



# **SOME PROBLEMS IN TIME SERIES ANALYSIS AND FORECASTING**

**M. SHELTON PEIRIS**  
B.Sc. (Hons), Postgrad. Dip., M.Sc.  
(Sri Lanka)

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## Summary

This thesis is concerned with various investigations relating to time series analysis and forecasting. Particular attention is given to fractional differencing and its applications to long memory time series models.

Chapter 1 entitled "Introduction", contains the summary of the basic time series theory required for the work carried out in the remaining six chapters. In addition to the general theory, the notion of fractional differencing, time reversibility and optimal experimental designs are included as special aspects of the thesis.

In Chapter 2, the general class of univariate ARMA models with time dependent coefficients is considered. Existence and uniqueness of a second-order solution to the model is established using certain AR and MA regularity conditions. A simple form of the solution~ the covariance structure and the associated model building problem are considered from the theoretical point of view. The prediction problem is solved using alternative approaches. Some recursive relations to the optimum linear predictors are obtained by the orthogonal projection in Hilbert spaces. Few examples are added in support of the general results derived in this chapter.

Chapter 3 considers the multivariate generalization of the results obtained in Chapter 2. The condition for the asymptotic stationarity of the associated process is obtained in terms of the spectral radii of the corresponding matrices. It is shown that the recursive relations of the predictors satisfy a matricial equation as in the univariate case. Assuming the predictive distribution to be multivariate normal, a simultaneous confidence interval for the predictors is derived.

The family of ARMA models with constant coefficients and nonstationary innovations is considered in Chapter 4. A particular form of the model is considered for further analysis. It is shown that the usual regularity conditions are necessary and sufficient to ensure the existence and uniqueness of second-order solutions. The covariance structure of the associated process is obtained. The prediction problem



is solved using the same procedures as in the stationary innovation case. The effect of nonstationarity in noise is shown to be insignificant in the parameter estimation.

In Chapter 5 the theoretical autocorrelation function of an ARIMA (0,d,q) process is obtained. An asymptotically unbiased estimator of the sample spectrum is given. The various relations of the predictors are obtained. In particular, the attention is paid to the minimum mean squared error prediction and it is shown that this predictor is not optimal from the theoretical point of view. Finally, a direct basic form of the predictors of ARIMA (0,d,q) ;  $d \geq 1$  model is obtained.

Fractional differencing and its applications to long memory time series are discussed in Chapter 6. This new class of ARIMA models aroused the interest of many time series since it has numerous applications in several scientific A proof for the stationarity and invertibility conditions ARIMA(p,d,q) ;  $d \in \mathbb{R}$  is given. Persistence in time series the Hurst phenomenon are examined for some actual situations. For the predictors of ARIMA (p,d,q) ; are given and it is shown that the minimum mean squared optimum properties when  $|d| < 1/2$  . Numerical examples are added. In particular the long memory characteristic of some Australian rivers are demonstrated a suitable model is fitted in each case.

Chapter 7 addresses itself to some miscellaneous problems in time series analysis. The first section is devoted to the discussion of some functions of ARMA models and generalizes the existing results for the multivariate case. Some contributions to bilinear time series models are considered in Section 7.2. Various new results regarding BL(p,q,p,s) and BL(p,q,r,l) are given. Section 7.3 considers the notion of time reversibility of a stochastic process and non Gaussian ARMA models from the theoretical point of view. The last section i.e. Section 7.4 discusses the optimal experimental design problems of time series regression model, when the coefficients are stochastic. It is shown that under certain conditions the effect of randomness of the coefficients is insignificant in the optimal design problem.

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M. SHELTON PEIRIS

B.Sc. (Hons), Postgrad. Dip., M.Sc.  
(Sri Lanka)



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patience and support of  
my wife and son.


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# CONTENTS

	<u>page</u>
SUMMARY	(i)
STATEMENT OF AUTHORSHIP	(iv)
LIST OF PAPERS SUBMITTED FOR PUBLICATION	(v)
ABBREVIATIONS AND NOTATION	(vi)
ACKNOWLEDGEMENTS	(viii)
CHAPTER 1	
INTRODUCTION	1
1.1	
Time Series Analysis	1
1.2	
Probabilistic Nature of Time Series Analysis	2
1.3	
Stationarity of a Stochastic Process	3
1.4	
Linear Prediction of a Stationary Stochastic Process	8
1.5	
Nonstationary Processes	11
1.6	
The General Linear Process and its Parsimonius Version	13
1.7	
The Model Building Problem in Linear Time Series Analysis	17
1.8	
Vector Linear Time Series Models	20
1.9	
Long Memory Time Series Models	22
1.10	
Some Functions of ARMA Processes	24
1.11	
Some Bilinear Models in Time Series	25
1.12	
Non Gaussian ARMA Models	26
1.13	
Optimal Experimental Design Problems for Time Series Regression Models	27
CHAPTER 2	
THE GENERALIZED CLASS OF UNIVARIATE TIME SERIES MODELS	29
2.1	
Introduction	29
2.2	
ARMA Models with Time Dependent Coefficients	31


	<u>page</u>	
2.3	AR and MA Regularity Conditions	34
2.4	The Explicit Solution and Inversion of a Model	38
2.5	The Covariance Structure	44
2.6	The Model Building Problem	48
2.7	Review of Some Related Topics on Time Dependent ARMA Models	53
2.8	On the Prediction with Time Dependent ARMA Models	57
CHAPTER 3	VECTOR LINEAR TIME SERIES MODELS WITH TIME DEPENDENT COEFFICIENTS	64
3.1	Introduction	64
3.2	Theoretical Models For Multiple Time Series	65
3.3	m-Variate Mixed Autoregressive Moving Average Models with Time Dependent Coefficients	67
3.4	The Explicit Solution of a Model	68
3.5	Generalized AR and MA Regularity Conditions and Wold-Cramer Decomposition	74
3.6	Theoretical Covariance Matrix Function	76
3.7	Various Aspects of Prediction in Nonstationary Multivariate ARMA Processes	82
CHAPTER 4	LINEAR TIME SERIES MODELS WITH NONSTATIONARY INNOVATIONS	93
4.1	Introduction	93
4.2	ARMA Models With Nonstationary Innovations	94
4.3	AR and MA Regularity Conditions and Some Related Results	96



		<u>page</u>
4.4	The Covariance Structure and Multivariate Stochastic Identification	100
4.5	The Prediction Problem	103
4.6	The Estimation Problem	110
4.7	Some Further Results on a Univariate Model	113
CHAPTER 5	THE EFFECT OF DIFFERENCING IN TIME SERIES ANALYSIS	115
5.1	Introduction	115
5.2	Background Theory and Some Related Results	118
5.3	Spectrum and Innovation Representation of an ARIMA(p,d,q) Process	120
5.4	 The Autocovariance Functions of ARIMA(0,d,q) and ARIMA(1,d-1,q) Processes	122
5.5	Prediction with ARIMA Models	131
CHAPTER 6	FRACTIONAL DIFFERENCING AND ITS APPLICATIONS TO LONG MEMORY TIME SERIES	141
6.1	Introduction	141
6.2	Some Basic Results	143
6.3	Persistence in Time Series and the Hurst Phenomenon	148
6.4	Fractionally Differenced ARIMA(0,d,0) Models and their Time Series Properties	152
6.5	Fractionally Differenced ARIMA(p,d,q) Models	168
6.6	Prediction with Fractionally Differenced ARIMA Models	173
6.7	Some Applications of Long Memory Time Series Models	179





	<u>page</u>
CHAPTER 7	
MISCELLANEOUS TOPICS IN TIME SERIES ANALYSIS	183
7.1	
On the Study of Some Functions of ARMA Processes	183
7.2	
Some Contributions to Bilinear Time Series Analysis	188
7.3	
Time Reversibility of a Stochastic Process and Non Gaussian ARMA Models	193
7.4	
Optimal Experimental Design for Linear Time Series Regression Models with Stochastic Coefficients	199
APPENDICES	
 University of Moratuwa, Sri Lanka. Electronic Theses & Dissertations APPENDIX 1 <a href="http://www.lib.mrt.ac.lk">www.lib.mrt.ac.lk</a>	A A1.1
APPENDIX 2	A2.1
REFERENCES	R1

## SUMMARY

This thesis is concerned with various investigations relating to time series analysis and forecasting. Particular attention is given to fractional differencing and its applications to long memory time series models.

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In Chapter 5 the theoretical autocorrelation function of an ARIMA(0,d,q) process is obtained. An asymptotically unbiased estimator of the sample spectrum is given. The various relations of the predictors are obtained. In particular, the attention is paid to the minimum mean squared error prediction and it is shown that this predictor is not optimal from the theoretical point of view. Finally, a direct basic form of the predictors of ARIMA(0,d,q) ;  $d \geq 1$  model is obtained.

Fractional differencing and its applications to long memory time series are discussed in Chapter 6. This new class of ARIMA models aroused the interest of many time series analysts, since it has numerous applications in several scientific areas. A proof for the stationarity and invertibility conditions for ARIMA(p,d,q) ;  $d \in \mathbb{R}$  is given. Persistence in time series and the Hurst phenomenon are examined for some actual situations. Some recursive relations for the predictors of ARIMA(p,d,q) ;  $d \in \mathbb{R}$  are given and it is shown that the minimum mean squared error predictor has its optimum properties when  $|d| < \frac{1}{2}$ . Some numerical examples are added. In particular the long memory characteristic of some Australian rivers are demonstrated and a suitable model is fitted in each case.

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## STATEMENT OF AUTHORSHIP

This thesis contains no material which has been accepted for an award of any other degree or diploma at any University. To the best of my knowledge, this thesis contains no material previously published, submitted for publication or written by any other individual than myself, except where due credit and comment is made in the text.

Some papers out of this thesis have been accepted for publication.

Peiris, M.S. (1986a). A note on the Predictors of differenced sequences.

(The Australian Journal of Statistics).

Peiris, M.S. (1986b). On Prediction with time dependent ARMA models. (Communications in Statistics).



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Singh, N. and Peiris, M.S. (1986). A note on the properties of some nonstationary ARMA processes.

(Stochastic Processes and their Applications).

The results of Chapters 2, 3, 5 and 6 are based on the above papers and the material of Chapters 4 and 7 has been submitted for publication.

M. Shelton Peiris  
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## LIST OF PAPERS SUBMITTED FOR PUBLICATION

1. Some Contributions to Bilinear Time Series Analysis.
2. Optimal Experimental Designs for Linear Time Series Regression Models with Stochastic Coefficients.  
(with Dr N Singh). (Under Revision)
3. Vector Linear Time Series Analysis when the Noise is Nonstationary. (Revised)
4. Analysis of IMA Processes. (with Dr N Singh). (Under Revision)
5. Sums of Multivariate ARMA Processes.
6. Analysis of Time Series when the Marginal Distribution is Non-Normal.  
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7. A Note on the properties of some Long Memory Time Series Models. (Revised)
8. On the Prediction of Multivariate ARMA processes with a Time Dependent Covariance Structure. (Revised)

## ABBREVIATIONS AND NOTATION

Throughout the thesis the equations are denoted by  $(X \cdot Y \cdot Z)$ , which means the equation  $Z$  of Section  $Y$  of Chapter  $X$ .

Similarly, definitions, theorems, corollaries, lemmas and examples are numbered. That is, Theorem  $X \cdot Y \cdot Z$  means Theorem  $Z$  of Section  $Y$  of Chapter  $X$ .

ARIMA	-	Autoregressive Integrated Moving Average
acf	-	Autocorrelation Function
pacf	-	Partial Autocorrelation Function
sdf	-	Spectral Density Function
wn	-	White Noise
BJ	-	Box and Jenkins
UBJ	-	Univariate Box and Jenkins
olp	-	Optimum Linear Predictor
mmsep	-	Minimum Mean Squared Error Predictor
fd	-	Fractional Differencing
fdwn	-	Fractionally Differenced White Noise
df	-	Degrees of Freedom
ie.	-	That is
cf.	-	Compare
w.l.o.g.	-	Without loss of Generality
mle	-	Maximum Likelihood Estimator
lse	-	Least Squares Estimator
mse	-	Mean Squared Error
det A	-	Determinant of square matrix A
$A^T$	-	Transpose of Matrix A

## Abbreviations and Notation (continued)

I	-	Identity Matrix
rhs	-	Right Hand Side
lhs	-	Left Hand Side
$N(\mu, \sigma^2)$	-	Normal Distribution with Mean $\mu$ and Variance $\sigma^2$ .
$N_m(\underline{\mu}, \Omega)$	-	m-Variate Normal Distribution with Mean Vector $\underline{\mu}$ and Variance-Covariance Matrix $\Omega$ .



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