SOME PROBLEMS IN
TIME SERIES ANALYSIS AND FORECASTING

M. SHELTON PEIRIS
B.Sc. (Hons), Postgrad. Dip., M.Sc.
(Sri Lanka)

A thesis submitted to
MONASH UNIVERSITY
for the degree of
DOCTOR OF PHILOSOPHY

DEPARTMENT OF MATHEMATICS, MONASH UNIVERSITY
MELBOURNE, AUSTRALIA

JANUARY 1987.

Summary

This thesis is concerned with various investigations relating to time series analysis and forecasting. Particular attention is given to fractional differencing and its applications to long memory time series models.

Chapter 1 entitled "Introduction", contains the summary of the basic time series theory required for the work carried out in the remaining six chapters. In addition to the general theory, the notion of fractional differencing, time reversibility and optimal experimental designs are included as special aspects of the thesis.

In Chapter 2, the general class of univariate ARMA models with time dependent coefficients is considered. Existence and uniqueness of a second-order solution to the model is established using certain AR and MA regularity conditions. A simple form of the solution~ the covariance structure and the associated model building problem are considered from the theoretical point of view. The prediction problem is solved using alternative approaches. Some recursive relations to the optimum linear predictors are obtained by the orthogonal projection in Hilbert spaces. Few examples are added in support of the general results derived in this chapter.

Chapter 3 considers the multivariate generalization of the results obtained in Chapter 2. The condition for the asymptotic stationary of the associated process is obtained in terms of the spectral radii of the corresponding matrices. It is shown that the recursive relations of the predictors satisfy a matricial equation as in the univariate case. Assuming the predictive distribution to be multivariate normal, a simultaneous confidence interval for the predictors is derived.

The family of ARMA models with constant coefficients and nonstationary innovations is considered in Chapter4. A particular form of the model is considered for further analysis. It is shown that the usual regularity conditions are necessary and sufficient to ensure the existence and uniqueness of second-order solutions. The covariance structure of the associated process is obtained. The prediction problem

is solved using the same procedures as in the stationary innovation case. The effect of nonstationarity in noise is shown to be insignificant in the parameter estimation.

In Chapter 5 the theoretical autocorrelation function of an ARIMA (0,d,q) process is obtained. An asymptotically unbiased estimator of the sample spectrum is given. The various relations of the predictors are obtained. In particular, the attention is paid to the minimum mean squared error prediction and it is shown that this predictor is not optimal from the theoretical point of view. Finally, a direct basic form of the predictors of ARIMA (0,d,q) ; d ≥1 model is obtained.

Fractional differencing and its applications to long memory time series are discussed in Chapter 6. This new class of ARIMA models aroused the interest of many time series since it has numerous applications in several scientific A proof for the stationarity and invertibility conditions ARIMA(p,d,q) ; d € R is given. Persistence in time series the Hurst phenomenon are examined for some actual situations. For the predictors of ARIMA (p,d,q) ; are given and it is shown that the minimum mean squared optimum properties when |d| $< 1/2$. Numerical examples are added. In particular the long memory characteristic of some Australian rivers are demonstrated a suitable model is fitted in each case.

Chapter 7 addresses itself to some miscellaneous problems in time series analysis. The first section is devoted to the discussion of some functions of ARMA models and generalizes the existing results for the multivariate case. Some contributions to bilinear time series models are considered in Section 7.2. Various new results regarding BL(p,q,p,s) and BL(p,q,r,l) are given. Section 7.3 considers the notion of time reversibility of a stochastic process and non Gaussian ARMA models from the theoretical point of view. The last section i.e. Section 7.4 discusses the optimal experimental design problems of time series regression model, when the coefficients are stochastic. It is shown that under certain conditions the effect of randomness of the coefficients is insignificant in the optimal design problem.

SOME PROBLEMS IN

TIME SERIES ANALYSIS AND FORECASTING

By

M. SHELTON PEIRIS

B.Sc. (Hons), Postgrad. Dip., M.Sc.

(Sri Lanka)

A thesis submitted to

MONASH UNIVERSITY

for the degree of

DOCTOR OF PHILOSOPHY

DEPARTMENT OF MATHEMATICS, MONASH UNIVERSITY

MELBOURNE, AUSTRALIA

JANUARY 1987.

DEDICATED TO :

The wisdom and sacrifice
of my parents and the
patience and support of
my wife and son.

CONTENTS

SUMMARY

This thesis is concerned with various investigations relating to time series analysis and forecasting. Particular attention is given to fractional differencing and its applications to long memory time series models.

Chapter 1 entitled "Introduction", contains the summary of the basic time series theory required for the work carried out in the remaining six chapters. In addition to the general theory, the notion of fractional differencing, time reversibility and optimal experimental designs are included as special aspects of the thesis.

In Chapter 2, the general class of univariate ARMA models with time dependent coefficients, is considered. Existence and uniqueness of a second-order solution to the model is established using certain AR and MA regularity conditions. A simple form of the solution, the covariance structure and the associated model building problem are considered from the theoretical point of view. The prediction problem is solved using alternative approaches. Some recursive relations to the optimum linear predictors are obtained by the orthogonal projection in Hilbert spaces. Few examples are added in support of the general results derived in this chapter.

Chapter 3 considers the multivariate generalizations of the results obtained in Chapter 2. The condition for the

asymptotic stationarity of the associated process is obtained in terms of the spectral radii of the corresponding matrices. It is shown that the recursive relations of the predictors satisfy a matricial equation as in the univariate case. Assuming the predictive distribution to be multivariate normal, a simultaneous confidence interval for the predictors is derived.

The family of ARMA models with constant coefficients and nonstationary innovations is considered in Chapter 4. A particular form of the model is considered for further analysis. It is shown that the usual regularity conditions are necessary and sufficient to ensure the existence and uniqueness of second-order solutions. The covariance structure of the associated process is obtained. The prediction problem is solved using the same procedures as in the stationary innovation case. The effect of nonstationarity in noise is shown to be insignificant in the parameter estimation.

In Chapter 5 the theoretical autocorrelation function of an ARIMA(0,d,q) process is obtained. An asymptotically unbiased estimator of the sample spectrum is given. The various relations of the predictors are obtained. In particular, the attention is paid to the minimum mean squared error prediction and it is shown that this predictor is not optimal from the theoretical point of view. Finally, a direct basic form of the predictors of ARIMA(0,d,q) ; $d \geq 1$ model is obtained.

Fractional differencing and its applications to long memory time series are discussed in Chapter 6. This new class of ARIMA models aroused the interest of many time series analysts, since it has numerous applications in several scientific areas. A proof for the stationarity and invertibility conditions for ARIMA(p,d,q) ; $d \in \mathbb{R}$ is given. Persistence in time series and the Hurst phenomenon are examined for some actual situations. Some recursive relations for the predictors of ARIMA(p,d,q) ; $d \in \mathbb{R}$ are given and it is shown that the minimum mean squared error predictor has its optimum properties when $|d| < \frac{1}{2}$. Some numerical examples are added. In particular the long memory characteristic of some Australian rivers are demonstrated and a suitable model is fitted in each case.

Chapter 7 addresses itself to some miscellaneous problems in time series analysis. The first section is devoted to the discussion of some functions of ARMA models and generalizes the existing results for the multivariate case. Some contributions to bilinear time series models are considered in Section 7.2 . Various new results regarding BL(p,q,p,s) and BL(p,q,r,1) are given. Section 7.3 considers the notion of time reversibility of a stochastic process and non Gaussian ARMA models from the theoretical point of view. The last section i.e. Section 7.4 discusses the optimal experimental design problems of time series regression model, when the coefficients are stochastic. It is shown that under certain conditions the effect of randomness of the coefficients is insignificant in the optimal design problem

STATEMENT OF AUTHORSHIP

This thesis contains no material which has been accepted for an award of any other degree or diploma at any University. To the best of my knowledge, this thesis contains no material previously published, submitted for publication or written by any other individual than myself, except where due credit and comment is made in the text.

Some papers out of this thesis have been accepted for publication.

Peiris, M.S. (1986a). A note on the Predictors of differenced sequences.
(The Australian Journal of Statistics).

Peiris, M.S. (1986b). On Prediction with time dependent ARMA models.
(Communications in Statistics).

Singh, N. and Peiris, M.S. (1986). A note on the properties of some nonstationary ARMA processes.
(Stochastic Processes and their Applications).

The results of Chapters 2, 3, 5 and 6 are based on the above papers and the material of Chapters 4 and 7 has been submitted for publication.

M. Shelton Peiris

January 1987
Melbourne.

LIST OF PAPERS SUBMITTED FOR PUBLICATION

1. Some Contributions to Bilinear Time Series Analysis.

2. Optimal Experimental Designs for Linear Time Series Regression Models with Stochastic Coefficients. (with Dr N Singh). (Under Revision)

3. Vector Linear Time Series Analysis when the Noise is Nonstationary. (Revised)

4. Analysis of IMA Processes. (with Dr N Singh). (Under Revision)

5. Sums of Multivariate ARMA Processes.

6. Analysis of Time Series when the Marginal Distribution is Non-Normal.

7. A Note on the properties of some Long Memory Time Series Models. (Revised)

8. On the Prediction of Multivariate ARMA processes with a Time Dependent Covariance Structure. (Revised)

ABBREVIATIONS AND NOTATION

Throughout the thesis the equations are denoted by (X•Y•Z) , which means the equation Z of Section Y of Chapter X .

Similarly, definitions, theorems, corollaries, lemmas and examples are numbered. That is, Theorem X•Y•Z means Theorem Z of Section Y of Chapter X .

ARIMA	-	Autoregressive Integrated Moving Average
acf	-	Autocorrelation Function
pacf	-	Partial Autocorrelation Function
sdf	-	Spectral Density Function
wn	-	White Noise
BJ	-	Box and Jenkins
UBJ	-	Univariate Box and Jenkins
olp	-	Optimum Linear Predictor
mmsep	-	Minimum Mean Squared Error Predictor
fd	-	Fractional Differencing
fdwn	-	Fractionally Differenced White Noise
df	-	Degrees of Freedom
ie.	-	That is
cf.	-	Compare
w.l.o.g.	-	Without loss of Generality
mℓe	-	Maximum Likelihood Estimator
ℓse	-	Least Squares Estimator
mse	-	Mean Squared Error
det A	-	Determinant of square matrix A
A^T	-	Transpose of Matrix A

Abbreviations and Notation (continued)

I	-	Identity Matrix
rhs	-	Right Hand Side
ℓhs	-	Left Hand Side
$N(\mu,\sigma^2)$	-	Normal Distribution with Mean μ and Variance σ^2.
$N_m(\underset{\sim}{\mu},\Omega)$	-	m-Variate Normal Distribution with Mean Vector $\underset{\sim}{\mu}$ and Variance-Covariance Matrix Ω .

ACKNOWLEDGEMENTS

I wish to express my deepest appreciation and love to Jayanthi and Udara for the strength, understanding and love which they have provided during the completion of the thesis.

I am indebted to my parents, teachers and all my friends who contributed to my development both as a statistician/ mathematician and as an individual.

I wish to express my gratitude to Dr N Singh for his supervision, encouragement, aid, valuable discussions, constant guidance and unfailing politeness throughout the course.

I am delighted to mention that the work I have done would not have had a chance to materialize if it were not for the kindness of Professor Maxwell L.King of the Department of Econometrics and Operations Research, Monash University. His fruitful comments and useful suggestions certainly improved the quality of the thesis.

I am also grateful to Dr Chris Perera of the Civil Engineering Department, Monash University for his useful suggestions and help in computer simulation studies and providing me sufficient data for the applications demonstrated in Chapter 6.

My sincere thanks are also due to Dr S G Loo (Monash University) and Dr Paul Kabaila (La Trobe University) for many helpful discussions and suggestions leading to the improvement of the quality of the thesis.

I acknowledge the help of the following individuals/ groups during the course of this project : Professors P D Finch and W J Ewens for their understanding the difficulties of an overseas student, help and kindness; members of the Statistics section for many discussions; Dr G Codner (Department of Civil Engineering) for some discussions and for providing me with data for an example in Chapter 6; Mr Dayawansa for his suggestions and help in the work of Chapter 6; Rallis family (now in Greece) for their warm welcome and for providing an attractive environment to study while they were in Melbourne; Mr Sumet Peiris and Mrs Manel Peiris for their nice correspondence and availability for any help; Mr G Taylor (HEARU) for his encouragement from the beginning; Gopalan, Rudy, Con and Magret for reading the typed version of the thesis and many other supports; Rajani for some discussions.

Also I am grateful to Monash University and the Department of Mathematics in particular for the financial support through a Monash Graduate Scholarship.

The ministry of higher education, University of Moratuwa and its Department of Mathematics in Sri Lanka are greatly acknowledged for granting study leave and the travel arrangements, which made this research program possible.

My sincere thanks also go to Mrs Billie Russell for her excellent typing of the thesis, a job which required great patience and care.