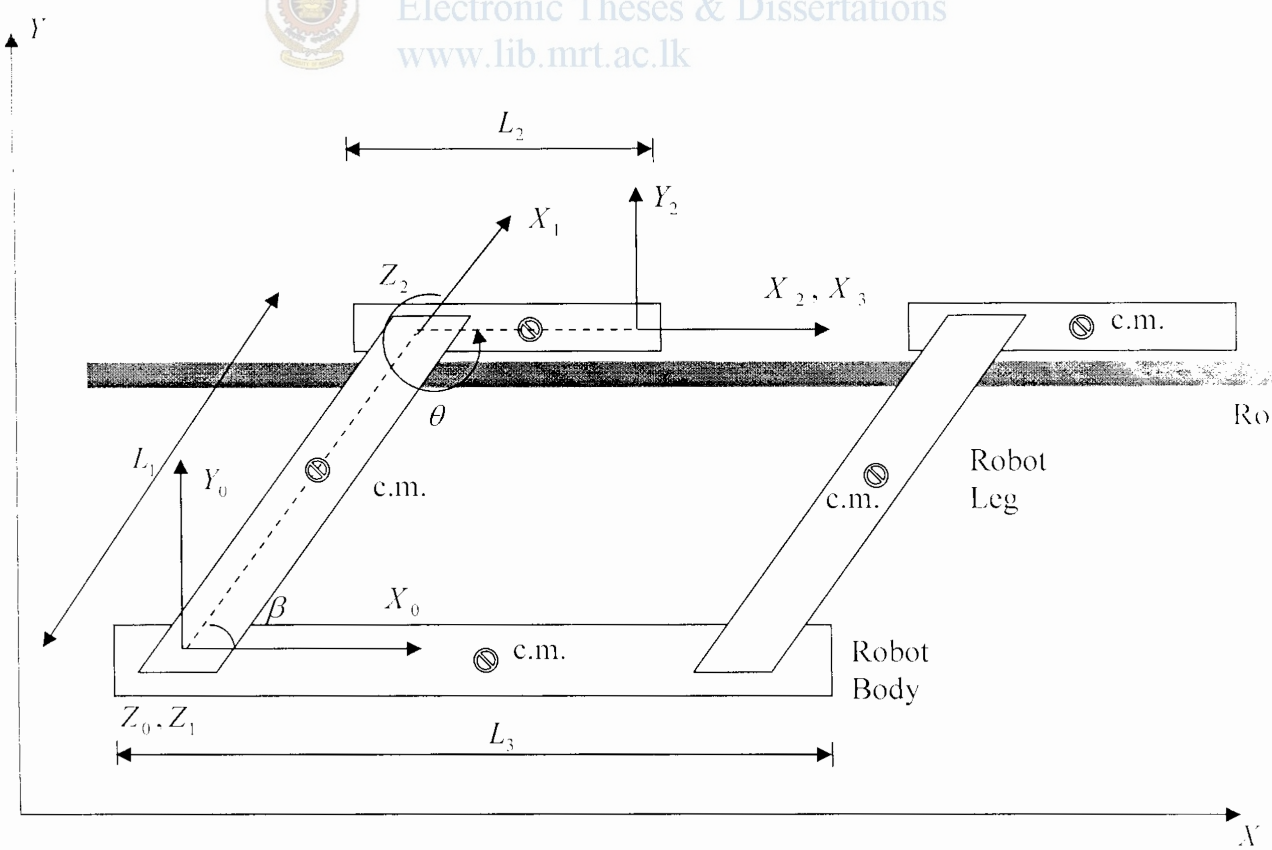

Figure 4.2: Robot configuration

4.4. Newton-Euler Method

For i = 1, 2

Assume the following initial conditions:

$$\underline{\omega}_0 = \underline{\dot{\omega}}_0 = \underline{v}_0 = \underline{0} \text{ and}$$

$$\underline{\dot{v}}_0 = (0, g, 0)^T = -g \qquad (4.5)$$

Compute angular velocities for revolute joints

$$^{i}(\underline{\omega_i}) = {}^{i}_{i-1}R\left[^{i-1}(\underline{\omega_{i-1}}) + \dot{q}_i\,{}^{i-1}(\hat{z}_i)\right]$$

So for i =1

$$^{1}(\underline{\omega_1}) = {}^{1}_{0}R\left[^{0}(\underline{\omega_0}) + \dot{\beta}\,{}^{0}(\hat{z}_1)\right]$$

$$^{1}(\underline{\omega_1}) = \dot{\beta}[0 \quad 0 \quad 1]^T \qquad (4.6)$$

For i = 2

$$^{2}(\underline{\omega_2}) = {}^{2}_{1}R\left[^{1}(\underline{\omega_1}) + \dot{\theta}\,{}^{1}(\hat{z}_2)\right]$$

$$^{2}(\underline{\omega_2}) = \begin{bmatrix} c\theta & s\theta & 0 \\ -s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \dot{\beta} \end{bmatrix} + \dot{\theta}\begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \dot{\beta}+\dot{\theta} \end{Bmatrix} \qquad (4.7)$$

Compute angular acceleration

$$^{i}(\underline{\dot{\omega}_i}) = {}^{i}_{i-1}R\left[^{i-1}(\underline{\dot{\omega}_{i-1}}) + \ddot{q}_i\,{}^{i-1}(\hat{z}_i) + {}^{i-1}(\underline{\omega_{i-1}}) * \left(\dot{q}_i\,{}^{i-1}(\hat{z}_i)\right)\right]$$

$$^{1}(\underline{\dot{\omega}_1}) = {}^{1}_{0}R\left[^{0}(\underline{\dot{\omega}_0}) + \ddot{\beta}\,{}^{0}(\hat{z}_1) + {}^{0}(\underline{\omega_0}) * \left(\dot{\beta}\,{}^{0}(\hat{z}_1)\right)\right]$$

$$^{1}(\underline{\dot{\omega}_1}) = \{0, 0, \ddot{\beta}\}^T \qquad (4.8)$$

$$^{2}(\underline{\dot{\omega}_2}) = {}^{2}_{1}R\left[^{1}(\underline{\dot{\omega}_1}) + \ddot{\theta}\,{}^{1}(\hat{z}_2) + {}^{1}(\underline{\omega_1}) * \left(\dot{\theta}\,{}^{1}(\hat{z}_2)\right)\right]$$

$$^{2}(\underline{\dot{\omega}_2}) = \begin{bmatrix} c\theta & s\theta & 0 \\ -s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \ddot{\beta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\beta} \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \ddot{\beta}+\ddot{\theta} \end{bmatrix} \qquad (4.9)$$

Compute the linear acceleration for revolute joint i

$$ {}^{1}_{0}R\dot{\underline{v}}_1 = {}^{1}\underline{a}_1 = {}^{1}_{0}R\left[{}^{0}(\underline{a}_0) + {}^{0}(\dot{\underline{\varpi}}_0) \times {}^{0}({}^{0}\underline{p}_1) + {}^{0}(\underline{\varpi}_0) \times \left[{}^{0}(\underline{\varpi}_0) \times {}^{0}({}^{0}\underline{p}_1)\right]\right] $$

$$ {}^{1}_{0}R\dot{\underline{v}}_1 = \begin{bmatrix} c\beta & s\beta & 0 \\ -s\beta & c\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ g \\ 0 \end{bmatrix} = \begin{bmatrix} gs\beta \\ gc\beta \\ 0 \end{bmatrix} \qquad (4.10) $$

$$ {}^{2}\underline{a}_2 = {}^{2}_{1}R\left[{}^{1}(\underline{a}_1) + {}^{1}(\dot{\underline{\varpi}}_1) \times {}^{1}({}^{1}\underline{p}_1) + {}^{1}(\underline{\varpi}_1) \times \left[{}^{1}(\underline{\varpi}_1) \times {}^{1}({}^{1}\underline{p}_2)\right]\right] $$

$$ {}^{1}({}^{1}\underline{p}_2) = [l_1 \quad 0 \quad 0]^T $$

$$ {}^{2}\underline{a}_2 = \begin{bmatrix} c\theta & s\theta & 0 \\ -s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \left[\begin{bmatrix} gs\beta \\ gc\beta \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \ddot{\beta} \end{bmatrix} \times \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\beta} \end{bmatrix} \times \left[\begin{bmatrix} 0 \\ 0 \\ \dot{\beta} \end{bmatrix} \times \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} \right] \right] $$

$$ {}^{2}\underline{a}_2 = \begin{bmatrix} c\theta(gs\beta - l_1\dot{\beta}^2) + s\theta(gc\beta + l_1\ddot{\beta}) \\ -s\theta(gs\beta - l_1\dot{\beta}^2) + c\theta(gc\beta + l_1\ddot{\beta}) \\ 0 \end{bmatrix} \qquad (4.11) $$

Linear acceleration of the center of mass for link 1 and link 2 is defined in (4.12) and (4.13).

$$ {}^{i}(\underline{a}_{ci}) = {}^{i}_{i-1}R.\underline{a}_{ci} = {}^{i}(\underline{a}_i) + {}^{i}(\dot{\underline{\omega}}_i) \times {}^{i}({}^{i}\underline{p}_{ci}) + {}^{i}(\underline{\omega}_i) \times \left[{}^{i}(\underline{\varpi}_i) \times {}^{i}({}^{i}\underline{p}_{ci})\right] $$

$$ {}^{1}(\underline{a}_{c1}) = {}^{1}(\underline{a}_1) + {}^{1}(\dot{\underline{\omega}}_1) \times {}^{1}({}^{1}\underline{p}_{c1}) + {}^{1}(\underline{\omega}_1) \times \left[{}^{1}(\underline{\varpi}_1) \times {}^{1}({}^{1}\underline{p}_{c1})\right] $$

$$ {}^{1}({}^{1}\underline{p}_{c1}) = \left[\frac{1}{2}l_1, 0, 0\right]^T $$

$$ {}^{1}(\underline{a}_{c1}) = \begin{bmatrix} gs\beta \\ gc\beta \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \ddot{\beta} \end{bmatrix} \times \begin{bmatrix} \frac{1}{2}l_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\beta} \end{bmatrix} \times \left[\begin{bmatrix} 0 \\ 0 \\ \dot{\beta} \end{bmatrix} \times \begin{bmatrix} \frac{1}{2}l_1 \\ 0 \\ 0 \end{bmatrix} \right] $$

$$ {}^{1}\left(\underline{a}_{c1}\right)=\begin{bmatrix} gs\beta-\frac{1}{2}l_1\dot{\beta}^2 \\ gc\beta+\frac{1}{2}l_1\ddot{\beta} \\ 0 \end{bmatrix} \qquad (4.12) $$

$$ {}^{2}\left(\underline{a}_{c2}\right)={}^{2}\left(\underline{a}_{2}\right)+{}^{2}\left(\underline{\dot{\omega}}_{2}\right)\times{}^{2}\left({}^{2}\underline{p}_{c2}\right)+{}^{2}\left(\underline{\omega}_{2}\right)\times\left[{}^{2}\left(\underline{\varpi}_{2}\right)\times{}^{2}\left({}^{2}\underline{p}_{c2}\right)\right] $$

$$ {}^{2}\left(\underline{a}_{c2}\right)=\begin{bmatrix} c\theta\left(gs\beta-l_1\dot{\beta}^2\right)+s\theta\left(gc\beta+l_1\ddot{\beta}\right) \\ -s\theta\left(gs\beta-l_1\dot{\beta}^2\right)+c\theta\left(gc\beta+l_1\ddot{\beta}\right) \\ 0 \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ \ddot{\beta}+\ddot{\theta} \end{bmatrix}\times\begin{bmatrix} \frac{1}{2}l_2 \\ 0 \\ 0 \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ \dot{\beta}+\dot{\theta} \end{bmatrix}\times\left[\begin{bmatrix} 0 \\ 0 \\ \dot{\beta}+\dot{\theta} \end{bmatrix}\times\begin{bmatrix} \frac{1}{2}l_2 \\ 0 \\ 0 \end{bmatrix}\right] $$

$$ {}^{2}\left(\underline{a}_{c2}\right)=\begin{bmatrix} gc\theta s\beta-l_1c\theta\dot{\beta}^2+gs\theta c\beta+l_1s\theta\ddot{\beta}-\frac{1}{2}l_2\left(\dot{\beta}+\dot{\theta}\right)^2 \\ -s\theta gs\beta+l_1s\theta\dot{\beta}^2+gc\theta c\beta+l_1c\theta\ddot{\beta}+\frac{1}{2}l_2\left(\ddot{\beta}+\ddot{\theta}\right)^2 \\ 0 \end{bmatrix} \qquad (4.13) $$

Backward equations of the algorithm are defined to calculate forces and torque values against the joint angles of the robot legs. Inertia tensor of link i about its center of mass referred to {i} is defined in (4.14).

$$ \bar{I}_i={}^{i}_{0}R.I_i\left({}^{i}_{0}R\right)^T $$

= Inertia tensor of link i about its center of mass referred to {i} (4.14)

Inertia tensor of link 1 and link 2 is defined in (4.15) and (4.16).

$$ \bar{I}_i=\begin{bmatrix} \bar{I}_{xx} & \bar{I}_{xy} & -\bar{I}_{xz} \\ -\bar{I}_{xy} & \bar{I}_{yy} & -\bar{I}_{yz} \\ -\bar{I}_{xy} & -\bar{I}_{yz} & \bar{I}_{zz} \end{bmatrix} $$ where frame {i} and frame of center of mass parallel.

<u>For i = 1</u>

$$ \bar{I}_1={}^{1}_{0}R.I_1\left({}^{0}_{1}R\right)=\begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{12}\left(m_1l_1^2+wl_3^2\right) & 0 \\ 0 & 0 & \frac{1}{12}\left(m_1l_1^2+wl_3^2\right) \end{bmatrix} \qquad (4.15) $$

For i = 2

$$\bar{I}_2 = {}^2_0R.I_2({}^0_2R) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{12}m_2l_2^2 & 0 \\ 0 & 0 & \frac{1}{12}m_2l_2^2 \end{bmatrix} \quad (4.16)$$

Forces and torque values can be derived using (4.17) and (4.18).

$$^i(\underline{F}_i) = {}_{i-1}^{i}R^{i+1}(\underline{F}_{i+1}) + m_i\,{}^i(\underline{a}_{ci}) \quad (4.17)$$

$$^i(\underline{n}_i) = {}_{i-1}^{i}R\begin{bmatrix} ^{i+1}(\underline{n}_{i+1}) + {}^{i+1}_{i}R\left[{}^i\left({}^i\underline{p_{ci}}\right)\times{}^i(\underline{F}_i) + \left[{}^i\left({}^i\underline{p}_{i+1} - {}^i\underline{p}_{ci}\right)\times{}^i(\underline{F}_{i+1})\right]\right] \\ + {}^{i+1}_{i}R\left[\bar{I}_i.{}^i(\underline{\dot{\omega}}_i) + {}^i(\underline{\omega}_i)\times\left(\bar{I}_i.{}^i(\underline{\omega}_i)\right)\right] \end{bmatrix} \quad (4.18)$$

Assuming no end effector load, forces and torque values calculation can be started from the end effecter. End effecter force and torque is shown in (4.19). Therefore.

$$\begin{aligned} ^3(\underline{F}_3) &= \underline{0} \\ ^3(\underline{n}_3) &= \underline{0} \end{aligned} \quad (4.19)$$

Recursively calculate forces and torque values for each link joint from backward direction.

For i = 2

$$^2(\underline{F}_2) = {}^2_3R^3(\underline{F}_3) + m_2\,{}^2(\underline{a}_{c2})$$

$$^2(\underline{F}_2) = \begin{bmatrix} m_2\left[gc\theta s\beta - l_1c\theta\dot{\beta}^2 + gs\theta c\beta + l_1s\theta\ddot{\beta} - \tfrac{1}{2}l_2\left(\dot{\beta}+\dot{\theta}\right)^2\right] \\ ,m_2\left[-s\theta gs\beta + l_1s\theta\dot{\beta}^2 + gc\theta c\beta + l_1c\theta\ddot{\beta} + \tfrac{1}{2}l_2\left(\ddot{\beta}+\ddot{\theta}\right)^2\right] \\ ,0 \end{bmatrix}^T \quad (4.20)$$

$$^2(\underline{n}_2) = {}^2_3R\left[{}^3(\underline{n}_3) + {}^3_2R\left[{}^2\left({}^2\underline{p_{c2}}\right)\times{}^2(\underline{F}_2) + \left[{}^2\left({}^2\underline{p}_3 - {}^2\underline{p}_{c2}\right)\times{}^2(\underline{F}_3)\right]\right] + {}^3_2R\left[\bar{I}_2.{}^2(\underline{\dot{\omega}}_2) + {}^2(\underline{\omega}_2)\times\left(\bar{I}_2.{}^2(\underline{\omega}_2)\right)\right]\right]$$

$$^2(\underline{n}_2) = {}^2\left({}^2\underline{p_{c2}}\right)\times{}^2(\underline{F}_2) + \left[{}^2\left({}^2\underline{p}_3 - {}^2\underline{p}_{c2}\right)\times{}^2(\underline{F}_3)\right] + \bar{I}_2.{}^2(\underline{\dot{\omega}}_2) + {}^2(\underline{\omega}_2)\times\left(\bar{I}_2.{}^2(\underline{\omega}_2)\right)$$

$$
{}^{2}(\underline{n}_2)=\begin{bmatrix}\frac{1}{2}l_2\\0\\0\end{bmatrix}\times\begin{bmatrix}{}^{2}F_{2x}\\{}^{2}F_{2y}\\0\end{bmatrix}+\left[\begin{bmatrix}l_2\\0\\0\end{bmatrix}-\begin{bmatrix}\frac{1}{2}l_2\\0\\0\end{bmatrix}\right]\times\begin{bmatrix}0\\0\\0\end{bmatrix}+\begin{bmatrix}0&0&0\\0&\frac{1}{12}m_2l_2^2&0\\0&0&\frac{1}{12}m_2l_2^2\end{bmatrix}\begin{bmatrix}0\\0\\\ddot{\beta}+\ddot{\theta}\end{bmatrix}+\begin{bmatrix}0\\0\\\dot{\beta}+\dot{\theta}\end{bmatrix}
$$

$$
+\left[\begin{bmatrix}0&0&0\\0&\frac{1}{12}m_2l_2^2&0\\0&0&\frac{1}{12}m_2l_2^2\end{bmatrix}\begin{bmatrix}0\\0\\\dot{\beta}+\dot{\theta}\end{bmatrix}\right]
$$

$$
{}^{2}(\underline{n}_2)=\begin{bmatrix}0\\0\\\frac{1}{12}m_2l_2^2(\ddot{\beta}+\ddot{\theta})+\frac{1}{2}l_2({}^{2}F_{2y})\end{bmatrix}\qquad(4.21)
$$

Second step of backward calculation and this step define individual forces and torques values for joint 1.

<u>For i = 1</u>

$$
{}^{1}(\underline{F}_1)={}_{2}^{1}R\,{}^{2}(\underline{F}_2)+m_1\,{}^{1}(\underline{a}_{c1})
$$

$$
{}^{1}(\underline{F}_1)=\begin{bmatrix}c\theta&-s\theta&0\\s\theta&c\theta&0\\0&0&1\end{bmatrix}\begin{bmatrix}{}^{2}F_{2x}\\{}^{2}F_{2y}\\0\end{bmatrix}+\begin{bmatrix}gs\beta-\frac{1}{2}l_2\dot{\beta}^2\\gc\beta+\frac{1}{2}l_2\ddot{\beta}\\0\end{bmatrix}(m_1+w)
$$

$$
{}^{1}(\underline{F}_1)=\begin{bmatrix}({}^{2}F_{2x})c\theta-({}^{2}F_{2y})+(m_1+w)(gs\beta-\frac{1}{2}l_2\dot{\beta}^2)\\({}^{2}F_{2x})s\theta+({}^{2}F_{2y})+(m_1+w)(gc\beta+\frac{1}{2}l_2\ddot{\beta})\\0\end{bmatrix}\qquad(4.22)
$$

$$
{}^{1}(\underline{n}_1)={}_{2}^{1}R\left[{}^{2}(\underline{n}_2)+{}_{1}^{2}R\left[{}^{1}({}^{1}\underline{p}_{c1})\times{}^{1}(\underline{F}_1)+\left[{}^{1}({}^{1}\underline{p}_2-{}^{1}\underline{p}_{c1})\times{}^{1}(\underline{F}_2)\right]\right]+{}_{1}^{2}R\left[\bar{I}_1.{}^{1}(\dot{\underline{\omega}}_1)+{}^{1}(\underline{\omega}_1)\times(\bar{I}_1.{}^{1}(\underline{\omega}_1))\right]\right]
$$

$$
{}^{1}(\underline{n}_1)=\begin{bmatrix}c\theta&-s\theta&0\\s\theta&c\theta&0\\0&0&1\end{bmatrix}\begin{bmatrix}0\\0\\{}^{2}n_{2z}\end{bmatrix}+\left[\begin{bmatrix}\frac{1}{2}l_1\\0\\0\end{bmatrix}\times\begin{bmatrix}{}^{1}F_{1x}\\{}^{1}F_{1y}\\0\end{bmatrix}+\left[\begin{bmatrix}l_1\\0\\0\end{bmatrix}-\begin{bmatrix}\frac{1}{2}l_1\\0\\0\end{bmatrix}\right]\times\begin{bmatrix}c\theta^2F_{2x}-s\theta^2F_{2y}\\s\theta^2F_{2x}+c\theta^2F_{2y}\\0\end{bmatrix}\right]
$$

$$
+\begin{bmatrix}c\theta&-s\theta&0\\s\theta&c\theta&0\\0&0&1\end{bmatrix}\begin{bmatrix}0\\0\\\ddot{\beta}\end{bmatrix}+\begin{bmatrix}0\\0\\\dot{\beta}\end{bmatrix}\times\left[\begin{bmatrix}0&0&0\\0&\frac{1}{12}(m_1l_1^2+wl_3^2)&0\\0&0&\frac{1}{12}(m_1l_1^2+wl_3^2)\end{bmatrix}\begin{bmatrix}0\\0\\\dot{\beta}\end{bmatrix}\right]
$$

$$^{1}(\underline{n}_1)=\begin{bmatrix}0\\0\\^{2}n_{2z}\end{bmatrix}+\begin{bmatrix}0\\0\\ \frac{1}{2}l_1\,^{1}F_{1y}\end{bmatrix}+\begin{bmatrix}0\\0\\ \frac{1}{2}l\left(s\theta^{2}F_{2x}+c\theta^{2}F_{2y}\right)\end{bmatrix}+\begin{bmatrix}0\\0\\ \frac{1}{12}\left(m_1l_1^2+wl_3^2\right)\ddot{\beta}\end{bmatrix} \quad (4.23)$$

Equations (4.20), (4.21), (4.22) and (4.23) are defined, forces and torque values of individual link joints. By cumulating individual joint torques, total torque can be calculated for the joint motor. Total torque for revolute joints can be computed using (4.24).

Torque 1 and torque 2 for link 1 joint and link 2 joint are represented in (4.25) and (4.26).

$$\tau_i=\left[^{i}(\underline{n}_i)\right]^{T}\left[^{i}(\hat{z}_i)\right] \quad (4.24)$$

$$\tau_1=\left[^{1}(\underline{n}_1)\right]^{T}\left[^{1}(\hat{z}_1)\right]=^{1}n_{1z}$$

$$\tau_1=^{2}n_{2z}+\tfrac{1}{2}l_1\left[^{1}F_{1y}+s\theta^{2}F_{2x}+c\theta^{2}F_{2y}\right]+\tfrac{1}{12}\left(m_1l_1^2+wl_3^2\right)\ddot{\beta} \quad (4.25)$$

$$\tau_2=\left[^{2}(\underline{n}_2)\right]^{T}\left[^{2}(\hat{z}_2)\right]=^{2}n_{2z}$$

$$\tau_2=\tfrac{1}{12}m_2l_2^2\left(\ddot{\beta}+\ddot{\theta}\right)+\tfrac{1}{2}l_2\left(^{2}F_{2y}\right) \quad (4.26)$$

By substituting $^{2}n_{2z},\,^{1}F_{1y},\,^{2}F_{2y},\,^{2}F_{2x}$ values into (4.25) and (4.26), we can calculate torque values for each leg joints.

$$\tau_1=\left(\frac{1}{3}m_2l_2^2+m_2l_1^2+m_2l_1l_2c\theta+\frac{1}{3}m_1l_1^2+\frac{1}{4}wl_1^2+\frac{1}{12}wl_3^2\right)\ddot{\beta}+$$
$$\left(\frac{1}{3}m_2l_2^2+\frac{1}{2}m_2l_1l_2c\theta\right)\ddot{\theta}-\frac{1}{2}m_2l_1l_2s\theta\dot{\theta}^2-m_2l_1l_2s\theta.\beta\theta+ \quad (4.27)$$
$$\frac{1}{2}m_2gl_1\left(c\theta c\beta-s\theta s\beta\right)+m_2gl_1c\beta-m_2l_1l_2s\theta s\beta+\left(m_1+w\right)gc\beta$$

$$\tau_2 = \left(\frac{1}{3}m_2 l_2^2 + m_2 l_1 l_2 c\theta\right)\ddot{\beta} + \left(\frac{1}{3}m_2 l_2^2\right)\ddot{\theta} + \frac{1}{2}m_2 l_1 l_2 s\theta\dot{\beta}^2 + \frac{1}{2}m_2 g l_2 (c\theta c\beta - s\theta s\beta) \quad (4.28)$$

Leg mass is considered as m_1, foot mass is m_2 and robot body weight is w. Equations (4.27) and (4.28) gives the torque profiles that need to control joint angles for robot dynamics.

Chapter 5

5. Proposed Solution

5.1. Simulation

Simulation model of the robot is developed using SimMechanics and Simulink toolboxes of MATLAB software application. Simulation model blocks are configured based on kinematics and dynamic equations derived in robot modeling. The robot simulation model consists with robot controller sub-system and robot model sub-system. The robot controller sub-system generates required control signals and toque profiles to control the robot. Robot model sub-system moves its robot leg based on the control signals and torque values generated by the robot controller sub-system.

As shown in Figure 5.1 simulation model consists with several applications together. SimMechanics tool is used to model the robot model based on the mathematical model derived in robot modeling. SimMechanics results can be feed into Simulink to further analysis of the robot movement. To simulate robot in virtual environment, VR Toolbox can be connected with the SimMechanics model. Using 3D modeling software such as SolidWorks, SolidEdge, and etc, VRML model can be easily developed. This VRML model should imported into VR Toolbox to animate it based on the output signals from the SimMechanics model.

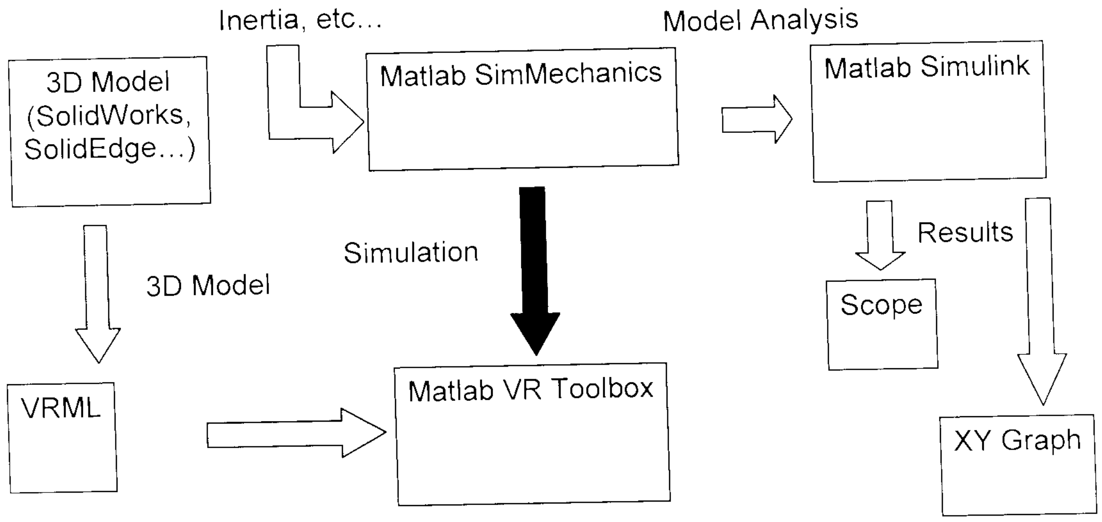

Figure 5.1: Simulation architecture

5.2. Two Links Robot Leg Simulation

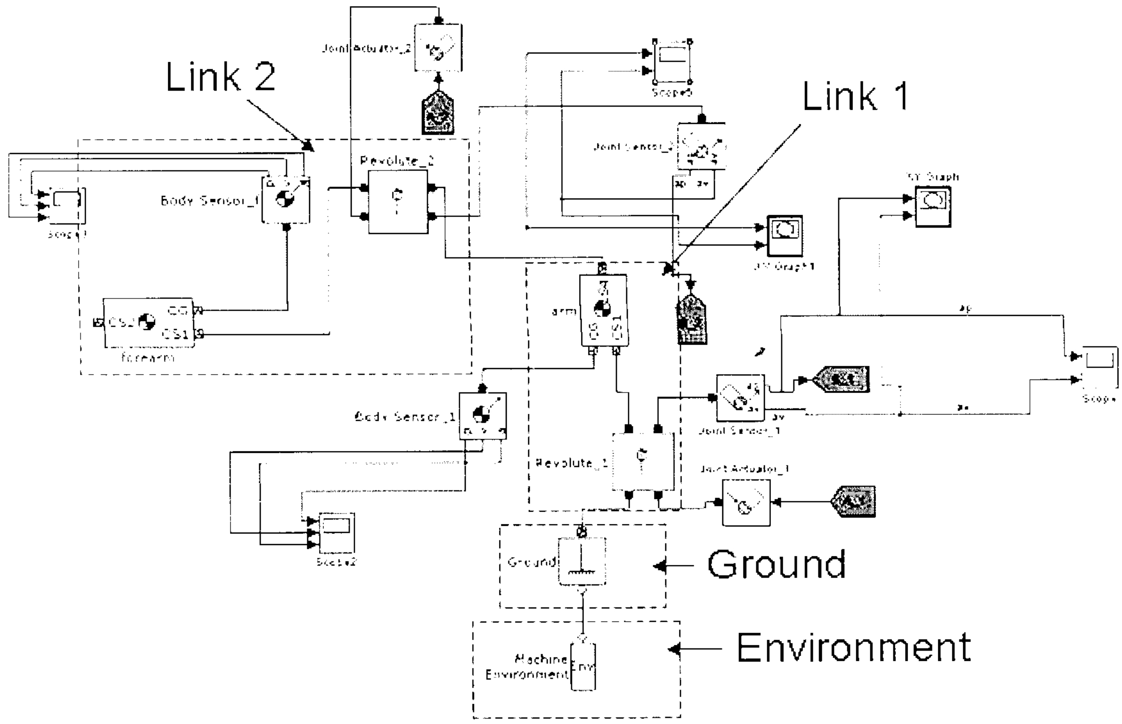

Figure 5.2: Robot model sub-system

The robot model sub-system (Figure 5.2) defines the environment where the simulation runs, world coordinate system and each revolute joint and links of the robot leg. Joint actuators and joint sensors are used to control and sense the each joint movement. Robot leg movement simulation is shown in Figure 5.3.

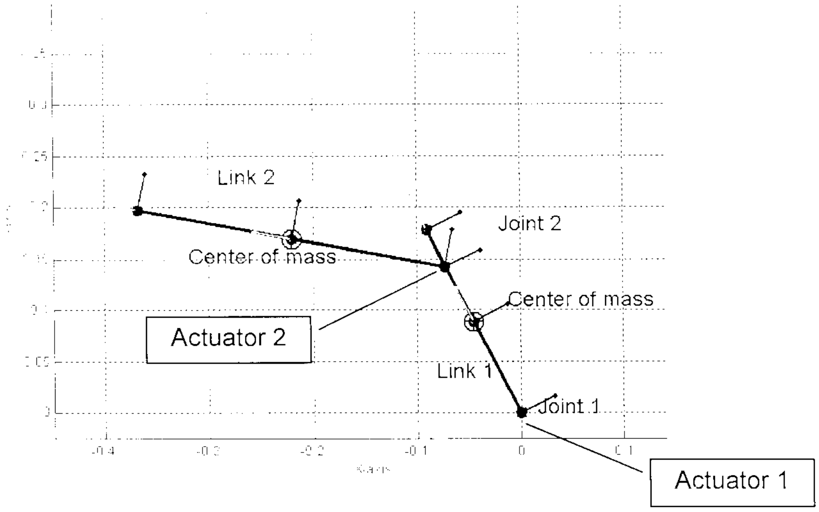

Figure 5.3: Robot leg movement simulation

Different robot movements and effect of environment factors to the robot movement can be simulated by changing the configuration settings of the model.

The robot model sub-system consists with environment, ground, link 1 and link 2 blocks. Link 1 and link 2 are actuated using joint actuator 1 and joint actuator 2 blocks. Each leg is constructed using a revolute joint block and body block.

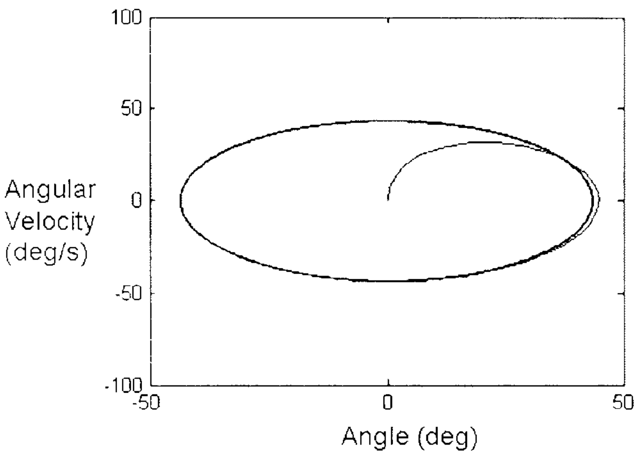

Figure 5.4: Phase space plot of joint 1

Phase space plot (Figure 5.4) of joint 1 shows the variation of the angular velocity against the angle of joint 1. The graph shows the smooth operation of joint 1 and small deviation at the maximum angle values.

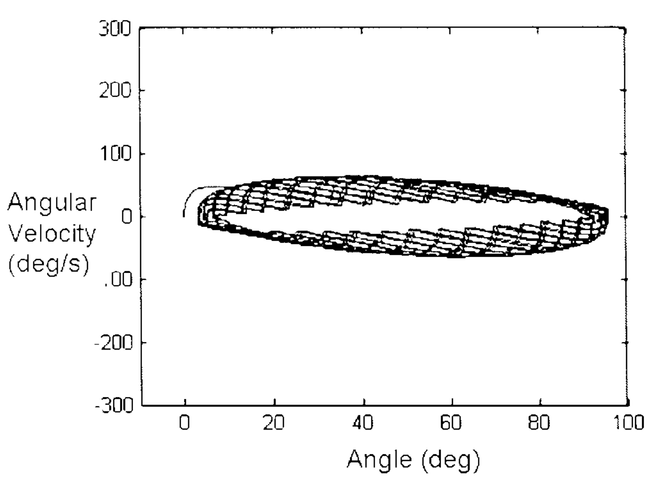

Figure 5.5: Phase space plot of joint 2

Figure 5.5 shows the phase space plot of joint 2 and the variation of the angular velocity against the angle of joint 2. Phase space plot of link 1 is similar to an ellipse, because of the conservation of total energy is constant and is the inertial moment of the link 1 about its pivot point. Small vibration on link 2 is resulted to small deviations on phase space plot of joint 2, but total energy conservation of link 2 is also stable. Deviations at phase space plot of link 2 can be reduced to elliptical shape by adjusting mass and center of mass of link 2. Small disturbances at links result to energy loss and as the amplitude of angular velocity and angle decrease, the phase diagram spirals inwards.

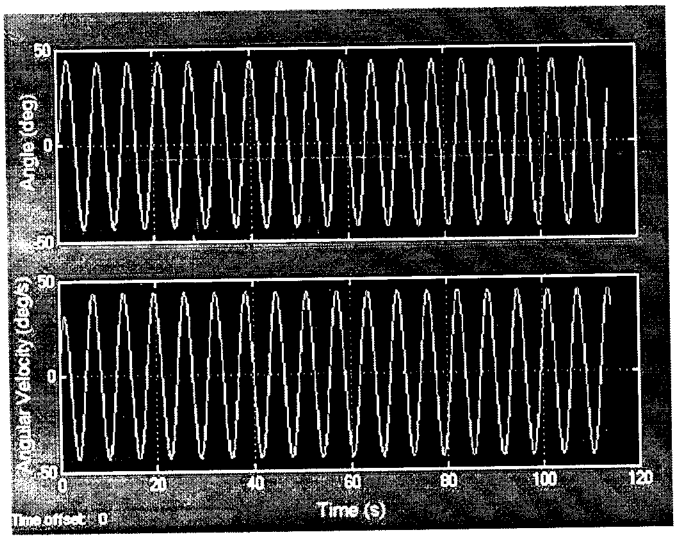

Figure 5.6: Variation of revolute joint angle and angular velocity of joint 1

Figure 5.6 shows the variation of revolute joint angle and angular velocity of joint 1. This graph illustrates that how robot leg joint functions very smoothly.

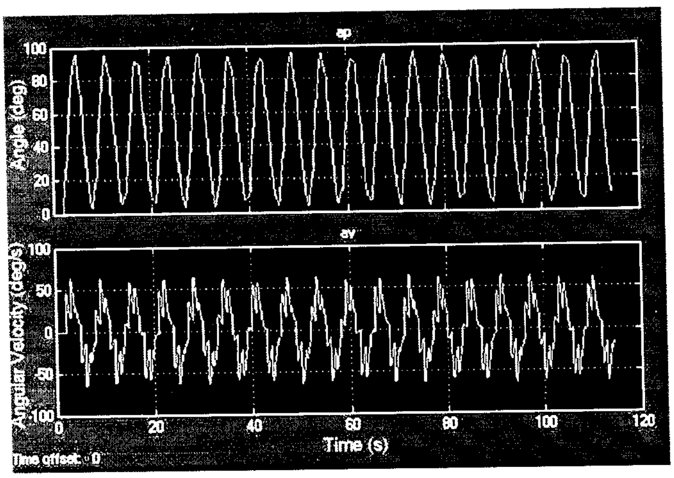

Figure 5.7: Variation of revolute joint angle and angular velocity of joint 2

Figure 5.7 shows the variation of revolute joint angle and angular velocity of joint 2. Joint sensor blocks are used to sense angular velocity and angle of each joint. Body sensor blocks are use to sense position, velocity and acceleration of center of mass of each link.

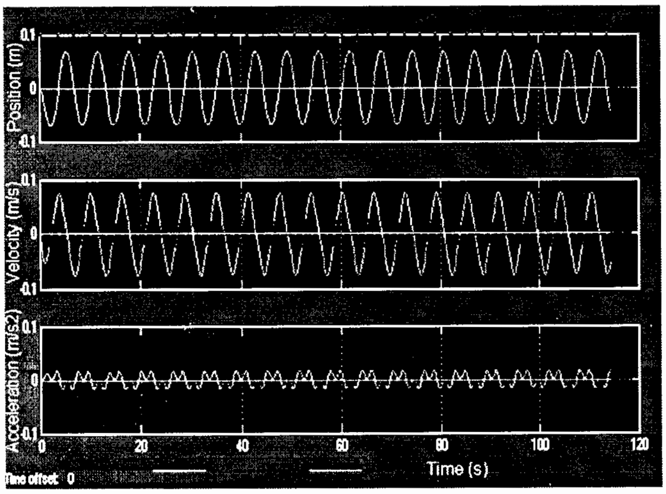

Figure 5.8: Behavior of the position, velocity and acceleration of each link 1 center of mass

Figure 5.8 and Figure 5.9 show the behavior of the position, velocity and acceleration of each link center of mass. The directionality of the revolute joint assumes that the rotation axis lies in the +z direction and movements on +z direction is zero. Figure 5.8 and Figure 5.9 show the movement of link 1 is smooth than movement of link 2. In general both links are moving steadily and smoothly without having jerky movements.

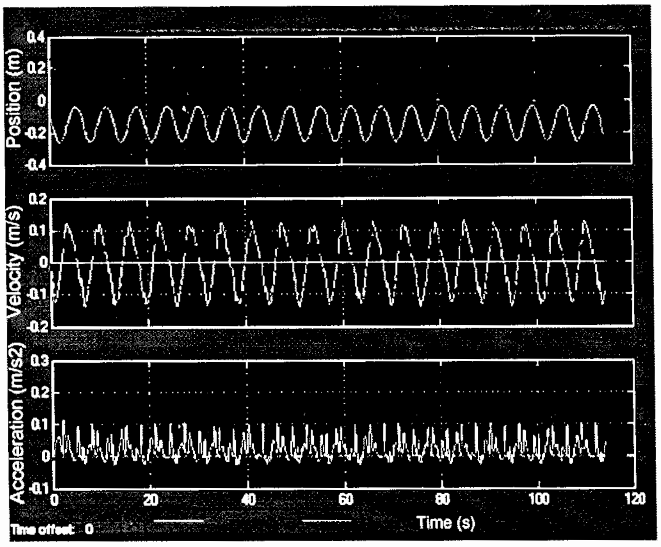

Figure 5.9: Behavior of the position, velocity and acceleration of each link 2 center of mass

5.3. Three Links Robot Leg Simulation

Two links robot leg simulation in previous section is about how control each robot leg using two actuators at each joint of the leg. The drawback of the two links robot leg design is the robot design is little complex and needed two actuators for a single leg. Also previous simulation results shows that link 2 and joint 2 are not actuated very smoothly. Figure 5.7 shows how angular velocity of joint 2 deviates from the smooth line and make some vibrations on link 2. Figure 5.9 shows how velocity and acceleration of center of mass of link 2 vibrating and make some noises against the smooth operation of robot leg.

By introducing third free link in between robot foot free edge and the robot body, it is possible to achieve much higher softness of operation of the robot leg. In addition to the smooth operation of the robot leg, this design lead to reduce the complexity of the robot design by avoiding one actuator for each leg.

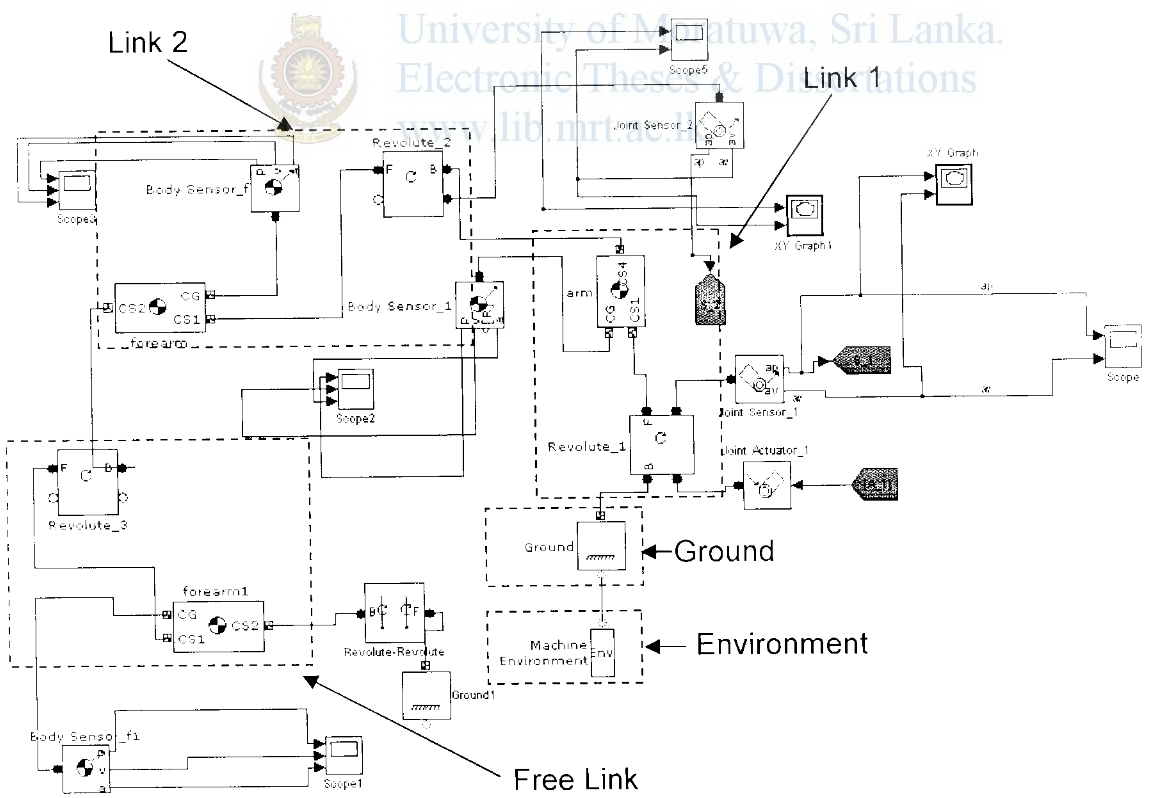

Figure 5.10: Three Links Robot model sub-system

Figure 5.10 shows environment, ground, link1, link2 and free link modules of robot module sub-system. Environment and ground blocks are used to parameterize the general parameters for entire model such gravitational forces…etc.

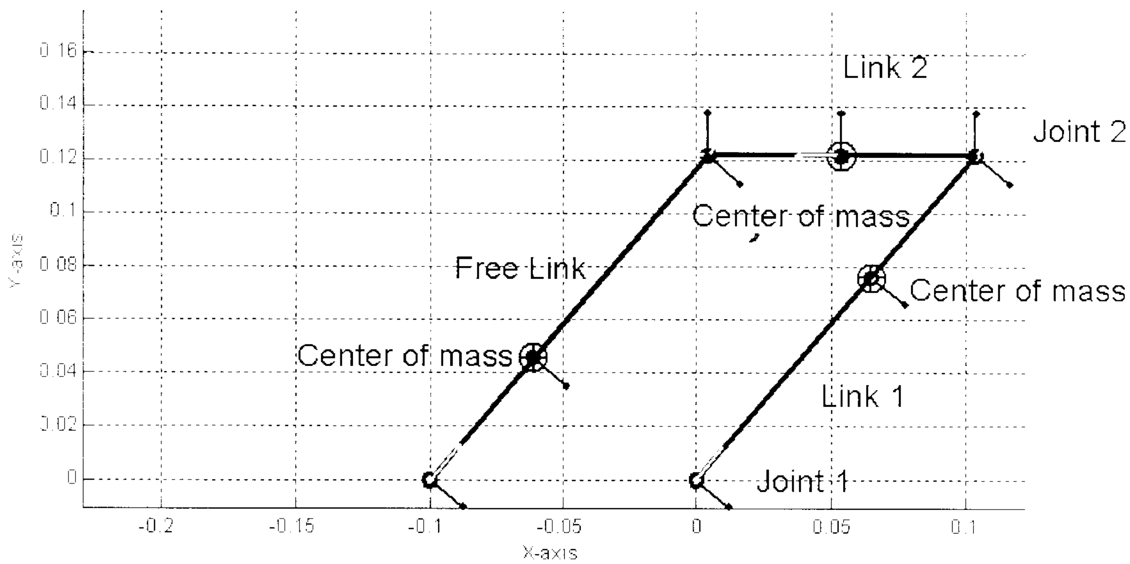

Figure 5.11: Three Links robot leg movement simulation

Figure 5.11 illustrates the simulation output of three links robot model sub-system. This shows link1, link2, free link and behavior of their center of mass. Additionally it simulates joint 1 and joint 2.

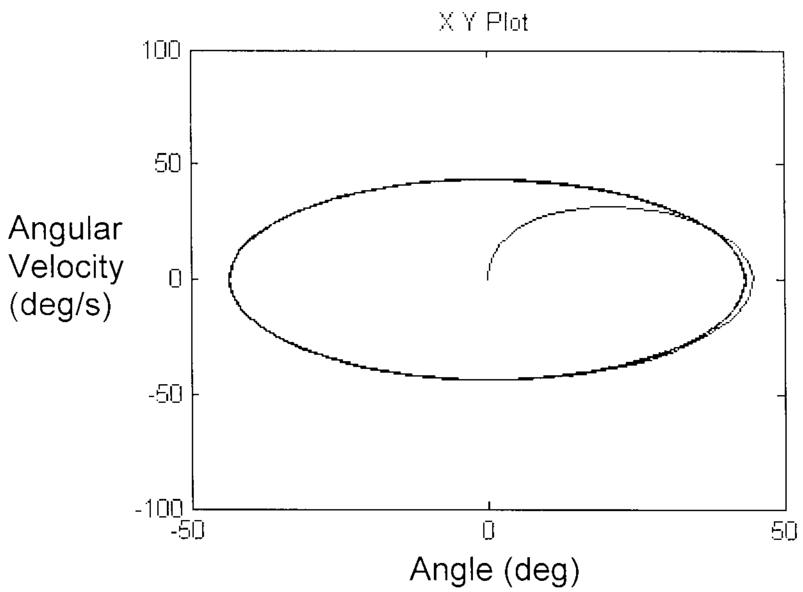

Figure 5.12: Phase space plot of joint 1

Figure 5.12 shows phase space plot of joint 1 and the elliptical shape of the graph prove that the total energy conservation of joint 1 is stable.

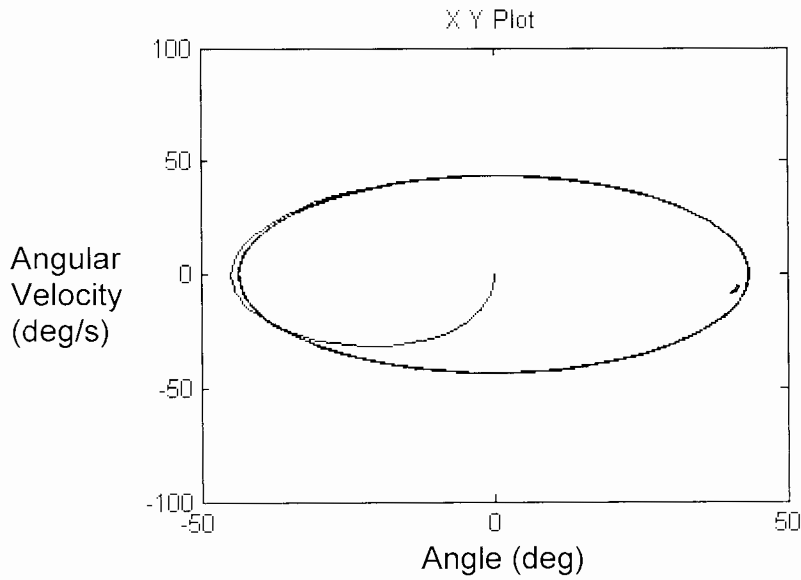

Figure 5.13: Phase space plot of joint 2

Figure 5.13 is also shows the phase space plot of joint 2 and same as previous we can conclude that total energy conservation at joint 2 is stable.

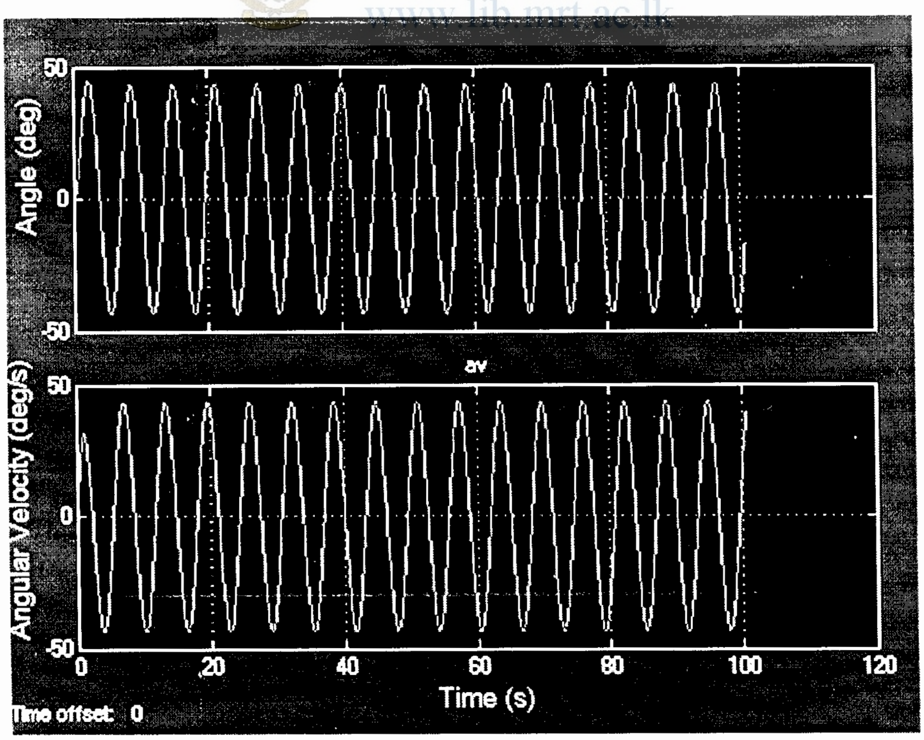

Figure 5.14: Variation of revolute joint angle and angular velocity of joint 1

Figure 5.14 shows the variation of revolute joint angle and angular velocity of joint 1. Sinusoidal shape of graph show how smoothly joint 1 is functioning.

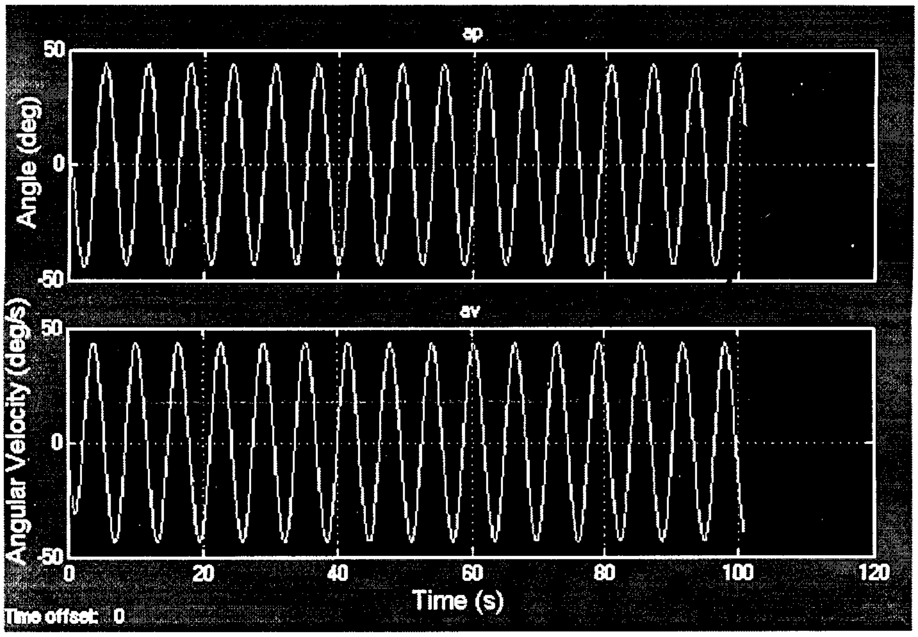

Figure 5.15: Variation of revolute joint angle and angular velocity of joint 2

Figure 5.14 shows the variation of revolute joint angle and angular velocity of joint 2. Sinusoidal shape of graph show how smoothly joint 2 is functioning.

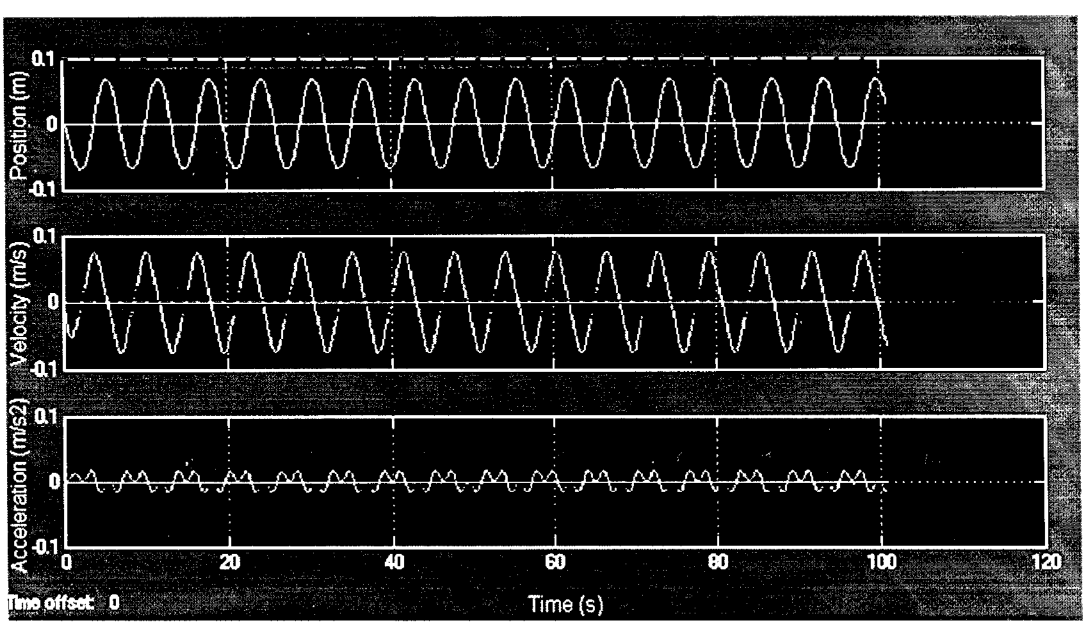

Figure 5.16: Behavior of the position, velocity and acceleration of link 1 center of mass

Figure 5.16 and Figure 5.17 show Behavior of the position, velocity and acceleration of link 1 center of mass and link 2 center of mass.

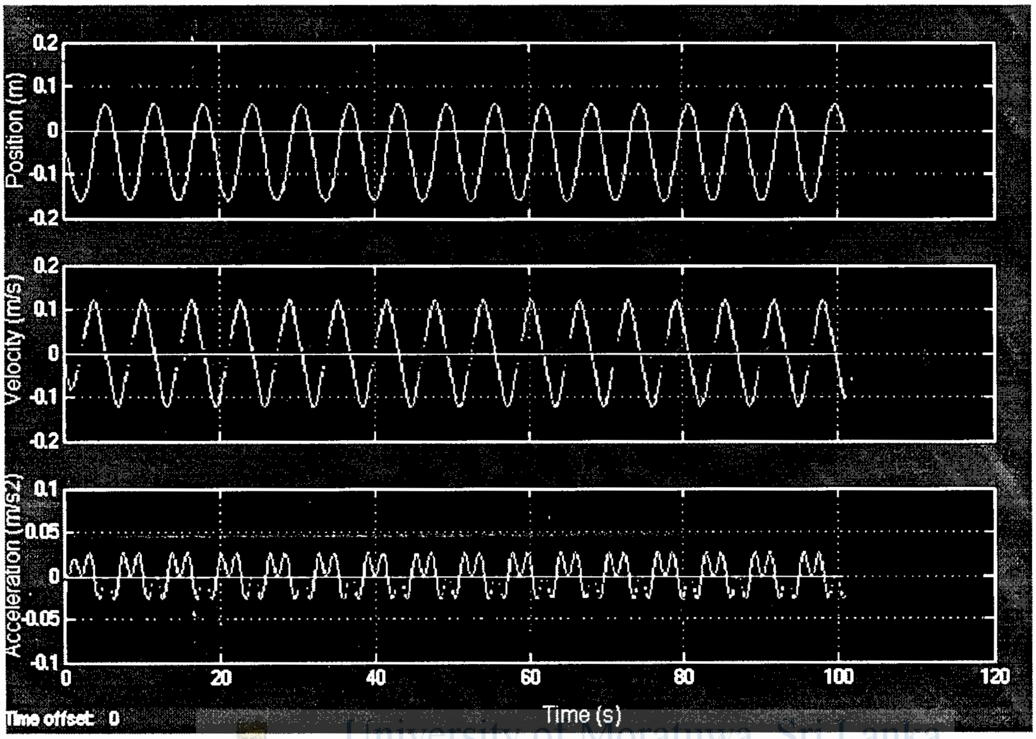

Figure 5.17: Behavior of the position, velocity and acceleration of each link 2 center of mass

Figure 5.18 shows how SimMechanics and VRML combine together using VR Sink block. Figure 5.19 illustrates SimMechanics blocks of front left leg and front right leg blocks. VR Sink block parameters should configured accordingly the simulation and Figure 5.20 shows the different configuration options. Figure 5.21 is about how to develop VRML model using V-Realm Builder tool which is built in with Matlab tool set. Figure 5.22 details how two leg robot simulation happening on VRML viewer.

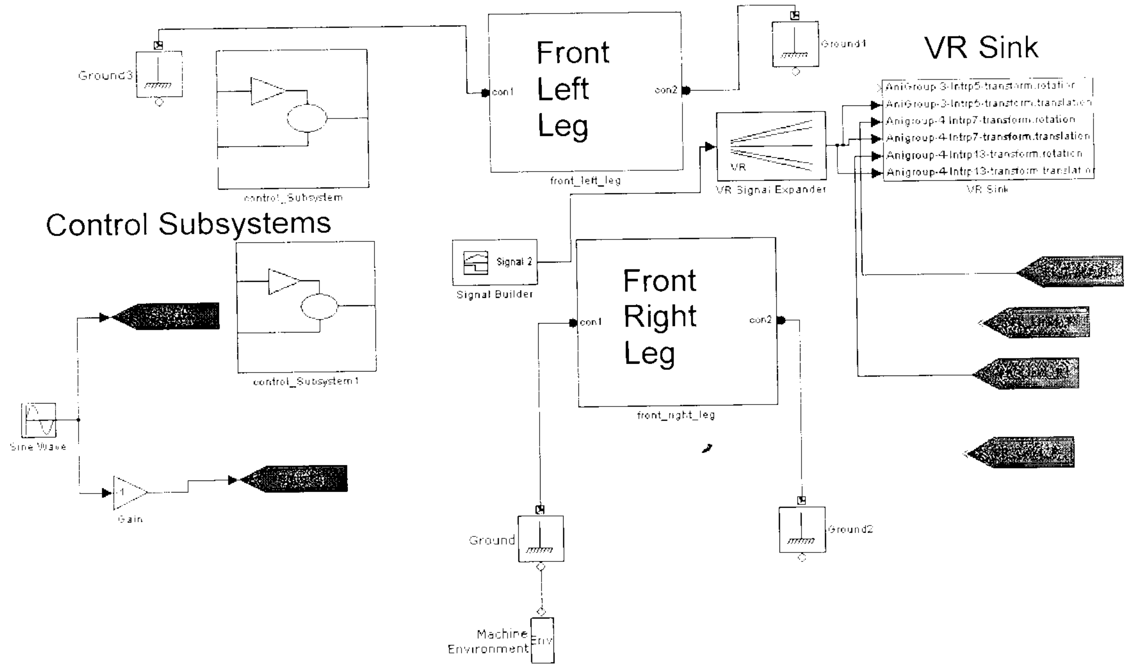

Figure 5.18: SimMechanics and VRML combine together using VR Sink block

Figure 5.18 illustrates how SimMechanics blocks and VRML model combined together using VR Sink block to simulate VRML 3D model based on SimMechanics outputs.

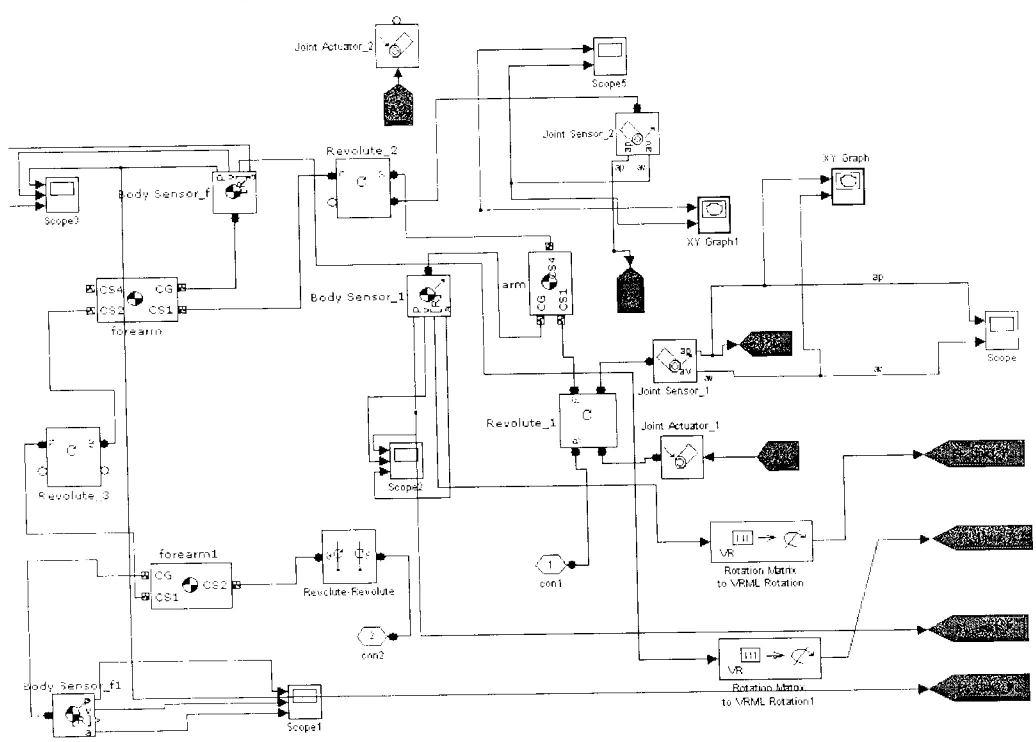

Figure 5.19: SimMechanics block of each leg of Figure 5.18

Figure 5.19 shows the SimMechanic blocks of each leg in the model described in Figure 5.18. Joint sensor blocks and body sensor blocks are used to read SimMechanics output signals and convert those output signals to matrix form using Rotation Matrix to VRML Rotation blocks.

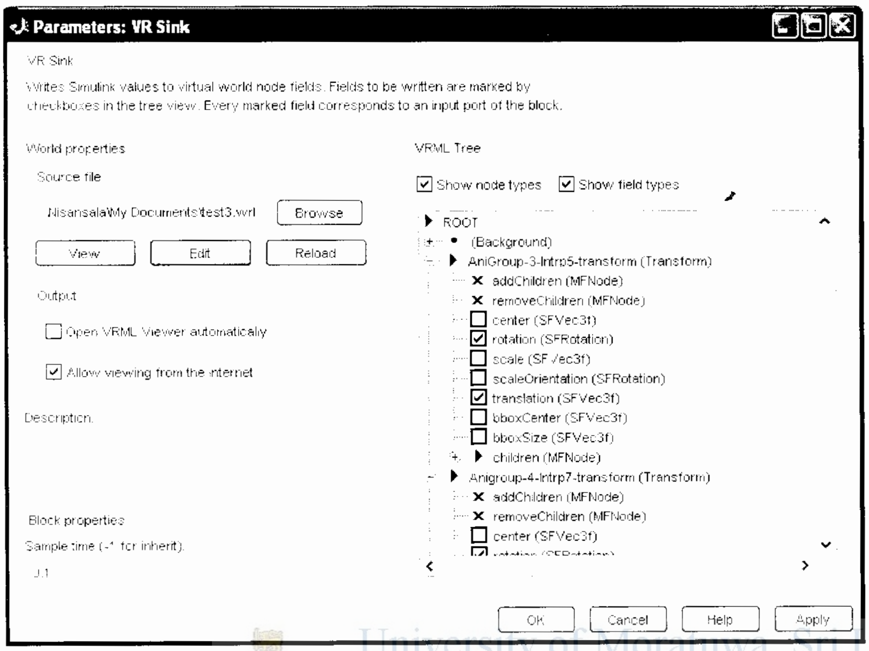

Figure 5.20: VR Sink block parameters

VR Sink parameter block is used to parameterize the VRML environment and VRML Tree nodes.

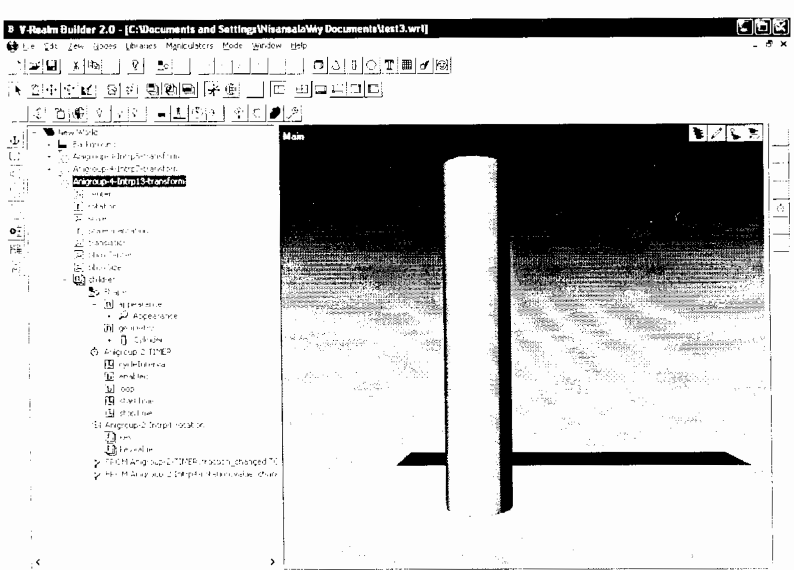

Figure 5.21: Develop VRML model using V-Realm Builder tool

V-Realm Builder tool can be used to model our VRML model. This is just a VRML modeling tool and not a simulation tool. But using VR Sink block we can connect SimMechanics output signals to VRML model to simulate it according to SimMechanics output signals.

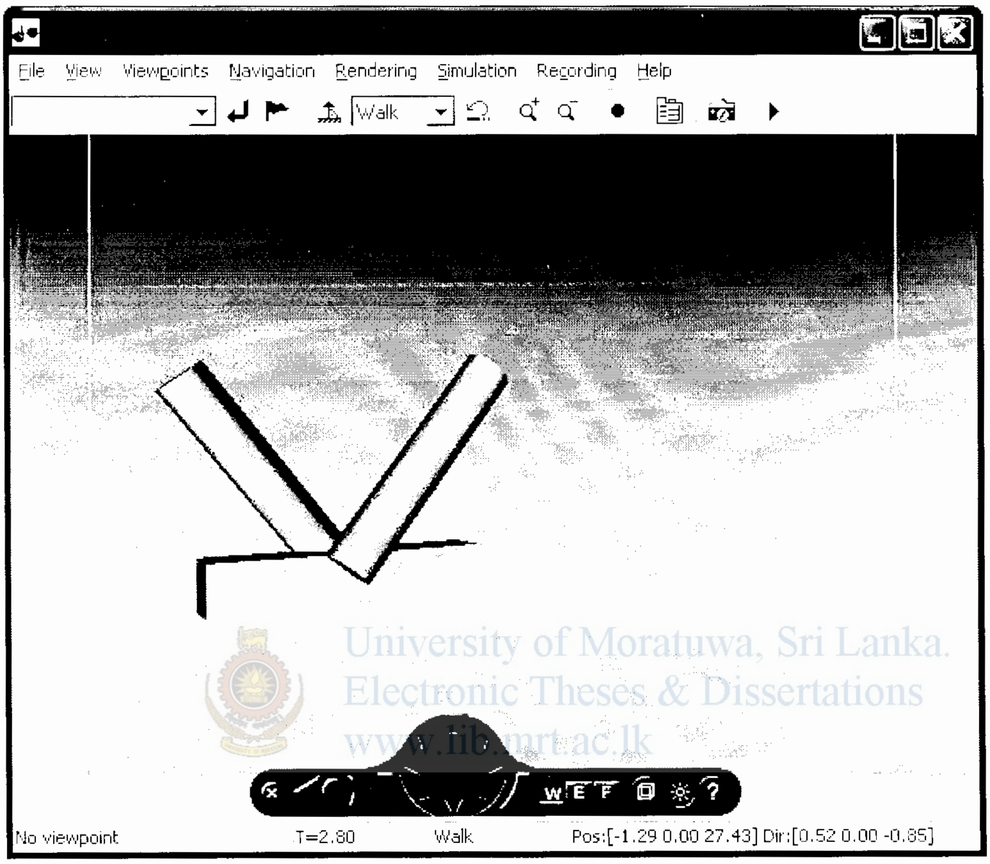

Figure 5.22: Two leg robot simulation on VRML viewer

Finally Figure 5.22 show how two legs robot is moving forward in VRML viewer in accordance to the SimMechanics output signals.

5.4. Robot Control System

The robot control system (Figure 5.23) is based on the PIC 16F8XX micro-controllers. Each leg is controlled by a dedicated motor and another is used to control a gripper of the leg. These two motors are controlled by a single controller board. The motor controller board is capable of controlling individual leg and its gripper. Control unit is used to synchronize four controller boards and perform entire robot forward and backward movements. Individual sensor signals from leg sensors are used as feedback signals to the control unit of the robot.

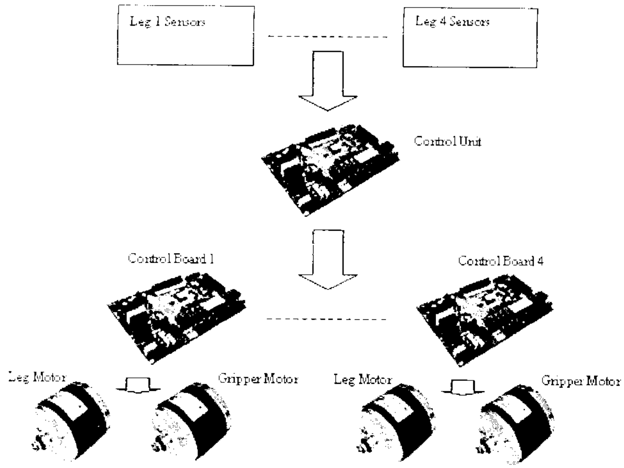

Figure 5.23: Robot control system

Individual motor controller board (Figure 5.24 and Figure 5.25) is capable of controlling leg motor and gripper motor of each leg. Control unit sends control signals to this board and based on them, motor controller board controls respective motors. Control unit sends only high level control signals to control each leg and controller board itself intelligence enough to execute low level functions for individual leg and its gripper. This decomposition of hardware and software components of the robot is necessary to reduce the complexity of the robot implementation. Complete robot is only needed one control unit board and four motor controller boards. All four motor controller boards are identical. Only two software pieces are needed and one for control unit and other for each controller board. All four motor controller boards are sharing same motor controller software algorithm. By decoupling the complexity of the control software algorithms, it is less complex to develop them and test them individually. Individually tested components can be integrated and perform integrated testing for overall performance of the robot.

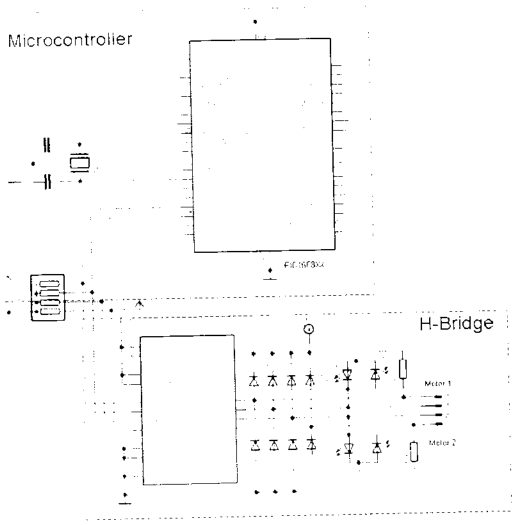

Figure 5.24: Circuit diagram of motor controller board

PIC16F877A micro-controller is used to develop the motor controller board. Four PWM signals are generating using micro-controller and feed into H-Bridge circuit to control two DC motors.

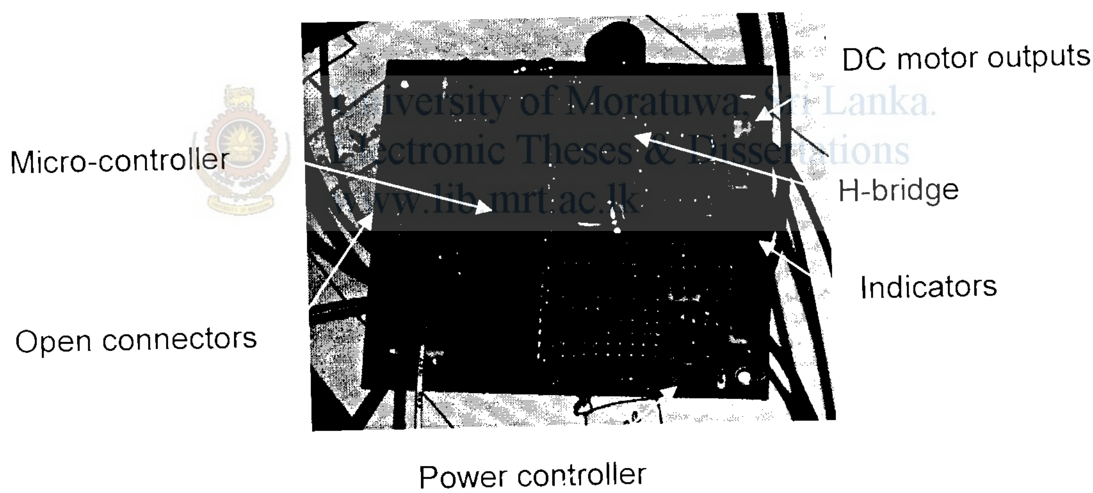

Figure 5.25: Motor controller board

Figure 5.25 shows the developed motor controller board and its components. This board is capable of control two DC motors. Five sensor signals can be feed into the board and behavior of two motors can be controlled accordingly.

5.5. Robot Implementation

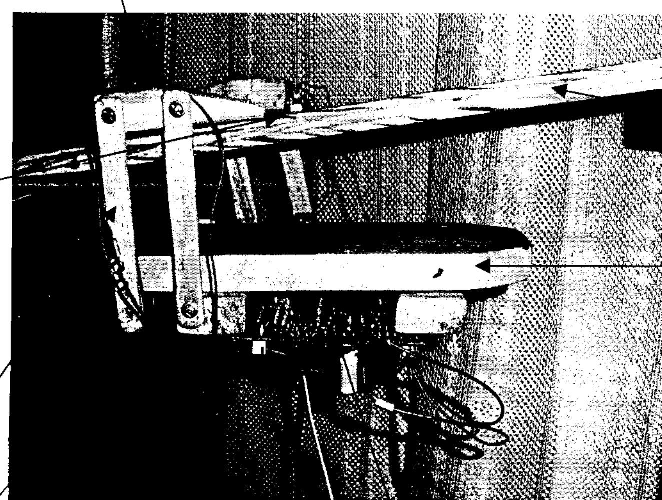

Figure 5.26: Two leg rope climbing robot prototype

Figure 5.26 shows two legs robot prototype developed for testing purposes. This prototype uses previous described motor controller board and two gear DC motors. Each leg has two sensor pair to detect position of the leg.

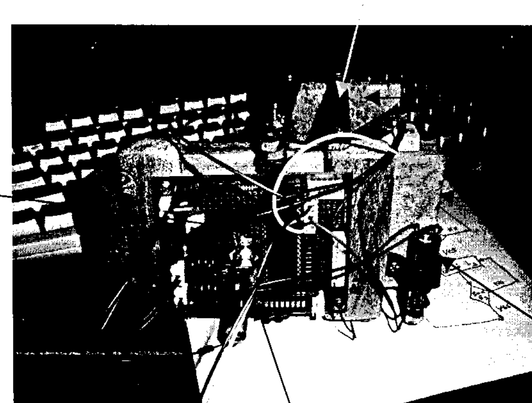

Figure 5.27: Robot prototype and its configuration

Figure 5.27 shows how the motor controller board and gear DC motors fix with the robot prototype. Also this figure clearly shows how free link has used.

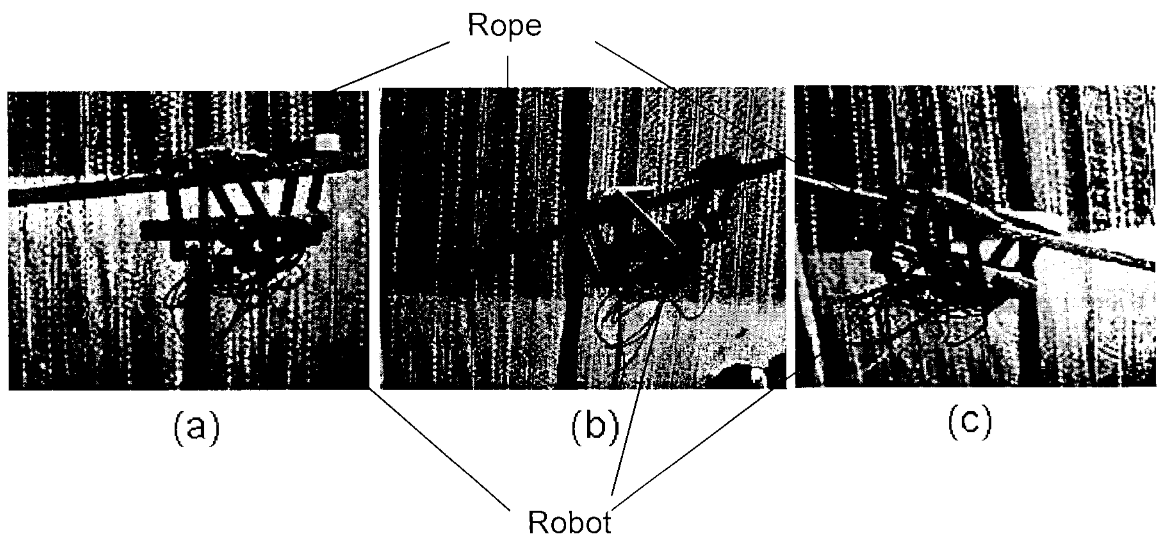

Figure 5.28: (a) Robot climb on horizontal rope, (b) Robot climb down and (c) robot climb up

Two legs robot prototype is developed and tested successfully to demonstrate the proposed solution. The robot prototype is developed using PIC16F877 micro-controller based controller board and gear DC motors. Figure 5.26 shows the developed robot prototype and Figure 5.27 shows how the motor controller board and motors fixed with the robot body. Figure 5.28 (a) is a snapshot of robot is moving horizontally on the rope. Robot can climb upward and downward as shown in figure 5.28 (b) and figure 5.28 (c).

Chapter 6

6. Applications and Results of the developed solution

6.1. Applications of the proposed solution

The proposed robot solution can be applicable in various practical situations to help humans in the means of reduce their work, protect them from insecure environments, increase productivity and research perspective. Although wheeled type robots are perform well in smooth environments, they are not effective on uneven or rough surfaces. One of the best alternative for wheeled type robots are legged type robots. But legged type robot is also having some limitations where rope climbing robots can operate on such scenarios much effectively. Practical applications of the proposed robot design discusses below.

6.1.1. Rescue operations

The proposed robot implementation can be effectively used in rescue operations while reduce the underline risk on the rescue officer. Robot could be able to climb using ropes or any media already exist in the environment that can be used to climb. Primarily this robot has designed to work with various types and sizes of ropes. This feature enables us to use various media that is already available in the environment for robot operation. Due to this reason this robot can be easily used in various practical situations. For example, if the rescue officer wants to go into fire effected building, but he is incapable of fixing their ladders. But if the building is already having some rope like media such as bundle of electricity cables or some pipe lines that the robot might be able to use them as a rope to climb and enter into the building. This method might have associated risk but still the operator can take the risk since human operator is in secure.

6.1.2. Work in hazardous environments

As discussed in previous section this robot is ideal for use in insecure environments. Not only for rescue operations but also the robot can be used to reach insecure environments and perform various activities such as take photographs, video out the scenario...etc. Volcano, toxic gas affected buildings, nuclear plants...etc can be covered by the robot.

6.1.3. Research activities

The similar robot designs are heavily used in various research activities in different geographical locations. Research perform in insecure locations, locations that difficult to reach can be covered by the robot and perform specific research activities. Also this kind of robot can be used to discover new locations and do research on them. It might be a huge risk to assign human than use similar robot at initial stages. Later if the location is secured human researches can go in to those locations.

6.1.4. Field operations

The proposed robot can be used to perform various field operations that can not be performed by wheeled vehicles. Various machinery works on rocks or mountains are much easier with rope climbing robots. Since the developed robot is four legged robot, its operations much smooth and it is capable of handling heavy operations. Wheeled type rope climbing robots might not be able used in some areas due to their restrictions.

6.1.5. Search operations

Various search operations can be carried out easily and effectively by using rope climbing robots than wheeled robots and some of the leg robots. The robot can be used in disaster situations, unsafe buildings and nuclear chemical effected areas for clearing and search operations.

6.1.6. Military operations

Different types of robots are heavily used in military operations around the world. Rope climbing robots can be used in military operations such as spying, search operations, area clearing and rescue operations...etc.

6.2. Results and Analysis

Based on the Robot model calculations, it was proofed that each robot leg can be controlled by a single actuator and each gripper needs another. For steady and smooth operation robot is fabricated with four legs and four grippers. Any given time frame minimum three legs are planned to contact with the rope and grippers of those legs should attach with the rope. This mechanism will improve the steady movement of the robot carrying with weight on it. Kinematics analysis concludes that relationship between Cartesian velocities and joint velocities depend on S_{β}. That mean by controlling only angle β, we can control the robot leg. Dynamic analysis is based on the Newton-Euler method with Luh-Walker-Paul's algorithm (Recur vise Newton-Euler). Torque profiles of each leg depend on $\theta, \dot{\theta}, \ddot{\theta}, \beta, \ddot{\beta}$ angles.

Simulation model of the robot is developed using SimMechanics and Simulink toolboxes of MATLAB software application. Phase plots of the simulation show the stability of the total energy conservation of the robot leg. By comparing simulation results for two links robot legs configuration and three links robot leg configuration with free link, we can conclude that three links robot leg can perform its operation as same as two links robot leg. But three links robot leg with free link help to reduce number of actuators required to fabricate the entire robot, further it facilitate to cost effective robot implementation. Finally 3D robot simulation is done using VRML robot model and VR Toolbox of Matlab application package.

The robot control system is based on the PIC 16F8XX micro-controllers. Each leg is controlled by a dedicated motor and another is used to control a gripper of the leg. These two motors are controlled by a single controller board. The motor controller board is capable of controlling individual leg and its gripper. Control unit is used to synchronize four controller boards and perform entire robot forward and backward movements. Individual sensor signals from leg sensors are used as feedback signals to the control unit of the robot.

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APPENDIX – A

PICBASIC code for 2 leg robot

```
TRISC = %00000000 'Output port
TRISB = %11111111 'Input port

direction var BIT
homePosition var BIT        '0 - not in home position, 1 - home position
leftLegForward var BIT      '0 - not forward, 1 - forward
rightLegForward var BIT     '0 - not forward, 1 - forward

PortB.1 = 0
PortB.2 = 0

    'HPWM 2, 250 , 1000         'PWM for CCPP1
    Low PortC.0          'Backward
    High PortC.1         'Backward

    Low PortC.3          'Backward
    High PortC.2         'Backward

    homePosition = 0
Start:

    if PortB.1 = 1 and PortB.2 = 1 and PortB.3 =1 and homePosition = 0 then
        High PortC.0  'Backward
        High PortC.1  'Backward

        High PortC.2          'Backward
        High PortC.3          'Backward

        High PortC.0  'Backward
        Low PortC.1   'Backward

        homePosition = 1
        leftLegForward = 1
        rightLegForward = 0
    endif

    if PortB.1 = 0 and PortB.2 = 1 and PortB.3 = 1 and leftLegForward = 1 then
        Low PortC.2           'Backward
        High PortC.3          'Backward
        leftLegForward = 0
        rightLegForward = 1
    endif

    if PortB.1 = 0 and PortB.2 = 1 and PortB.3 = 1 and rightLegForward = 1 then
        Low PortC.0           'Backward
```

```
            High PortC.1          'Backward

            Low PortC.3           'Backward
            High PortC.2          'Backward

            homePosition = 1
        endif

Goto Start
```