RATIONALIZATION OF PRESTRESSED CONCRETE SPINE BEAM DESIGN PHILOSOPHY FOR EXPERT SYSTEMS

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Abstract

The most important aim of expert systems is to emulate the expert. The majority of existing expert systems for design try to achieve this by integrating the phases of the design process within one software environment thus achieving an overall automation. These integrated systems tend to support design by numerous repeated analysis due to their inability to suggest good preliminary solutions. The feedback from numerical analyses is needed to modify the preliminary solutions.

It is argued here that human experts have a different approach to design problems. They try to minimise the iterative nature of design by suggesting preliminary solutions which have a higher chance of succeeding at the subsequent detailed design stage. Expert systems should be able to do the same. Ideally, good preliminary solutions should be tailored to the requirements; this means that they should take account of the majority of constraints and structural behaviours quantitatively while selecting the values for key design parameters. It is suggested here that the numerical processing power of the computer should be used to obtain good preliminary solutions by developing design algorithms, which can take account of governing factors at an early stage of the design process. These in turn can be used to encapsulate knowledge in the expert systems instead of the 'heuristics' which are used to incorporate past experience in existing expert systems.

In order to develop these design algorithms, it is necessary to unravel the rationale behind each decision made during the preliminary design stage. In this thesis, the work carried out to rationalize the philosophy of the design process of prestressed concrete spine beams is explained in detail. The main advantage of this approach is that the expert system is compact and fast in execution. It is also capable of guiding the designer in a consultation session either by suggesting appropriate values or allowable ranges for key design parameters, as is done by a human expert.

Keywords: Prestressed Concrete, Spine Beams, Bridges (structures), Expert Systems, Prolog, Deep Knowledge

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Declaration

This thesis is a report of research work carried out in the Department of Engineering , University of Cambridge, between October 1988 and January 1991. Except where references are made to other work, the content of this thesis is original and includes nothing which is the outcome of work done in collaboration. The work has not been submitted in part or in whole to any other university. This dissertation is 250 pages.

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Notation

A_b	Area of the bottom flange
A_{b-min}	Minimum area of the bottom flange allowed to prevent cracking
A_{b-span}	Required area of bottom flange at span critical section
$A_{b-support}$	Required area of bottom flange at support critical section
A_c	Area of the concrete section
A_t	Area of the top flange
A_w	Area of the web
b_z	Breadth of the section at a height z from the bottom
c	Ratio between concrete cover required and depth of the section
c_1	Position of top fibre (measured from centroid, always -ve)
c_2	Position of bottom fibre (measured from centroid, always +ve)
COR	Cantilever Overhang Ratio as defined on page 105
d	Depth of the section
e	Eccentricity of prestressing cable (measured +ve downwards from centroid)
e_{k1}, e_{k2}	Eccentricity at kern points (Z_2/A_c) and (Z_1/A_c)
e_{min}	Minimum eccentricity allowed considering cover limits
e_{max}	Maximum eccentricity allowed considering cover limits
e_p	Eccentricity of line of thrust
e_{p-min}	Minimum eccentricity of the line of thrust (upper bound)
e_{p-max}	Maximum eccentricity of the line of thrust (lower bound)
e_s	Eccentricity of actual cable profile
e_1	Distance to centroid of idealised top flange from the centroid of section
e_2	Distance to centroid of idealised bottom flange from the centroid of section
E	Young's modulus of the section
E_z	Young's modulus at a height z from the bottom fibre
f_c	Permissible stress of concrete in compression
f_{ct}	Permissible stress of concrete in compression at transfer
f_{cu}	Characteristic cube strength of concrete
f_{cw}	Permissible stress of concrete in compression at working load
f_t	Permissible stress of concrete in tension
f_{temp}	Temperature stresses due to direct strain and curvature
f_{tt}	Permissible stress of concrete in tension at transfer
f_{tw}	Permissible stress of concrete in tension at working load
I	Second moment of area about centroid

$l_{l(i)}$	Distance measured to left change point from i^{th} support
$l_{r(i)}$	Distance measured to right change point from i^{th} support
M	External moments acting on the section
M_a	Minimum working load moment
M_b	Maximum working load moment
M_f	Moment range in one span (mid-span sagging less pier hogging)
M_n	Moment due to notional loads
M_u	Ultimate state moment acting on the cross section
M_{2-min}	Minimum reactant moment due to prestressing effects
M_{2-max}	Maximum reactant moment due to prestressing effects
$(M_2)_j$	Reactant moment at internal support j
$(M_t)_j$	Continuity moment at internal support j due to temperature
n	Number of supports
N	Number of webs of a box girder
P	Horizontal component of the prestressing force in cable
P_B	Cable force corresponding to point B of Magnel diagram
$P_{n(i)}$	Cable force in the new cable at the i^{th} cable force change point
$P_{r(i)}$	Cable force in the running cable at i^{th} cable force change point
$P_{su(i)}$	Cable force over i^{th} support
P_t	Force in prestressing cable at transfer
P_1	Minimum prestress to satisfy moment range
P_2	Minimum prestress to satisfy lever arm
P_3	Minimum prestress for existence of line of thrust
P_4	Minimum prestress for existence of a line of thrust and maximum cable range
R	(Cable force at service)/(Cable force at transfer),
RMR	Reactant moment ratio as defined on page 64
t_b	Thickness of bottom flange
t_l	Linear transformation at left pier
t_m	Linear transformation at mid-span
t_r	Linear transformation at right pier
t_t	Thickness of top flange
t_w	Thickness of web
t_z	Temperature at a height z above the bottom of the section
w_b	Width of the bottom flange
w_s	Clear spacing between webs
w_t	Width of the top flange

x	Distance measured to cross section from the left support
z	Distance measured from the bottom fibre
$\bar{z}$	Distance to centroid from the bottom fibre
Z_1	Section modulus for upper fibre, I/c_1 (always -ve)
Z_2	Section modulus for lower fibre I/c_2 (always +ve)
α	Load factor for the ultimate limit state
α_c	Coefficient of expansion of concrete
α_i	($i = 1, 2$ and 3) Factors used to represent the idealised section as defined on page 84
$\alpha_{n(i)}$	Inclination of the anchor for new cable
$\alpha_{r(i)}$	Inclination of the running cable at force change point i
α_z	Coefficient of thermal expansion at height z
β	(Lever arm at ultimate)/(depth of the section)
β_j	Distribution coefficient for M_2
$\beta_{ll(i)}$	Inclination of lower bound of cable profile at left hand side of the i^{th} cable force change point
$\beta_{ur(i)}$	Inclination of upper bound of cable profile at right hand side of the i^{th} cable force change point
δ	(Maximum allowable concrete stress)/(Characteristic strength)
δ_i	($i = 1, 2$ and 3) Factors used to represent the idealised section as defined on page 84
ϕ	Diameter of the prestressing duct
ϕ_e, ϕ_l	Diameter of stirrups and longitudinal reinforcement in webs
ψ	Curvature due to temperature
ϵ_0	Direct strain due to temperature
κ	Total curvature caused by prestressing effects

In addition to these symbols, a number of others symbols are defined and used locally.

Chapter 1

Introduction

The ultimate goal of expert systems is to emulate the human expert. The main objective of the work described in this thesis is to achieve this goal at one of the most important stages of engineering design; preliminary design. The application domain is the design of prestressed concrete spine beams.

Engineering design is a process of integrating the constraints imposed by the problem, the medium and the designer to produce a description of an artifact. Analysis is often closely related to design. However, there is a fundamental difference between them. While design deals with an unknown product, at least in part, analysis implies investigating a proposed or a known product.

At the present time, most engineering designs are performed interactively; a designer works with a computer in an attempt to solve a design problem. Hence, there is a human-computer interaction; expert systems are expected to enhance the involvement of the computer in this interaction. The same concept is applicable to structural design, a branch of engineering design.

The structural design process is generally divided into four stages:

1. Conceptual design
2. Preliminary design
3. Detailed design
4. Design documentation.

The boundaries between each of these stages are not well defined.

In general, conceptual design is the initial phase of the structural design process; this is mainly concerned with the selection of construction materials, shape and construction technique from a large number of possible alternatives.

The detailed design is mainly concerned with checking the preliminary solutions for adequacy and hence is deeply involved in analysis; analysis is used to check the design and also to provide feedback for modifications.

The preliminary design links the gap between the conceptual and detailed designs. It is a very important stage of the overall design process, since the designer selects values for most of the key design parameters; the subsequent detailed design will be based on these. Therefore, selecting proper values for the key design parameters is a prerequisite for an *'efficient design'*; one needing the least number of modifications which cause redesign.

The key design parameters are selected by human experts, based mainly on their past experience and judgement; generally the skill and experience gained over a number of years are deployed. The chief strategy is to find a set of values that give the maximum chance of compensating in case of failure at the subsequent detailed design stage (Inder *et al.* (1989)).

When expert systems were first introduced, the idea was that 'knowledge engineers' would interview human experts to find what they did, and incorporate these as rules into a program specifically designed to handle rules in the form,

If A then B.

In practice, this means that the knowledge that the human experts use at the preliminary design stage is elicited and expressed as *heuristics,* which are rules encapsulating a piece of knowledge, often a 'rule of thumb'. The information gathered from text books and research papers is usually used to supplement the knowledge from heuristics. Most of the information gathered in this way will be optimum values for design parameters under certain circumstances, which may or may not be explicitly identified.

At the preliminary design stage, expert systems use this knowledge to infer solutions which comply with the constraints. The success of the expert systems depends on whether these solutions are comparable with those of the human experts.

Encapsulating all the knowledge needed for preliminary design in the form of heuristics is a complex task. Many design problems may have constraints specific to them. At the hands of human experts, these will be only minor irritations. However, expert systems will not be able to take account of these unless explicitly represented as a piece of knowledge. Hence, the following question can be raised; *'Is it possible to obtain good specific design solutions with a general set of rules?'*

The knowledge expressed as heuristics alone is not sufficient to produce good preliminary solutions because they explain only *'what'* should be done under a certain set of circumstances. They often fail to explain *'why'* a certain action is needed. The fundamental design principles underlying these heuristics should be unravelled and should be used to replace heuristics wherever possible.

1.1 Scope and aim

In this thesis, a detailed is study carried out of the following topics, as applied to the design process of prismatic prestressed concrete spine beams used for bridge construction:

1. Explicit identification of the key design parameters, and the determination of various structural behaviours and constraints which govern each key design parameter.
2. Development of design algorithms for the automated determination of key design parameters to satisfy the appropriate constraints. These algorithms are dedicated to finding values for the design parameters, and so are different from analytical algorithms.
3. Provision of adequate allowance in the values selected for key design parameters to cater for aspects which cannot be determined at the preliminary design stage.
4. Development of an overall design technique, which minimises the need for redesign (complicated modifications at the intermediate design stages) by producing better preliminary solutions.

5. Identification of all the possible situations in which redesign is inevitable and the corresponding options available to the design engineer. This minimises iteration in the design process, and allows these redesign tasks to be performed at the preliminary design stage.

Based on these ideas, a number of design techniques have been proposed for the selection of design parameters for prismatic prestressed concrete spine beams. Most of the techniques are expressed as design algorithms which incorporate the fundamental underlying design principles. These techniques are included in the expert system to produce preliminary designs that have a higher chance of succeeding at the subsequent detailed design stage.

The work presented in this thesis can therefore be visualised as an attempt to solve some of the problems that arise in expert systems developed for design, by *rationalizing the design process itself*, rather than relying on software tools to solve them.

Prestressed concrete spine beam bridges

As pointed out by Rowe & Somerville (1971), the essential difference between spine beam bridges and other bridge structures is mainly due to differences in plane geometry. Spine beam bridges may be defined as members whose breadth and depth are small in relation to their length. Generally they are used for medium span bridges of span $20 - 100m$. As far as overall behaviour is concerned, they are subjected to longitudinal flexure, transverse shear and torsion, where usually the flexural behaviour dominates. There are a number of possible cross sectional shapes for spine beams as shown in Figure 1.1 (Lee (1978)).

1.2 Arrangement of the thesis

This thesis is arranged in eight chapters with this introductory chapter being the first.

Chapter 2: A detailed literature review is undertaken into expert systems that have been developed for structural design. The various reasons for the limitations of the existing expert systems are discussed. The methodology that is proposed to overcome these limitations is briefly outlined.

Chapter 3: The current design techniques available for the design of prestressed concrete beams are reviewed. The shortcomings of these techniques, which need to be rectified by developing alternative techniques, are identified. A review of research that is applicable to the development of alternative design techniques is also described briefly.

Chapter 4: A design technique is developed for the determination of section dimensions of statically determinate beams. The principles of this design technique are then extended to cover statically indeterminate beams. The possibility of using reactant moments to generate alternative solutions is demonstrated with examples. The corresponding design algorithms are also presented.

Chapter 5: Two design techniques are developed; one for the determination of the limits on the cable forces for continuous beams, and the other for the automatic selection of a cable profile while satisfying a number of practical considerations. These techniques allow the prestressing force to vary along the beam. It is shown that the ability to select the cable force within the specified limits is essential for the success of the automatic procedure for the cable profile. Once combined, these two techniques simplify the process for the selection of cable forces and the profile considerably, thereby minimising the number of iterations.

Chapter 6: The encoding of the knowledge module, PREDEX, which carries out the preliminary design for the proposed expert system, BRIDEX, is explained. It is shown that the explicit representation of knowledge plays a central role in simplifying the implementation of PREDEX, which is developed using the Edinburgh Prolog Blackboard Shell. In order to suggest accurate preliminary solutions, use is made of the design techniques developed in Chapters 4 and 5.

Chapter 7: Possible extensions to the design techniques suggested in Chapters 4 and 5 are described. These are intended to produce better preliminary solutions by integrating a number of extra constraints and structural behaviours at the preliminary design stage. Certain aspects are earmarked for further investigation.

Chapter 8: The conclusions and the topics for future work are discussed.

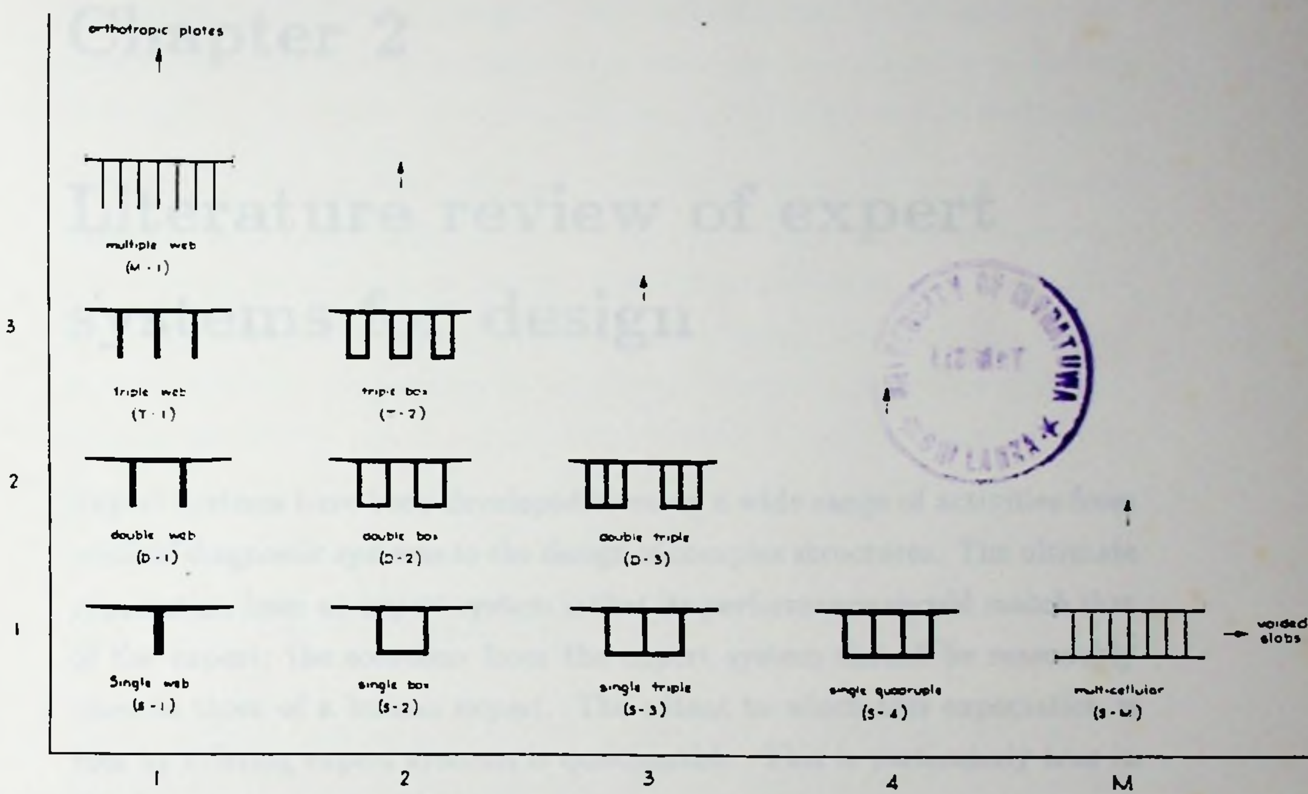

Figure 1.1: Possible cross-sectional shapes of spine beams. Ordinate: number of boxes. Abscissa: number of webs in each box. (taken from the classification of bridge deck cross sections by Lee (1978))

Chapter 2

Literature review of expert systems for design

Expert systems have been developed to cover a wide range of activities from medical diagnostic systems to the design of complex structures. The ultimate expectation from an expert system is that its performance should match that of the expert; the solutions from the expert system should be reasonably close to those of a human expert. The extent to which this expectation is met by existing expert systems is questionable. This is particularly true in the domain of structural design.

The expert systems developed for *conceptual design* are similar to medical diagnostic systems; they are mainly concerned with suggesting qualitative solutions, and thus can play an important role in a complete expert system for bridge design. They are, however, not directly relevant to the work presented in this thesis, which is concerned with the preliminary design. Therefore, the expert systems covered in this literature review are those developed for preliminary and detailed design.

The underlying conceptual model of most of the existing expert systems has been to consider the structural design process as an iterative feedback process; knowledge from analysis is used to modify the preliminary solutions generated at an earlier stage in the process. Hence, all these models have tried to integrate the whole design process and tend to support *'design by repeated analysis'*, where the decisions made during the early stages are modified on the basis of feedback from subsequent stages. It is argued here that this approach does not emulate the human experts.

It is shown in this chapter that the expert system developers have considered design as an iterative feedback process because of the difficulties of suggesting *good preliminary solutions*; ones which need minimal modifications at the subsequent detailed design stages. This difficulty has arisen because the knowledge modules for preliminary design are unable to encapsulate sufficient knowledge as *'heuristics'* to tackle many intricacies of the design problems.

A number of existing expert systems which tackle design in various domains related to structural design are presented in this chapter. The reasons why these expert systems rely on heuristics, and their failure to gain acceptance as viable tools in industry, are described. The limitations of existing expert systems with respect to reasoning knowledge, domain factual knowledge and solution progression are also presented. Possible remedial actions are outlined.

2.1 Expert systems

Expert systems have been developed in many areas covering a wide range of applications. Most of these applications fall into one or more of the following categories:

- Diagnosis
- Design
- Data interpretation
- Planning
- Education.

A number of definitions of 'expert systems' can be found in Artificial Intelligence literature, as listed by Adeli (1988). However, the following is the formal definition approved by the British Computer Society (Naylor (1983)):

> "An expert system is regarded as the embodiment within a computer of a knowledge-based component from an expert skill, in

such a form that the system can offer intelligent advice or take intelligent decisions about a processing function. A desirable additional characteristic, which many would consider fundamental is the capability of the system, on demand, to justify its own line of reasoning in a manner directly intelligible to the enquirer."

Many researchers favour the name, *'knowledge based expert systems'* instead of *'expert systems'*. This is mainly because there are very few expert systems which can truly mimic the expert. In this thesis however, they are referred to as *expert systems* for two reasons.

1. This name conveys a very important idea behind this technology; that is, the ultimate goal of matching the performance of human experts.
2. The arguments presented in this thesis outline an approach to the development of versatile expert systems for structural design which moves towards emulating the expert.

2.1.1 Difference between conventional computer programs and expert systems

It is difficult to point out the exact differences between conventional programs and expert systems. It could be a matter of considerable debate to prove whether a particular expert system is something really different from a conventional program. The following features, however, have been stated by Adeli (1988).

1. Expert systems are *knowledge intensive* programs.
2. Expert knowledge is usually divided into many separate independent rules or entities. The knowledge representation is transparent; that is, it is easy to read and understand.
3. The knowledge base used in an expert system is usually separated from the methods of applying the knowledge to the current problem, which is referred to as the inference mechanism.
4. Expert systems are usually highly interactive.

5. The output of an expert system can be qualitative rather than quantitative.
6. Expert systems tend to mimic the decision making and reasoning process of human experts.

The extent to which these features can be found in a particular expert system depends on the domain for which it has been developed and its software implementation.

2.1.2 Architecture of an expert system

According to Adeli (1988), there are three main components for an expert system together with other desirable features. A possible architecture is illustrated in Figure 2.1.

The three main components are:

1. **Knowledge base:** This is a collection of information about a particular domain. The knowledge base may consist of well-established and documented definitions, facts and rules.
2. **Inference mechanism:** This is also known as the *inference engine* or the *reasoning mechanism.* It controls the reasoning strategy of the expert system by making assertions, hypotheses and conclusions.
3. **Working memory:** This is also known as the global database. It is a temporary storage area for the current state of the specific problem being solved. Its contents change dynamically and include information provided by the user about the problem and information derived by the system.

The desirable additional components are:

- **Intelligent interfaces:** The user interface allows the user to interact with the expert system. It may include menus, multiple windows and graphical facilities.

- **Explanation facility:** This is a feature which enhances the user interface by providing answers to questions (*e.g.* why a certain question has been asked) and providing justifications for answers (*e.g.* why a specific conclusion or recommendation is made).
- **Help facility:** It helps and guides the user during a session in which the expert system is consulted.
- **Debugging facility:** This is mainly to provide assistance during the development stage of the expert system.

2.1.3 Expert system development tools

Expert system development tools can be divided into three categories.

1. Declarative languages
2. Expert system shells
3. Development environments

2.1.3.1 Declarative languages

Declarative languages are considered to be more suitable than procedural languages for Artificial Intelligence (AI) applications. Procedural languages are commonly used in traditional algorithmic programming; examples are Fortran, Pascal, C *etc.* For computer programs written in procedural languages, it is essential to predetermine the sequence of steps involved in the program. In addition, the author of the program has to think of all possible routes available while the user has little control over the program. The advantage is efficiency, but this results in so-called *opaque knowledge* which is embedded in the programming code.

In declarative languages, knowledge is encoded as data and normally expressed as facts and relationships; the order of execution is not important. The knowledge is more understandable and easier to modify, and can also be collected at one place (the knowledge base). Prolog and Lisp are the most widely used declarative languages. Prolog is an acronym for PROgramming in LOGic. Lisp is an acronym for LISt Processing.

Prolog

The idea of using logic as a basis for a programming language was developed in the early 1970s, originally by Alain Colmerauer at the University of Marseilles and subsequently by Robert Kowalski at the University of Edinburgh. A logic program is a set of *'facts'* or *'rules'* defining relationships between objects. Derivations performed by a logic program usually consist of deduction of conclusions that can be drawn from the knowledge encoded in the program (Sterling & Shapiro (1987)). An example is the *'fact'*

father(abraham,isaac).

This fact states that 'abraham' is the father of 'issac'.

A typical *'rule'* is

son(X,Y) < −− father(Y,X), male(X).

This is a rule expressing the son relationship, where X is son of Y if Y is father of X and X is male.

'Queries', also called *'goals'*, can be used to retrieve information from a logic program. An example is

son(isaac,abraham)?

For this query, Prolog would return *'yes'*, if the knowledge base also contains the fact male(issac).

Prolog has the advantages that it:

- uses syntax and semantics that are much closer to formal logic, and
- provides automatic backtracking, a feature very useful for searching for a solution, and hence has a built-in inference mechanism. In backtracking, the goal is matched with the available facts or rules. In the case of failure to match, an alternative fact or rule is tried automatically until all possibilities are exhausted.

Lisp

Lisp is a language initially developed by John MacCarthy during the 1960s for non-numeric computation. Lisp has undergone considerable development since then, and it has been used in a number of applications in Artificial Intelligence.

The fundamental structure of Lisp is a *'list'*, which is made up of a combination of abstract symbols called *'atoms'*. An *'atom'* has a name, and a *'value'* attached to it, which is like a variable having a value in a procedural language.

'Lists' are made up of some combination of atoms and hence can be used to represent relationships. For example, the list:

(FATHER ABRAHAM ISAAC)

represents a list of three atoms: FATHER, ABRAHAM and ISAAC, which can be used to store information and also to deduce other results.

Lisp is widely used for Artificial Intelligence applications in the U.S.A. Lisp does not implicitly include inference mechanisms, but they have been written for it.

Detailed descriptions of Lisp can be found, for example, in Jones *et al.* (1990).

2.1.3.2 Expert system shells

The basic component of an expert system is explained in Section 2.1.2. An expert system shell can be considered to have all these components with an empty knowledge base. Hence the task of the expert system developer has been enormously simplified because it is possible to concentrate on the knowledge component of the expert system rather than getting involved with intricacies of the programming languages. The disadvantage is that the user generally does not have access to the source code of the software and hence may be restricted in enhancing certain capabilities of the expert system.

2.1.3.3 Programming environments

These are large, flexible and powerful expert system shells. Since they are built on top of declarative languages, many features can be extended by the user. Hence they are classified as Artificial Intelligence Programming Environments (Adeli (1988)). Examples are KEE (Knowledge Engineering Environment) and ART (Automated Reasoning Tool); both are based on Lisp and need dedicated Lisp machines.

In this context, the expert system shell used for the present project, the

Edinburgh Prolog Blackboard Shell (EPBS), also can be considered as a programming environment. The user has free access to Prolog and to procedural languages like 'C' through Prolog.

EPBS is based on a *blackboard architecture* where the knowledge modules communicate via a global database called *the blackboard.* Thus a knowledge module can contribute to a solution on the basis of the information available on the blackboard, independent of the other knowledge modules. A more detailed description of the blackboard can be found in Section 6.2.

2.2 Some early expert systems

Some of the successful early expert systems are DENDRAL, MYCIN, MOLGEN, PROSPECTOR, and HEARSAY *II*.

- DENDRAL determines the molecular structure of chemical components. In the expert system, heuristic knowledge has been used to solve the problem of *combinatorial explosion,* which is caused by the large number of possible permutations that should be searched to obtain a solution (Buchanan (1975)).

- MYCIN is a medical diagnosis system. Knowledge expressed as heuristics has been used to handle the problems associated with narrow specialised knowledge (Shortliffe (1976)).

- MOLGEN is a knowledge based expert system that assists in the planning of experiments in MOLecular GENetics. This has made an important contribution to the constraint handling in problem solving, where constraints are used to represent the interaction between subproblems (Stefik (1981)).

- PROSPECTOR is an expert system developed to assist geologists to find mineral deposits. It has been successful in finding a molybdenum deposit worth 100 million U.S. dollars (Duda *et al.* (1979)).

- HEARSAY *II* is a speech recognition system. This was successful in introducing blackboard architecture (Erman (1980)).

A common feature of all these systems is the use of heuristic knowledge as the main source of information.

2.3 Expert systems for design

Expert systems have been developed for a number of disciplines in engineering design. As explained by Mostow (1985), the purpose of design is to construct a description of a structure so that it:

- Satisfies a given functional specification (*e.g.* provides a safe ride across an obstruction such as a valley, a waterway or a motorway).
- Conforms to the limitations of available resources (*e.g.* limitations in spans, construction technique, speed of construction).
- Meets implicit or explicit requirements of performance and resource usage (*e.g.* serviceability and cost of structure).
- Satisfies implicit or explicit design criteria on the form of the artefact (*e.g.* style, aesthetics, constructability, maintainability and durability).
- Satisfies restrictions on the design process itself (*e.g.* the time taken for the design, cost of design or tools available for design).

As described by Boyle (1987), engineering design is a process of integrating constraints imposed by the problem, the medium and the designer, to produce a description of an artefact from an initially incomplete and general set of objectives. In this process, the designer is grappling with both uncertainty (lack of information) and complexity (too much information).

2.3.1 Coupled expert systems

The fundamental difference between the early expert systems and the expert systems for engineering design is that the latter use well-established theories and principles, thus requiring numerical computation in addition to heuristics. Hence, they are called *'coupled expert systems'*; these promise to integrate the explanation and problem solving capabilities of expert systems with the precision of traditional numerical computing (Kitzmilar & Kowalik (1987)).

There are two types of coupled expert systems, namely *shallow* and *deep*.

2.3.1.1 Shallow coupled system

The most common approach is to develop coupled systems that treat numerical routines as black boxes. Shallow coupled systems have little knowledge of the process involved. Typically, they set up data for numerical routines as part of the solution process and interpret the results of those calculations. The expert system by Cameron and Grierson (1989) presented in Section 2.3.2.3 is a good example of a shallow coupled expert system.

2.3.1.2 Deep coupled system

The deep coupling approach utilises extensive knowledge about the numerical algorithms used. Depending upon the purpose of the application, they explicitly represent the algorithms' functions, inputs and outputs, usage constraints and limitations. This information is integrated with the knowledge base and used directly during problem solving. Therefore the knowledge base provides an intelligent interface to numerical routines. Owing to this explicit representation, deep coupled systems are more robust than shallow coupled systems.

The major disadvantage of deep coupled over the shallow coupled systems is the overhead associated with the deep coupling and the time required to develop the initial applications.

2.3.2 Expert systems for structural design

Expert systems have been developed for structural engineering (Maher (1984), Sriram (1986), Kumar & Topping (1988a), Adeli & Balasubramanyam (1988b), Cameron & Grierson (1989) and Miles & Moore (1989)); architecture (Oxman & Gero (1988), Rosenman & Gero (1989)); control engineering (Boyle (1987)) and mechanical engineering (Brown & Chandrasekaran (1986))). The expert systems developed for structural design are the most pertinent for the research described in this thesis. A few of them are described briefly in the following subsections.

2.3.2.1 Expert systems by the research group at Carnegie-Mellon University

The research group at the Department of Civil Engineering and the Engineering Design Research Centre of Carnegie-Mellon University has been actively involved in the development of expert systems for the preliminary design of buildings. This research was initiated with the HI-RISE project (Maher (1984)) followed by DESTINY (Sriram (1986)). DESTINY is discussed in more detail later.

Gero & Maher (1988) suggest that the preliminary design process has identifiable phases or sub-processes. Although they may not be addressed hierarchically for the entire design cycle, there is an inherent order in which the designers approach the design problem. The following phases have been identified:

- **Design analysis or formulation**

  This is the phase where the goals and requirements set out in the client's brief are identified in order to produce a detailed specification for the design synthesis. This phase is highly subjective and uses personal knowledge and experience.

- **Design synthesis**

  Design synthesis is the phase where components and subsystems are identified and combined to form alternative design configurations. The associated knowledge is expressed as heuristics; solutions are found by searching through these heuristics. Since the search space is large, blind or exhaustive search is rarely used. The alternative is a guided search in which strategies for controlling the search are contained in *control knowledge* modules. EDESYN is a framework developed by Maher & Longinos (1986) to be utilised in expert systems for design synthesis. According to Maher (1989), this has been used for a variety of applications such as expert systems for designing an industrial robot, heat exchanger configurations and stairwells.

- **Design evaluation**

  Design evaluation involves testing a partially or completely specified design description for conformance with goals and expected performance. This is a process which designers apply implicitly throughout the design process. The evaluation can be either based on qualitative or quantitative reasoning. Evaluation during the early stages of design is usually based on a subjective assessment of relevant criteria. Quantitative reasoning often requires a mathematical model of the design in order that an analysis may be performed. The input to an evaluation program is a partially or completely specified design and the output is either satisfaction or recommendations for modifications.

After a number of years of research, this group has identified three problems which need attention with respect to design synthesis (Westerberg *et al.* (1990)).

1. How to deal effectively with the broad objectives and constraints in synthesis (*e.g.* economy, feasibility and constructability).
2. What is the role of qualitative and quantitative knowledge in the choice of design alternatives?
3. How to deal with knowledge at the time of design synthesis, because the knowledge is large and not well defined.

The ultimate aim is to identify synthesis methodologies which are based on sound design principles so that they can be applied effectively to major classes of design problems.

DESTINY by Sriram

The main aim of the structural design process has been considered as producing a detailed specification of a structural configuration capable of transmitting the loads with appropriate levels of safety and serviceability. The design process has been viewed as a sequence of three major stages.

1. **Preliminary design** - The synthesis of a potential configuration satisfying a few key constraints.

2. **Analysis** - The process of modeling the selected structural configuration and determining its response to external effects, such as gravity and lateral loads.

3. **Detailed design** - The selection and proportioning of the structural components such that all applicable constraints are satisfied.

There may be a significant discrepancy between the properties of the components assumed at the analysis stage and those actually needed, which would necessitate a re-analysis. An evaluation of a design is often performed to determine its feasibility. This synthesis–analysis–evaluation–detail design cycle is typical of many design processes.

Based on this view of the design process, a conceptual model, DESTINY, has been presented; this provides a framework for integrating the preliminary design, analysis, detailed design and evaluation. These stages are executed by distinct knowledge bases that communicate through a blackboard.

The DESTINY model is an example of *'design by repeated analysis'*. In this model, the design process starts with a preliminary solution in which the quantities are assumed on the basis of previous experience; experience is generally expressed as heuristics and/or incorporated in databases. The preliminary solution is modified until relevant evaluation criteria are satisfied. The DESTINY framework is intended to enhance the facilities for design by repeated analysis.

2.3.2.2 INDEX by Kumar & Topping

INDEX is a rule based expert system developed at Edinburgh University for the design of industrial buildings. This work has been greatly influenced by HI-RISE and DESTINY (see Section 2.3.2.1). INDEX is based on the DESTINY model with the incorporation of *non-monotonic reasoning* (see Section 6.1.2) as an extension (Kumar (1989)). One of the knowledge modules which is most relevant to the present work is ALTSEL, which stands for ALTernative SELection. Almost all the knowledge contained in ALTSEL is expressed as heuristics which are obtained either from experts or from literature. The tasks of ALTSEL are as follows:

1. Selection of feasible alternative structural systems using the SYNTHESIS sub-module.

2. Performing preliminary analysis of the structural system using the PREANALYSIS sub-module. The rules in this sub-module are mostly analysis formulae for different types of structural systems. Also included are rules to decide which type of analysis should be undertaken (*e.g.* a plastic analysis for portal frames or an elastic analysis for trusses).

3. Preliminary sizing and proportioning of the components of the alternative systems which have been generated by the SYNTHESIS sub-module. This is done by the PREDES sub-module which finds the feasible section details from a database on steel sections.

4. Suggesting recommendations for producing economical designs for selected structural systems by using the ECONOMICS sub-module. This sub-module is based solely on heuristics obtained from various research projects.

5. Posting relevant constraints for each of the alternative systems on the blackboard for subsequent use. The DESIGN sub-module has been developed for this.

6. Evaluation of preliminary alternative systems generated by the SYNTHESIS sub-module to find which ones most closely match the specifications, by using the PREVALUATOR sub-module.

A number of examples are given by Kumar & Topping (1988a) to illustrate the process of generating alternative solutions and validating them in ALTSEL.

The other appropriate modules of this system are STRuctural ANalysis EXpert (STRANEX), DETail Design EXpert (DETDEX) and DESign CONsultant (DESCON) which are concerned with the detailed design of the structure; this is the stage after the preliminary design. Detailed design involves the detailed analysis of a chosen structure from the preliminary design, the sizing and proportioning of its components and the satisfaction of different constraints; this often leads to an iterative cycle in which initial component sizes and proportions are changed after detailed analysis, if certain constraints are not satisfied.

2.3.2.3 An expert system by Cameron & Grierson

The development of an expert system for the least-weight design of structural steel buildings has been considered by Cameron & Grierson (1989) as a matter of replacing the control structure of an existing design and optimisation package, by using the routines written in the rule-based programming language OPS 83. Thus this forms a shallow coupled system, explained in Section 2.3.1.1. The knowledge base of this expert system is divided into three distinct sets of rules corresponding to three stages of the design process.

1. **A Preliminary stage** that establishes an initial design using the rules in the knowledge base to suit the structural topology (the bay widths, storey heights, connections and support types) and the design loading (dead, live, snow, wind, thermal and settlements).

2. **A Solution stage** that establishes the corresponding least weight design using optimisation techniques applied to the solution generated at the preliminary stage.

3. **A Critique stage** that evaluates the current design and suggests a redesign with suitable modifications. This stage uses a variety of *ad hoc* techniques to identify any design deficiencies or violations of designer preferences. The critique stage is open-ended and ever-changing as new design scenarios are encountered and designer preferences are modified.

2.3.2.4 BTEXPERT by Adeli & Balasubramanyam

Another example of the extensive use of optimisation techniques for expert systems is Bridge Truss EXPERT developed by Adeli & Balasubramanyam (1988a), (1988b). BTEXPERT can be used for the optimum detailed design of four types of bridge trusses.

An important consideration of BTEXPERT is the use of expert systems to create the missing knowledge through interactive consultation sessions performed for different types of structures. This is called *'machine experimentation'*. This knowledge could be incorporated in the expert system to produce better initial designs and control parameters for the optimisation process. The optimum design process starts with a preliminary solution generated on the basis of heuristics, which is gradually changed in order to satisfy an optimisation function.

Re-analysis of the structure has to be undertaken in order to obtain stresses and displacements each time new values are selected for design parameters. The procedures for repetitive re-analysis of a structure may be either exact or approximate, depending on a number of factors such as the extent of change in the value of design variables in a particular iteration, the size of the problem, or the number of loading conditions. The numerical optimisation routines used to generate the final solutions are dedicated to produce minimum weight designs, but other objective functions could also be included (*e.g.* minimum cost).

2.3.2.5 An expert system for bridge design by Burgoyne & Sham

Expert systems have been considered by Burgoyne (1987a) as tools that offer the structural engineer the potential for a revolutionary change in design, similar to that which occurred in structural analysis with the advent of Fotran and the matrix methods in the early 1960s.

An expert system has been proposed by Burgoyne & Sham (1987) for the design of prestressed concrete bridges. Three distinct areas where expert systems may be used with the current design techniques have been identified by them. These areas are illustrated in Figure 2.2 and can be described as follows.

- **Expert system as a tool for decision support:** The first phase is the initial or conceptual stage of the design process. The rules used are largely qualitative with a considerable amount of subjective preference built in by the domain expert. There is a possibility of substantial difference in the rule bases established by different experts. This is the area covered by the expert system, QED, developed by Sham (1989a) for the conceptual design of prestressed concrete bridges. The output of this part of the design process is feasible structural forms. Only approximate limits on major dimensions are likely to be given, as a refined analysis is needed for accurate sizing.

- **Black box routines under the expert system control:** The second phase of the design process mainly concerns the numerical calculations, some of which require iteration, while others require the solution of sets of simultaneous equations. These calculations are carried out by *'black box'* routines written in a procedural language. The role of the expert system in this phase of the design process is to suggest the type of calculations to be carried out and to specify their order. The expert system is also expected to make decisions using the results of the analysis. This phase corresponds to a *shallow coupled expert system* (see Section 2.3.1) in which the black box routines written in procedural languages are manipulated using declarative knowledge in the expert system.

- **Expert system for decision support:** The final phase needs the determination of the cable profiles, parasitic moments, ultimate moment capacity and shear strengths. If reasonable decisions have been made in the earlier stages, these tasks could be performed in a logical sequence. However, an expert system must be able to make rational decisions on the course of actions to be taken when the design fails to satisfy any of the evaluation criteria. Therefore it may require *extensive redesign knowledge* (see Section 2.4.2.3).

2.3.3 Discussion of existing expert systems

There are three important characteristics which can be emphasised about the existing expert systems:

- The expert systems for design (except those concentrating only on conceptual design) try to integrate the preliminary design, detailed design and design evaluation in one expert system, although they may have different knowledge modules concentrating on each of these stages.

- All these expert systems have been developed by considering structural design as an iterative feedback process; the design process starts with a preliminary solution, and analysis is used as a means of obtaining the feedback for design modifications. Design modification, which is often called redesign, is carried out using the redesign knowledge incorporated in separate knowledge modules. Hence, all these expert systems support 'design by repeated analysis'; priority is given to enhance the capabilities of software to support this.

- Preliminary solutions have been derived on the basis of past experience incorporated as heuristics or as a database in the knowledge module for preliminary design; the appropriate solution is found by searching through this knowledge. Hence, the suitability of the preliminary solution, which determines the extent of modifications needed at the subsequent detailed design stage, is dependent on the applicability of knowledge incorporated. The emphasis placed upon design by repeated analysis in the existing expert systems is clear evidence that knowledge incorporated as heuristics or databases alone is not sufficient to produce good preliminary solutions which lead to efficient designs.

2.4 Philosophy of expert systems

Despite the number of expert systems reported in the literature, they have still failed to find acceptance in industrial applications. A number of factors can be reviewed to find the possible causes. Among them are:-

1. the suitability of searching through heuristics to generate preliminary solutions,
2. the suitability of the models used to represent design,
3. the limitations due to the methods used to represent knowledge and the software used to implement expert systems, and
4. how explicitly represented knowledge can be used to produce better models.

These factors are considered in more detail in the following subsections.

2.4.1 Suitability of heuristic search for design

The process of obtaining a solution by searching through a database or heuristics is considered as suitable for well-structured problems. As suggested by Simon (1973), a well structured problem should possess a number of properties as collated below.

- Definite single criterion: there should be a definite single criterion for testing any proposed solution.
- Representation of the problem: the statement of the problem and the goals of the problem should be known at the start.
- Accurate prediction of the problem: the effects of external factors such as constraints should be known.
- Practical amount of computation: the procedure for solving the problem involves only a practicable amount of searching through available data.
- Acceptable information cost: all information required should be available so that all the data can be gathered economically.

Conversely, ill-structured problems can be defined as those which fail to satisfy some or all these properties. Most real design problems fall into this category. The following aspects have been emphasised by Williams (1990) as the reasons for the ill-structured nature of the design process:

- **Problems with design goals** – What is the objective of the search?

  In many design situations, it is extremely difficult to specify the goals firmly at the outset of a design. A designer will try to establish a set of outcomes that are both acceptable to the client and realizable. There is uncertainty involved in the setting of acceptable goals that are achievable.

- **Problems with design representation** – What constitutes the search space?

  The design is considered in terms of different search spaces. The basis of these spaces is the inner and outer environments with a functional boundary between them. The inner environment corresponds to the object under consideration and its properties. The functional boundary represents the way that the objects designed will respond to the requirements of the outer environment.

An example from the domain of spine beam design is that the properties of the materials and the section dimensions of the beam will form the inner environment. The requirements of the outer environments are the load carrying capacity, serviceability, maintainability and durability. The function boundary is the behaviour of the beam in flexure, shear and torsion.

A major source of complexity in modern design problems is the number of attributes in these search spaces and their interactions.

- Problems with the design process – How to proceed with the search?

  A rudimentary method that is available to reach the goal is a blind search which runs the risk of combinatorial explosion. Therefore, all design problems involve some form of knowledge to reduce the amount of search that has to be performed. In the case of search techniques, knowledge is often used to manipulate the problem space into a form compatible with the technique used. Thus design has become a trade-off process between overcoming computational complexity and the uncertainty of design. Knowledge is used to guide the search.

The foregoing discussion highlights the aspects of design which make it ill-structured and unsuitable for heuristic search. Nevertheless, searching through heuristics has been widely used in the majority of the design expert systems as a chief solution strategy at the preliminary design stage. The reasons for this are found by considering the way that experts and novices approach design problems.

Experts and novices at design

According to Hoeltzel and Chieng (1989), when experts find solutions to a problem, they routinely use their experience in the form of heuristics to develop a preliminary model of the problem. While some small degree of numerical calculation for concept clarification may accompany this process, it is predominantly a symbolic reasoning process. Until a clearly conceptualised symbolic working model has been formulated, the detailed numerical calculations are not carried out.

Conversely, novice designers perform design in a different manner, in that the relative time spent in the symbolic and numerical domains depends on their domain knowledge. If the available knowledge is small, then the time spent in the symbolic domain is less, and they are thus required to spend a longer time in the numerical domain; this generally involves manipulating equations to obtain trade-offs between various design parameters, in order to make judgements concerning the viability of various design alternatives. Furthermore, they need to undertake numerical calculations earlier in the design process.

Inder *et al.* (1989) observed that the experts' way of tackling problems is different from that of novices or near-experts. They deploy their skill to balance the influences of a number of factors that affect the course of the process, often choosing a strategy that gives them the maximum chance of compensating in the event of failures at the subsequent design stages. Therefore the key feature is the adoption an overall strategy that would work despite the number of obviously relevant factors that are not analysed.

This observation is supported by Maher (1987), who noted that designers may not use an exhaustive organised approach to the synthesis of alternative preliminary designs, but they enumerate the alternatives that appear to be the best. This ability to identify feasible designs with partial information has not been formalised, so there is a difficulty in passing this knowledge to new engineers or in developing intelligent computer aids for this stage of the design process.

The work of Hoeltzel & Chieng (1989) shows that the knowledge of the expert is vast and is best represented symbolicly. This has tempted many expert system developers to express the design knowledge as heuristics and to adopt a solution strategy compatible with heuristics, such as controlled search.

The success or failure of this approach is decided by the quality of the preliminary solutions generated (based on heuristics). If the knowledge incorporated is applicable and the search strategy is efficient then the quality of the solutions may be comparable with those of experts. If not, the solutions suggested by the expert system will require a number of modifications, and hence may need extensive evaluation and redesign; this is equivalent to the expert system adopting the novices' approach. Thus, any expert system showing this characteristic is not (strictly speaking) a true expert system.

2.4.2 Models for design

A number of models and classifications of design methods have been presented by several researchers; these are for developing computer tools such as expert systems for design, in order to increase the contributions of the computer to the human-computer interaction in the design process.

2.4.2.1 Iterative model of design

Design is viewed by Dixon *et al.* (1984) as an iterative activity of generation and evaluation of candidate designs. The process begins with a designer being given a set of specifications for the problem at hand. Based upon previous experience or other knowledge, the designer generates a first candidate design. This candidate design is subjected to various analysis and evaluation methods and the results are interpreted. At this stage analytical tools are extensively used for analysis and evaluation. Next, a decision has to be taken, based upon these results, as to whether the candidate design is acceptable. If not, the particular failings of the candidate design are examined and the design is altered; this is called redesign. This cyclic process is repeated until an acceptable design is found; it can be illustrated as in Figure 2.3.

Two questions have been raised by Simmon & Dixon (1986):

1. Why is the process of iterative design so common?
2. Are there quantitative methods for directly generating good designs?

According to them, there are only a few simple examples of analytical methods that can be inverted. In these cases, a direct design procedure is possible and design is trivial; an example is the design of a determinate prestressed concrete beam with a rectangular cross section. The more common situation in engineering design is that good algorithms are available for analysing a candidate design once it exists, but there are no known algorithms for generating good designs.

2.4.2.2 A classification of design methods

A classification of design methods has been presented by Boyle (1989), on the basis of the amount of analytical knowledge that can be applied to the process of obtaining design solutions. Three broad methodologies are identified: analytical, procedural and experimental search.

Analytical design method

The analytical nature of the design activity means that, given an object and an appropriately defined compatible set of objectives, the computer can be used to synthesise a design solution automatically using analytical procedures. The analytical procedures transform the design process into a single activity in which the calculations are performed with the appropriate design solutions as the output. The role of the designer is to set up realistic design objectives. Analytical design procedures are those which either

- allow analytical methods to be inverted to form design procedures, or
- allow an understanding of design principles to produce algorithmic solutions for design.

Procedural design method

The designer does not have an exact prescription for a design solution, but has a set of procedures. These can be used to perform operations in order to transform the object to one with desirable attributes. Procedural design is a trade-off process. The design objectives are rarely fixed and tend to be modified as the designer obtains more information about what can realistically be achieved.

Procedural design goes through the following cyclic process:

- Problem definition: to obtain a workable model of the system and a realistic set of design objectives
- Design generation: to generate trial designs

- Design analysis: to evaluate trial designs
- Redesign: to modify designs.

The procedural design method is similar to the iterative model of design described by Dixon *et al.* (1984).

Experimental search design method

Experimental search involves working through an available set of designs in order to find the design whose attributes best match the design objectives. This relies on search rather than on formal procedures.

This design method is widely used by most of the existing expert systems owing to the use of heuristics to encapsulate knowledge. However, more explicit use of this design method can be found in the use of analogical reasoning. In the expert system called STRUPLE, developed by Zhao & Maher (1988), a database containing data on previous experience is recalled, related and selected to solve a new problem. Another example of experimental search design is the interactive graphics system based on a database, developed by Fan & Chan (1989), for concrete box girder bridges.

The solution selected on the basis of experimental search requires modifications to take account of constraints which are either unforeseen or deliberately overlooked. These modifications are another form of redesign.

2.4.2.3 Redesign as proposed by Boyle

As described in Section 2.4.2.1, a failure to satisfy some of the constraints leads to redesign. This becomes a complex task due to the number of possible options available to the designer.

Redesign consists of two parts:

1. Identifying the possible redesign decisions available in a particular situation.
2. Implementing one or more of the options to rectify the failure.

According to Boyle (1989), the decisions available for redesign can be any of the following:

- Modifying one or more parameters of the designs.
- Investigating alternative design options.
- Selecting a different design technique.
- Modifying one or more of the current design objectives.

The outcome of the redesign is a modified design.

2.4.2.4 Redesign model by Simmon & Dixon

The redesign model of design by Simmon & Dixon (1986), which considers the iterative nature of design, has been adopted for a number of expert systems. In this model, redesign is closely connected with design evaluation. The outcome of the design evaluation is the identification of the failure to meet important constraints. Designers often use purely symbolic reasoning at this step, generally based on previous experience, without recourse to deep interpretation of the reason for failure. This type of surface knowledge has therefore been used in expert systems in the form of heuristics. This heuristic knowledge can be applied to data obtained from quantitative analysis of candidate designs in order to make redesign decisions.

The knowledge required for the redesign step is primarily the knowledge of dependencies in the problem. Dependancy is a relation between goals and design variables that expresses how the changes in a design variable are likely to alter the degree of satisfaction of a goal. The strategy employed in expert systems for finding dependencies is *'dependency directed backtracking'* [1]. As has been pointed out by Mittal & Araya (1986), dependency directed backtracking is able to backtrack to a relevant point to establish the dependencies, but the shallow knowledge included in heuristics is of little value in deciding how to modify designs.

2.4.2.5 Application of fundamental dependencies in redesign by Kumar & Topping

In order to overcome the drawbacks of shallow redesign knowledge, fundamental dependency relationships accompanied by dependency directed back-

[1] see Section 6.1.3

tracking have been used by Kumar & Topping (1988b) to obtain redesign decisions for INDEX, the INdustrial Design EXpert. This has been done as an extension to INDEX, which is explained in Section 2.3.2.2. This knowledge module is called DEtailed Structural Design EXpert (DESDEX). The task of DESDEX is to find alternatives to the current solution when changes are made to the design requirements. DESDEX consists of three knowledge modules:

STABILITY consist of knowledge regarding dependencies and relationships between different structural design entities such as stiffnesses, stresses, moments and section sizes. A portion of the dependency tree is shown in Figure 2.4.

PROSOL is responsible for suggesting alternative solutions. In order to establish the cause of failure, dependency-directed backtracking is used.

GENPRO includes general knowledge about manipulating relationships between design entities such as direct and inverse proportionality. An example is 'stiffness is directly proportional to the size of the section'.

Although this approach to redesign shows promise in reducing the number of rules required to find the type of modifications needed, DESDEX has been prohibitively slow in execution for use as an interactive design tool.

2.4.2.6 Discussion on design models

It is clear form the preceding discussion that there are models and classifications for design. A topic which is fundamental to these is redesign, because it fulfils a vital task in the design process by allowing modification of previous design decisions. In summary, research on redesign can be classified as those concentrating on

1. identifying the nature of redesign: Boyle (1989),
2. presenting models based on redesign: Simmon & Dixon (1986),
3. adopting strategies such as dependency directed backtracking: Kumar & Topping (1989b) and Mittal & Araya (1986), and

4. using mathematical optimisation and heuristics as the redesign strategy: the expert systems by Cameron & Grierson (1989) and Adeli & Balasubramanyam (1989b) described in Section 2.3.

However, none of these methods have been successful in handling redesign sufficiently well, so any design model that relies heavily upon redesign is unlikely to produce a good expert system. Therefore, for the present study, it is considered that redesign must be minimised, from which it follows that it is necessary to produce good preliminary solutions that succeed at subsequent detailed design stages.

It is considered here that the procedural design method is fundamental for the *overall* design process of complex design problems. Procedural design becomes a straightforward task if the number of possibilities for redesign can be minimised. Within the procedural design strategy there will be a number of separate tasks to be performed. These sub-tasks themselves can be carried out by any of the available methods: analytical, experimental search and procedural.

The amount of redesign needed in the overall process can be reduced if the number of sub-tasks performed analytically is increased. These analytical design methods, however, are not normally available, so detailed studies are needed to develop them. They should take account of as many structural behaviours and constraints as possible, and should also be able to make adequate allowance for those effects that are too complicated to be calculated at the initial stage. The experimental search technique should be used only for those aspects for which proper justification is available. If the sub-tasks are not amenable to either method, then a procedural design method should be used. In this case also, detailed studies must be carried out to identify all the possible redesign decisions.

2.4.3 Limitations of existing expert systems

Some of the limitations of the existing expert systems are considered by Keravnou & Washbrook (1989) as being due to inadequacies of three components, which are in turn due to inadequacies in the knowledge represented in expert systems and of the software used for implementations. These three components are the following:

1. The reasoning knowledge: the component that carries out inferencing.
2. The domain factual knowledge: the component that embodies the domain facts.
3. The solution progression: the component that registers the development of the solution.

These are interrelated in an expert system and inadequacies in one will be reflected in the others. Some of the inadequacies most relevant for the expert systems used for design are briefly described below.

- **Inadequacies in reasoning knowledge**

  1. The reasoning knowledge is not a complete representation of the human experts' knowledge: When experts tackle design problems, they are able to make quick calculations and to select key design parameters in such a way as to make sure that the design will have to be repeated a minimum number of times (Hoeltzel & Chieng (1989)). This process is not fully understandable by observing the experts at work or eliciting knowledge from them, because most of these decisions and calculations are based on years of experience, and which the experts themselves are sometimes unable to explain rationally. Therefore, an expert system which reasons entirely deductively may be unable to match the human expert because the knowledge incorporated is incomplete. In essence, this knowledge is unable to explain *'why'* certain decisions have been taken.
  2. The reasoning knowledge is incompatible: Human experts employ a number of alternative strategies for achieving a goal. Many expert systems contain rules which are derived from a number of different strategies, without the compatibility of the rules, or the interactions between them, being validated.
  3. The reasoning knowledge is not properly abstracted: There is no explicit representation of the justifications that lie behind the reasoning knowledge.

- **Inadequacies in domain factual knowledge**

    1. The factual knowledge structure is not compatible with the knowledge organisation used by human experts. The knowledge organisation of the expert is difficult to understand, as explained by Maher (1987), because experts carry out certain checks implicitly, which are vital for the success of the solutions.
    2. The factual knowledge is incomplete: Usually common sense background knowledge is necessary to solve a problem efficiently or credibly. This kind of knowledge is best embodied explicitly. As an example, the expert system should not attempt to design a spine beam for an impracticable span. It should rather identify the limit as soon as the span is known.

- **Inadequacies in solution progression**

    1. The database support: the database support during solution progression is important in keeping a track of the solution history. This facilitates the progression of a solution over a number of temporally distinct sessions.
    2. Temporal reasoning: the expert system should represent time (*i.e.* about how the design evolves in time) and support reasoning about time (*i.e.* what a parameter may represent at a particular time - the representation of the idealised cross section at the preliminary design stage may differ from that at the detailed design stage.)
    3. Truth maintenance: the truth maintenance system keeps a record of the solution progression; these records are used to maintain the consistency of the current decisions when some early decisions are changed.

The solution advocated by Keravnou & Washbrook (1989), for overcoming most of these limitations and inadequacies, is to represent the knowledge explicitly. Reasoning knowledge should be distinctly different from the factual knowledge. They have further suggested that future expert systems should be able to reason from fundamental design principles, thus exhibiting more flexibility, and also should support evolutionary development of solutions.

2.4.4 Explicit knowledge representation for better models of design

Mostow (1985) identified the importance of representing the knowledge explicitly for producing better models of the design process. The important considerations that are applicable to the present work are collated below, with examples from the domain of the design of prestressed concrete spine beams.

Making the state of the design explicit: The intermediate stages in the design should be represented explicitly; this means the ability to represent the partial solutions of a design where all the design decisions have not yet been made. Hence, the constraints that should be satisfied at each intermediate design stage should be identified, for which a detailed study of the overall design process is essential.

Making the goal structure explicit: In the design process, goals guide the decision making at each point. Hence, goals should be distinguished from preparatory steps. The identification of key design decisions helps to describe goals explicitly. The following kinds of goals can be identified in design:

- Functionality goals: The functional requirements such as strength, serviceability, and safety should be satisfied by the structure. Of these, only some will be the governing conditions for a particular type of design, while others will be secondary in nature.

  (*e.g.* A functional requirement such as serviceability needs the satisfaction of flexural, torsional, shear and deflection criteria along the beam. Of these, the flexural behaviour in the longitudinal and transverse direction often governs the cross sectional layout in the case of right spine beam bridges.)
- Performance goals: seek to satisfy requirements like economy, maintainability, and reliability for the structure. These can often be used as criteria for selecting from alternative solutions which satisfy a given functional specification.

  (*e.g.* For the same spine beam, different reactant moment distributions can be selected which result in different section dimensions and different cable force distributions. In this case, economy could be a criterion for selecting between the alternative solutions.)

- Knowledge goals: seek to gather information needed to carry out the design. Approximate calculations are performed at the preliminary stages to obtain values for key design parameters. The subsequent detailed designs will be based on these.

  (*e.g.* The knowledge goals for the design of prestressed concrete spine beams are the selection of the reactant moments, the selection of the cross sectional layout, and the selection of the cable force distribution.)

- Design process goals: govern the route taken to arrive at a design, not the end product itself. The route taken for a particular design depends on the nature of the problem, the time, and the tools available for design.

  (*e.g.* The route taken for the selection of section dimensions of a prismatic beam may differ considerably from that of a non-prismatic beam. Hence, different routes should be explicitly identified, based on the characteristics of the problem.)

Making the design decisions explicit: Given a goal, there may be several ways to achieve it. Design decisions represent choice among them, which means underlying design assumptions should be made explicit.

(*e.g.* When selecting the web thickness for a spine beam, if the simplicity of construction is important, then a constant web thickness should be specified even though this may be uneconomical in certain cases. The final solution depends on these assumptions.)

Another important aspect is the identification of sub-goals to achieve primary goals and how to achieve these sub-goals.

(*e.g.* If the goal is the selection of dimensions, then the sub-goals are the selection of dimensions for top and bottom flanges and webs. This process may continue for several levels of sub-goals.)

Making the design rationale explicit: An understanding of the fundamental reasoning about why a particular plan achieves a goal helps to explain the reasons for the success of this particular plan. The same understanding can also be used to explain why a particular design is unsatisfactory.

(*e.g.* When selecting the cable forces of prestressed concrete spine beams, there are a number of bounds which need to be satisfied. If any of these are not satisfied, then it is not possible to find a cable profile.)

Understanding how to control the design process: Selection of design goals in the proper order makes a big difference to the amount of iteration required in design.

(*e.g.* The goals should be accomplished in a specific order so that subsequent decisions should not violate early decisions. Failure to achieve this will force the designer to resort to iterative techniques.)

These points highlight the characteristics of explicit knowledge representation. It is evident from the examples that identification of key design parameters, as well as the determination of the influence of structural behaviours and constraints on them, plays an important role in representing the knowledge explicitly; this will also undoubtedly lead to a better understanding of the design.

2.5 Discussion

There are a number of expert systems developed for structural design. A few notable applications have been presented to identify the implementation, capabilities and limitations of the existing expert systems.

A notable feature of most of these expert systems is that the structural design process has been viewed as an iterative feedback process. Hence, they have tried to reach a suitable design solution by modifying the preliminary solutions until some acceptance criteria is satisfied. This approach often leads to design by repeated analysis. It does not help to meet the basic and most important requirement of expert systems; that of emulating the expert.

The reason for the iterative nature of the design process stems from the deficiencies of the preliminary solutions, because they lack some important characteristics. The preliminary solutions should either take account of all the key constraints accurately, or be able to make adequate provision for any effects which are too complicated to be taken into account accurately at the preliminary design stage. The majority of existing expert systems have failed in this respect, due to inappropriate knowledge being used to obtain preliminary solutions. The main tools used to represent knowledge in expert systems, the heuristics, are not capable of taking into account the many intricacies of design.

Heuristics are often unable to delve into the fundamental design principles; they are unable to answer the question, "*why* is a certain decision taken under certain circumstances?". Heuristics normally tend to be general rules. In order to take account of specific cases, a large number of them are required. The ability to take account of the peculiarities of the problem is a prime requirement in minimising the iterative nature of design.

Therefore, it can be concluded that knowledge expressed as heuristics alone is not sufficient to take account of the intricacies of design problems. Heuristics must be supplemented; the way to do this for the design of prestressed concrete spine beams is a key area of the research presented in this thesis. The techniques suggested for supplementing the knowledge expressed as heuristics are given below.

The proposed technique

The solution to any non-trivial design problem is generally not found in one step but tends to go through an evolution process. Therefore the procedural design method explained in Section 2.4.2.2 is inevitable for the overall design process. The procedural design method becomes a complex process when the designer uses the information obtained in intermediate stages to modify previous decisions. This invariably leads to iterative cycles. Designers using the procedural design technique would be much better off if they could engage in design as a logically evolving series of tasks.

Engineering design becomes a complex process, mainly due to a lack of understanding about the influence of constraints on the key design parameters. These influences are of two types; direct and indirect (due to interactions of constraints). Therefore, the crux of the approach stems from the premise that the engineer has to determine which parameters need to be calculated and which can be found on the basis of experience. This can be done by identifying the key design parameters and then determining the influences and interactions of constraints on them. This approach helps to select the method most suitable for determining the magnitudes of key design parameters during the design process. Those which can be calculated are suitable for an analytical design method. Those which require experience or those for which analytical design methods are not yet available are suitable for experimental search. Those not amenable for either are suitable for procedural design. The key feature is the use of localised iterative loops, backed by precise redesign knowledge at the preliminary design stage.

The following propositions explain the basis of the design technique:

1. The technique adopted for the preliminary design is hybrid in nature. It uses analytical, procedural and experimental search design methods to generate a preliminary design solution.

2. All the key design parameters and their characteristics must be identified explicitly. The influences of constraints and also the interaction of constraints on key design parameters should be determined to enhance the understanding of the design.

3. The determination of the nature of the key parameters (whether they can be calculated or should be based on experience) helps to select the appropriate design method (*e.g.* analytical or experimental search).

4. An understanding of the design philosophy plays an important role in adopting redesign at the preliminary design stage; this can help to keep redesign localised, thus relieving the overall design process of any cyclic nature. *Explicit knowledge representation* is useful in incorporating an understanding of design philosophy into expert systems.

5. The use of the analytical design method for calculable parameters needs the development of new algorithms for design. These should be capable of determining the magnitudes required for the design parameters to satisfy the constraints. These design algorithms will be different from the conventional analytical algorithms, since they are dedicated to the production of values for design parameters.

6. The use of design algorithms to generate part of the design knowledge calls for the development of *'deep coupled expert systems'*, which use

numerical processing along with heuristics and also delve into fundamental principles of structural design.

Figure 2.3: Components of an expert system

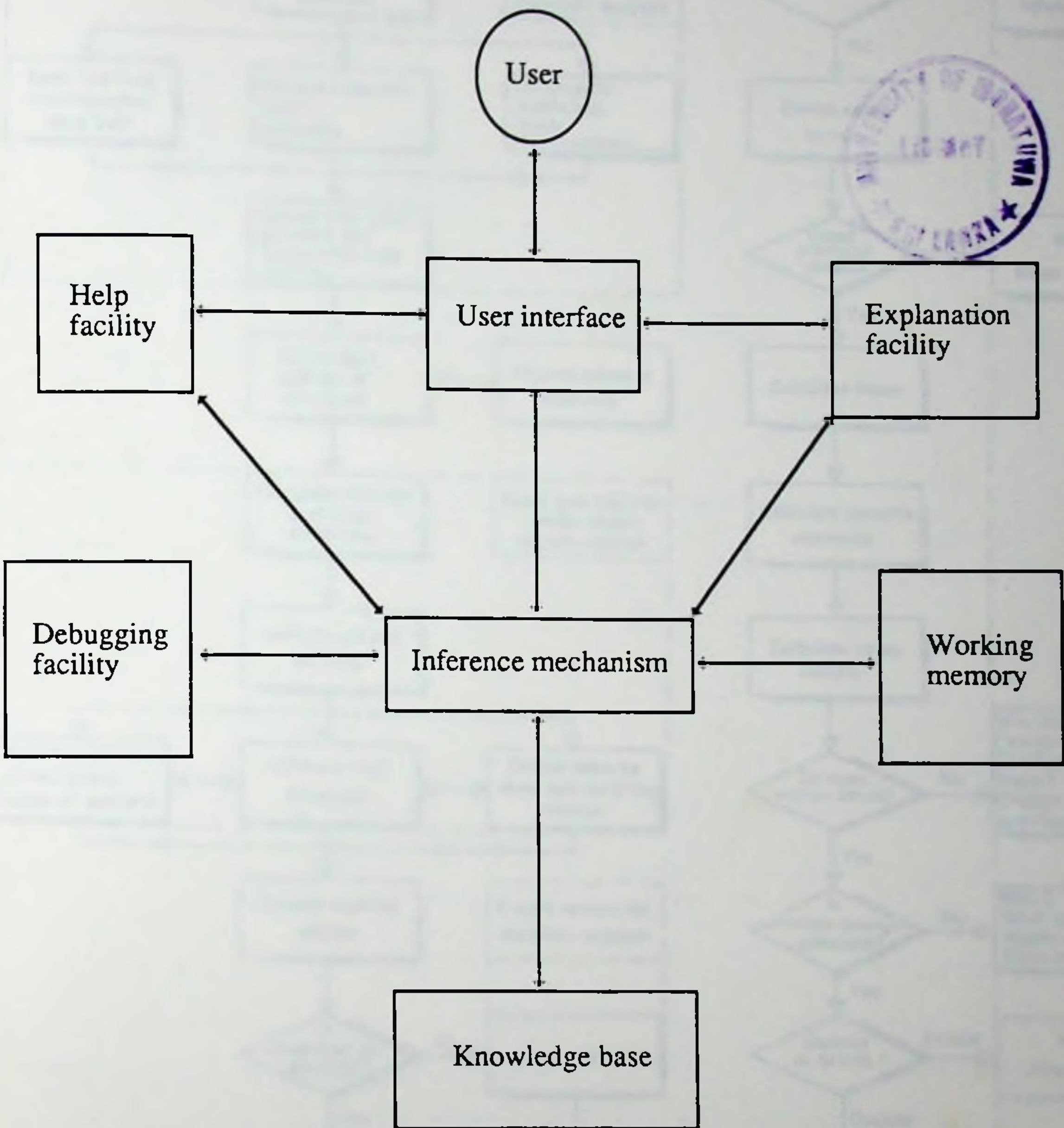

Figure 2.1: Components of an expert system

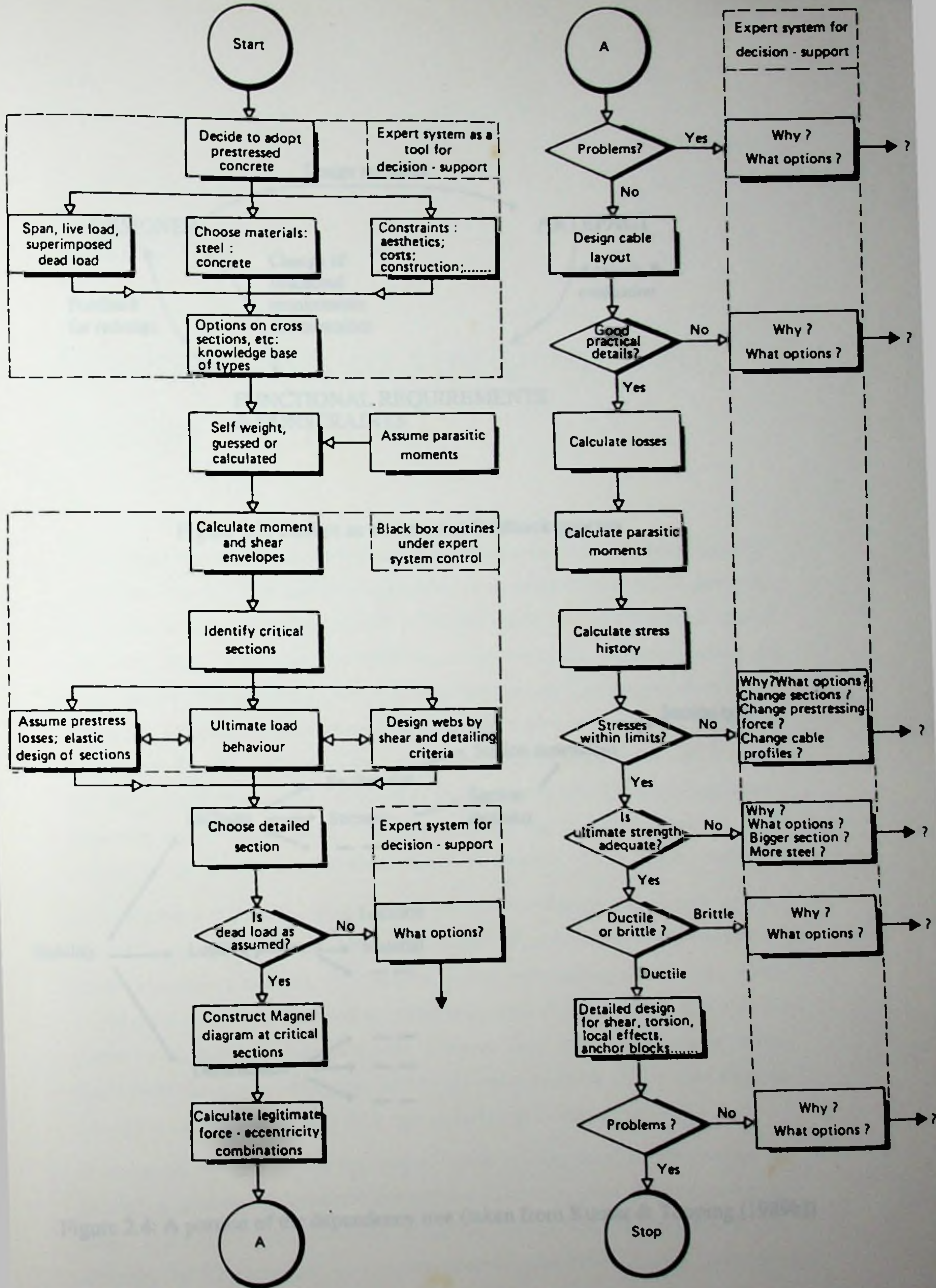

Figure 2.2: An expert system for design of bridges (taken from Burgoyne and Sham (1987))

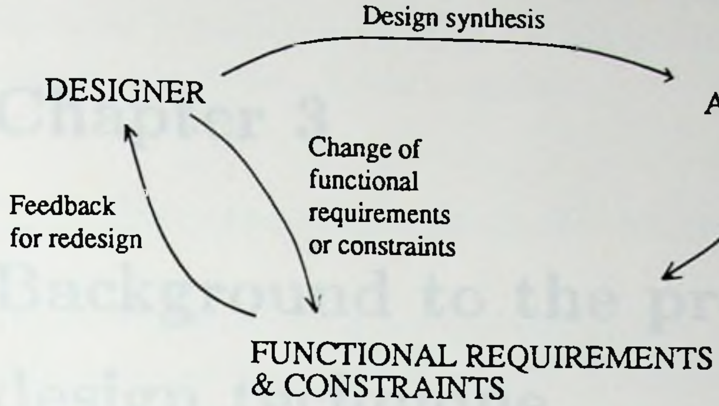

Figure 2.3: Design as an iterative feedback process

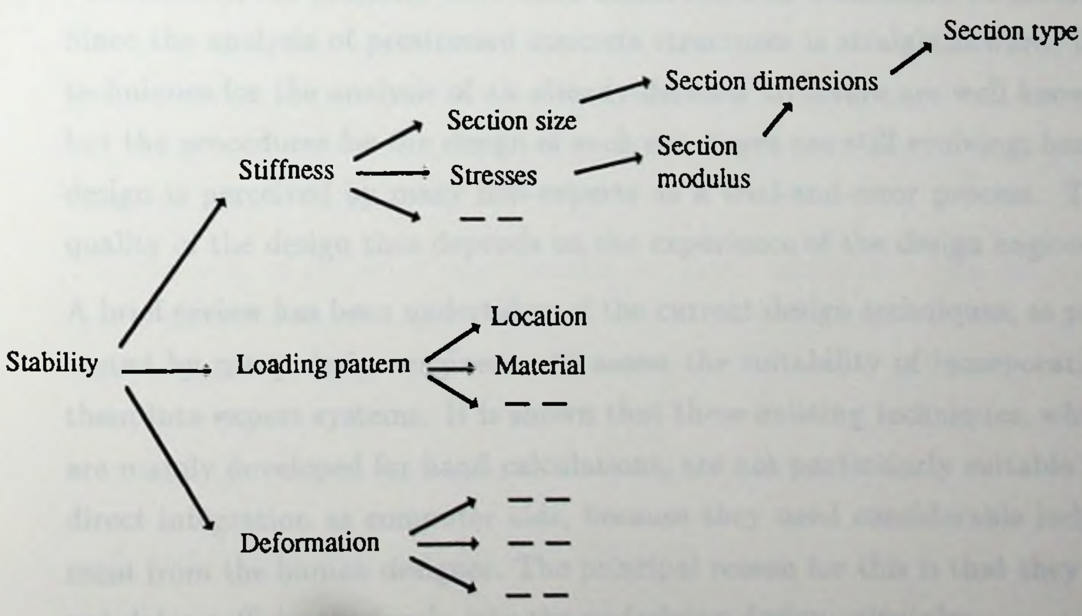

Figure 2.4: A portion of the dependency tree (taken from Kumar & Topping (1989b))

Chapter 3

Background to the proposed design technique

Prestressed concrete has been widely used in highway and railway bridge construction. Many spine beam bridges have been constructed world-wide, for the medium range spans (20-100m). With the realisation of the advantages of continuity with respect to serviceability, maintainability, durability and constructability, the majority have been constructed as continuous structures. Since the analysis of prestressed concrete structures is straightforward, the techniques for the analysis of an already-detailed structure are well known, but the procedures for the design of such structures are still evolving; hence design is perceived by many non-experts as a trial-and-error process. The quality of the design thus depends on the experience of the design engineer. A brief review has been undertaken of the current design techniques, as presented by many design engineers, to assess the suitability of incorporating them into expert systems. It is shown that these existing techniques, which are mainly developed for hand calculations, are not particularly suitable for direct integration as computer aids, because they need considerable judgement from the human designer. The principal reason for this is that they do not delve sufficiently deeply into the underlying design principles.

It would be useful if design techniques could be suggested which harness the numerical processing power of computers to obtain good preliminary solutions. It is argued here that these design algorithms should integrate the underlying design principles. In order to propose alternative design techniques for preliminary design, a number of aspects of the prestressed concrete design process are earmarked which have not been addressed sufficiently in current design techniques. The aim is to take account of them in the best possible way at the preliminary design stage. Certain research that has already been done on some of these aspects is described.

3.1 Prestressed concrete bridges

Prestressed concrete differs from reinforced concrete in its use of high-strength steel coupled with high strength concrete, thus improving the behaviour and strength of the elements. By prestressing, high strength steel can be used at high strain without causing extensive cracking in the concrete; this allows the full strength of steel to be utilised. Precompressing the concrete also prevents cracking at normal load levels. Prestressed concrete has been used extensively for the construction of buildings, bridges, silos, liquid retaining structures, nuclear reactor containment structures and off-shore structures (Sawko (1978)).

The main use of prestressed concrete spine beams is found in the construction of bridges and viaducts for highways and railways. A large number of prestressed concrete spine beam bridges have been built to date. The details of 173 such structures were reported by Swann (1972), since which time many more have been built.

The majority of the spine beam bridges are made continuous with the realisation that structural continuity leads to the following benefits (Low (1982) and Naaman (1982)):

1. savings in joint maintenance and enhanced durability,
2. improvements in ride for the road user,
3. higher rigidity during construction and against lateral loads, and
4. savings in concrete and steel due to reduced moments and increased stiffness.

3.1.1 Design of prestressed concrete bridges

The design of prestressed concrete bridges is not a strictly ordered activity which leads directly from the specification to a solution. It is rather a procedure comprising loosely-knit activities with several possible interacting routes to a solution. It is characterised by the need to make tentative decisions on the basis of incomplete data and with the growth of understanding of a solution as the design proceeds (Cope & Bungey (1975)).

ACI Committee 443 (1976) have stated that continuous beam structures should be designed to possess satisfactory service load behaviour and specified safety factors at the ultimate load. They have suggested that the behaviour should be determined by elastic analysis taking into account the reactions, moments, shears and axial deformations, restraints of attached structural elements and foundation settlements.

3.2 Current design techniques

The techniques which are currently used for the design of determinate and indeterminate beams are described briefly; four such methods are presented. The following points should be noted about this discussion:

1. The design techniques for an I-section can be adapted for other shapes like rectangular-, T- and inverted T-sections. Thus, the design techniques are presented as applied to I-sections.

2. For statically determinate beams, the main design task is finding a suitable section. Once this has been selected, the determination of legitimate force-eccentricity combinations is a trivial exercise; the construction of Magnel diagrams[1] at a number of cross sections along the beam is sufficient.

   The design procedures for the selection of section dimensions are explained as applied to cross sections which have the highest moments or moment ranges; these cross sections are often called the critical sections.

[1] see Appendix B.2

3. For statically indeterminate beams, the presence of reactant moments (see Section 3.3.1) causes additional problems. These affect the choice of section dimensions and may impose severe restrictions on cable forces and the cable profile.

3.2.1 Design technique by Abeles & Bardhan-Roy

Abeles & Bardhan-Roy (1981) are of the view that the structural dimensions are governed by the need to have sufficient strength in case of overloading; this is particularly true in structures where a certain amount of tensile stresses is allowed under service conditions.

3.2.1.1 Statically determinate beams

A number of design techniques have been suggested, giving a comprehensive coverage of a wide range of possible applications, based on the following criteria:

- the top flange area is determined by the need to resist the compressive forces caused by the ultimate loads (see Section 4.1.2).
- the bottom flange area is determined so that it is large enough to ensure the existence of a valid Magnel diagram under service load conditions (see Section 4.1.3).

A graphical design technique, which can be very useful for hand calculations, is provided but this requires an initial estimate of self weight.

3.2.1.2 Statically indeterminate beams

No direct guide lines are given for the selection of section dimensions.

Selection of cable forces and profile

Cable forces and profile are selected on the basis of three guide-lines:

1. The appropriate cable forces should be selected to counteract the moments acting at working loads; thus the cable forces could be based on the feasible region of the Magnel diagram.

2. The eccentricity, which depends on the magnitude of moments acting on the section, should be as large as possible at critical sections.
3. The profile of curved cables should be such as to produce the smallest possible frictional losses.

After selecting the cable forces and profile, the magnitudes of the reactant moments are evaluated and the resulting stresses are calculated. The cable profile may need readjustment if there are violations of the allowable stresses. It is suggested that for a well-chosen cable profile, the reactant moments should not be excessive. The importance of checking the design for the ultimate condition is also emphasised.

3.2.2 Design technique by Gilbert & Mickleborough

Gilbert & Mickleborough (1990) are of the view that at each stage of the lifespan of a prestressed concrete structure, it is necessary to satisfy both the serviceability and the strength requirements. However, the level of prestressing is determined by the serviceability condition because the flexural strength may be readily increased, if necessary, by the addition of non-prestressed conventional reinforcements. The important serviceability requirements considered are cracking and deflection.

3.2.2.1 Statically determinate beams

1. The required value of the section modulus of the bottom flange, Z_2, is found by considering the stress range between the transfer- and working load-conditions. An initial estimate of self weight is required, but the final solution is not unduly sensitive to this value. The expressions that can be used are given in Appendix B: B.14, B.15 and B.16.
2. For a member containing a parabolic cable profile, another guide on section dimensions can be obtained by considering the deflection requirements of the member. This results in an appropriate value for the second moment of area, I.

The trial section is selected using trial and error to give the required values of Z_2 and I.

3.2.2.2 Statically indeterminate beams

1. The trial dimensions are found on the same criteria as for determinate beams; the design moments at critical sections should be found based on an initial estimate of the self weight.

2. Based on this trial section, the bending moment envelopes are established.

3. The cable forces and the corresponding bounds on eccentricity along the beam are selected with the aid of Magnel diagrams. The resulting cable profile should not cause any reactant moments; a cable profile showing this property is called a concordant profile. This concordant profile can be subsequently adjusted to obtain the actual cable layout, which fits within the bounds of the section.

3.2.3 Design technique by Lin & Burns

Lin & Burns (1981) emphasise that the balanced-load concept for design of statically indeterminate prestressed concrete structures is often the simplest approach, although it does not have a particular advantage for statically determinate structures.

3.2.3.1 Statically determinate beams

- The top flange area should be sufficiently large to resist the working load moment range and ultimate limit state moments.

- The web area should be large enough to provide the required shear resistance and also to encase the prestressing cables.

- The bottom flange area should be based on elastic considerations, especially to resist transfer stresses.

A number of approximate formulae have been suggested for the selection of the cross sectional dimensions, which cover a number of shapes. Sizing of the cross section on the basis of previous experience is also recommended.

3.2.3.2 Statically indeterminate beams

No guide-lines are given on the selection of cross sectional dimensions.

Selection of cable forces and profile

For the selection of cable profiles, two methods have been suggested:

The method based on concordant profiles: The cable profile is selected so that it is concordant, which means that the selected cable profile will not cause any reactant moments. This concordant cable profile is linearly transformed[2] to fit within the bounds of the section and also to satisfy the cover required for steel cables.

The method based on load balancing: In load balancing, prestressing of concrete is seen as an attempt to balance a proportion of the load of the structure. Generally, this is selected as a portion of the permanent load. The force required in the cable to resist this permanent load is calculated by considering an appropriate cable profile (parabolic for uniformly distributed loads and linear for concentrated loads). Once the force and the cable profile are found, the prestressed concrete beam can be considered as an ordinary beam subjected to a uniform prestress and by definition there can be no reactant moments. Hence, the main advantage of load balancing is that it relieves the designer from the complications due to the existence of reactant moments. However, there are shortcomings associated with it:-

- In order to select which portion of the load is to be balanced, the designer has to perform a few trial calculations, which needs some intuitive judgement by the designer. Hence, this is a good hand calculation method, but is not suitable as a computer technique.
- The resulting cable profile needs considerable adjustment over the supports to obtain a practicable cable profile which complies with the allowable curvatures. This can have considerable consequence for the reactant moments, which the method does not, otherwise, need to take into account.

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[2](see Section 3.3.1.5)

3.2.4 Design technique by Naaman

Naaman (1982) considers that the most difficult part of the design process is the selection of the proper section dimensions, which therefore involves a considerable amount of trial and error. The designer starts the process by assuming some practical values for the unknowns.

3.2.4.1 Statically determinate beams

The unknown dimensions of four different shapes of cross section (rectangular, T-, box- and I-sections) are listed. It is recommended that the designer uses standard sections if possible. If standard sections are not feasible, then a number of *'rules of thumb'*, which are based on previous experience, have been presented.

3.2.4.2 Statically indeterminate beams

Selection of section dimensions

If the cross section is not provided, the section dimensions are found as if the beam is simply supported. Then the depth of the section is reduced by 20%.

Selection of cable forces and profile

The following procedure is suggested:

1. The minimum and maximum moments at each section due to external loads are determined.

2. A cable profile is selected so as to have the maximum eccentricity at each critical section (but no guidance is given about what profile is to be adopted).

3. The reactant moments corresponding to a unit cable force acting on this cable profile are determined.

4. The cable forces are selected at the critical sections so that the resulting stresses satisfy the permissible stresses. The highest of these cable forces is adopted throughout the beam.

Emphasis has been placed on the possibility of redistributing the support moments by using large reactant moments, thus enhancing the structural efficiency.

3.2.5 Summary of current design techniques

3.2.5.1 Selection of section dimensions for statically determinate beams

1. Since the range of cross sectional shapes that can be selected for determinate beams is large, it is not possible to suggest general guide-lines or techniques for all the possible section shapes. This difficulty has forced the design engineers to seek assistance from previous experience (Lin & Burns (1981)), or standard sections (Naaman (1982)). If guide-lines are suggested, then they will deal with some special cases which are most commonly encountered. As an example, the design technique by Abeles & Bardhan-Roy (1981) is suitable for I- or T- sections.

2. Both Abeles & Bardhan-Roy (1981), and Lin & Burns (1981) suggest that both service and ultimate load conditions should be used in the selection of section dimensions. This is an important consideration for the majority of determinate beams for the following reasons:

   - The ultimate moments at critical sections can be calculated directly from the service moments if the dead load is accurately known, by using the load factors.
   - A sufficient compression flange area should always be provided in prestressed concrete beams, especially if the web is narrow and can make only a little contribution to the moment carrying capacity. According to Leonhardt (1962), the ultimate moment carrying capacity of a section cannot be increased by providing additional unstressed compressive reinforcement in the compression flange. Any unstressed steel tends to yield due to large creep and shrinkage deformations of the prestressed concrete members; thus this reinforcement will not be available for carrying compression at ultimate state.

Therefore, provision of adequate compression area of concrete can be used as a governing design criterion at the preliminary design stage for statically determinate beams.

3.2.5.2 Selection of section dimensions for statically indeterminate beams

It is clear from the procedures suggested in the current design techniques that there is no rational basis for the determination of the section dimensions of continuous beams. For example, methods which use a section similar to determinate beams (Gilbert & Mickleborough (1990)), or reduce the section depth by 20% from that required for a determinate beam (Naaman (1982)), do not have any proven justification for doing so, and are thus good examples of rules that are used for heuristics. Thus, many designers prefer to seek guidance from previous designs when selecting section dimensions. The guide-lines presented by Swann (1972), which are based on parametric studies of a large number of spine beam bridges already constructed, have been used by many practising engineers for the selection of preliminary section dimensions. But since these rules are nearly 20 years old, they become a self-fulfilling prophecy; a new survey would find many structures which matched, simply because they used rules suggested by Swann (1972).

3.2.5.3 Selection of cable forces and profile for statically indeterminate beams

1. Neither Abeles & Bardhan-Roy (1981) nor Gilbert & Mickleborough (1990) identify the advantages of reactant moments in enhancing the prestressing effects. Hence, cable profiles which are either concordant or those causing small reactant moments are recommended. On the other hand, Lin & Burns (1982), and Naaman (1982) emphasise the importance of reactant moments in enhancing the prestressing effects. But the design techniques they suggest do not explicitly show the effect of reactant moments on section dimensions, cable forces and cable profile, because the reactant moments can be calculated only after selecting all the above parameters. Therefore, any benefit is achieved implicitly.

2. The design techniques suggested for finding a line of thrust by Gilbert & Mickleborough (1990), and Lin & Burns (1981) are essentially the same. Although these techniques seem to be straightforward, there are two issues that need to be addressed.

   - The selection of a cable force and eccentricity, from the bounds given by the Magnel diagram, is not sufficient to ensure that the cable profile is concordant. There are additional limits on the cable forces as explained in Section 3.3.2.3.
   - Once a concordant cable profile is found, it should be linearly transformed. However, there is a possibility that the linearly transformed profile may not comply with the bounds of the section (it may lie outside at certain locations, see Section 3.3.2.2).

3. The design technique of Naaman (1982) is straightforward. The designer selects the cable profile himself. Then, the minimum cable force required at each critical section is calculated considering the effects of reactant moments; the maximum of these cable forces is adopted. However, there is a possibility that some of these force-eccentricity combinations may lie outside the feasible regions of the Magnel diagrams; this may need some readjustment of the cable profile and cable forces for which Naaman (1982) gives no guidance.

4. A common feature of all these current design techniques is their iterative nature. The designer starts by selecting a suitable section using experience or guide-lines set by various parametric studies, such as that of Swann (1972). The cable forces and profile are selected without regard to many of the complexities that can be involved. Hence, the efficiency of the design process (*i.e.* the number of iterations involved before reaching an acceptable solution) depends on the experience of the designer.

3.3 Proposed revisions of the design techniques

The existing design techniques are developed with the aim of providing techniques for engineers performing hand calculations. Hence, they rely on the experience of the design engineer to come up with good solutions. However, it is argued here that direct integration of these design techniques into computer packages will not produce efficient preliminary solutions unless there is considerable intervention by the user. Computer packages are usually used merely to perform calculations rapidly, thus reducing the time that is needed to arrive at the solution; they are not used in the decision making process. This is not an acceptable situation in an expert system environment because the expert system should guide the designer to a suitable design solution.

To overcome this problem, it proposed here that the numerical processing power of the computer should be used for a better purpose at the preliminary design stage, by using the underlying design principles to calculate parameters in such a way that they satisfy constraints. The design parameters calculated in this way should cut down the number of iterative loops that may be involved in the design, by suggesting efficient preliminary design solutions. In view of this, a detailed review has been undertaken of the design process in order to recognise the factors which give rise to its iterative nature, especially with respect to continuous spine beams. Some of them are due to peculiarities of prestressed concrete itself (*e.g.* structural behaviour such as long term creep or reactant moments), whereas others are due to inadequacies of the current design techniques (*e.g.* there is no rational technique to select the section dimensions). All these factors are collated below; only some of them are dealt with in detail in this thesis while others are highlighted for further work.

1. **The selection of cross sections:**

   The cross section suitable for a particular situation depends on a number of factors such as the spans, traffic volume, width and number of traffic lanes, position of expansion joints, fixity at supports, and the construction technique; it is almost impossible to find identical highway bridges. It follows that the selection of cross sections based on previous experience inadvertently leads either to the overlooking of important constraints, or to the consideration of unnecessary constraints. Rational techniques, which can generate values for the design parameters of the cross section to suit constraints, should be developed; one such technique developed in the present project is presented in Chapter 4.

2. **The selection of cable forces:**

   For determinate prestressed concrete beams, the calculation of legitimate force-eccentricity combinations can be a simple matter of constructing a Magnel diagram[3] at each section. However, this is not the case with continuous beams. In addition to the moments due to dead and live loads, there are reactant moments acting at each section. These reactant moments are unknown until the actual cable force and profile are known, but valid force-eccentricity combinations depend on the magnitudes of the reactant moments. Hence, in addition to the limits set on the prestressing force by the Magnel diagram, there are extra restrictions due to continuity. A technique developed for the determination of limits on the cable forces for continuous beams is presented in Section 5.2.

3. **Methods of dealing with reactant moments:**

   There are two different methods of dealing with reactant moments (Burgoyne (1988a)):

   (a) The reactant moments are considered as prestressing effects. The designer finds a concordant profile which fits into the allowable line of thrust zone and then uses *linear transformations* [4] to find a practical cable profile that satisfies the requirements imposed by the cover needed for steel cables.

   (b) The reactant moments are considered as loads. The designer estimates a value for the reactant moments which are then included in the total moment envelope. The problem becomes one of finding an actual cable profile which causes the assumed reactant moments and also satisfies the stress limits.

[3] see Appendix B.2

[4] see Section 3.3.1.5

The selection of which method to use is up to the designer. Thus the one offering the highest flexibility should be selected, and this is discussed in more detail in Section 4.3.6.

4. **The effects of construction technique and sequence:**

   As described by Gee (1987), a particular construction sequence and technique should be assumed and built into the design of a continuous concrete bridge (*e.g.* span-by-span construction on false work or balanced cantilever method with precast segments). Any stresses existing in the structure at the completion of each phase of construction are locked-in and carried forward to the next phase. The final condition at the completion of construction will then become the initial self weight condition. This means that the construction technique and sequence affect the stresses, and in turn affect the section dimensions and the cable forces. This is discussed in more detail in Section 7.3.

   The construction sequence can also affect the reactant moments due to prestressing (Lee (1971)). This effect should be taken into account at the preliminary design stage and is discussed in more detail in Section 7.5.

5. **The effects of long term creep:**

   For indeterminate concrete beams, there is a long term redistribution of moments due to the effects of creep, which tend to modify the beam in the direction of the fully continuous state (Gee (1987)). This means that the moments due to the dead load of the beam change with time. The initial dead load moments include any moments trapped in the beam due to the construction sequence. A long time after construction, these trapped moments will gradually become small. Hence, continuous beams have to resist dead load distributions that vary with age; this leads to some loss of structural efficiency, since the beams have to be designed to resist initial and final moments. Therefore, at the preliminary design stage, sufficient provision should be made for long term effects due to creep. This is explained in Section 7.4.

6. **The effect of temperature:**

   Bridge structures are generally subjected to temperature stresses. These stresses depend on the factors such as shape, thickness, colour and location of the structure. Although comprehensive calculations can be done only at the detailed design stage, an allowance has to be made for these stresses at the preliminary design stage. The ways to take account of effects of temperature at the preliminary design stage are described in Section 7.6.

7. **The effect of shear lag:**

   In simple terms, shear lag affects the width of the flange that is effective for resisting flexural stresses. According to Hambly & Pennels (1975), shear lag is most significant at the intermediate supports for continuous beams. This effect should either be taken into account when selecting section dimensions or adequate provision be made to cater for it during the preliminary design stage. The difficulties arising due to shear lag effects are explained in Section 7.8.

8. **The effect of transverse load distribution:**

   Prestressed concrete spine beam bridges are generally prestressed in the longitudinal direction. In the transverse direction, they are either reinforced or prestressed. If prestress is present, it is generally confined to the top slab and/or the webs. Certain components of the cross section, such as the top flange and the webs, must be designed to resist the transverse distribution of loads (local effects) in addition to longitudinal requirements (global effects), and in certain instances, these criteria govern.

9. **The governing criteria for the preliminary design:**

   Bridge structures should satisfy the serviceability limit state and the ultimate limit state. Whether to place emphasis on one or both of these has to be determined.

It is clear from the range of points highlighted that preliminary design parameters selected on the basis of previous experience, expressed either as empirical formulae, rules or databases, are unlikely to be able to take account of the majority of these factors sufficiently to minimise the iterative nature of design (redesign). This is especially true when some of the effects such as those due to construction sequence and temperature may affect the design parameters in a way specific to a particular structure.

Hence a detailed study is needed to propose an alternative design technique for producing better preliminary designs. A certain amount of research which aims at producing better preliminary designs has already been documented on the following topics.

1. A method to deal with the reactant moments.
2. The selection of the prestressing force.
3. The effects of transverse load distribution.
4. The criteria for preliminary design.

3.3.1 A method for dealing with the reactant moments

An indeterminate prestressed concrete beam is subjected to a set of reactant moments, M_2, in addition to the external moments, M. The prestressing cable placed at an eccentricity e_s tries to induce a curvature Pe_s/EI at each cross section of the beam, where Pe_s is the moment due to the cable force. If the displacement due to these curvatures is not compatible with zero displacement at the supports, a set of self-equilibrating reactions are developed, which in effect redistribute the reactions due to the dead load. These reactions induce moments in the beam, which vary linearly between the supports.

These moments are often called secondary or parasitic moments. However, they are neither necessarily small (as implied by secondary) nor necessarily deleterious (as implied by parasitic). Therefore the more appropriate term, *reactant moments*, is used in this thesis.

3.3.1.1 The benefits of reactant moments

The main benefit of reactant moments is in the redistribution of moments. A continuous spine beam is subjected to high hogging moments close to supports. However, the large top flange area needed to accommodate highway traffic means that the cross section layout of a spine beam is particularly suitable for resisting sagging moments efficiently.

The reactant moments induced in the continuous spine beams used in highway construction generally show a sagging moment distribution, since the draped tendon profiles tend to lift the beam from the internal supports. The redistribution results in increased sagging moments in the span region and reduced hogging moments over the supports, which enhances the structural efficiency of the overall structure.

3.3.1.2 Handling reactant moments

There are two ways of dealing with the reactant moments for design purposes (Burgoyne (1988a)). They are:

1. Line of thrust design.
2. Actual cable profile design.

Any cable has an *actual profile* which is denoted by the eccentricity e_s. The cable exerts forces on the beam which may redistribe the support reactions, causing the reactant moments. When these reactant moments are added to the moments due to the prestressing effects, the effect is as if the cable had been placed at a different position e_p, known as the *line of thrust.* If a cable were placed at the line of thrust, it would cause no reactant moments and would be known as a concordant profile (for example see Lin & Burns (1981)).

3.3.1.3 Line of thrust design

In line of thrust design, the reactant moments are treated as prestressing effects; this means that the reactant moments are unknown until the cable forces and profile are found, thus the reactant moments are not considered directly in the design. As described in Appendix B, there are a number of stress conditions which need to be satisfied by the cable forces and eccentricities. If the horizontal component of the cable force is P, the area of concrete is A_c, the section modulus is Z, the permissible stress of concrete in compression is f_c, and the permissible stress of concrete in tension is f_t, then these stress conditions can all be presented in the form:

$$f_c \leq -\frac{P}{A_c} - \frac{P\,e_p}{Z} + \frac{M}{Z} \leq f_t \tag{3.1}$$

In this expression M is an externally applied moment, thus the corresponding eccentricity is e_p. This expression can be arranged to give bounds on the line of thrust in the form of Expression 3.2. Since there are upper and lower bounds on e_p at each cross section, these bounds create a *'line of thrust zone'*, when considered along the length of the beam.

$$-\frac{Z}{A_c} - \frac{f_c\,Z}{P} + \frac{M}{P} \geq e_p \geq -\frac{Z}{A_c} - \frac{f_t\,Z}{P} + \frac{M}{P} \tag{3.2}$$

Therefore, a profile must be found, which not only fits within the bounds of the line of thrust zone throughout the length of the beam, but is also concordant. It is also necessary to ensure that this concordant profile can be linearly transformed[5] to fit within the bounds imposed by the concrete cover required for steel cables. The designer does not have to know the magnitude of the reactant moments until the actual cable profile is found.

3.3.1.4 Actual cable profile design

In actual cable profile design, the reactant moments (M_2) are assumed initially and treated as loads. Therefore, they appear in the eccentricity equation, which is now written in terms of the actual cable profile.

$$-\frac{Z}{A_c} - \frac{f_c\,Z}{P} + \frac{M + M_2}{P} \geq e_s \geq -\frac{Z}{A_c} - \frac{f_t\,Z}{P} + \frac{M + M_2}{P} \tag{3.3}$$

Since there are two bounds at each cross section, this will result in a *'cable profile zone'*. A cable profile then has to be found which not only satisfies the limits set on e_s, but also causes the assumed values of the reactant moments, M_2.

In this case, the corresponding line of thrust of the actual cable profile will also satisfy Expression 3.2 and will of course satisfy the concordancy requirement. A design which satisfies one condition will also satisfy the other, so the choice of which method to adopt is dependent only on the requirements of the techniques used to determine other design parameters.

[5]see Section 3.3.1.5

3.3.1.5 Relationship between M_2, e_s, and e_p

In Expressions 3.2 and 3.3, two eccentricity terms, e_p and e_s, are introduced. At each cross section there is a relationship between these eccentricities, the reactant moment and the cable force.

An indeterminate beam is subjected to an external moment, M, a reactant moment, M_2, and a moment due to the prestressing cable, Pe_s. Owing to the reactant moments, the cable effectively acts at the line of thrust, e_p. Since the moments due to the prestressing force and reactant moments are in opposite senses, the following relationships can be established.

$$P\,e_p = P\,e_s - M_2$$

$$e_s - e_p = \frac{M_2}{P} \tag{3.4}$$

If the reactant moments are known, a cable profile can be transformed into a line of thrust and vice versa. The reactant moments are caused by the point forces acting at the supports, thus the reactant moments vary linearly between adjacent supports. If the cable forces are constant throughout the beam, then the difference between e_s and e_p also varies linearly between supports; this is called a *'linear transformation'*.

However, with varying cable forces, the transformation is no longer linear, although the reactant moments themselves will continue to vary linearly. At the points where the cable forces change, there will be a sudden change in the profile.

3.3.1.6 Representation of the reactant moments

The bending moment envelope for a continuous structure consists of four parts:

1. The bending moment due to dead load
2. The bending moment envelope due to superimposed dead load
3. The bending moment envelope due to live load

4. The bending moment due to the reactant moments.

The total bending moment envelope is the superposition of the bending moment envelopes due to superimposed dead load and live loads together with the bending moment diagrams due to dead load and reactant moments.

The effect of the reactant moments is to shift the origin of the dead load moment diagram, thus shifting the origin of the total moment envelope. The reactant moments can be expressed conveniently as a ratio of the dead load bending moments at intermediate supports by a parameter called the *Reactant Moment Ratio (RMR)* defined in Equation 3.5. This is illustrated in Figure 3.1.

$$RMR = \frac{reactant\ moment\ at\ support\ i}{dead\ load\ moment\ at\ support\ i} \tag{3.5}$$

3.3.2 Selection of cable force

The limits on the cable force for the internal spans of a multispan spine beams have been determined by Low (1982). Four governing conditions have been distinguished to determine the limits on the cable force. This work has been generalised by Burgoyne (1988a) highlighting the underlying principles. Low (1982) assumed that each condition was governed by tensile stresses, and illustrated the failure mechanism by the crack patterns that would exist if the tensile stresses were marginally not satisfied. In the generalisation by Burgoyne (1988a), tensile stresses do not necessarily govern, although the same underlying principles remain valid. The following discussion summarises the principles for each condition, where marginally satisfied tensile stresses are represented as potential cracks in the associated figures. Only the working load moments are considered in this discussion; moments at transfer are ignored but they can easily be taken into account.

3.3.2.1 Case 1: Cable force governed by the moment range

The allowable tensile stresses are marginally satisfied at the bottom of the section under the maximum sagging moment and at the top of the section under the minimum sagging moment; this occurs at the cross section with the maximum moment range, as shown in Figure 3.2. The resulting cable force is equal to the minimum value allowed by a valid Magnel diagram and thus corresponds to point C on the Magnel diagram of Figure 3.3.

3.3.2.2 Case 2: Cable force governed by the lever arm

In this case, the prestressing cable is considered to be at the most advantageous position at the critical sections; this means that it is at the minimum acceptable cover limits at the critical sections as shown in Figure 3.4. The limiting case will occur when the allowable tensile stress limits are marginally satisfied at the bottom of the section at mid-span and at the top of the section over the supports.

Based on this condition, a minimum limit on the cable force, P_2, has been derived by Low (1982), in terms of the full moment range of an internal span M_f (see Figure 3.5), and the maximum lever arms, l_s and l_h, within the section (see Figure 3.6).

$$P_2 = \frac{M_f}{(l_s + l_h)} \tag{3.6}$$

As described by Burgoyne (1988a), this value of P_2 is a special case of a more general condition that the line of thrust, e_p, must be capable of being linearly transformed to fit within the minimum and maximum limits on the cable profile. In order to satisfy this condition, as shown in Figure 3.7, the linear transformation required at the left pier, t_l, and that at the right pier, t_r, must be large enough to bring the tendon zone within the minimum e_s over the supports. However, the linear transformation required close to the mid-span, t_m, must be small enough to keep the tendon within the maximum e_s limit at the critical section in the span. The essence of case 2 is that the required range of eccentricity (see Figure 3.7) must be less than the permitted range of eccentricity governed by the cover. If this is satisfied, then the cable can be moved into the correct position by a linear transformation.

3.3.2.3 Case 3: Cable force for the existence of a concordant profile

In this case, the cable is placed near the top of its allowable zone everywhere as shown in Figure 3.8. Hence the allowable tensile stress is marginally satisfied throughout the length of the beam.

As shown by Burgoyne (1988a), this is the condition for the existence of a concordant cable profile which fits within the e_{p-min} limit; e_{p-min} represents the upper bound of the line of thrust zone. The principle underlying this condition can be explained as follows.

If the limits on the eccentricity e_p have been selected for a prestressing force compatible with the Magnel diagram, then it follows that $e_{p-min} \leq e_{p-max}$ for all positions. These limits are the boundaries of the line of thrust zone. The line of thrust, which fits within these limits, must be a concordant profile. If the cable is placed at e_{p-max}, then it will cause the maximum possible sagging reactant moments, M_{2-max}. Conversely, if placed at e_{p-min}, it will cause the minimum possible sagging reactant moments, M_{2-min}. If a concordant profile is to exist between these, then M_{2-min} should be hogging and M_{2-max} should be sagging. The dividing line between existence and non-existence of a line of thrust occurs when M_{2-min} itself defines a concordant profile, because for a beam subjected to gravity loads M_{2-max} generally tends to be sagging.

Based on this principle, expressions have been derived by Burgoyne (1988a) which shows that this effectively puts a lower limit on the prestressing force; this bound on the cable force is referred to as P_3.

3.3.2.4 Case 4: A special case

This is a special case derived on the same basis as case 3, but limited by the condition that the cable cannot be placed at the top of its allowable zone over the supports, since this would fail to satisfy the cover requirements (see Figure 3.9). A limiting value, P_4, which is more complicated to calculate, results from these restrictions. A detailed account of case 4 is presented in Section 5.2.4.

3.3.3 The effects of transverse load distribution

The majority of existing spine beam bridges are designed by considering that they are prestressed in the longitudinal direction and reinforced in the transverse direction. If prestress is provided in the transverse direction, the top slab is prestressed to improve the transverse flexural behaviour and the webs are prestressed vertically to improve the shear resistance.

In addition to stresses due to flexural moments, spine beam bridges are subjected to shear forces and torsional moments throughout the length of the beam. The shear forces are predominantly resisted by the webs of the spine beam. Spine beams with cells show good torsional behaviour due to the closed frame layout. For medium span bridges with little or no skew, shear and torsional stresses are of a secondary nature except at certain localities (*e.g.* shear stresses can be critical close to supports); hence, these effects are often taken into account by the provision of additional reinforcement. This view is supported by a number of practising designers (Low (1982), Lewis *et al.* (1983)).

The top flange serves an important role in addition to resisting the stresses due to longitudinal behaviour, by resisting the transverse flexural moments caused by traffic. The transverse behaviour is in fact the governing case which determines the shape and the thickness of the top flange (Swann (1972)).

The technique adopted for the design of the approach viaducts of the Orwell Bridge in Ipswich has been explained by Lewis *et al.* (1983), as consisting of four steps:

1. The idealisation of the bridge for global analysis and determination of longitudinal flexural stresses at each stage of construction.

2. Analysis of the spine beam for transverse bending, torsion and distortion, including local effects of wheel loads, followed by reinforcement of webs and flanges for shear, torsion and transverse flexure.

3. Determination of the ultimate strength in flexure and shear.

4. Analysis of the diaphragms over each pier.

A similar approach has been adopted for the proposed design techniques explained in Chapters 4 and 5.

3.3.4 The governing criteria for preliminary design

There are two limit states on which the design of prestressed concrete structures should primarily be based. They are:-

1. The serviceability limit state
2. The ultimate limit state.

According to Low (1983), for structures designed for zero tension under service loads (as is often done for highway bridges), the ultimate limit state seldom governs the design. Moreover, prestressing a tendon does not enhance its contribution to the ultimate carrying capacity significantly.

The prime purpose of prestress is to enhance the behaviour of the beam at the serviceability limit state. In particular, prestress is used to restrict cracking of the concrete to an acceptable level and the need to improve the resistance to fatigue loading. Hence, any rational procedure for prestressed concrete beam design should be developed with respect to the serviceability limit state.

Where possible, however, measures should be taken which ensure that subsequent ultimate strength checks can be satisfied.

3.4 Discussion

Prestressed concrete has been widely used in highway and railway bridge construction as well as in a number of other applications. One such application in bridge construction is the use of continuous spine beams.

The design of spine beams is not a strictly ordered activity which leads directly from specification to solution. It is characterised by the need to make tentative decisions based on incomplete data. Therefore, the conventional design techniques often force designers into a trial and error process, in which the amount of iteration depends on the quality of the preliminary solution.

Any design technique which supports the iterative development of designs does not fulfil the needs of versatile expert systems, which should be able to suggest good design solutions directly, while covering a wide number of possible designs. A brief study has been carried out into the current design techniques to investigate their suitability for the proposed expert system, which revealed a number of their inadequacies. A distinct feature is that these techniques have been developed for hand calculation and are not suitable for direct integration into an expert system. There are also a number of factors which have not been adequately addressed in the current design techniques; some relate to the structural behaviour of prestressed concrete, while others may be specific to a particular structure. Nine such factors have been earmarked for detailed study as the most likely causes for iteration. The aspects that have already been covered in the literature are also described briefly.

In order to overcome these inadequacies, the importance of developing alternative design techniques which minimise the need for design by repeated analysis, is emphasised; these techniques, in essence, should concentrate on producing preliminary designs which succeed at the subsequent detailed design stage. The following points can be the basis of the proposed design technique.

- It should support the incremental development of preliminary solutions.
- The preliminary solutions should take account quantitatively of as many constraints and structural behaviours as possible.
- The preliminary solutions should make adequate allowance for any constraints or structural behaviours that are too complicated for accurate quantitative determination at the preliminary design stage.
- The solutions generated should be comparable with those of an expert. This means that solutions should show characteristics like economy, constructability, serviceability and maintainability.

A detailed study has been undertaken into the design of prestressed concrete spine beams to unravel the design principles. As a result of this study, the following design techniques have been developed.

1. A design technique has been suggested for the selection of cross sectional dimensions of a statically determinate I-beam. An I-beam has

been selected because the same principles can be extended for rectangular beams and T-beams. The basis of the technique stems from the explicit identification of the governing criteria for each section dimension. The same principles have been extended to statically indeterminate beams to give the required cross sectional dimensions. This work is explained in detail in Chapter 4.

2. The technique suggested for the determination of cross sectional dimensions of a statically indeterminate beam needs the reactant moments to be assumed at the start of the design process. Therefore, it is necessary to find an actual cable profile, which satisfies the appropriate bounds and generates the assumed reactant moment distribution. Since the existing techniques tend to go through a number of iterative cycles before obtaining an acceptable cable profile, a technique has been suggested that is suitable for use as a computer tool. This technique consists of two steps:

   (a) Determination of all the possible bounds on the cable forces and selecting the appropriate values in each region.

   (b) Automated determination of an actual cable profile for the selected cable forces.

   This work is explained in Chapter 5.

3. The above techniques are presented within a framework of assumptions described in Chapter 4. Therefore, the implications of these assumptions, and the possible ways to take account of them in the proposed design techniques, are explained in Chapter 7.

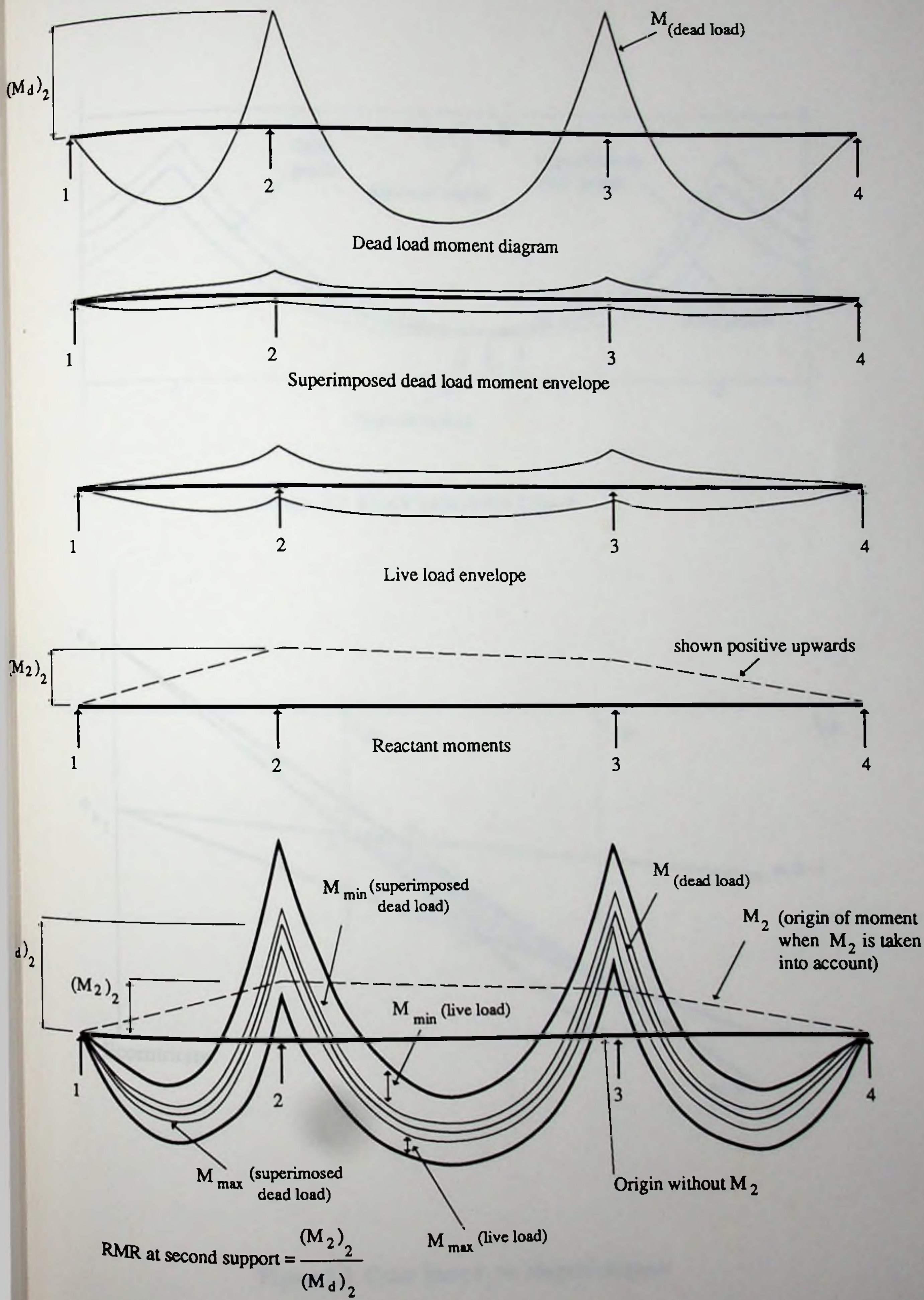

Figure 3.1: Components of the bending moment envelope

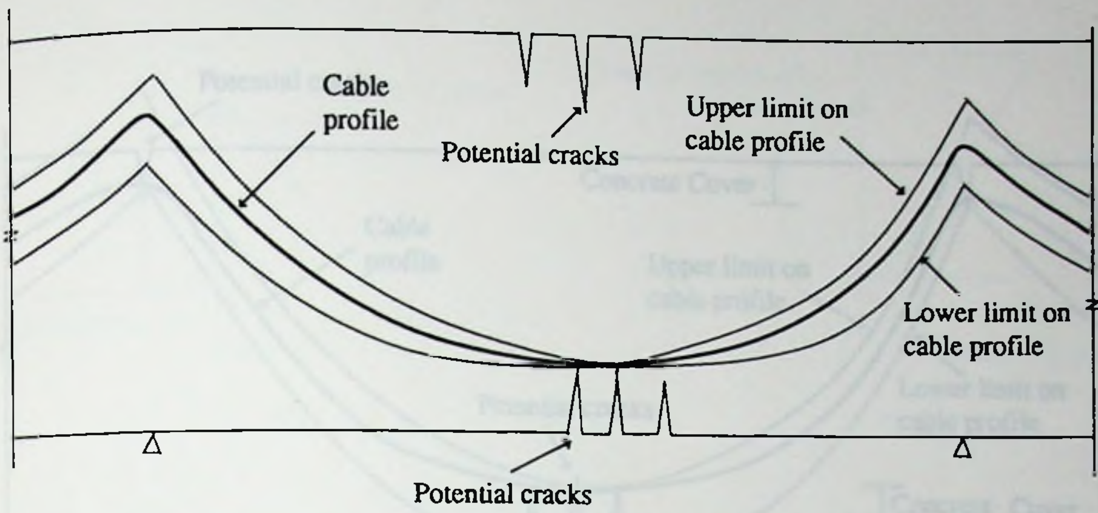

Figure 3.2: Crack pattern for Case 1

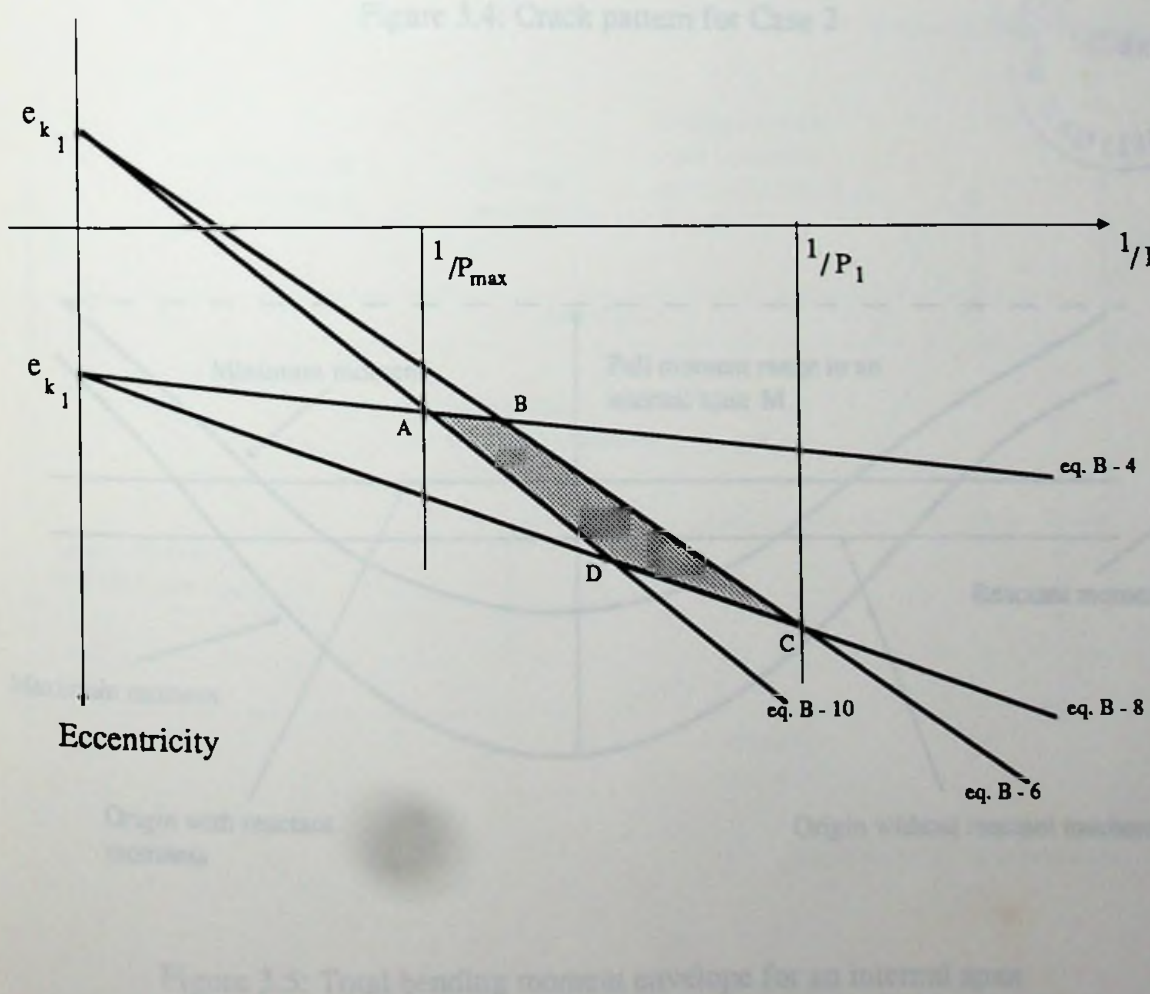

Figure 3.3: Cable force P_1 on Magnel diagram

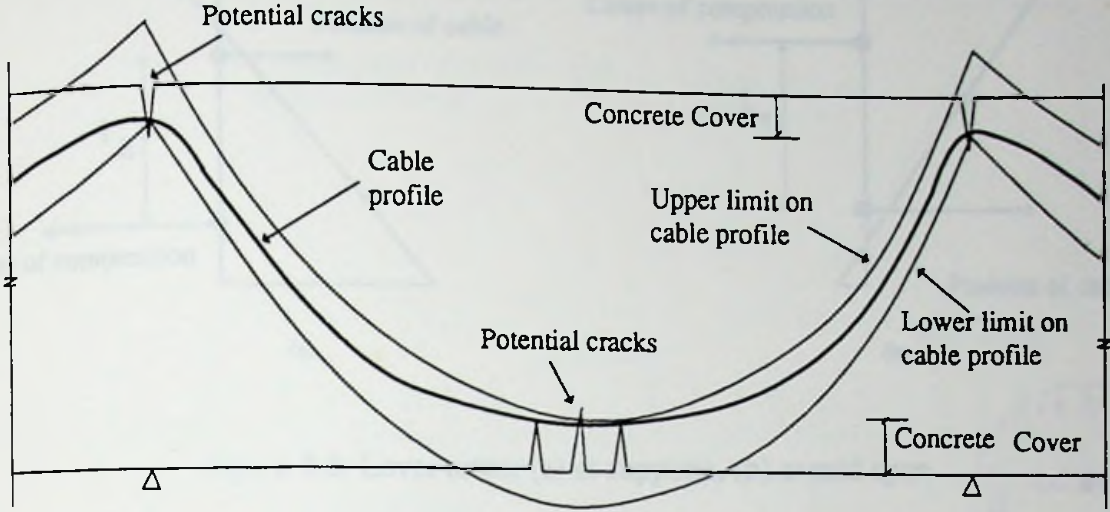

Figure 3.4: Crack pattern for Case 2

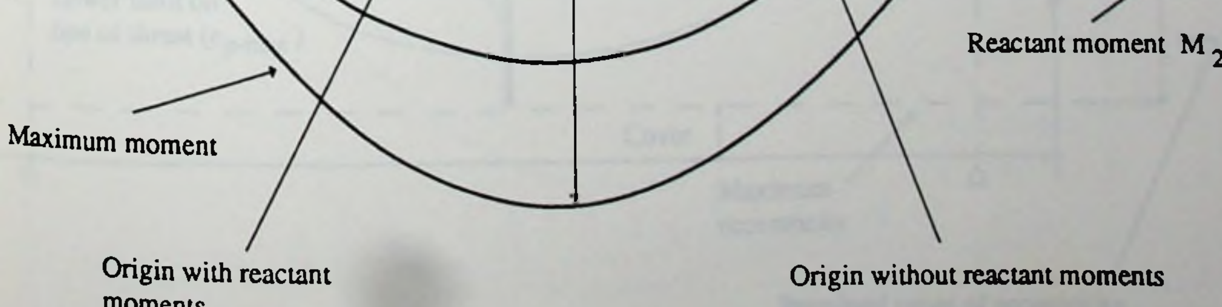

Figure 3.5: Total bending moment envelope for an internal span

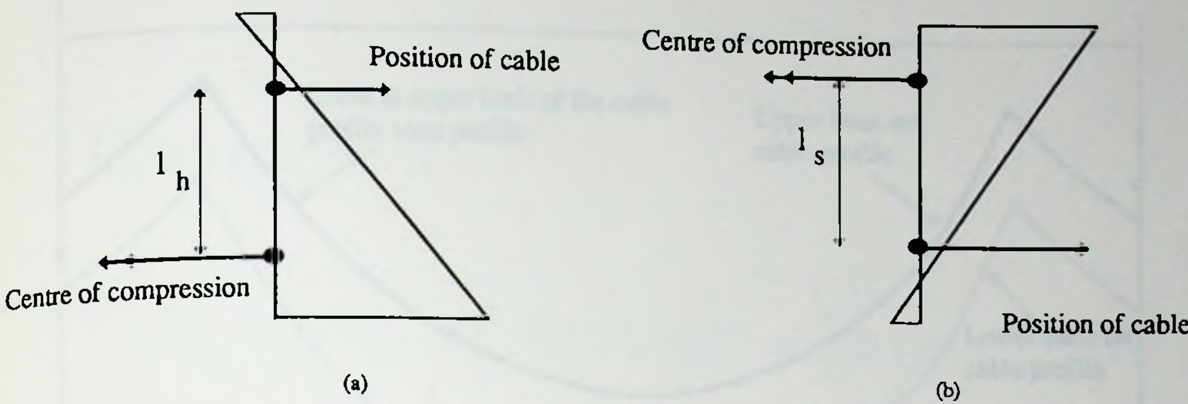

Figure 3.6: Lever arms: (a) at supports, (b) at mid span

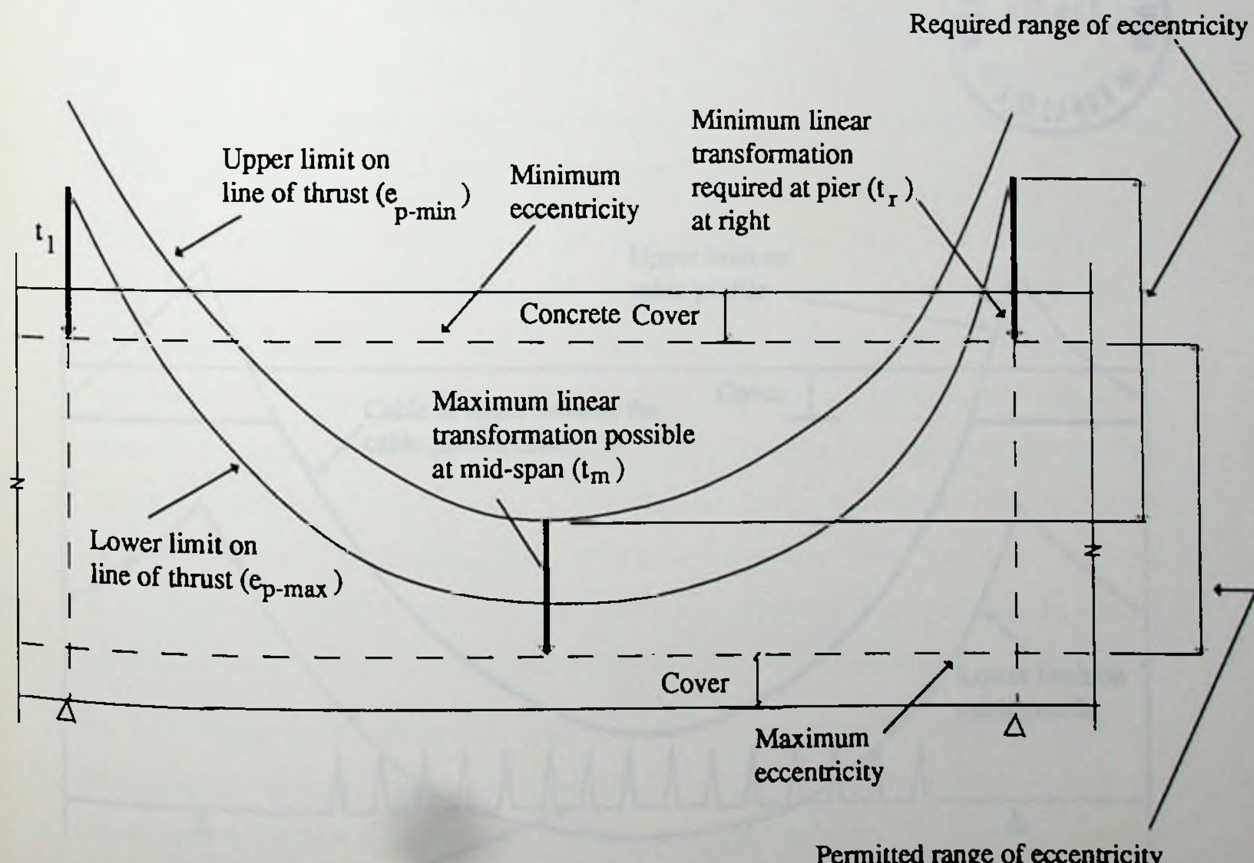

Figure 3.7: Bounds on cable profile

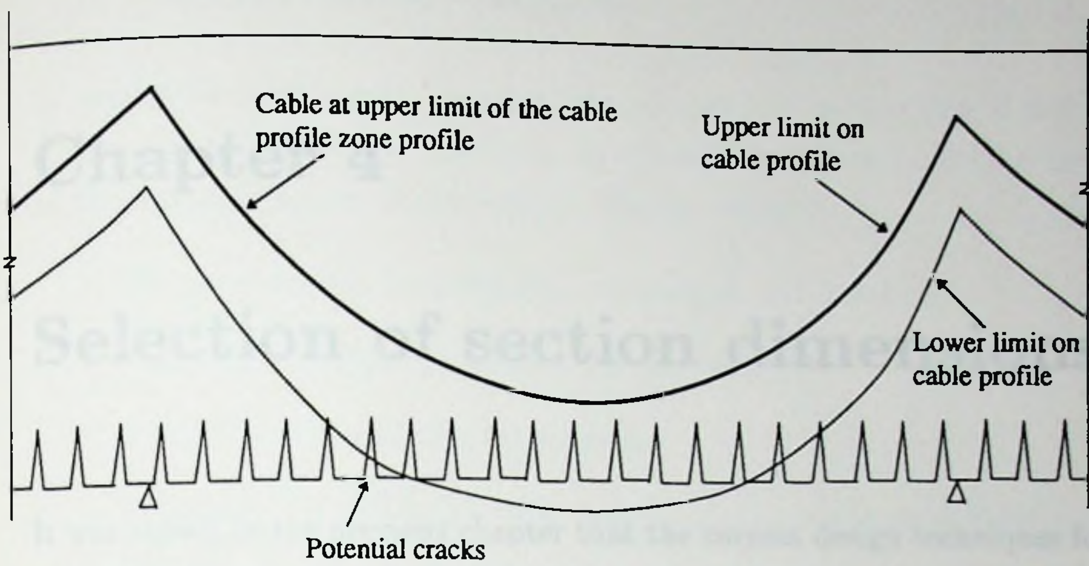

Figure 3.8: Crack pattern for Case 3

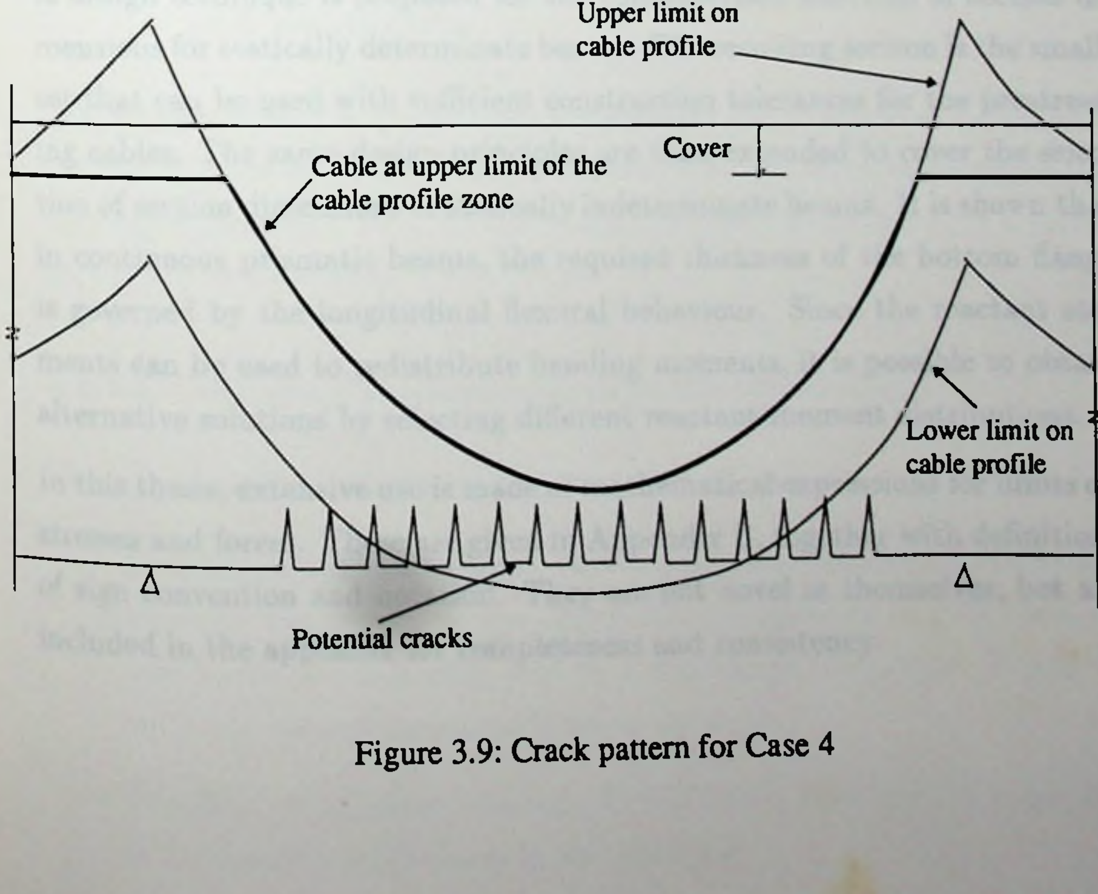

Figure 3.9: Crack pattern for Case 4

Chapter 4

Selection of section dimensions

It was shown in the previous chapter that the current design techniques for the selection of section dimensions are primarily developed as methods for hand calculation. They are not suitable for direct integration into expert systems to produce computer aids. It is argued here that in order to harness the numerical processing power of the computer to produce good preliminary solutions, design techniques should be developed which delve into the underlying fundamental design principles.

A design technique is proposed for the computerised selection of section dimensions for statically determinate beams. The resulting section is the smallest that can be used with sufficient construction tolerances for the prestressing cables. The same design principles are then extended to cover the selection of section dimensions of statically indeterminate beams. It is shown that in continuous prismatic beams, the required thickness of the bottom flange is governed by the longitudinal flexural behaviour. Since the reactant moments can be used to redistribute bending moments, it is possible to obtain alternative solutions by selecting different reactant moment distributions.

In this thesis, extensive use is made of mathematical expressions for limits on stresses and forces. These are given in Appendix B, together with definitions of sign convention and notation. They are not novel in themselves, but are included in the appendix for completeness and consistency.

4.1 Design criteria for section dimensions - Determinate beams

A number of criteria must be taken into account for the selection of section dimensions of statically determinate structures; some need prior knowledge of the moments acting on the section. The full list is:-

1. Minimum section thickness for construction and durability.
2. Provision of sufficient ultimate moment capacity.
3. Existence of a valid Magnel diagram.
4. Existence of a sufficiently large feasible region in the valid Magnel diagram.

Each of these will be described in the following subsections with reference to a typical I-section, which can include $T-$ and rectangular sections as special cases.

4.1.1 Minimum thickness required for construction and durability

The minimum section thicknesses required for construction and durability are two independent criteria. However, they are considered together because they can collectively influence the minimum thickness required for a certain part of the cross section.

4.1.1.1 Minimum thickness required for construction

The construction requirements determine the following.

- Provision of sufficient spacing between adjacent reinforcing bars or prestressing tendons for proper compaction of concrete (*e.g.* If there are two layers of reinforcing steel in a top flange, then there should be sufficient vertical clearance between the layers).
- Provision of sufficient clearance for vibrators, especially in webs; this is discussed in more detail in Section 4.3.4.

4.1.1.2 Minimum thickness required for durability

When prestressing tendons and reinforcement are embedded in concrete, they should be protected from environmental elements which cause corrosion. This is usually achieved by providing sufficient concrete cover; these cover requirements are rigorous for highway structures. Hence, the durability of the structure can impose limits on the minimum thicknesses of the structure.

The thickness of the web required is a good example of a section thickness governed by the conditions for construction and durability, as explained in Section 4.3.4. However, these conditions may not be the governing criteria for the thicknesses of top and bottom flanges. Nevertheless, minimum thickness for construction and durability can be used to specify the appropriate minimum thicknesses of top and bottom flanges.

In this thesis, the values of minimum thickness required for construction and durability are not determined for statically determinate beams because these are dependent on the construction techniques and the form of beam to a large extent. For the design example, nominal thicknesses of $0.15m$ are used for the top and bottom flanges, and $0.2m$ is used for webs.

4.1.2 Provision of sufficient ultimate capacity

The provision of sufficient ultimate moment carrying capacity for beams at the preliminary design stage has been recommended by Guyon (1972), Abeles & Bardhan-Roy (1981) and Lin & Burns (1981). The compressive flange area is selected so as to provide adequate compressive force carrying capacity under ultimate moments. According to Abeles & Bardhan-Roy (1981), provision of sufficient compression area is a governing criteria for the area of the compression flange, since it often overrides the requirements of the elastic conditions. There are two possible cases as described below:

1. Only the full area of compressive flange is effective in resisting the ultimate moment.
2. In addition to the full area of the compressive flange, part of the web also assists in resisting the ultimate moment.

The first case is straightforward. It is a simplification of the more rigorous second case, but should be applied only if the designer can safely ignore the contribution from the web; this is possible when the compression flange area is substantially larger than the associated web area.

The following mathematical equations can be derived for a member subject to sagging moments, by assuming a rectangular stress distribution for concrete at the ultimate state (see Figure 4.1).

If a load factor α is required, then the required ultimate moment capacity is αM_b.

The maximum lever arm available can be expressed as a proportion of the actual depth, thus can be written as βd

If the characteristic cube strength of concrete is f_{cu}, then the stress in the compression zone is taken as δf_{cu}.

Case 1: Ignoring the web

By taking moments about the centroid of the steel, the following expression can be obtained for the top flange area.

$$A_t = \frac{\alpha M_b}{\delta f_{cu} \beta d} \tag{4.1}$$

According to Lin & Burns (1981), the lever arm varies with the shape of the section and β is usually between 0.6 and 0.9.

Case 2: Considering the web

The area of the top flange required is estimated using Equation 4.1. If the designer wants a thin top flange, he can choose the position of the neutral axis in such a way that part of the web contributes to the compressive force, thus allowing a reduction in area of the top flange. A slight reduction in the initial estimate of lever arm may be required. This is a very approximate procedure, but it is not possible to be more accurate because of the difficulties in determining the position of the neutral axis, which depends on the amount of reinforcement and prestressing steel.

4.1.3 Existence of a Magnel diagram

The Magnel diagram is a graphical representation of the various conditions imposed on the eccentricity and the prestressing force; these conditions are imposed by allowable stress limits, section dimensions and the moments acting on the section. This graphical representation is named after the Belgian engineer, Gustave Magnel, who introduced the idea of using this graphical representation for design (Magnel (1954)). In order that legitimate force-eccentricity combinations for a concrete section can be found, a valid Magnel diagram must exist for the given bending moments.

4.1.3.1 Governing equations

If the minimum working load moment M_a and the maximum working moment M_b are considered as the governing conditions, then there are four governing equations. If the moment at transfer is M_t then another four governing equations can be derived (Burgoyne (1988a)).

These conditions can be rearranged to give conditions on the permissible eccentricities, as a function of the applied moments, the section properties and the reciprocal of the prestressing force. These equations are given in appendix B, and two of them are considered here.

$$e \geq -\frac{Z_2}{A_c} - \frac{Z_2 f_{tw}}{R P_t} + \frac{M_b}{R P_t} \tag{4.2}$$

$$e \leq -\frac{Z_2}{A_c} - \frac{Z_2 f_{cw}}{R P_t} + \frac{M_a}{R P_t} \tag{4.3}$$

4.1.3.2 Conditions for valid Magnel diagram

The Expressions 4.2 and 4.3 can be rearranged in the following form.

$$e \geq -\frac{Z_2}{A_c} + \frac{(M_b - Z_2 f_{tw})}{R} \frac{1}{P_t} \tag{4.4}$$

$$e \leq -\frac{Z_2}{A_c} + \frac{(M_a - Z_2 f_{cw})}{R} \frac{1}{P_t} \tag{4.5}$$

On the Magnel diagram, which is drawn as the eccentricity versus the reciprocal of the prestressing force, these expressions represent straight lines; they intersect on the eccentricity axis at $-Z_2/A_c$. This intercept, $-Z_2/A_c$, represents one of the *kern points* of the cross section, which is denoted as e_{k1}. If a resultant force acts within the kern points of a section, there will be no tensile stresses in the section.

If a legitimate force-eccentricity combination exists between these two lines, then the following requirement should be satisfied by the gradients of the lines:

$$Gradient\ of\ line\ 4.4 \leq Gradient\ of\ line\ 4.5 \tag{4.6}$$

The following condition can thus be derived.

$$Z_2 \geq \frac{M_b - M_a}{f_{tw} - f_{cw}} \tag{4.7}$$

By considering similar criteria, five other inequalities can be derived for the section moduli Z_1 and Z_2 (these are given in Appendix B); these inequalities will form the limits on the section moduli Z_1 and Z_2. If any of these limits are violated, there will be no valid Magnel diagram. Hence any cross section selected for resisting the applied moments should satisfy all six expressions.

4.1.4 Existence of a sufficient feasible region

In Section 4.1.3, the conditions for the existence of a feasible region in a Magnel diagram were considered. However, this region may lie outside the practical limits imposed by the eccentricity conditions, as illustrated in Figure 4.2. Thus additional criteria are needed to ensure the existence of a *sufficient feasible region* within the practical limits.

If the depth of the section has already been selected, the practical limits on the eccentricity are dependent only on the concrete cover required for prestressing steel at top and bottom fibres of the section. These limits on eccentricity are called e_{max} and e_{min}.

The condition on the sufficient feasible region is explained for *dominantly sagging* bending moments, in which case only e_{max} will impose any limit on the feasible region. Figure 4.3 shows the strict condition for the existence of a sufficient feasible region. This figure is drawn for a beam where $e_{k_2} \leq e_{max}$ (which is normal). The strict condition is $e_A \leq emax$. However, for a variety of reasons, a more restrictive condition, $e_B \leq emax$, is preferred.

These reasons are the following:

1. When $e_A = e_{max}$ the feasible region is reduced to a point. Hence, it is not possible to provide any construction tolerances.

2. As can be seen on Figure 4.3, when $e_A < e_{max}$, the minimum cable force is governed by Equation B.4. Since, the gradient of this line is low, it is possible to achieve a much lower prestressing force if e_B also lies within the Magnel diagram, with only a small penalty in section dimensions. Therefore, increasing the bottom flange thickness so that $e_B \leq e_{max}$ is a preferred criterion, on the grounds of economy. However, any further increase in the bottom flange thickness will not have the same advantage, because then the minimum cable force is governed by Equation B.6 which has a steep gradient.

3. If $e_B \leq e_{max}$ is satisfied, then there will be a sufficient feasible region, which is very useful in practice to provide construction tolerances. This preferred Magnel diagram is shown in Figure 4.4.

If $e_{k_2} > e_{max}$, then the preferred criterion is $e_A \leq e_{max}$. The resulting Magnel diagram is similar to the one shown in Figure 4.5.

If the moments acting on the section are *dominantly hogging*, then there will be a different criterion for the existence of a Magnel diagram. The shape of the resulting Magnel diagram is shown in Figure 4.6. As with the sagging case, the strict condition $e_A \geq e_{min}$ is replaced by the more restrictive condition $e_D \geq e_{min}$.

4.1.5 Discussion of the design criteria

It has been shown that there are four governing criteria for the selection of the section dimensions of a statically determinate beam. Of these, two are based on the Magnel diagram. They are the conditions for the existence of a Magnel diagram itself (expressed as limits on Z), and the existence of a sufficient feasible region in it (expressed as a limit on eccentricity). Therefore, any Magnel diagram which satisfies these criteria is called a *suitable Magnel diagram.* The mathematical expressions for the existence of a suitable Magnel diagram are derived on the basis of working load moments M_a and M_b. A similar set of expressions could be derived for the transfer moments M_t. They are not discussed in detail because they will produce a large number of equations that will mask the explanations.

4.2 Automated determination of section dimensions - Statically determinate beams

In order to automate the design process for the selection of the required section dimensions, it is necessary to mathematically express the criteria for the existence of a Magnel diagram and the existence of a sufficient feasible region. The section moduli must be expressed as functions of flange and web areas, which are shown for an idealised section in Figure 4.7.

4.2.1 Section moduli as a function of flange and web areas

With reference to Figure 4.7, the total area, A_c, is the summation of the areas of the individual parts, namely areas of the top-flange, the bottom-flange and the web.

$$A_t = (w_t - t_w)t_t \tag{4.8}$$

$$A_w = t_w d \tag{4.9}$$

$$A_b = (w_b - t_w)t_b \tag{4.10}$$

$$A_c = A_t + A_w + A_b \tag{4.11}$$

If the second moment of area of the flanges about their own centroids is ignored, then a simplified expression for the overall second moment of area (I) is given by

$$I = A_t e_1^2 + A_w(d/2 - c_2)^2 + \frac{1}{12}A_w d^2 + A_b e_2^2 \tag{4.12}$$

where e_1 is the distance to the centroid of the idealised top flange from the centroid of the section and e_2 is the distance to the centroid of the idealised bottom flange from the centroid of the section.

This can be rearranged to:-

$$I = A_t e_1^2 + A_w e_3^2 + A_b e_2^2 \tag{4.13}$$

$$\text{where} \quad e_3^2 = (d/2 - c_2)^2 + \frac{1}{12}d^2$$

If non-dimensional parameters γ_i are introduced such that

$e_1 = \gamma_1 d$
$e_2 = \gamma_2 d$
$e_3 = \gamma_3 d$
$c_1 = \gamma_4 d$
$c_2 = \gamma_5 d$
and
$\alpha_1 = \frac{\gamma_1^2}{\gamma_5}$ $\qquad \delta_1 = \frac{\gamma_1^2}{\gamma_4}$

$\alpha_2 = \frac{\gamma_2^2}{\gamma_5}$ $\qquad \delta_2 = \frac{\gamma_2^2}{\gamma_4}$

$\alpha_3 = \frac{\gamma_3^2}{\gamma_5}$ $\qquad \delta_3 = \frac{\gamma_3^2}{\gamma_4}$

Then the section moduli Z_1 and Z_2 are given by:

$$Z_1 = A_t\delta_1 d + A_w\delta_3 d + A_b\delta_2 d \tag{4.14}$$

$$Z_2 = A_t\alpha_1 d + A_w\alpha_3 d + A_b\alpha_2 d \tag{4.15}$$

4.2.2 Expressions for the existence of a Magnel diagram

Equation 4.15 can be substituted into Expression 4.7, to give,

$$A_t\alpha_1 d + A_w\alpha_3 d + A_b\alpha_2 d \geq \frac{M_b - M_a}{f_{tw} - f_{cw}} \tag{4.16}$$

The equivalent relationship for the top flange (Expression B.13 of Appendix B) gives,

$$A_t\delta_1 d + A_w\delta_3 d + A_b\delta_2 d \leq \frac{M_b - M_a}{f_{cw} - f_{tw}} \tag{4.17}$$

These expressions will be used later to derive the required area of the bottom flange, A_b, once the other areas are known.

4.2.3 Expressions for the existence of sufficient feasible region

As described in Section 4.1.4, there are different criteria for the existence of a sufficient feasible region, depending on the sign (sagging or hogging) of the dominant moment. The aim is to use these different criteria to obtain expressions for A_b in terms of A_t and A_w.

4.2.3.1 Sections with dominantly sagging moments

In these expressions, c represents the minimum cover required for the prestressing cable expressed as a ratio of the actual cover to overall depth.

$$c = \frac{minimum\ cover}{overall\ depth} \tag{4.18}$$

Therefore the following bounds on eccentricity can be derived:

$$e_{max} = c_2 - c\,d \tag{4.19}$$

$$e_{min} = c_1 + c\,d \tag{4.20}$$

Case 1: Expressions when $e_A \leq e_{max}$

A is the point of intersection of the two lines on the Magnel diagram, represented by Equations B.4 and B.10 on the Magnel diagram of Figure 4.3 and corresponds to the maximum prestressing force. From these equations, e_A can be expressed as:

$$e_A = \frac{(M_b Z_2 - M_a Z_1)}{A_c[(M_a - M_b) + f_{cw}(Z_1 - Z_2)]} \tag{4.21}$$

By considering the criterion, $e_A \leq e_{max}$ and substituting for Z_1 and Z_2 from Equations 4.14 and 4.15:

$$\begin{aligned} M_b(A_t\alpha_1 d + A_w\alpha_3 d + A_b\alpha_2 d) - M_a(A_t\delta_1 d + A_w\delta_3 d + A_b\delta_2 d) \\ \leq e_{max}[A_t + A_w + A_b]\{(M_a - M_b) + f_{cw}[(\delta_1 - \alpha_1)A_t + \\ (\delta_3 - \alpha_3)A_w + (\delta_2 - \alpha_2)A_b]d\} \end{aligned} \tag{4.22}$$

By defining:

$$\begin{aligned} W_1 &= M_b(A_t\alpha_1 + A_w\alpha_3)d && (4.23)\\ W_2 &= M_b\alpha_2 d && (4.24)\\ W_3 &= M_a(A_t\delta_1 + A_w\delta_3)d && (4.25)\\ W_4 &= M_b\delta_2 d && (4.26)\\ W_5 &= A_t + A_w && (4.27)\\ W_6 &= (M_a - M_b) + f_{cw}[(\delta_1 - \alpha_1)A_t + (\delta_3 - \alpha_3)A_w]d && (4.28)\\ W_7 &= f_{cw}(\delta_2 - \alpha_2)d && (4.29) \end{aligned}$$

Expression 4.22 can be rearranged to give a quadratic in A_b:

$$\begin{aligned} e_{max}W_5W_7{A_b}^2 + [e_{max}(W_5W_7 + W_6) - (W_2 - W_4)]A_b \\ + e_{max}W_5W_6 - W_1 + W_3 \geq 0 \end{aligned} \tag{4.30}$$

If the top flange and web areas are known, then the required area of the bottom flange can be determined.

This second order expression, which is of the form $f(x) = ax^2 + bx + c \geq 0$ has two roots. Since $W_5 > 0$, $W_7 > 0$ and $e_{max} > 0$, the shape of the function is as shown in Figure 4.8. Hence, it could either have one positive root or two positive roots. The highest positive root of the quadratic gives the required area of the bottom flange. If the roots are negative or imaginary, then this criterion does not govern the section dimensions.

Case 2: Expressions when $e_B \leq e_{max}$

B (the preferred case) is the point of intersection of the lines represented by equations B.4 and B.6 of the Magnel diagram of Figure 4.4; e_B can be represented in the following form.

$$e_B = \frac{M_b(Z_2 - Z_1) - Z_1 Z_2 (f_{cw} - f_{tw})}{A_c(Z_1 f_{cw} - Z_2 f_{tw})} \tag{4.31}$$

By considering the criterion, $e_B \leq e_{max}$ and following a similar procedure to Case 1 gives

$$\begin{aligned}(e_{max}W_9 + W_4W_6)A_b^2 + [e_{max}(W_7W_9 + W_8) + (W_3W_6 - W_4W_5) - \\ W_2]A_b + e_{max}W_7W_8 - W_1 + W_3W_5 \geq 0\end{aligned} \tag{4.32}$$

where

$$W_1 = M_b[(\alpha_1 - \delta_1)A_t + (\alpha_3 - \delta_3)A_w] \tag{4.33}$$
$$W_2 = M_b(\alpha_2 - \delta_2) \tag{4.34}$$
$$W_3 = (\delta_1 A_t + \delta_3 A_w)(f_{cw} - f_{tw})d \tag{4.35}$$
$$W_4 = \delta_2(f_{cw} - f_{tw})d \tag{4.36}$$
$$W_5 = \alpha_1 A_t + \alpha_3 A_w \tag{4.37}$$
$$W_6 = \alpha_2 \tag{4.38}$$
$$W_7 = A_t + A_w \tag{4.39}$$
$$W_8 = (\delta_1 f_{cw} - \alpha_1 f_{tw})A_t + (\delta_3 f_{cw} - \alpha_3 f_{tw})A_w \tag{4.40}$$
$$W_9 = \delta_2 f_{cw} - \alpha_2 f_{tw} \tag{4.41}$$

The higher positive root of this second order expression is the required value of A_b.

4.2.3.2 Expressions with dominantly hogging moments

In a similar way to the dominantly sagging moment case, there are two points on the Magnel diagram which determine the governing criteria depending on the cover required for the prestressing cables. They are the points A and D shown on the Magnel diagram of Figure 4.6. Two sets of expressions can be derived based on these two points.

Case 1: Expressions when $e_A \geq e_{min}$

This expression can be deduced directly from Expression 4.30 by substituting e_{min} for e_{max} and reversing the inequality.

$$\begin{aligned} e_{min} W_5 W_7 {A_b}^2 + [e_{min}(W_5 W_7 + W_6) - (W_2 - W_4)]A_b \\ + e_{min} W_5 W_6 - W_1 + W_3 \leq 0 \end{aligned} \tag{4.42}$$

The higher positive root of this second order expression is the required magnitude of A_b.

Case 2: Expressions when $e_D \geq e_{min}$

D (the preferred case) is the point of intersection of the lines represented by Equations B.8 and B.10. Therefore e_D can be represented in the following form.

$$e_D = \frac{M_a(Z_2 - Z_1) + Z_1 Z_2 (f_{cw} - f_{tw})}{A_c(Z_1 f_{tw} - Z_2 f_{cw})} \tag{4.43}$$

By considering the criterion, $e_D \geq e_{min}$ and following a procedure similar to Case 1 with dominantly sagging moments:

$$\begin{aligned} (e_{min} W_9 - W_4 W_6){A_b}^2 + [e_{min}(W_7 W_9 + W_8) - (W_3 W_6 + W_4 W_5 \\ + W_2)]A_b + e_{min} W_7 W_8 - W_1 - W_3 W_5 \leq 0 \end{aligned} \tag{4.44}$$

where

$$W_1 = M_a[(\alpha_1 - \delta_1)A_t + (\alpha_3 - \delta_3)A_w] \tag{4.45}$$

$$W_2 = M_a(\alpha_2 - \delta_2) \quad (4.46)$$

$$W_3 = (\delta_1 A_t + \delta_3 A_w)(f_{cw} - f_{tw})d \quad (4.47)$$

$$W_4 = \delta_2(f_{cw} - f_{tw})d \quad (4.48)$$

$$W_5 = \alpha_1 A_t + \alpha_3 A_w \quad (4.49)$$

$$W_6 = \alpha_2 \quad (4.50)$$

$$W_7 = A_t + A_w \quad (4.51)$$

$$W_8 = (\delta_1 f_{tw} - \alpha_1 f_{cw})A_t + (\delta_3 f_{tw} - \alpha_3 f_{cw})A_w \quad (4.52)$$

$$W_9 = \delta_2 f_{tw} - \alpha_2 f_{cw} \quad (4.53)$$

The higher positive root of this second order expression is the required magnitude of A_b.

All these expressions can be recast in terms of A_t, the area of the top flange, by a similar process.

4.2.4 Automated Selection of section dimensions

In this section an algorithm which can automate the determination of section dimensions for a statically determinate structure is presented. This algorithm is based on the criteria presented in Section 4.1 and the mathematical expressions derived in Section 4.2. The algorithm is developed for a typical I-section with a large top flange width for supporting live-loads and hence subjected to predominantly sagging moments. For a different cross section, different algorithms may be needed, but could be based on the same principles as those presented here.

All the expressions derived for the various criteria for selection of section dimensions have a common feature; they all assume that the moments acting on the cross section are known. However, the dead load moment acting on the section is unknown until the section dimensions are selected. Hence, it is impossible to automate the selection of section dimensions without going through an iterative loop; thus an efficient algorithm is required, which converges rapidly to an acceptable solution.

An over-riding criterion for any prestressed concrete beam is that it should be constructable and durable. Hence, the minimum section thicknesses imposed by construction and durability of the structure must always be satisfied.

These dimensions provide a good starting point for an iterative procedure.

The aim of this automated procedure is to find the top- and bottom-flange areas required at critical sections. If the widths of these are known, then the required thicknesses can be found.

The following conditions are assumed in this process;

- the section can be idealised as shown in Figure 4.7,
- the widths of the top- and bottom-flanges, the thickness of the web and the depth of the section are known.

The following iterative design algorithm is adopted.

1. Suitable dimensions for the widths of the top- and bottom-flanges and the depth of the section are specified.

2. The thicknesses of web, top- and bottom-flanges are determined so as to satisfy the minimum requirements for construction and durability. This allows the dead weight to be calculated from which the moment at transfer (M_t), moment at minimum working load (M_a), and the moment at maximum working load (M_b) can be calculated for the starting point of the iteration.

3. The bottom flange area required to satisfy the criteria for the existence of a Magnel diagram and for a sufficient feasible region are found using expressions presented in Sections 4.2.2 and 4.2.3. Additional expressions can be included to take account of the transfer conditions. In this case the values α_i and δ_i, which are used to determine the section moduli, can be obtained based on the current estimate of the section dimensions.

4. The top flange area required to resist ultimate state moments is found using Equation 4.1. The appropriate ultimate state moment can be obtained from the service state moments.

5. Adopt the new sections. If the new section dimensions are significantly different from previous iteration, then calculate the new moments and start with 2.

The section selected in this way is based on the criteria which usually govern the section dimensions of a determinate beam. However, it is essential to check that the section satisfies all the possible criteria (*i.e.* the actual section moduli must be sufficient for both top and bottom fibres).

The resulting section will be the smallest possible which has a sufficient feasible region in the Magnel diagram, a sufficient ultimate strength and is constructable for a given section depth and live loads. It is, however, not possible to say whether the resulting section is the most economical, because this depends on the relative costs of prestressing steel and concrete.

4.2.5 Design example

These examples are obtained using a computer program written for the design algorithm presented above in Section 4.2.4. The following points should be noted.

1. The beam considered is an I-beam spanning $30m$ with top and bottom flanges of equal width. The width of each flange is $1.0m$ and the thickness of the web is $0.2m$.

2. The imposed load acting on the section is obtained from the BS 5400/2. It is considered that the beam is used for a bridge carrying carriageways of width $2.5m$ and hence the imposed load on each beam is $12.0kN/m$. A superimposed dead load of $3.0kN/m$ is assumed.

3. The concrete used is of Grade 50, so the permissible compressive stress at working loads is $-20.0N/mm^2$, the permissible compressive stress at transfer is $-18.0N/mm^2$, the permissible tensile stress at working loads is zero and the permissible tensile stress at transfer is $2.54N/mm^2$.

4. The load factors at service are 1.0 for the dead load, 1.2 for the superimposed dead load, and 1.2 for the imposed load. The load factors at the ultimate loads are 1.15 for dead load, 1.75 for the superimposed dead load, and 1.5 for the imposed load.

5. The cover needed at the centroid of the prestressing cables is $0.2m$.

6. At transfer conditions, the self weight is assumed to be effective.

7. The design is started with the minimum thicknesses for the top and the bottom flanges. The minimum thickness for each flange for construction and durability is $0.15m$ (a nominal value). Thus the minimum top and bottom bottom flange areas are equal to $0.12m^2$.

Span/depth ratios varying from 20-25 are used to get solutions for the required top and bottom flange areas which are defined for an idealised I-beam in Section 4.2.1. These results are given in Table 4.1. The required value of top flange is calculated using the ultimate state requirements. Three values are calculated for the bottom flange area. $(A_b) - 1$ is the bottom flange area required for the existence of a Magnel diagram for working load moments (M_a and M_b), $(A_b) - 2$ is the bottom flange area required for a sufficient feasible region in the Magnel diagram at working load moments, and $(A_b) - 3$ is the bottom flange area required for the existence of the Magnel diagram by considering transfer and maximum working load moments (M_t and M_b).

The table shows that negative areas are required for $(A_b) - 1$ and $(A_b) - 2$ for a number of span/depth ratios. This means that the conditions for a sufficient Magnel diagram do not govern the bottom flange area for these simply supported beams.

The solutions presented in Table 4.1 are reached iteratively. The values of α_i and δ_i at the start and end of iteration for span/depth of 20 is presented in the Table 4.2.

It is clear, even from this simple example, that a penalty is paid for making the section too shallow. As the overall depth decreases, the total area of concrete increases, which will require extra prestress, and the available eccentricity will decrease, so an even larger prestress will be needed to resist the loads.

span/depth ratio	20	22	23	24	25
depth of the section (m)	1.5	1.36	1.30	1.25	1.2
M_a at start (kNm)	1838.7	1764.4	1734.6	1705.9	1679.4
M_a at end (kNm)	1900.3	1877.1	1908.6	1986.2	2080.4
M_b at start (kNm)	3458.7	3384.4	3354.6	3325.9	3299.4
M_b at end (kNm)	3520.4	3497.1	3528.6	3606.2	3700.4
M_t at start (kNm)	1433.7	1359.4	1329.6	1300.9	1274.4
M_t at end (kNm)	1495.3	1472.1	1503.6	1581.2	1675.4
M_u at start (kNm)	4264.4	4178.8	4144.6	4111.7	4081.2
M_u at end (kNm)	4335.3	4308.5	4344.7	4433.9	4542.3
Required $A_t(m^2)$	0.143	0.163	0.174	0.190	0.209
Required $(A_b) - 1\,(m^2)$	-0.127	-0.09	-0.087	-0.096	-0.108
Required $(A_b) - 2\,(m^2)$	-0.06	-0.021	0.003	0.041	0.155
Required $(A_b) - 3\,(m^2)$	0.057	0.109	0.132	0.155	0.187
Adopted $t_t(m)$	0.179	0.204	0.218	0.238	0.262
Adopted $t_b(m)$	0.150	0.150	0.163	0.194	0.234
Number of iterations	1	1	3	3	2

Table 4.1: The selection of top and bottom flange thicknesses for a statically determinate beam

	α_1	α_2	α_3	δ_1	δ_2	δ_3
At the start	0.3915	0.3915	0.1667	-0.3915	-0.3915	-0.1667
At the end	0.2846	0.3336	0.1635	-0.2963	-0.3473	-0.170

Table 4.2: The values of α_i and δ_i at the start and the end of the iterative cycle for span/depth of 20

4.3 Selection of section dimensions for statically indeterminate beams

It is clear from the summary on the current design techniques (see Section 3.2.5.2) that the existing design techniques do not suggest any rational procedures for the determination of section dimensions of statically indeterminate beams. Thus a new technique is developed based on the principles applied to statically determinate beams.

Since it is difficult to generalise a technique for application to any type of continuous beam, attention is focused on prismatic spine beams with a single box. However, based on the same principles, it would be possible to develop similar techniques for other types of structures such as multi-cell spine beams, or double-T beams.

The cross section required for a spine beam for highway construction depends on a number of considerations such as the span, the traffic volume, the width and number of traffic lanes, position of expansion joints, fixity at supports and construction technique. In essence, the cross section selected for the spine beam should fulfil the following requirements:

- It should have a top flange wide enough to accommodate the number of traffic lanes, foot and/or cycle paths, central reserves and crash barriers,
- It should have a bottom flange if it is necessary to support services,
- It should demonstrate a satisfactory behaviour under serviceability conditions such as flexure, torsion, shear and deflection, and
- It should be constructable, maintainable and durable.

The assumptions on design process

In order to simplify the explanation of the technique, a number of assumptions have been made. Chapter 7 explains qualitatively how the design process would be modified if these assumptions are not made.

1. The flexural behaviour of continuous spine beams without skew can

adequately be represented as a continuous beam without considering transverse distribution effects.

2. The structure is constructed instantly and monolithically. Therefore the structure is free of trapped moments [1].
3. The effect of construction sequence on reactant moments is ignored.
4. Moment redistribution due to long term creep deformation is ignored.
5. The effects of temperature are ignored.
6. The effects of shear lag are ignored.
7. The selected spine beam structure is a prismatic single cell box girder.
8. At the preliminary design stage, the cross section dimensions are based on the average thicknesses ignoring the variations due to chamfers.

Since the cross section consists of components such as top- and bottom-flanges, and webs, the structural behaviour which governs the sectional dimensions of each of these components should be identified explicitly. The following discussion concentrates on this with respect to a single cell box girder as shown in Figure 4.9.

4.3.1 Selection of top flange width

This depends on the number of traffic lanes, the width of each traffic lane, the number of foot and/or cycle paths and their widths, the width of central reserves, if any, and the clearances for crash barriers. These requirements are generally provided in the client's brief and thus are known at the beginning of the design. Hence, the selection of top flange width neither poses any difficulty to the design engineer nor gives any freedom.

4.3.2 Selection of top flange thickness

As described by Swann (1972), the top flange thickness is seldom varied in the longitudinal direction. In almost all the cross sections covered in his survey,

[1] see Section 7.3

the top flange is shaped so as to imply that the transverse flexural behaviour due to self-weight, imposed load and cross-sectional distortion dominates the design.

Another significant observation is that the area of the top flange is almost always sufficiently large that it is not critical for longitudinal bending. This is due to the great width required to accommodate the carriageway and the minimum thickness required to resist punching shear. Accordingly, the following points can be collated about the top flange.

1. The minimum thickness of the top flange is governed by the punching shear requirements to resist concentrated wheel loads. This thickness is dependent on the perimeter used to calculate the actual punching shear stress, which varies for different codes of practice such as BS 5400/4 (1984) and ACI 318 (1983).

2. The actual thickness is generally governed by transverse flexural behaviour. If the longitudinal flexural behaviour becomes predominant, then the thickness can conveniently be adjusted to suit the longitudinal behaviour (see Section 4.3.6.1).

3. The thickness in the transverse direction depends on the following factors:

    - the type of the slab in the transverse direction, *i.e.* whether it is transversely reinforced or prestressed, and

    - the spacing of the webs, which act as supports for the top slab.

4. The top flange often has two cantilevers which may accommodate the foot or cycle paths and part of the carriageway. The type of slab in the transverse direction determines the maximum overhang of the cantilever.

5. The top flange is shaped to suit the transverse flexural behaviour, but can be approximated to a rectangle to determine the longitudinal flexural behaviour at the preliminary design stage.

A detailed study of the transverse flexural behaviour to determine the optimum shapes with the minimum cross sectional area would be rewarding because of the differences of loading types and magnitudes applicable in various countries. However, this is beyond the scope of the work covered in this thesis. For the present study, the average thickness of the top flange for the preliminary design is based on experience.

The following thicknesses, t_t (Figure 4.10), can be drawn from the survey by Swann (1972), for the top flange as a function of W_s, the clear spacing between webs. These are used for the preliminary design stage in the present study.

Clear Spacing $W_s(m)$	t_t - transversely reinforced (m)	t_t - transversely prestressed (m)
up to 3.00	0.250	0.200
3.00 - 4.50	0.300	0.225
4.50 - 6.00	0.350	0.275
6.00 - 7.50	-	0.300

Table 4.3: Required minimum thicknesses of the top flange

The selection of W_s is a trade-off process. According to Mathivat (1983), the number of cells which make up the cross section should be minimised since:-

- each cell needs additional formwork which may be cumbersome during construction, and
- a reduction of the number of cells is followed by a reduction in number of webs which consequently reduces the self-weight.

Conversely, a reduction of the number of cells increases the transverse span of the top flange which results in an increased thickness. Nevertheless, the preference is for reducing the number of cells considering the ease of construction and detailing.

The allowable cantilever overhang, which depends on the type of top flange, should be restricted to limit the local moment transferred to the junction of the web and the top flange. The following guide lines have been used, which are based on the survey by Swann (1972).

- If the top flange is reinforced in the transverse direction, then the maximum overhang is $3.0m$.
- If the top flange is prestressed in the transverse direction, then the maximum overhang is $4.0m$.

4.3.3 Selection of the overall depth of the section

The overall depth of the section is often specified in the client's brief since modern highway bridges need to cater for the road alignment while providing adequate clearances for navigation. If it is not given, then the designer is free to select a reasonable value for the span/depth ratio. This generally varies between 14 and 25.

Gee (1987) states that it is economical to use larger depths and a reduced prestress if allowed. Bigger depths allow a reduction of the bottom flange area, but carry the penalty of increased weight due to increased depth of web, which may outweigh the benefit from improved flexural behaviour. The best option is to generate a few alternative designs with different overall depths, which cover the possible range, so that the cost of the structure could be a criterion for selecting the best depth.

4.3.4 Selection of web thickness

Podolny & Muller (1982) give three conditions that should be used to determine the thickness of the web for spine beams:

1. Shear stresses due to shear forces and torsional moments should be kept within allowable limits.
2. Concrete must be properly placed, particularly when draped cables are present in the web.
3. Anchors for new or terminating cables, when located in the web, should properly distribute the high prestress load concentrated at anchorages.

By considering the type of prestressing ducts available in the web and the minimum clearances necessary for proper concreting, minimum thicknesses have been suggested by Podolny & Muller (1982).

Type of ducts	web thickness(m)
no prestressing ducts in webs	0.200
small ducts for vertical prestressing	0.250
ducts for the prestressing cables	0.300
anchors for the prestressing cables	0.350

Table 4.4: Required web thickness depending on the type of ducts

As described by Gee (1987), for cast *in situ* webs up to $4.0m$ height, the minimum thickness should not be less than $400mm$ to accommodate prestressing ducts and two planes of reinforcement. This thickness is also considered as a maximum since unnecessary concrete in webs creates additional self weight, which does not significantly improve the section properties. Furthermore, the additional cross sectional area needs to be prestressed and reinforced, so that it attracts a double penalty.

A set of formulae, which determine the constructable thickness of the webs based on the diameter of cable ducts and the height of webs have been suggested by Guyon (1972) and Mathivat (1983).

According to Mathivat, the arrangement on the webs can be illustrated as shown in Figure 4.11.

The minimum web thickness is $a \geq \varphi + 2(20 + \varphi_e + \varphi_l + 60)\, mm$, where the concreting shafts are assumed to have a minimum thickness of $60mm$ and a $20mm$ cover to reinforcement.

Guyon (1972) suggested an empirical formula for webs with a depth, d, less than $6.0m$.

$$a \geq \frac{d}{36} + 50 + \phi\,(mm) \tag{4.54}$$

According to Mathivat (1983), for $d > 7m$, the Guyon formula should be replaced by the following formula:

$$a \geq \frac{d}{22} + 80 + \phi\,(mm) \tag{4.55}$$

In view of the preceding discussion, the constructability of the webs can be considered as the determining factor for the thickness of web due to two reasons:

1. Any deficit in shear and torsional strengths could be overcome by using additional unstressed reinforcements or by introducing vertical prestress.
2. If the above option does not work, local increases in web thickness could be a solution without paying much penalty for the increased self-weight.

The empirical formulae by Guyon and the guide lines by Podolny & Muller (1982) address the underlying design principles. The use of empirical formulae would need the integration of a database containing information about various types of cables available. Thus, for the present study, the recommendations by Podolny & Muller (1982) have been adopted.

4.3.5 Selection of the bottom flange width

The width of the bottom flange is dictated by two factors:-

1. The spacing of the webs: This is influenced by the support spacing required for the top flange.
2. The inclination of the webs: This is a trade-off between ease of construction and aesthetics.

By the time the bottom flange width is determined, the clear spacing between webs, depth of the section and the inclination of the webs should have been chosen either by the designer or the expert system (see Figure 4.12). Thus the required width of the bottom flange is easily calculated.

4.3.6 Selection of the bottom flange thickness

The bottom flange is not very effective flexurally in span sections[2], since these parts of the beam are predominantly subjected to sagging bending moments. Nevertheless, the bottom flange is often used to support services, which prevents its complete elimination. Therefore, the tendency is to keep the thickness of the bottom flange to the minimum constructable and serviceable, thus reducing the self-weight moments (Podolny & Muller (1982)). Conversely, the bottom flange serves an important role at the support sections by acting as the compression flange; this usually becomes the governing criterion on the thickness of the bottom flange (as will be demonstrated in Section 4.3.6.1).

Podolny & Muller (1982) state that the minimum constructable thickness of the bottom flange is $125mm$, but this minimum value should not be adopted due to difficulties of preventing cracking caused by horizontal shear. The recommended minimum is $175mm$. But, in practice, the actual thickness required should be selected to satisfy the criteria for the existence of a suitable Magnel diagram.

Hence, if the moments acting on each cross section can be obtained, the expressions derived in Section 4.2 can be used to obtain the required bottom flange thickness. Since the minimum thickness of the bottom flange is known, it is possible to use an iterative procedure as used for statically determinate beams. However, there is a difficulty caused by the reactant moments. At the start of the design, the values of reactant moments are not known, so it is not possible to evaluate the total moments (which include reactant moments) acting at each cross section.

The principles of the technique

At this point, it is necessary to consider the method by which the cable profile will be determined. Two methods have been identified, which are based on the line of thrust e_p and the actual cable profile e_s (see Section 3.3.1.2).

Superficially, e_p design is easier, since no knowledge of reactant moments is

[2]the cross sections close to mid span are called *the span sections* and those close to supports, *the support sections*. This distinction is necessary because these cross sections are subjected to different kinds of dominant flexural moments (sagging or hogging), as shown in Figure 3.1.

required, but most practising engineers never bother with e_p and assume a starting value for the reactant moments (Burgoyne (1988a) and private discussion with engineers). This allows the full moment acting on the section to be considered at an early stage in the design process, which normally allows a smaller bottom flange area to be used; this is because the sagging reactant moments can be used to reduce the hogging moments over the internal supports.

Thus, a decision was taken to use the *actual cable profile design method* (see Section 3.3.1.4) because it gives freedom to select reactant moments and also to adopt a design procedure similar to the statically determinate case. The reactant moments acting at each intermediate support can be selected by using the reactant moment ratio (RMR) defined in Section 3.3.1.6.

The actual cable profile must now satisfy the following requirements:-

- it should generate reactant moments which are exactly the same as those assumed,
- it should satisfy all the stress conditions at each cross section along the beam and also comply with the limits imposed on the eccentricity by the requirements of concrete cover to steel cables, and
- the cable should follow a smooth profile.

Traditional methods of achieving the desired e_s have relied upon a 'trial and error' process. However, a rational method of selecting e_s, which has been developed in the present project, is described in Chapter 5.

4.3.6.1 Selection of the actual thickness of the bottom flange for a continuous prismatic box girder

For continuous prismatic box girders, the bending moment envelope consists of peaks at the span (maximum sagging) and support (maximum hogging) sections. Therefore, if valid Magnel diagrams exist at these cross sections for a particular section layout, then the section is generally satisfactory everywhere else. Thus, the designer can concentrate only on these critical sections. In certain cases, however, there may be problems with the cable profile close to the support where the bending moment is varying very rapidly; these complications are not considered here when explaining the principles because they may be taken into account as extensions to the proposed methods.

If the moments acting on the critical sections are known, then it is possible to use the criteria for the existence of a suitable Magnel diagram to select the required thickness of the bottom flange. But these moments are not known at the start of the design. Therefore, some iteration is inevitable, although some positive steps can be taken to minimise its extent. The design procedure can be explained as follows.

Start by specifying a reactant moment distribution by means of the RMR. Assume a minimum thickness of the bottom flange (see Section 4.3.6), which allows dead load moment to be calculated. It also allows the initial values of α_i and δ_i, which define the section moduli of the cross section (see Section 4.2.1), to be calculated. Then iterate to find the required area of the bottom flange while keeping the dead load moment constant using the expressions presented in Sections 4.2.2 and 4.2.3. This is apparently paradoxical since some effects of changing the section are being considered, but not others. The reason for this is to present the designer with options for choice when the iterations for α_i and δ_i have converged. This process of determining the required value of bottom flange thickness based on α_i and δ_i can be considered as a local, self-contained iterative cycle, which is easy to automate.

The designer will then be presented with three values for A_b. They are A_{b-min} (bottom flange area based on the minimum thickness), A_{b-span} (the bottom flange area required at span critical sections) and $A_{b-support}$ (the bottom flange area required at support critical sections).

The designer now has to choose what to do. One option is to keep the chosen RMRs and take the largest of the values of the required area of the bottom flange, $A_{b-required}$, if $A_{b-required} \geq A_{b-min}$. Then the dead load should be modified which will need further iterative cycles.

However, at this stage, the designer can alternatively decide to alter the reactant moments or other assumed dimensions. The logical choices would be:-

1. If $A_{b-support} \leq A_{b-span}$ and $A_{b-span} \geq A_{b-min}$, then the reactant moments are too large. Thus A_{b-span} governs the choice of A_b. So consider

reducing the reactant moments.

2. If $A_{b-support} \geq A_{b-min}$ and $A_{b-span} \leq A_{b-min}$, then the reactant moments are too small. Thus the hogging support moments are too high so consider increasing the sagging reactant moments.

3. If $A_{b-support}$ and A_{b-span} are both $\geq A_{b-min}$, then consider increasing the top flange.

All these options would require a recalculation of the required area of the bottom flange.

4.4 Rules used for the expert system

In the preceding section, a number of governing design criteria were explicitly identified for the selection of the dimensions of the cross section. These criteria should be expressed as rules in order to include them in the expert system. It is equally important to express the logic behind these rules in plain English, because expert system languages tend to be rather transitory. Once explained in English, these rules could be readily understood and criticised by any structural engineer who is not familiar with expert system languages. The numbering of these rules depicts the sequence in which the decisions should be taken.

1. **The depth of the section** (d) should be determined by the design engineer as explained in Section 4.3.3. If there is no restriction on the depth, then a number of possible depths can be selected to produce alternative designs.

2. **The width of the top flange** (w_t) should be determined as explained in Section 4.3.1, and is a straightforward task.

3. **The thickness of the webs** (tw) depends on whether there are ducts in the webs for cables and whether they are anchored. This is a decision that the designer has to take considering the construction technique and ease of construction.

4. **The thickness of the top flange** (t_t) depends on whether it is reinforced or prestressed in the transverse direction and the clear spacing between webs. The clear spacing between webs in turn depends on the width of the top flange, cantilever overhang of the top flange and the thickness of the webs.

   Therefore, the designer should determine the ratio between the cantilever overhang and the clear spacing between the webs (cantilever overhang ratio (COR)). For the present study, it is considered that COR should be less than 0.5, irrespective of the type of the top flange due to following reasons.

   - With the knowledge available presently, it is not possible to suggest rational guide lines for the cantilever overhang ratio.
   - The maximum value of 0.5 agrees reasonably well with the maximum allowable values for the clear span of webs and the cantilever overhangs for both reinforced and prestressed concrete top flanges.

   There is no restriction imposed on the clear spacing between webs other than that due to the maximum spacings. Thus, the number of webs should be determined to be the smallest possible, subject to a minimum of two.

   Thus, Equation 4.56 can be used to determine the clear spacing between the webs (w_s), where the number of webs (N) is increased in steps of 1, until w_s becomes less than the maximum value allowed for a particular type of top flange.

$$w_s = \frac{w_t - N t_w}{2 * COR + (N - 1)} \tag{4.56}$$

   Once the clear spacing between the webs is determined, it is possible to use Table 4.3 to obtain the minimum thickness required for the top flange.

5. **The width of the bottom flange** (w_b) depends on the inclination and number of webs, the clear spacing between webs and the thickness of the webs. The only decision that the designer should make is the inclination of the webs (see Section 4.3.5). Once this is determined, then the bottom flange thickness can be determined. In the present

expert system, the designer has to specify the required ratio between the width of the bottom flange and the width of the top flange, if the webs are inclined.

6. **The thickness of the bottom flange** (t_b) depends on the cover required at the centroid of the tendons and the overall flexural behaviour of the beam. The cover required is a decision that the designer should take by considering the diameter of the available cables and ducts. The required thickness of the bottom flange has to be determined as explained in Section 4.3.6.1, starting with a minimum thickness of 175mm.

As can be seen from the order of these rules, the decision making process has taken into account the interdependencies by making the decisions in a particular order. The explicit identification of the governing criteria helps to identify these interdependencies.

4.5 The design example

The design example selected is one of the simplest possible to demonstrate the characteristics and the behaviour of a continuous structure. The preliminary section dimensions are selected in accordance with the guide lines described in Section 4.3. The structure adopted is a single cell prismatic continuous box girder. This is illustrated in Figure 4.13. The following properties are used:

1. The number of spans = 3

2. The length of each span:

span	length (m)
1	35
2	50
3	40

3. The number of traffic lanes = 2

4. The width of each traffic lane = 3.5m

5. The number of cycle paths = 2
6. The width of each cycle path = $1.0m$
7. The number of foot paths = 2
8. The width of each foot path = $1.5m$
9. The superimposed dead load = $3.0kN/m^2$
10. The live load (HA only[3]) = $27.9kN/m$ per lane
11. The knife edge load[4] = $120.0kN$ per lane
12. The live load on cycle/foot paths[5] = $4.28kN/m^2$
13. The load factor for dead loads[6] = 1.0
14. The load factor for super-imposed dead loads[7] = 1.2
15. The load factor for live loads[8] = 1.2
16. The characteristic cube strength of concrete = $40.0N/mm^2$
17. The permissible compressive stress in concrete[9] = $-16.0N/mm^2$
18. The permissible tensile stress in concrete[10] = $0.0N/mm^2$

4.5.1 Selection of section dimensions for the example

Since sufficient data has been specified, the following dimensions can be selected for the key dimensions of the section, based on the guidelines explained in Section 4.3. The order in which the key dimensions are selected depicts the decision making process.

[3]The HA load corresponding to the shortest span is used, BS5400/2 Cl 6.2

[4]BS5400/2 Cl 6.2

[5]BS5400/2 Cl 7.1

[6]BS5400/2 Table 1

[7]BS5400/2 Cl Table 1

[8]BS5400/2 Cl Table 1

[9]BS5400/4 Cl 6.3.2.2

[10]BS5400/2 Cl 6.3.2.4

The depth of the section: The span/depth ratio is chosen to be approximately 19, hence the depth of the section is selected as $2.65m$. Other values between 14-25 could have been chosen for span/depth ratio, but this value allows the demonstration of other effects more clearly.

The width of the top flange: This is equal to the sum of the widths required for carriageways, cycle paths and footpaths ($= 12.0m$). The widths of crash barriers and parapets are ignored.

The cantilever overhang of the top flange: This should not exceed $3.0m$ for a top flange which is transversely reinforced. The cantilever overhang ratio, (COR), is selected as 0.5.

The thickness of each web: This beam is for *in situ* construction with draped anchored cables in webs, hence the thickness of each web should be $0.35m$.

The number of webs: This should be selected so that it minimises the number of webs and also keeps the clear spacing between webs within $6.0m$. In this case there should be two webs which result in a clear spacing of $5.65m$. Thus the corresponding cantilever overhang is $2.825m$ (COR is 0.5).

The thickness of the top flange: The thickness of the top flange can be selected from the Table 4.3. For a clear spacing of $5.65m$, it is selected as $0.35m$.

The inclination of webs: The webs are considered as vertical (designer's choice).

The width of the bottom flange: This can be calculated since the width of the top flange, the overhang of the top flange and the inclination of the webs are known, and is equal to $6.35m$.

The minimum thickness of the bottom flange: The minimum thickness required for the bottom flange is $0.175m$.

The minimum cover required at the centre of the cable: This is selected as $0.150m$, in order to provide sufficient construction tolerances.

4.5.2 Calculation of bending moments for the design example

The bending moment envelopes are calculated using a computer program for continuous beam analysis based on Macaulay's method. This program was initially written by Burgoyne (1987b) in Basic to run on a BBC micro computer. The bending moment envelopes due to highway loading (both HA and HB) can be determined, for prismatic beams of up to five spans. The program has been modified by Chi (1990) to take account of stiffnesses that vary along the length of the beam and was also re-written in Fortran. The computer program by Chi (1990) has been adopted for the expert system with the necessary modifications to determine the bending moment envelope due to dead and live loads.

4.5.3 Design example on selecting bottom flange area

This example is based on the design parameters given in Section 4.5.1. A few design examples are considered to highlight the different possibilities explained in Section 4.3.6.1 by considering different values for the reactant moment ratio, (RMR), especially the effect on the thickness of the bottom flange. The required thickness of the bottom flange is determined to satisfy the criteria for a suitable Magnel diagram.

The area of the bottom flange needed to resist hogging moments at ultimate load has not been considered over the supports, as this masks the effect of varying the RMR.

Case 1:

Minimum thickness of bottom flange = $0.175m$

Bottom flange thickness assumed for 1^{st} iteration= $0.175m$

The RMR is taken as 0.25 and 0.4 at the two internal supports.

The design parameters and results are given in Table 4.5.

Position	1^{st} span	2^{nd} supp	2^{nd} span	3^{rd} supp	3^{rd} span
Chainage	14.0	35.0	60.0	85.0	108.0
RMR		0.25		0.4	
M_{max} (kNm)	32343.5	-27639.8	52766.0	-27788.9	45134.8
M_{min} (kNm)	12204.5	-51690.5	28454.8	-52373.7	21897.43
Bottom flange thickness required (m)	-0.004	0.165	0.019	0.168	0.013

Table 4.5: The design parameters and results for Case 1

The minimum thickness of the bottom flange is sufficient.

Case 2:

1^{st} step:

Minimum thickness of bottom flange = $0.175m$

Thickness of bottom flange assumed for 1^{st} step = $0.175m$

The RMR is taken as 0.1 and 0.25 at the two internal supports.

The design parameters and results are given in Table 4.6.

Position	1^{st} span	2^{nd} supp	2^{nd} span	3^{rd} supp	3^{rd} span
Chainage	14.0	35.0	60.0	85.0	108.0
RMR		0.1		0.25	
M_{max} (kNm)	30534.6	-32161.9	47920.8	-32957.5	42938.2
M_{min} (kNm)	10395.0	-56212.7	23609.4	-57542.4	19700.7
Bottom flange thickness required (m)	-0.004	0.187	0.019	0.194	0.013

Table 4.6: The design parameters and results for Case 2

Thickness of bottom flange required after 1^{st} step = $0.194m$

This is larger than the minimum; the reactant moment ratio can be increased, thus reducing the moment at the support and increasing the moment in the span region. Thus the design should head towards case 1.

Case 3:

1^{st} step: -

Minimum thickness of bottom flange = $0.175m$

Thickness of bottom flange assumed for 1^{st} step = $0.175m$

The RMR is taken as 0.1 and 0.25 at the two internal supports.

The design parameters and results are as same as for Case 2.

Thickness of bottom flange required after 1^{st} step is $0.194m$. For the second step, the bottom flange thickness is changed accordingly while keeping the reactant moment ratio fixed.

2^{nd} step:

New bottom flange thickness = $0.200m$.

Position	1^{st} span	2^{nd} supp	2^{nd} span	3^{rd} supp	3^{rd} span
Chainage	14.0	35.0	60.0	85.0	108.0
RMR		0.10		0.25	
M_{max} (kNm)	29328.8	-35176.8	42106.2	-41571.9	39987.7
M_{min} (kNm)	9189.7	-59227.6	17794.8	-66156.8	16747.0
Bottom flange area required (m)	-0.004	0.187	0.019	0.195	0.013

Table 4.7: The design parameters and results for Case 3, Step 2

The thickness of the bottom flange provided is $0.200m$. Hence, this is satisfactory.

Case 4:

In this the reactant moments are set to zero.

1^{st} step:

Minimum thickness of bottom flange = $0.175m$

Thickness of bottom flange assumed for 1^{st} step = $0.175m$

The design parameters and results are given in Tables 4.8 and 4.9.

Position	1^{st} span	2^{nd} supp	2^{nd} span	3^{rd} supp	3^{rd} span
Chainage	14.0	35.0	60.0	85.0	109.0
RMR		0.0		0.0	
M_{max} (kNm)	29328.8	-35176.8	42106.2	-41571.9	39987.7
M_{min} (kNm)	9189.7	-59227.6	17794.8	-66156.8	16747.0
Bottom flange thickness required (m)	-0.004	0.202	0.019	0.236	0.011

Table 4.8: The design parameters and results for Case 4, Step 1

Thickness of bottom flange required after 1^{st} step $= 0.236m$

Hence, this is not satisfactory; the bottom flange thickness is changed accordingly while keeping the reactant moments as zero. After two more iterations, the following result is obtained.

3^{rd} Step:

New bottom flange thickness $= 0.260m$

Position	1^{st} span	2^{nd} supp	2^{nd} span	3^{rd} supp	3^{rd} span
Chainage	14.0	35.0	60.0	85.0	109.0
RMR		0.0		0.0	
M_{max} (kNm)	30171.2	-37306.2	43426.0	-44005.8	40727.1
M_{min} (kNm)	10032.2	-61357.0	19114.8	-68590.6	17986.6
Bottom flange thickness required (m)	-0.004	0.213	0.019	0.260	0.001

Table 4.9: The design parameters and results for Case 4, Step 3

The thickness of the bottom flange provided is $0.260m$. Hence, this is satisfactory.

These examples show that the reactant moments can be used to change the required thickness of the bottom flange. In Case 1, the minimum thickness of $0.175m$ is satisfactory whereas in Case 4, a thickness of $0.260m$ is required. This is a clear indication that the reactant moments can be used to obtain smaller cross sections.

4.6 Discussion

The technique explained for the selection of section dimensions highlights some important aspects which are described below; some of these aspects are related to expert systems, while others are related to the development of rational design techniques.

- The importance of considering principles when the opinions of experts differ: In order to select the width of individual webs, different experts suggest a number of guidelines, each marginally different. If the knowledge is elicited directly from the experts, then it would be difficult to come to a compromise unless the underlying design principles are considered. The principle underlying the experts' guide lines is straightforward, in that the thickness of web should be sufficient to allow the proper concreting while accommodating prestressing tendons. If it is necessary to obtain accurate rules for the required thickness of the web, then it is possible to concentrate on the principle that the construction requirements govern the web thickness.

- The identification of the dominant criteria explicitly: Each section dimension for the idealised section is selected by considering the dominant criteria for that dimension and any interdependencies. For example, the thickness of the top flange depends on the web spacing, which in turn is determined by the maximum clear spacing between the webs and the need to minimise the number of cells.

- Development of design algorithms for partial solutions: The design algorithms are developed for the generation of design parameters such as bottom flange thickness. This partially automates the design process. This is an example of the use of the analytical design method (see Section 2.4.2.2) in the design process.

It must be emphasised that only some of the conditions that govern the section dimensions have been covered here. Extra rules would be needed to take account of effects that are not considered in this study and certain designers might consider that some of the rules presented here should be modified. The principle of applying the rules in this way, however, remains valid.

Of the design criteria explained in Section 4.1, that based on the ultimate carrying capacity has not been used to determine any dimensions for statically indeterminate beams. This is due to a particular behaviour at the ultimate state of statically indeterminate beams.

As the applied load is increased, the ultimate moment of resistance will be reached at some points in the structure forming hinges, but in statically indeterminate structures, a mechanism will not form due to these hinges. If some rotation can take place at these points, then *plastic hinges* will form, which will allow additional load to be carried by the beam. The ability of prestressed concrete members to undergo plastic rotation depends on the position of the neutral axis, and the amounts of prestressing or reinforcing steels available at the section (Hurst (1988)); these details will not be available at the preliminary design stage.

However, the condition for the formation of plastic hinges itself can be a governing criteria for the bottom flange area. In order for the plastic hinge to form, the section should have adequate ductility, thus there should be sufficient concrete area to carry compression when the plastic hinge is formed. In order to apply this condition, a detailed study is needed into the ultimate state behaviour of continuous beams, which is not pursued in this thesis.

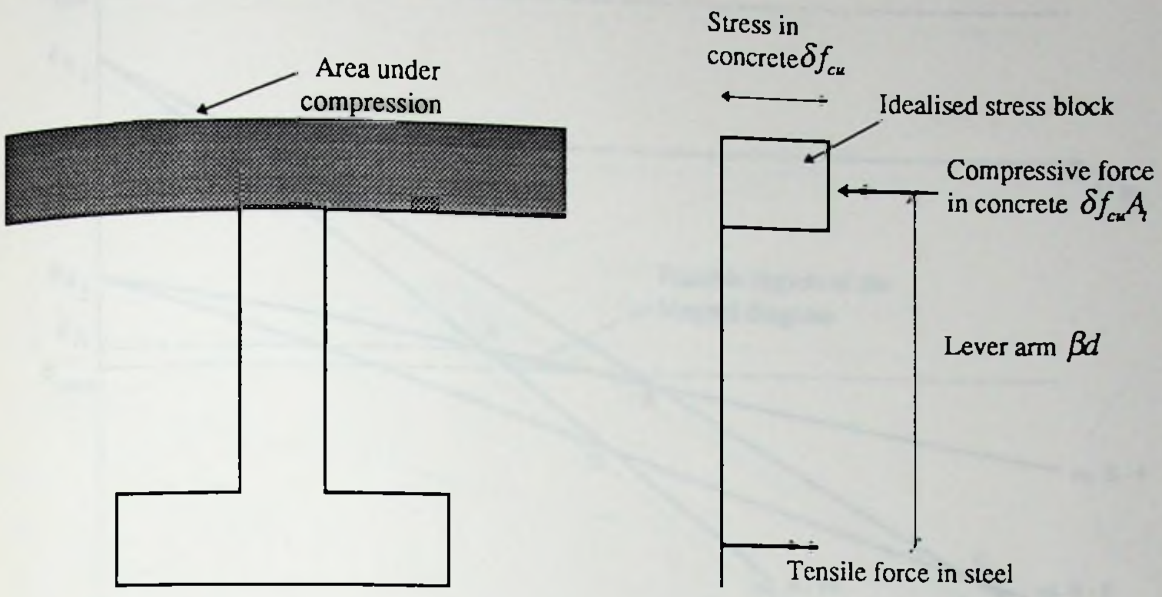

Figure 4.1: Force diagram at ultimate state

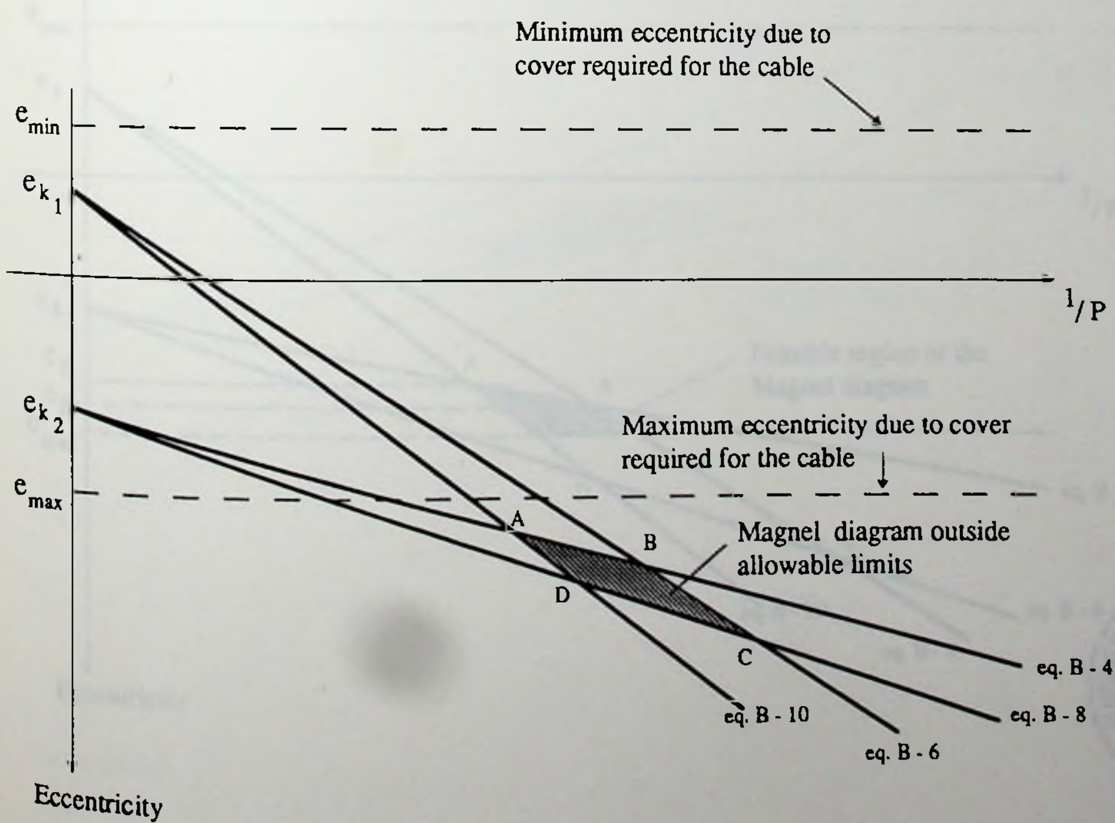

Figure 4.2: Magnel diagram with feasible region outside the eccentricity limits

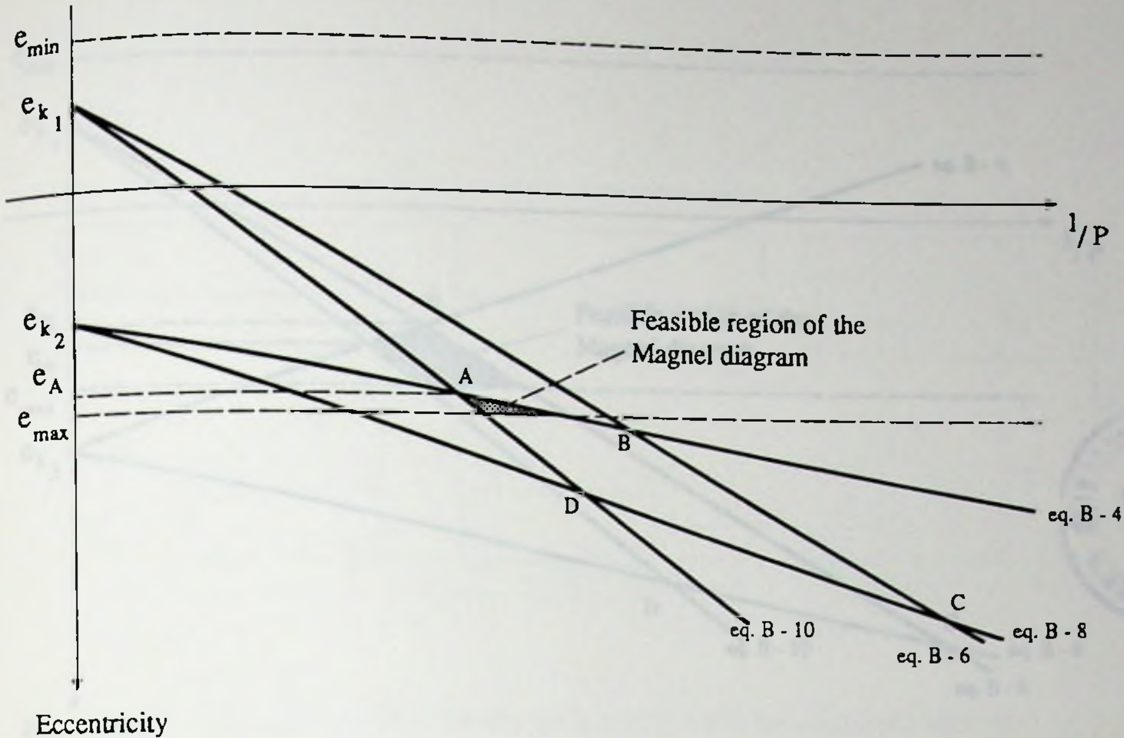

Figure 4.3: Magnel diagram with feasible region when $e_{k_2} \leq e_{max}$ and $e_A \leq e_{max}$

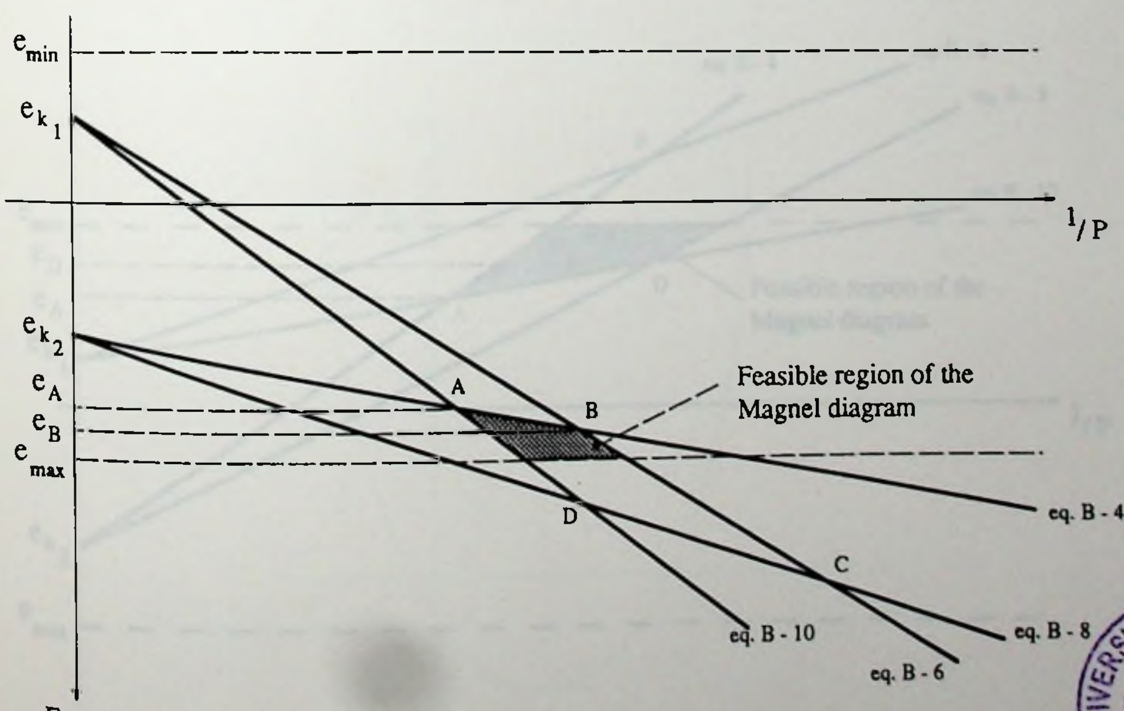

Figure 4.4: Magnel diagram with feasible region when $e_{k_2} \leq e_{max}$ and $e_B \leq e_{max}$

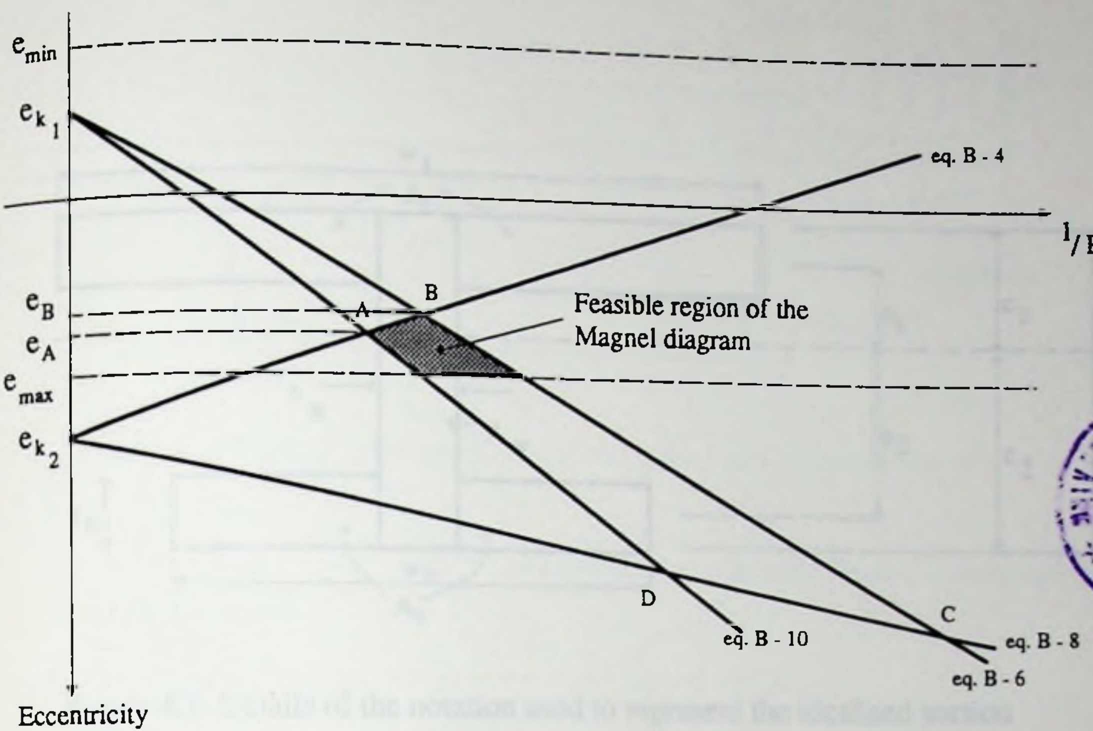

Figure 4.5: Magnel diagram with feasible region when $e_{k_2} \geq e_{max}$ with dominantly sagging moments

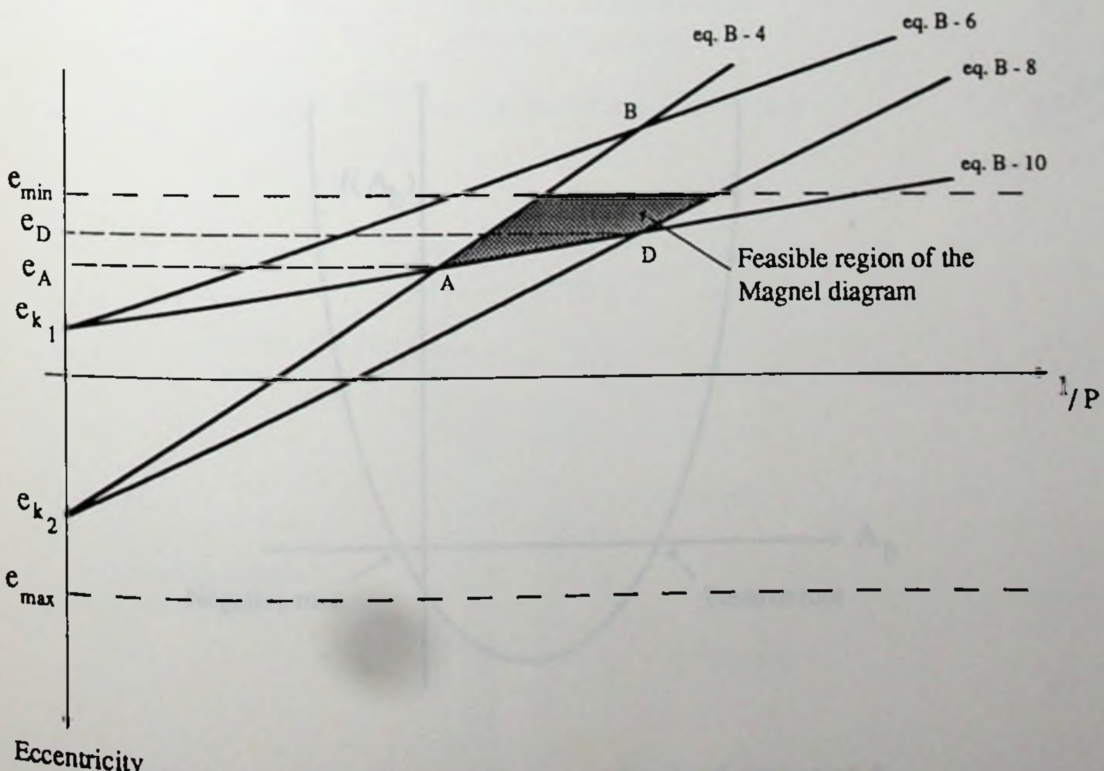

Figure 4.6: Magnel diagram with feasible region when $e_{k_1} \geq e_{min}$ with dominantly hogging moments

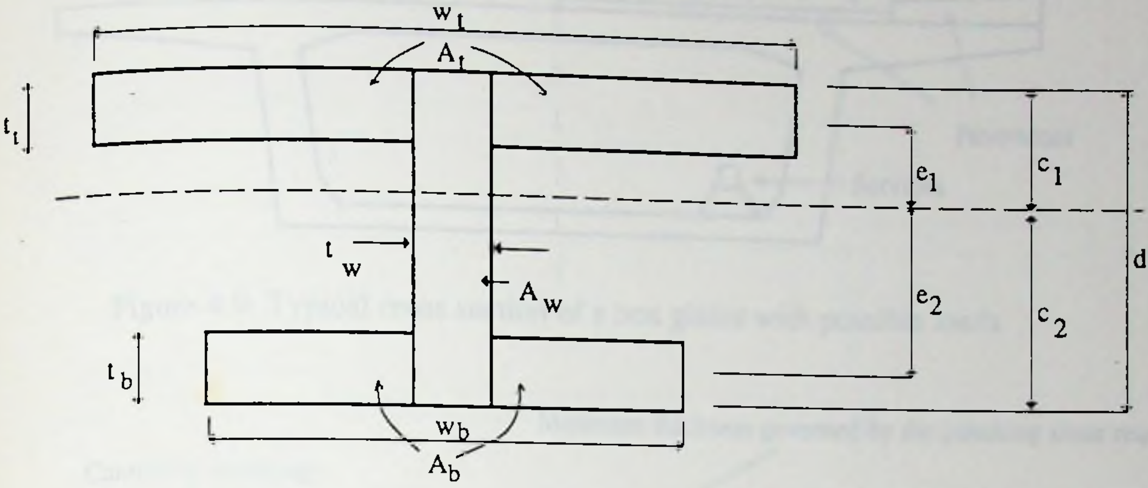

Figure 4.7: Details of the notation used to represent the idealised section

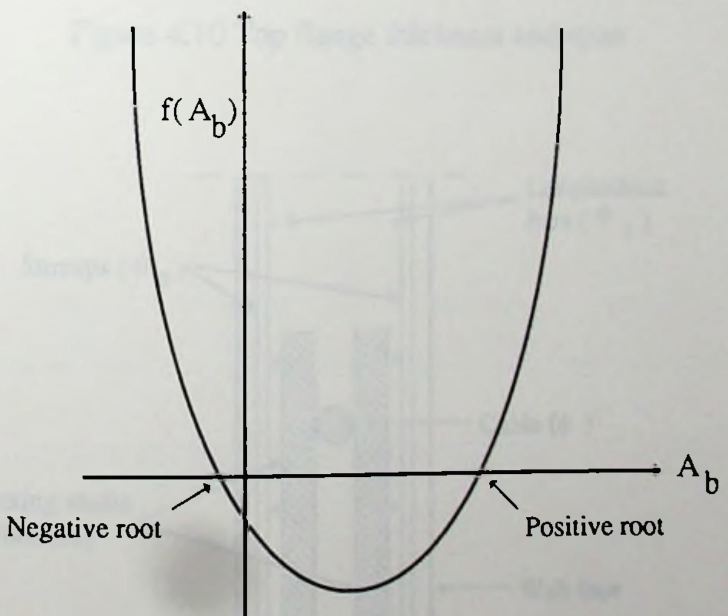

Figure 4.8: The shape of the second order function of A_b

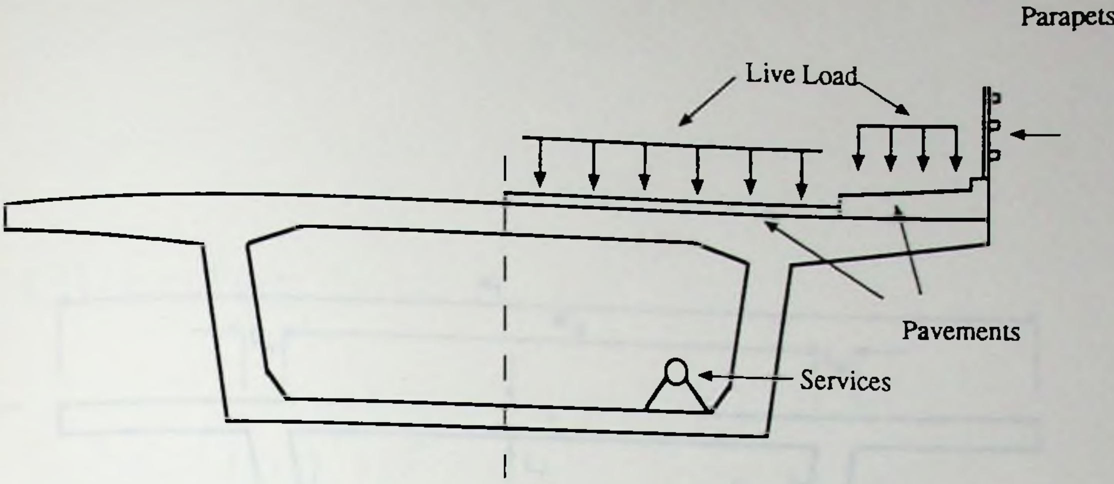

Figure 4.9: Typical cross section of a box girder with possible loads

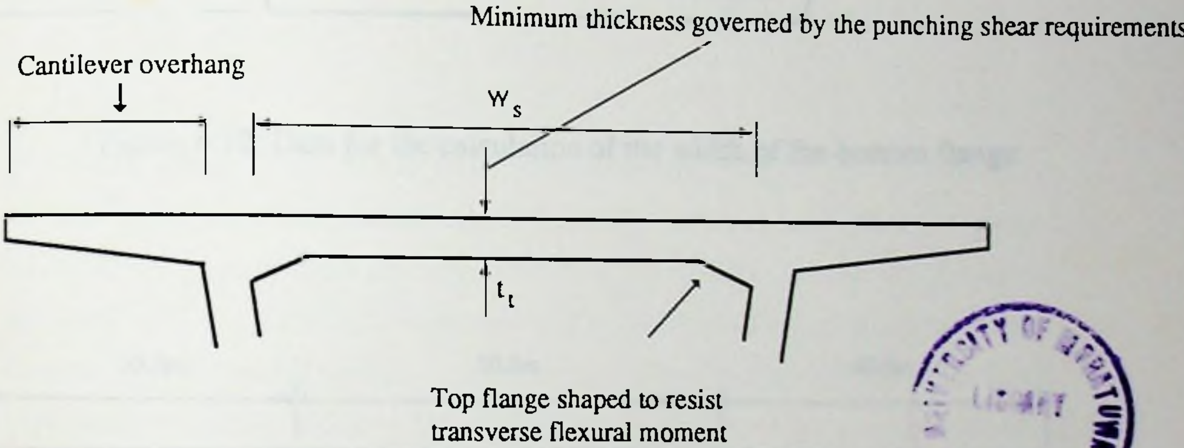

Figure 4.10 Top flange thickness and span

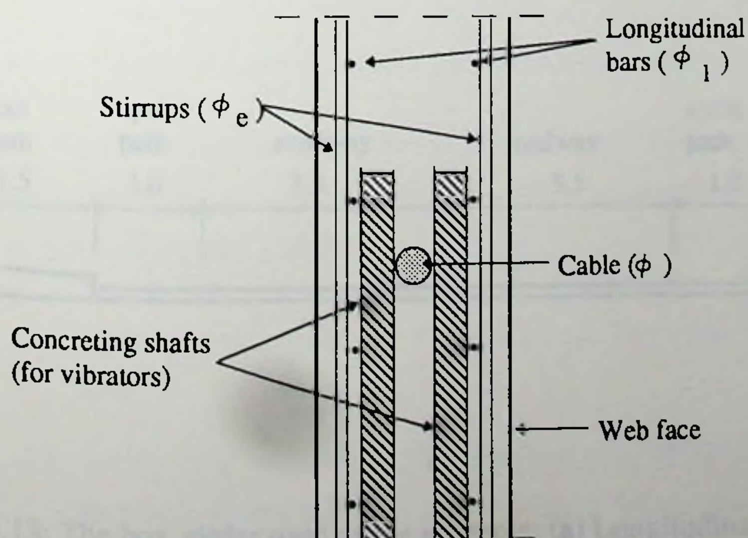

Figure 4.11: Minimum thickness of the web

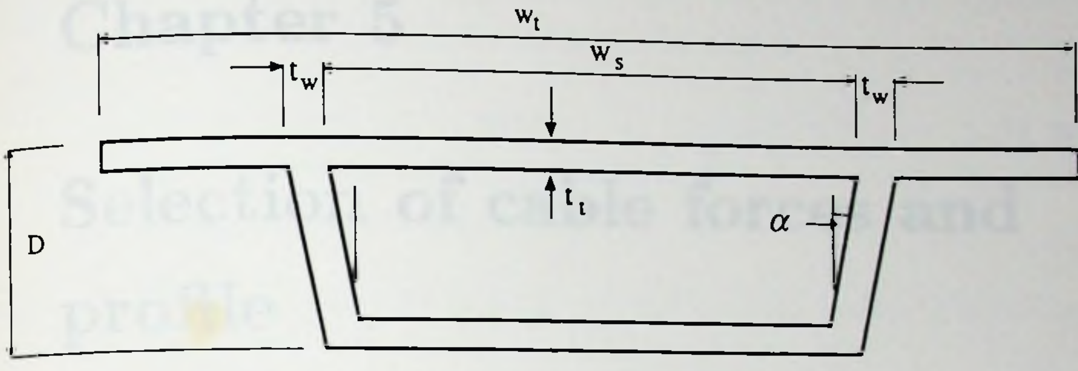

Figure 4.12: Data for the calculation of the width of the bottom flange

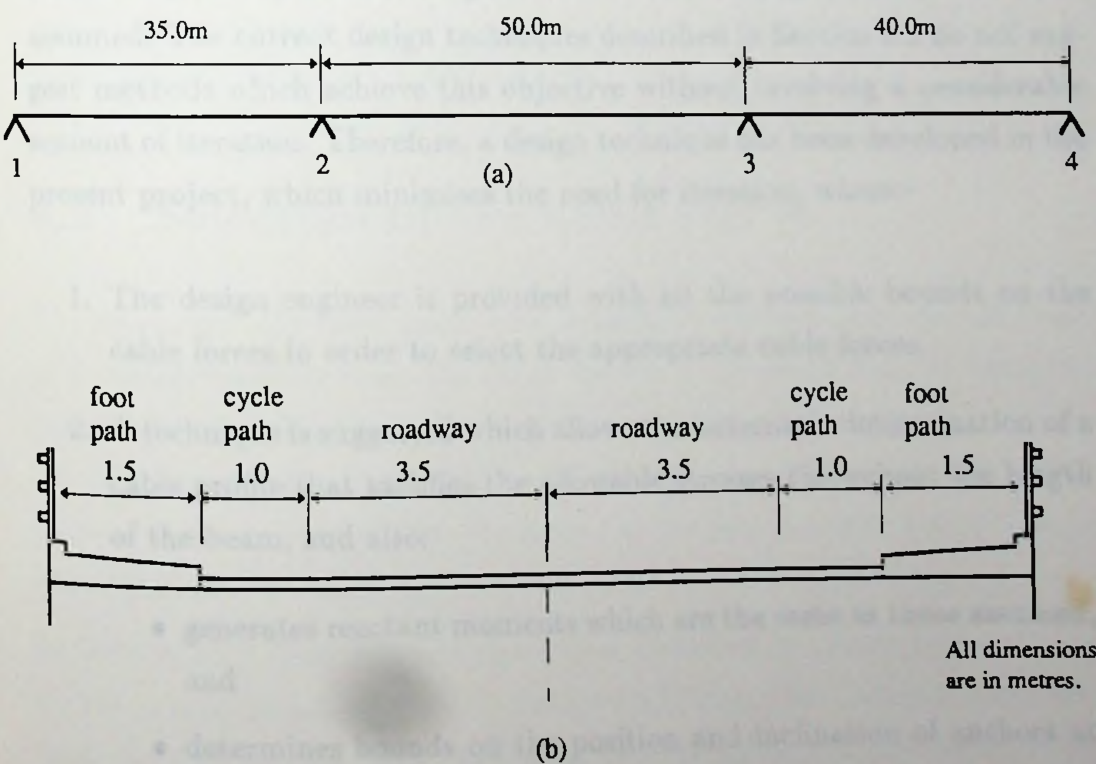

Figure 4.13: The box girder used as the example: (a) Longitudinal layout, (b) Cross sectional layout that is known at the start

Chapter 5

Selection of cable forces and profile

The design technique introduced in the previous chapter for the determination of section dimensions needs the reactant moments to be assumed initially; this means that the *cable forces* and the *cable profile* should be selected so that they are able to generate the same reactant moments as those assumed. The current design techniques described in Section 3.2 do not suggest methods which achieve this objective without involving a considerable amount of iteration. Therefore, a design technique has been developed in the present project, which minimises the need for iteration, where:-

1. The design engineer is provided with all the possible bounds on the cable forces in order to select the appropriate cable forces.
2. A technique is suggested which allows the automatic determination of a cable profile that satisfies the allowable stresses throughout the length of the beam, and also:
   - generates reactant moments which are the same as those assumed, and
   - determines bounds on the position and inclination of anchors at sections where the cable forces vary in magnitude, thus giving guidance to the designer when selecting the actual position and inclination of anchors.

5.1 Steps of the algorithm

The steps of the algorithm are as follows:

1. Determine all the bounds on the cable forces and select appropriate values for the cable forces within these bounds.

2. For the chosen forces, establish the corresponding *cable profile zone*, and then transform it to get the equivalent *line of thrust zone*, using the assumed reactant moments.

3. Find a concordant cable profile which fits within the line of thrust zone.

4. Transform this concordant cable profile back to the actual profile using the assumed reactant moment distribution and the selected cable forces. The profile will now fit within the bounds of cable zone and also generate the assumed reactant moment distribution.

The algorithm is straightforward and is not iterative overall, although iterations are involved in Step 3. A self contained design algorithm is used in Step 3 which will converge to a solution.

5.2 Selection of the cable forces

The selection of the cable forces for a determinate prestressed concrete beam is straightforward because the cable force can be found directly from the Magnel diagram. However, this is not the case with continuous beams due to the existence of reactant moments; the cable force is additionally constrained because it has to ensure that the assumed reactant moment distribution is obtainable.

In Section 3.3.2, four different cases were identified as governing conditions for the minimum cable forces in continuous beams. The required minimum cable force at transfer, P_t, is the highest of these minimum cable forces (Low (1982)). These four cases are considered again in order to incorporate them into the proposed design technique, with the necessary extensions.

5.2.1 Cable force governed by the moment range (P_1)

The cable force P_1 corresponds to the point C on the Magnel diagram of Figure 5.1. If the conditions for the *existence of a Magnel diagram* govern (see Section 4.1.3), then the full Magnel diagram may lie within the limits on eccentricity (e_{min} and e_{max}) as shown in Figure 5.2.

5.2.2 Cable force governed by the lever arms (P_2)

The cable force limit P_2 has been derived by Low (1982) for a complete internal span of a continuous beam. However, it is shown here that P_2 can be satisfied at each cross section along the beam thus giving more freedom for the design engineer to select cable forces.

As described in Section 3.3.2.2, P_2 is governed by the condition that a profile in the line of thrust zone should be capable of being linearly transformed to fit into the actual limits of e_s. In essence, this means the existence of a cable profile zone throughout the length of the beam, within the bounds of e_{min} and e_{max}, which are governed by the cover required for the prestressing cables. Such a zone will exist if the section dimensions are selected as described in the previous chapter. The cable force P_2 at each cross section is given by the minimum cable force allowed by the Magnel diagram. These values of P_2 are shown in the Magnel diagrams of Figures 5.1 and 5.2.

If the condition for the *existence of a sufficient feasible region* governs the section dimensions (see Section 4.1.4) at critical sections, then point C will be outside the bound set by the eccentricity; P_2 is larger than P_1 and thus governs.

5.2.3 Cable force governed by the existence of a concordant profile (P_3)

As described in Section 3.3.2.3, P_3 is the condition for the existence of a concordant cable profile which fits onto the e_{p-min} limit (e_{p-min} is the upper bound of the line of thrust zone). In the present project, the work by Burgoyne (1988a) has been extended to cover the following aspects:

- Application to varying prestressing cable forces.

- Identification of the stress limit which governs the minimum value of e_p.

A theoretical investigation has been carried out in order to address these issues.

5.2.3.1 Calculation of reactant moments by virtual work

The technique that is suggested for the to determination of the value of P_3 is based on the method employed for the calculation of reactant moments by virtual work (Burgoyne (1988a)), because the analysis can be performed symbolically. The following virtual work systems have been employed for a beam consisting of n supports.

The equilibrium system

An equilibrium system consisting of a fictitious moment system and associated reactions is shown in Figure 5.3. The reactions R'_{ik} are unknown. The quantity β_i represents a function that is continuous over the total length of the beam, but it is non-zero only in two spans next to the i^{th} support. There is one such equilibrium system for each internal support.

The compatibility system

The compatibility system consist of the actual curvature κ of the beam, which consists of two components:

1. The curvature (+ve sagging), $-Pe/EI$, at each cross section due to horizontal component of the prestressing cable force (P) that acts at an eccentricity e. Since the curvature due to the cable force with positive eccentricity is hogging, a negative sign should be introduced.

2. The reactant moment acting on any cross section of a continuous beam can be represented as $\sum_j \beta_j (M_2)_j$, where β_j represents the value of β with respect to the j^{th} support. This is shown in Figure 5.4. These reactant moments vary linearly between the supports. Thus the reactant moments give rise to a curvature $\sum_j \beta_j \, (M_2)_j / EI$ at each cross section.

The virtual work equation

The virtual work equation is

$$\int_0^l M\,\kappa\,dx = \sum W\,\Delta \tag{5.1}$$

The only point forces on the beam in the equilibrium system are the reactions R'_{ik}, but these are applied at internal supports, where the deflections Δ caused by the prestress are zero. Hence the right hand side of Equation 5.1 is zero.

There is one virtual work equation for each equilibrium system in the following form (all subsequent integrals are over the full length of the beam).

$$\int \beta_i\,(M_2')_i \Big(\sum_j \beta_j\,(M_2)_j - P\,e\Big)\frac{1}{EI}\,dx = 0 \quad for\; i = 2, 3, \ldots, n-1 \tag{5.2}$$

For any particular equation $(M_2')_i$ is a constant, so can be cancelled from the equation without loss of generality. The equation can be rearranged in the form of Equation 5.3, where $(M_2)_j$ refers to the unknown reactant moment at each support.

$$\sum_j (M_2)_j \int \frac{\beta_i \beta_j}{EI}\,dx = \int \frac{\beta_i P\,e}{EI}\,dx \quad for\; i = 2, 3, \ldots, n-1 \tag{5.3}$$

This gives $n-2$ equations in terms of $n-2$ unknowns (the values of $(M_2)_j$). Hence, the unknown reactant moments can be obtained if the cable forces and the profile are known. However, in this case, Equation 5.3 is used to obtain the conditions for a concordant profile.

5.2.3.2 Condition for a concordant cable profile

If the cable profile is concordant, then the reactant moment at each support, (M_2), is zero. It is both a necessary and sufficient condition for this that the terms in the right hand side of Equation 5.3 are all separately zero. Thus a cable profile is concordant if:

$$\int \frac{\beta_i\,P\,e\,dx}{E\,I} = 0 \quad for\;\; all\;\; i. \tag{5.4}$$

Both P and e are unknowns in Equation 5.4. As described in Section 3.3.2.3, the limiting case on a line of thrust being concordant is given by finding the cable forces that generate an e_{p-min}, which is itself concordant. Since limits on e_{p-min} can be expressed as functions of the cable force (P), the bending moments (M_a and M_b) and the section moduli (Z_1 and Z_2), the cable forces required to satisfy the condition of Equation 5.4 can be determined.

Under working load conditions, there are two equations which can govern e_{p-min} as shown in Figure 5.5. As explained in Appendix B, under working load conditions, P_B is independent of the moments acting on the section and dependent only on the section properties.

When $P \leq P_B$;

$$e_{p-min} = -\frac{Z_2}{A_c} - \frac{Z_2\, f_{tw}}{R\, P_t} + \frac{M_b}{R\, P_t} \tag{5.5}$$

When $P > P_B$;

$$e_{p-min} = -\frac{Z_1}{A_c} - \frac{Z_1\, f_{cw}}{R\, P_t} + \frac{M_b}{R\, P_t} \tag{5.6}$$

Hence, the choice of governing equation for e_{p-min} depends only on the magnitude of the cable force. However, without knowing the magnitude of the cable force in each region, it is difficult to determine which is the governing equation. This complexity can be resolved, as explained in the next section, in order to apply the condition of Equation 5.4 in a meaningful way.

5.2.3.3 Determination of P_3 for a beam with n spans

The following assumptions are made for the determination of P_3 in this discussion:

- Determination of P_3 is explained as applicable to a prismatic spine beam constructed by the *'span by span'* construction technique. In span by span construction, each span and part of the following span are built on false work in each phase.
- The cable forces are assumed to vary at specific locations in the beam, which are called the *cable force change points*. These locations are

considered to be on either side of each intermediate support, where the construction is stopped at each phase. Between two consecutive force change points, the cable force is constant.

- The locations of the cable force change points on either side of the intermediate support are determined by the designer, taking into account the constraints imposed by construction.
- No account is taken of any variation of cable force due to friction.

For a beam of n supports, there are $n-1$ spans. Thus there are $2n-3$ regions where the cable force is constant; these form $2n-3$ unknown cable forces.

By applying the conditions for the existence of a concordant cable profile (Equation 5.4), $n-2$ equations can be derived. Thus, $n-1$ unknown cable forces need to be found. Equations 5.5 and 5.6 define two other conditions for each region, so the determination of P_3 could be considered as an optimisation problem. There could be a number of objective functions such as minimising the total prestressing force acting over the total length of the beam ($\int Pds$, which will be considered here) or an objective function that takes account of the true cost of cables, anchorages *etc.*

By considering certain characteristics of the problem, a strategy can be suggested which eliminates the need for mathematical optimisation and also gives some control to the designer in selecting the cable forces.

The minimum value for $\int Pds$ can ideally be achieved by adopting the minimum cable force allowed by the Magnel diagram in each region. However, this may not assure the existence of a concordant cable profile. Therefore, higher cable forces may be needed in some places and a strategy should be developed to determine them.

For the existence of a cable profile, Equation 5.4 should be satisfied; *i.e.* $\int \frac{\beta_i P e\, dx}{EI}$ should be equal to zero. For a given internal support i, β_i, which is defined as in Figure 5.3, has a maximum value of unity at the support, reducing to zero at the adjacent supports. The value of the integral $\int \beta_i\, P\, e\, dx$ is thus *dominated by the value of P close to that support.*

Hence, the following strategy is adopted to obtain the cable force distribution:

- Select a cable force for each span region so that it is very close to the minimum value required to provide sufficient construction tolerances in both force and position. This gives $n-1$ cable forces, one for each span region.

- Substitute these known cable forces into the $n-2$ equations obtained from Equation 5.4, which can then be solved to find the required cable forces at the supports.

This strategy can be adopted for designs where the designer is free to select the cable forces used over the internal supports. However, there may be cases where the designer has to fulfil additional requirements such as a specified ratio between the force over the support and the force in the adjacent span regions. An example is where the cable force over support should be one and a half times that in spans to satisfy a particular cable arrangement. These are special cases and need special attention.

In order to derive the $n-2$ equations, it is necessary to determine which stress criteria will govern the value of e_{p-min}. The function $\frac{\beta_i P e}{EI}$ is generally positive in the span regions for a beam with a draped cable profile. Hence, the values of $\frac{\beta_i P e}{EI}$ over the support regions should be as negative as possible.

For a prismatic beam, the values of the function $\frac{\beta_i P e}{EI}$ are dependent only on P and e. Based on Equations 5.5 and 5.6, two equations can be derived for Pe, for a particular section.

From Equation 5.5, when $P \leq P_B$:

$$P\, e_{p-min} = -\frac{Z_2}{A_c} P - \frac{Z_2 f_{tw}}{R} + \frac{M_b}{R} \tag{5.7}$$

and

$$\frac{d(P\, e_{p-min})}{dP} = -\frac{Z_2}{A_c} \tag{5.8}$$

According to the sign convention, Z_2 is always positive, thus when P increases, $P\, e_{p-min}$ decreases.

From Equation 5.6, when $P > P_B$;

$$P\,e_{p-min} = -\frac{Z_1}{A_c}\,P - \frac{Z_1\,f_{cw}}{R} + \frac{M_b}{R} \tag{5.9}$$

and

$$\frac{d(P\,e_{p-min})}{dP} = -\frac{Z_1}{A_c} \tag{5.10}$$

According to the sign convention, Z_1 is always negative, thus when P increases, $P\,e_{p-min}$ increases.

Figure 5.6 shows the variation of $P\,e_{p-min}$ versus P; the function $P\,e_{p-min}$ is a minimum when $P = P_B$. As explained in Appendix B, P_B depends only on the section properties, and hence is constant along the beam for a prismatic section.

Since the object is to minimise $\int P ds$, the corresponding support cable forces should be found so that they are between P_{min} and P_B; this means that a solution should be sought where e_{p-min} will be given by Equation 5.5. Once the support cable forces are determined based on Equation 5.5, it is necessary to ensure that the limits on the cable forces at the support regions are less than P_B. If the required cable forces are greater than P_B, then it can be shown that there is no concordant profile for the selected span cable forces. If this happens, then the design engineer can increase the selected cable forces in span regions and re-calculate another set of bounds on P_3, but this may lead to a trial and error procedure. The other option is to increase the thickness of the bottom flange and start afresh.

Mathematical equations

For the existence of a cable profile

$$\int \frac{\beta_i\,P\,e\,dx}{E\,I} = 0 \quad \text{for all } i = 2, 3, \ldots, n-1 \tag{5.11}$$

For a prismatic section, EI is constant. Hence, if the cable force at transfer is P_t and the loss ratio is R:

$$\int \beta_i\,R\,P_t\,e\,dx = 0 \quad \text{for all } i = 2, 3, \ldots, n-1 \tag{5.12}$$

In Equation 5.12, RP_t is constant in each region as shown in Figure 5.7a. The unknowns in this equation are the cable forces over the support regions. Therefore, it should be possible to obtain a relationship of the form $A_{ij} P_{support-i} = B_i$, which can be used to determine the unknown cable forces over the supports.

The quantity $\int \beta_i \, RP_t \, e \, dx$ represents the area covered by the function $\beta_i \, RP_t \, e$. These areas are illustrated in Figure 5.7d along with the shape of β_i and the eccentricity on either side of the i^{th} support.

From Figure 5.7d, the following equation can be derived for each intermediate span.

$$A_{l(i-1)} + A_{sp(i-1)} + A_{r(i-1)} + A_{l(i)} + A_{sp(i)} + A_{r(i)} = 0 \qquad (5.13)$$

Knowledge of the fact that e_{p-min} is governed by Equation 5.5, allows Equation 5.13 to be rewritten:-

$$\begin{aligned}
RP_{su(i-1)} & \int_0^{l_{l(i-1)}} \frac{x}{l_{r(i-1)}} \left(\frac{-Z_2}{A_c} - \frac{Z_2 \, f_{tw}}{R \, P_{su(i-1)}} + \frac{M_b}{R \, P_{su(i-1)}} \right) dx \\
+RP_{su(i)} & \int_{l_{sp(i-1)}}^{l_{r(i-1)}} \frac{x}{l_{r(i-1)}} \left(\frac{-Z_2}{A_c} - \frac{Z_2 \, f_{tw}}{R \, P_{su(i)}} + \frac{M_b}{R \, P_{su(i)}} \right) dx \\
+RP_{su(i)} & \int_0^{l_{l(i)}} \left(1 - \frac{x}{l_{r(i)}} \right) \left(\frac{-Z_2}{A_c} - \frac{Z_2 \, f_{tw}}{R \, P_{su(i)}} + \frac{M_b}{R \, P_{su(i)}} \right) dx \\
+RP_{su(i+1)} & \int_{l_{sp(i)}}^{l_{r(i)}} \left(1 - \frac{x}{l_{r(i)}} \right) \left(\frac{-Z_2}{A_c} - \frac{Z_2 \, f_{tw}}{R \, P_{su(i+1)}} + \frac{M_b}{R \, P_{su(i+1)}} \right) dx \\
& = -A_{sp(i-1)} - A_{sp(i)} \qquad (5.14)
\end{aligned}$$

Separation of all the quantities which contain support cable forces to the left hand side of the equation, gives

$$
\begin{aligned}
& RP_{su(i-1)} \int_{0}^{l_{l(i-1)}} \frac{x}{l_{r(i-1)}} \frac{Z_2}{A_c} dx + RP_{su(i)} \int_{l_{sp(i-1)}}^{l_{r(i-1)}} \frac{x}{l_{r(i-1)}} \frac{Z_2}{A_c} dx \\
+ \; & RP_{su(i)} \int_{0}^{l_{l(i)}} \left(1 - \frac{x}{l_{r(i)}}\right) \frac{Z_2}{A_c} dx + RP_{su(i+1)} \int_{l_{sp(i)}}^{l_{r(i)}} \left(1 - \frac{x}{l_{r(i)}}\right) \frac{Z_2}{A_c} dx \\
= \; & \int_{0}^{l_{l(i-1)}} \frac{x}{l_{r(i-1)}} (-Z_2 f_{tw} + M_b) dx + \int_{l_{sp(i-1)}}^{l_{r(i-1)}} \frac{x}{l_{r(i-1)}} (-Z_2 f_{tw} + M_b) dx \\
+ \; & \int_{0}^{l_{l(i)}} \left(1 - \frac{x}{l_{r(i)}}\right) (-Z_2 f_{tw} + M_b) dx + \int_{l_{sp(i)}}^{l_{r(i)}} \left(1 - \frac{x}{l_{r(i)}}\right) (-Z_2 f_{tw} + M_b) dx \\
+ \; & A_{sp(i-1)} + A_{sp(i)}
\end{aligned}
\tag{5.15}
$$

In these equations i refers to the middle support of the three under consideration.

If there are n supports, there will be $n-2$ equations which can be written in the following form:

$$
\sum_{i=2}^{n-1} \sum_{k=2}^{n-1} A_{ik} \, P_{su(i)} = B_i \tag{5.16}
$$

Representing the equation 5.16 in matrix notation:

$$
\begin{pmatrix}
A_{22} & A_{23} & 0 & 0 & 0 & \cdots & 0 \\
A_{32} & A_{33} & A_{34} & 0 & 0 & \cdots & 0 \\
0 & A_{43} & A_{44} & A_{45} & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & \cdots & A_{(n-1)(n-1)}
\end{pmatrix}
\begin{pmatrix}
P_{su(2)} \\ P_{su(3)} \\ P_{su(4)} \\ \vdots \\ P_{su(n-1)}
\end{pmatrix}
=
\begin{pmatrix}
B_2 \\ B_3 \\ B_4 \\ \vdots \\ B_{(n-1)}
\end{pmatrix}
$$

from which the values of P_{su} can be obtained.

5.2.4 Cable force governed by P_4

This is a special case derived on the same basis as in Section 5.2.3, but limited by the condition that the cable cannot be placed at the top of its allowable zone over the supports as this would fail to satisfy the cover limits. In this case, the minimum eccentricity allowed for the cable profile is governed by e_{min}, not by Equation 5.5, for a certain region over the supports. A Magnel diagram corresponding to a case for which P_4 governs is shown in Figure 5.8.

For the section layout selected in accordance with the guide-lines given in Section 4.3, P_4 is often applicable, since the cover limits are likely to govern the cable profile over the supports. A possible shape of the Pe diagram with limits imposed by the cover is shown in Figure 5.9b. The special feature is the additional limits imposed on eccentricity by the requirements of cover. These limits are not horizontal because they represent the actual limits on cable profile, e_{min}, ($e_s \leq e_{min}$) transformed into line of thrust terms (e_p).

The condition for the existence of a concordant cable profile is given by Equation 5.12. However in this case the total area under the function Pe is not effective. Hence Equation 5.12 should be modified using the shaded area A_{sh} of Figure 5.9c, in the following manner:

$$\int \beta_i \, RP \, e \, dx = A_{sh} \text{ for all } i = 2, 3, \ldots, n-1 \tag{5.17}$$

A_{sh} can be written as follows:

$$A_{sh(i)} = A_{sh_{l(i-1)}} + A_{sh_{r(i-1)}} + A_{sh_{l(i)}} + A_{sh_{r(i)}} \tag{5.18}$$

The same quantities A_{ik} of Equation 5.16 are valid as was used to determine P_3, but the right hand side requires modification by the addition of terms representing A_{sh}. The new equation can be written as follows:

$$\sum_{i=2}^{n-1} \sum_{k=2}^{n-1} A_{ik} \, P_{su(i)} = B_i + A_{sh(i)} \tag{5.19}$$

The shaded areas $A_{sh(i)}$ are dependent on the cable forces over the intermediate supports. Hence, the solution should be reached iteratively, and the procedure can be described as follows:

- Use the cable force given by P_2 at the support to transform the limits on the cover into the line of thrust zone (P_2 over the support can be determined using the Magnel diagram). If these limits do not intersect with the bounds on e_p, then P_4 will not govern. Normally, however, with a section selected to be as small as possible, P_4 must be determined. The area $A_{sh(i)}$ should be calculated numerically and this value used to obtain the first estimate of P_4. It is possible to stop here, because only an estimate is necessary. If P_4 governs, this first estimate of P_4 is an overestimate, since the area $A_{sh(i)}$ was calculated using a lower value (P_2). The calculation can be repeated using the new value of P_4 until convergence.

5.2.5 Design example

The cable forces are found for the moments obtained with the design example presented in Section 4.5.3 for Case 1. The cable force change points are assumed to be at $8.0m$ from each internal support. The limits on the cable force P_{max} and P_{min} are obtained from the Magnel diagram. P_{min} refers to either P_1 or P_2 depending on the shape of the Magnel diagram at the critical sections. The selection of cable forces is then done in two steps. In the first step, the span cable forces are selected. These span cable forces are used to derive the limits P_3 and P_4 as they are applied in Sections 5.2.3 and 5.2.4 to give limits on the forces at the internal supports. Then the cable forces over the supports are selected such that all the bounds are satisfied.

Position	1^{st} span	2^{nd} support	2^{nd} span	3^{rd} support	3^{rd} span
P_{max}	126638.0	125069.1	124795.2	124625.3	124976.4
P_{min}	19846.8	38551.9	32392.1	39062.1	27704.3
$P_{selected}$	22500.0		34000.0		29000.0
P_3		20796.0		19203.0	
P_4 using P_2		46186.0		40703.0	
P_4 after iteraion		44490.0		40200.0	
$P_{selected}$	22500.0	46000.0	34000.0	42000.0	29000.0

Table 5.1: The selection of cable forces to satisfy the bounds

The value of P_4 is obtained after three iterative cycles where different values are tried for the cable force over the support. The value of P_B for this case is $99256.0kN$ everywhere.

5.2.6 Summary on the selection of cable forces

There are four conditions which govern the cable forces in a continuous prestressed concrete beam. P_1 and P_2 are automatically determined from the Magnel diagram once the section has been determined as described in Chapter 4. The techniques required for the calculation of P_3 and P_4 have been explained for a continuous beam with a cable layout suitable for span-by-span construction; the same principles can be applied for other cable arrangements. The required minimum cable force at a particular section is the highest value of these four cable forces. This is the first step in eliminating the possible iterations involved in the selection of cable forces and profiles. These limits are derived on the basis of service load conditions, but should also be extended to cover transfer conditions.

In the present study, the condition for the existence of a concordant profile is used to determine the minimum limits on the cable forces. However, a similar condition can be used to determine maximum limits on the cable forces as well, but this is not pursued in this thesis because it would produce uneconomic designs. However, these maximum limits should be derived and included in the design algorithms used for commercial expert systems, because these expert systems may be used to obtain solutions for problems which can only be satisfied by the maximum possible force.

5.3 Automated determination of cable profile

Once the cable forces have been obtained as described in the preceding section, the cable profile has to be found. The most important requirement is that this profile should generate reactant moments that are the same as those assumed, while at the same time satisfying the stress limits at each cross section. The principles of the technique can be explained as follows.

In the design procedure adopted so far, the reactant moments have been assumed and considered to act as loads. This allows the bounds on the *actual cable profile* (e_s) to be determined at each cross section. Some of these bounds are due to stress limits, while others are due to cover required for prestressing cables. A cable profile is sought which fits within these bounds and also generates the required reactant moments. This is not a trivial task. Methods do exist, however, which allow the determination of concordant profiles that fit within the bounds of the *line of thrust zone.* In order to use these methods, the bounds on the actual profile, including those due to cover requirements, are transformed to a line of thrust using the relationship derived in Section 3.3.1.5.

$$e_p = e_s - \frac{M_2}{P} \tag{5.20}$$

A concordant profile is now sought which lies within these bounds, which can then be transformed back to the actual cable profile again using Equation 5.20.

Thus, the main task in finding the appropriate cable profile will be the determination of a concordant profile which fits within the bounds of the line of thrust zone; for this, use is made of an important property of concordant profiles. Any bending moment diagram, due to a set of loads acting on the beam, can be scaled to form a concordant profile. This follows from the fact that loads acting on the beam give rise to curvatures, which are compatible with zero deformations at the internal supports. If a cable profile gives rise to the same curvature, there is no tendency to change the support reaction; thus it will not cause any reactant moments, and the profile is concordant (Lin & Burns (1982)).

Hence, the determination of a concordant profile is a matter of finding an appropriate distribution of *'notional loads'*, which give rise to a suitable bending moment diagram. These notional loads, which can act upwards or downwards, are not real loads, and there is no direct relationship to the external loads acting on the section.

One technique available for the designer is to try various notional loads until a profile is found which satisfies the bounds on the line of thrust throughout the length of the beam. However, this is not appropriate in an expert system environment. Therefore, a method is needed which can be automated; one such method has been suggested by Burgoyne (1988b). For the present project this method has been extended to cover the following aspects:

- Determination of concordant profiles when the cable forces vary at specific locations along the length of the beam.
- Selection of the position and orientation of anchors for the cables which cause the change in cable forces.

5.3.1 Concordant cable profiles with a constant cable force

The method suggested by Burgoyne (1988b) for the determination of a concordant cable profile, aims to find a set of notional loads which cause a bending moment diagram that fits within the bounds of the line of thrust. The method is iterative and the principles can be explained in the following manner.

If the bending moment that is caused by the set of notional loads lies outside the bounds of the line of thrust zone at certain regions, then the notional loads should be corrected. A set of changes to the notional loads is sought which corrects the largest error in the bending moment, but does not cause major changes elsewhere. To do this, use is made of the influence line for bending moment at the point in question. Notional loads are applied to the structure in proportion to the height of the influence line, on the assumption that loads distributed in this way are those most likely to cause a suitable distribution of moments (see Figure 5.10). Experience has shown that this is a valid statement for most parts of a continuous beam except at certain regions on either side of an internal support. The actual magnitude required is determined by the need to bring the profile due to existing notional loads inside the bounds of the line of thrust at the point in question.

Once the magnitudes are determined these notional loads can be added to the existing notional loads. The process is repeated until the bounds are satisfied throughout the beam. At the start of the process, the notional loads are set to zero.

The profile resulting form this method causes kinks at the internal supports. This profile can be refined to have continuous curves over the supports by distributing the reactions due to the notional loads over a region on either side of the support.

In this method, a degree of 'over-compensation' is useful for improving convergence, by applying a larger correction than necessary. In the iterative algorithm, this over-compensation factor is predetermined and kept constant; it is called the *accelerating factor*, because it improves the convergence rate.

5.3.2 Concordant cable profiles with varying cable forces

When the cable forces vary in a known way, a refinement is necessary for the above method. What is important is that the *moment* caused by the concordant profile Pe_p is the same as the *moment* caused by the notional loads. This is because the curvatures due to the concordant profile, $\frac{Pe_p}{EI}$, should be the same as curvatures due to the notional loads, $\frac{M_n}{EI}$. Thus, notional loads are sought which generate a bending moment diagram that falls within the appropriate bounds of a *force-eccentricity zone* (Pe_p). The bounds of this force-eccentricity zone can be obtained by multiplying the limits of the line of thrust zone by the cable force in each region.

5.3.2.1 Steps of iteration

The steps involved in the iteration can be listed as follows:

1. Analyse the structure under the influence of the current set of notional loads to find the bending moment, M_n.
2. Find the position where M_n is most outside the bounds on the force-eccentricity zone, Pe_p.
3. Calculate the influence line for bending moment at that point.
4. Determine the magnitude and distribution of the notional loads required to correct the error in the moment at the worst position, multiplied by a suitable accelerating factor. Add this to the existing notional loads.
5. Distribute support reactions as applied loads.
6. Repeat steps $1 - 4$ until M_n lies within the bounds on Pe_p everywhere.

5.3.3 Location of anchor blocks

When the cable forces vary at specific locations along the beam, there is an important practical consideration which needs to be taken into account; the location of the anchor blocks. In order to start a new set of cables or to curtail some of the cables, anchorages are required. For practical reasons, these anchor blocks should be located away from the continuous cables.

In order to fulfil this requirement, the resulting cable profile will not be smooth, but should have sharp changes at the sections where the cable force varies. Hence, a technique has been developed that will find a cable profile to suit the positions of the anchors selected by the design engineer.

5.3.3.1 Principles of the technique

The situation at a point where the cable force changes can be visualized as shown in Figure 5.11. If $P_{r(i)}$ is the force in the running cable, and $P_{n(i)}$ is the force in the new cable, then on the left of the i^{th} change point, the cable force is $P_{r(i)}$ and on the right, it is $P_{r(i)} + P_{n(i)}$. The centroid of $P_{r(i)} + P_{n(i)}$ will represent the location of the resultant cable profile. This situation is equivalent to the application of a point moment equal to $P_{n(i)}e_{sn(i)}$ and a vertical force equal to $P_{n(i)} \sin \alpha_{n(i)}$ at the cable force change point, where $e_{sn(i)}$ is the eccentricity of the new cable and $\alpha_{n(i)}$ is the inclination at which the new cable is started. The sign convention used is that point moments acting in the anti-clockwise sense and forces acting downwards are positive.

$P_{r(i)}$ and $P_{n(i)}$ are already known, so the design engineer should select the following:

1. The position in the cross section at which the cable is anchored, thus fixing the eccentricity of the anchor.
2. The inclination at which the anchor block is located.

This will allow the determination of the point force and the moment induced at each anchor point. The final cable profile in the force-eccentricity zone should also have discontinuities at each change point, corresponding to the same point forces and moments.

5.3.3.2 Limits on eccentricity

Since the cable forces are already selected, the anchors *cannot* be placed at any eccentricity within the section. There are maximum and minimum eccentricities that are imposed by the shape that the cable profile can possibly take at the change points. These shapes can be visualised in terms of the force-eccentricity zone and the bending moment diagram which fits into it, as shown in Figures 5.12 and 5.13.

Figure 5.12 shows that the cable is placed at e_{p-min} to the left of anchor point and e_{p-max} to the right of it, which gives rise to a maximum point moment. Figure 5.13 shows that the cable is placed at e_{p-max} to the left of anchor point and e_{p-min} to the right of it; this gives rise to a minimum moment. These figures refer to a point where a new cable starts. Thus, the maximum moment represents a maximum eccentricity and the minimum moment represents a minimum eccentricity. Similar relationships are needed at a point where the cable is terminated. The minimum and maximum limits on the eccentricity can be obtained by dividing the corresponding moments by the force in the new cable.

5.3.3.3 Limits on inclination

The limits on the inclination are not as rigorous as the limits on the eccentricity of the anchorage. The limits on the inclination become critical only when the eccentricity is selected close to a limiting value, which gives rise to a bending moment of the shapes shown in Figures 5.12 and 5.13. This can be explained as follows.

When the eccentricity is selected as a limiting value, then the inclination of the resulting bending moment must be *parallel* to the bounds of force-eccentricity zone; the possible maximum or minimum inclinations will be restricted by this condition. These limits on the inclination can be obtained mathematically by considering the vertical forces induced by the individual cables and the resultant cable.

One limit on the inclination is derived here to illustrate the principles. The limits on the inclination of the new cable, $\alpha_{n(i)}$, have to be determined at a point where a new cable is started. The limits depend on two known inclinations (as in Figure 5.14); that of the lower bound on the cable profile zone to the left of force change point and the upper bound on the cable profile zone to the right of the force change point. By considering the point force caused by the cable force at this cross section, Equation 5.21 can be written, where $\beta_{ur(i)}$ represents the inclination of the upper bound of the cable profile zone to the right of the i^{th} force change point, and $\beta_{ll(i)}$ represents the inclination of the lower bound of the cable profile zone to the left of the i^{th} force change point.

$$(P_{r(i)} + P_{n(i)}) \sin \beta_{ur(i)} - P_{r(i)} \sin \beta_{ll(i)} = P_{n(i)} \sin \alpha_{n(i)} \qquad (5.21)$$

The left hand side represents the force due to the resultant cable. The right hand side represents the force due to the new cable. Thus a limit on $\alpha_{n(i)}$ can be determined. Similarly, another bound can also be determined by considering that the cable is placed at the upper bound of the cable profile zone on left hand side and at the lower bound of the cable profile zone on right hand side of the i^{th} anchor point.

5.3.3.4 Detailed method

A set of cables with anchorages in the beam causes two sets of forces on the structure. There are a set of point forces and moments at the anchorage points (which are now known), and a set of distributed loads caused by the cable curvatures which are unknown. The method described in Section 5.3.1 can be used to determine the smooth cable profile, if the bounds of force-eccentricity zone are adjusted by an amount that corresponds to the anchorage forces. This is done in the following manner:-

1. Determine the bending moment diagram which is caused by the point forces and moments acting at the anchorages (using a continuous beam analysis).
2. Superimpose the inverse of this bending moment diagram on the bounds of the force-eccentricity zone to find the modified force-eccentricity zone.
3. Find the notional loads which cause a smooth bending moment diagram lying within the bounds of the modified force-eccentricity zone.

4. Superimpose the bending moment diagram due to point forces and moments to the bending moment diagram due to notional loads.
5. Obtain the actual concordant profile by dividing the bending moment at each section by the corresponding cable force.
6. Linearly transform this concordant cable profile as explained in Section 3.3.1.5, by using the assumed reactant moment distribution.

The resulting cable profile will cause reactant moments that are the same as those assumed and will also lie within the limits of the cable profile zone, thus satisfying the allowable stress limits throughout the length of the beam.

5.3.4 The design example

This design example is the same as that used to illustrate the selection of cable forces in Section 5.2.5. The cable forces used are given in Table 5.2.

Position	1^{st} span	2^{nd} support	2^{nd} span	3^{rd} support	3^{rd} span
P_{max}	126638.0	125069.1	124795.2	124625.3	124976.4
P_{min}	19846.8	38551.9	32392.1	39062.1	27704.3
$P_{selected}$	22500.0	46000.0	34000.0	42000.0	29000.0

Table 5.2: The cable forces in span and support regions

The maximum eccentricity due to cover limits e_{max} is $1.664m$ and minimum eccentricity due to cover is $-0.686m$. Since the cable forces are known, the cable profile zone can be obtained (see Figure 5.19). The line of thrust zone (see Figure 5.15) that is used is of a special shape because it includes the additional limits imposed by the minimum and maximum eccentricities. These additional limits are obtained by linearly transforming the maximum and minimum limits on eccentricity by using the assumed reactant moment distribution and the cable forces in each region. The corresponding force-eccentricity zone can be obtained by multiplying the limits of the line of thrust zone by the cable force in each region.

The anchors are assumed to be situated in the webs with an eccentricity of $-0.20m$ from the centroid of the section. The inclinations of the anchors are assumed to be 358.0^0, where new cables are started, and 182.0^0, where cables are terminated. The angles are measured from the horizontal to the right in a clockwise sense. The resulting point forces and the moments at each of the change points are given in Table 5.3. The cable force change points are selected $8.0m$ away from each internal support.

Position	1^{st} change point	2^{nd} change point	3^{rd} change point	4^{th} change point
Chainage	27.0	43.0	77.0	93.0
Force in anchor	23500.0	12000.0	8000.0	13000.0
Maximum eccen.	0.711	3.574	3.782	2.541
Minimum eccen.	-0.344	-3.305	-3.321	-1.836
Selected eccen.	-0.20	-0.20	-0.20	-0.20
Selected incli.	358.0	182.0	358.0	182.0
Point force	-820.1	-418.8	-379.5	-453.7
Point moment	-4700.0	2400.0	-1600.0	2600.0

Table 5.3: The point forces and moments due to anchorages

The bending moment diagram resulting from these point forces and moments are shown in Figure 5.16.

Once the inverse of this bending moment diagram is superimposed on the limits of the force-eccentricity zone, a modified force-eccentricity zone can be obtained. This modified zone will be used as the reference in the successive iterations.

A set of notional loads which causes a bending moment diagram complying with the limits of the modified force eccentricity zone is found as described in Section 5.3.2. This bending moment diagram fits within the bounds of the modified force-eccentricity zone as shown in Figure 5.17.

The bending moment diagram due to the anchorage forces is superimposed to obtain the bending moment diagram which fits into the limits of the actual force-eccentricity zone as shown in Figure 5.18.

Finally, the actual cable profile is obtained by dividing the values of the bending moment by the corresponding cable forces in each region and then transforming this profile into the cable profile zone using the assumed reactant moment distribution. The final cable profile is shown in Figure 5.19. This profile is that of the centroid of all tendons, and hence has a discontinuity at the anchorage positions. At the detailed design stage, the engineer (or another expert system) will have to find a suitable profile for each individual tendon.

The initially assumed reactant moments are compared with those resulting from this cable in Table 5.4. They are identical for all practical preliminary design purposes.

Position	2^{nd} support	3^{rd} support
M_2 assumed (kNm)	7536	13783
M_2 actual (kNm)	7524	13774

Table 5.4: Assumed and actual reactant moments

The following points can be noted about this algorithm. The design example is a case where the cross section is as small as possible. The cable forces are also close to the minimum allowed in each region. These conditions impose strict limits on the cable profile. The reactant moments caused by a cable placed at the upper and lower bounds of the line of thrust zone are shown in the Table 5.5.

Position	2^{nd} support	3^{rd} support
M_2 for upper bound (kNm)	-1741.9	-1025.9
M_2 for lower bound (kNm)	20312.2	18665.4

Table 5.5: The reactant moments corresponding to the upper and lower bounds of line of thrust zone

The reactant moments due to the upper bound are small which implies that a concordant cable must lie very close to the upper bound of the line of thrust zone. However, the upper bound of the line of thrust is not a smooth curve so it is difficult to obtain a smooth curve that lies within limits throughout the beam and is also concordant. As can be seen on Figure 5.19, the actual cable profile violates the bounds of the cable profile zone at certain locations with a maximum error of $0.014m$ close to the chainage $60.0m$.

These violations can be minimised by specifying higher cable forces over the support regions, and a more stringent limit of P_4 can be determined on the basis of the curvature allowed over the support. This would be an extension to the present work.

In Section 5.3.3.2, it was emphasised that the limits on eccentricity must be satisfied at the anchor points to obtain a continuous force-eccentricity zone. Figure 5.20 shows a modified zone, where the limits on eccentricity for anchor blocks are violated, showing that there is no continuous force-eccentricity zone. The eccentricity selected at the first change point is $-0.5m$, which violates the allowable limits of the Table 5.3.

5.3.5 Summary on the automated determination of cable profile

A technique is described for the automated determination of the cable profile which can generate the initially assumed reactant moment distribution. This automated technique is very useful for designers because considerable time is spent finding cable profiles if done manually (Low (1982)). Since this is an iterative technique it is essential to ensure that the designer has selected a cable force distribution for which a cable profile exists. This emphasises the importance of finding the limits on the cable forces.

The automated determination of the cable profile is a very important step in the proposed alternative design technique. It eliminates an enormous amount of complications that would be needed if heuristics were used to determine the cable profiles in an expert system environment.

5.4 Conclusions

A design technique has been proposed to determine the limits on cable forces. For the cable forces selected within these limits, another method is described to find a cable profile automatically. This cable profile will generate reactant moments that are equal to those initially assumed at the internal supports. The technique explained for the determination of the cable profile is an example of the use of the analytical design method. The determination of all the limits on the cable forces is an essential prerequisite for obtaining valid cable profiles, since it sets realistic targets for the analytical design component.

These design techniques demonstrate the degree of simplification that a designer can achieve if he is willing to harness the fundamental design principles and an understanding of the structural behaviours to derive design algorithms. One important feature of these design algorithms is the knowledge encapsulated in the form of numerical routines.

These design algorithms require a large amount of numerical calculations to generate solutions. Hence, they are ideally suited for computer aided design. This is another step forward in the process of increasing the involvement of the computer in human-computer interaction for the structural design of spine beam bridges.

Another important point is that these rational design techniques can be used for the teaching of prestressed concrete design to give an insight to the underlying design principles, as described by Burgoyne (1990a).

The design techniques explained in this and the previous chapters are essentially simplified descriptions of design principles which have wider implications in other closely related domains, such as applications to other types of beams constructed using different construction techniques. A number of simplifying assumptions were made at the beginning of Chapter 4, the implications of which will be discussed in Chapter 7.

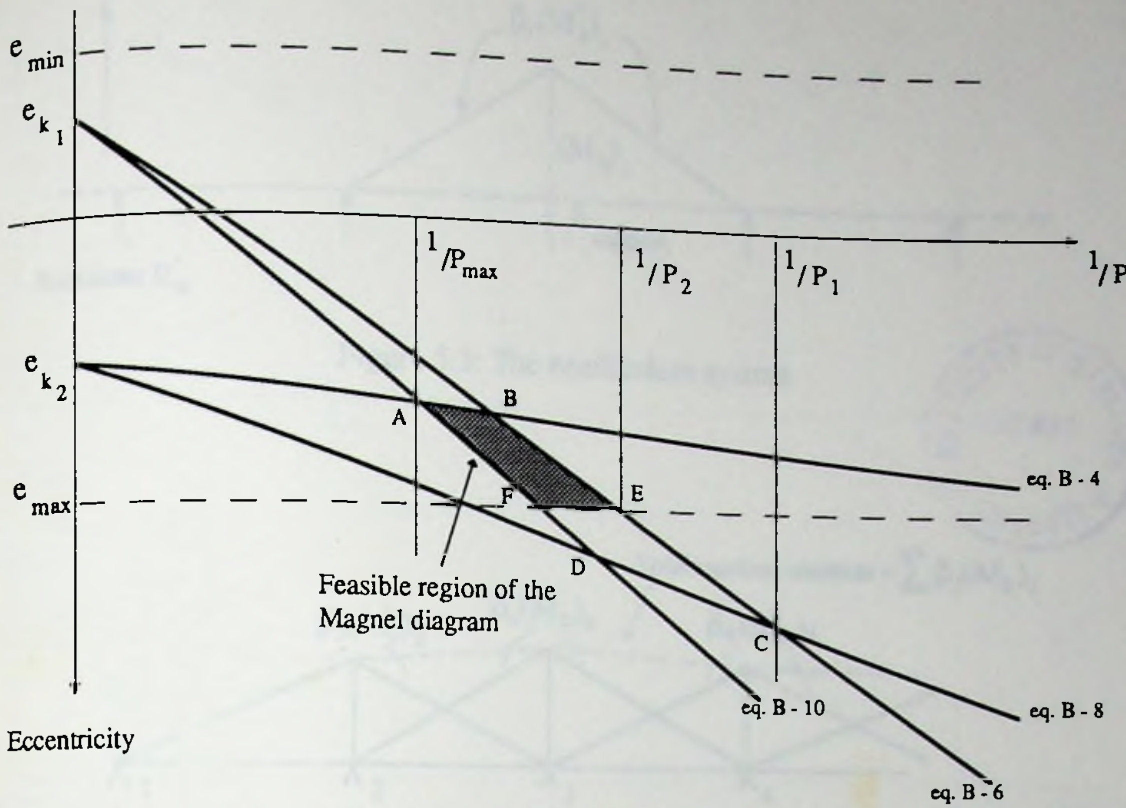

Figure 5.1: Magnel diagram showing the limits on cable forces (P_1 and P_2)

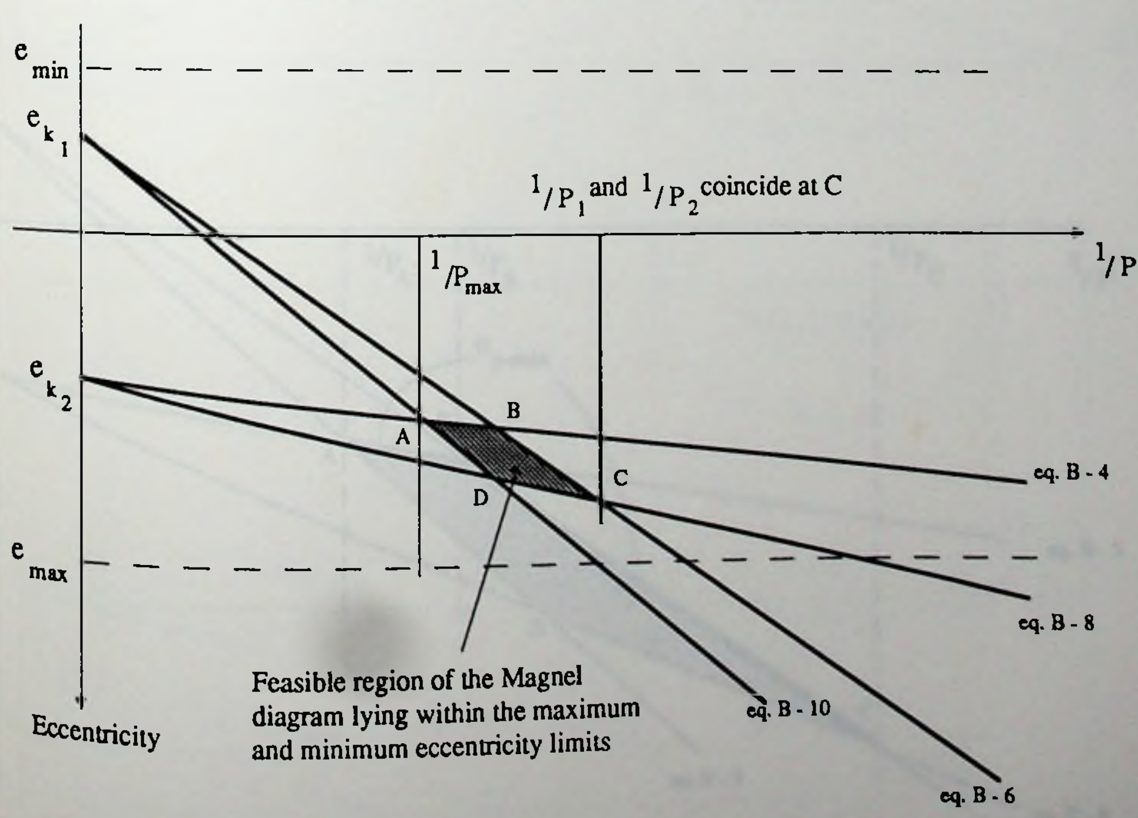

Figure 5.2: Magnel diagram showing the special case where P_1 and P_2 coincide

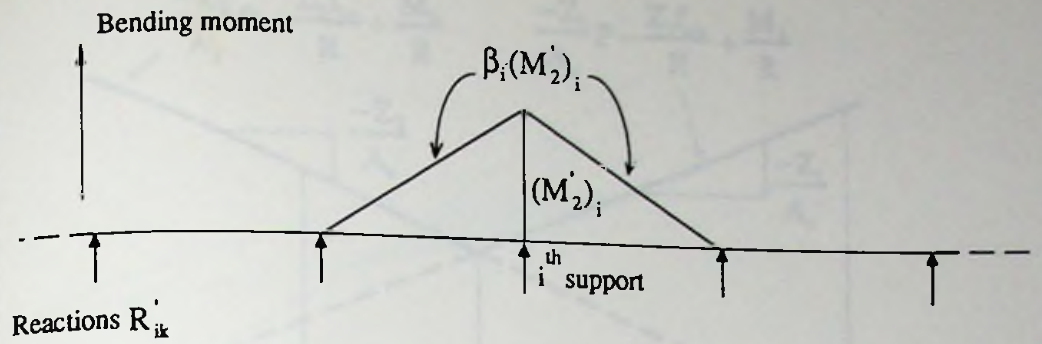

Figure 5.3: The equilibrium system

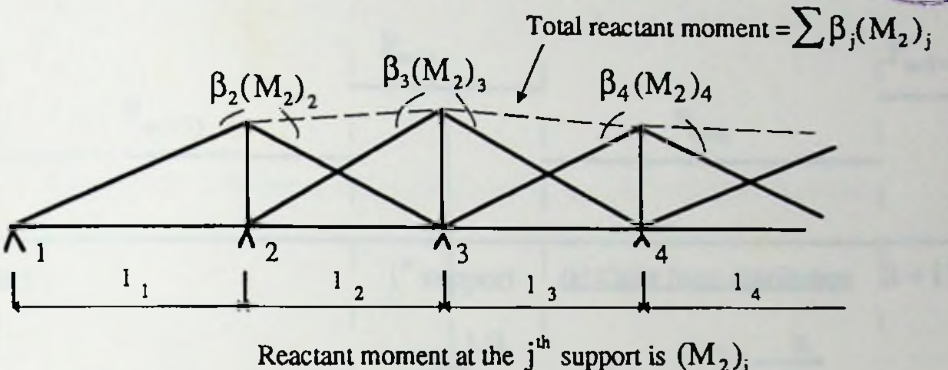

Figure 5.4: The reactant moments at supports

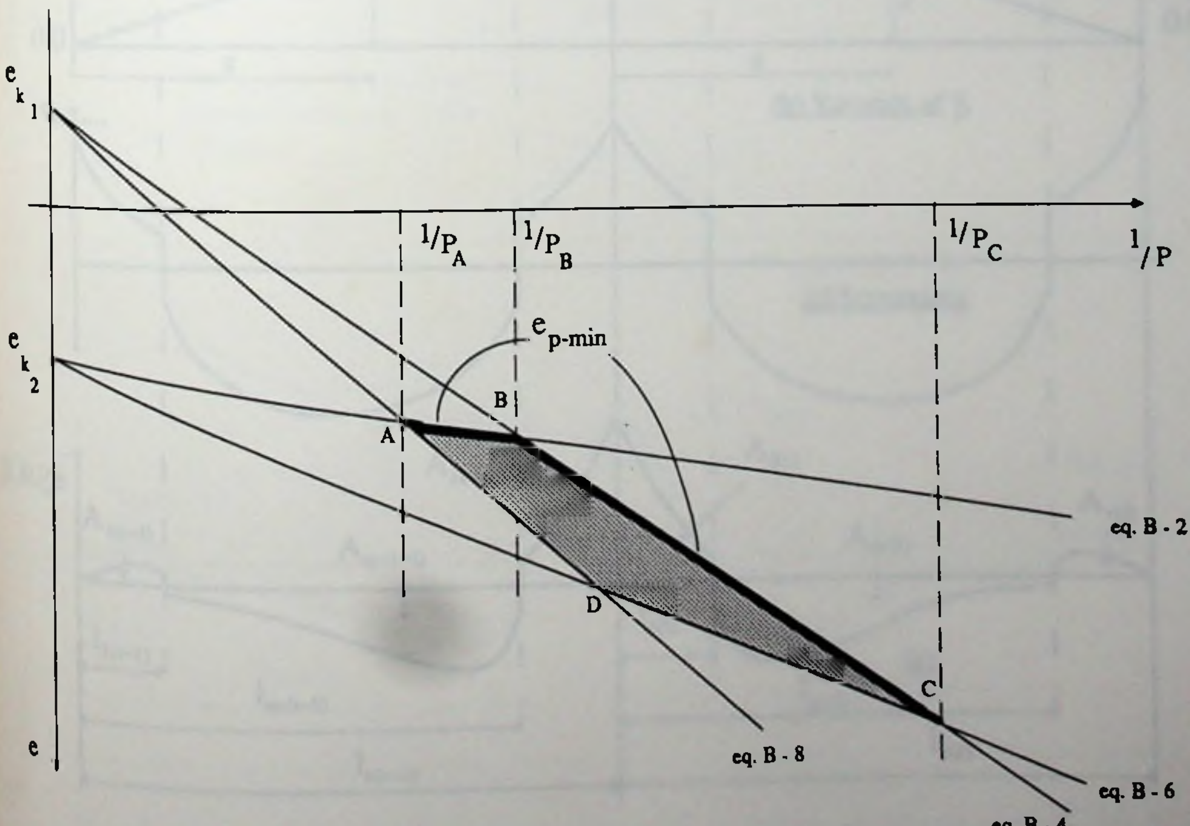

Figure 5.5: Magnel diagram on the limits of $e_{p\text{-min}}$

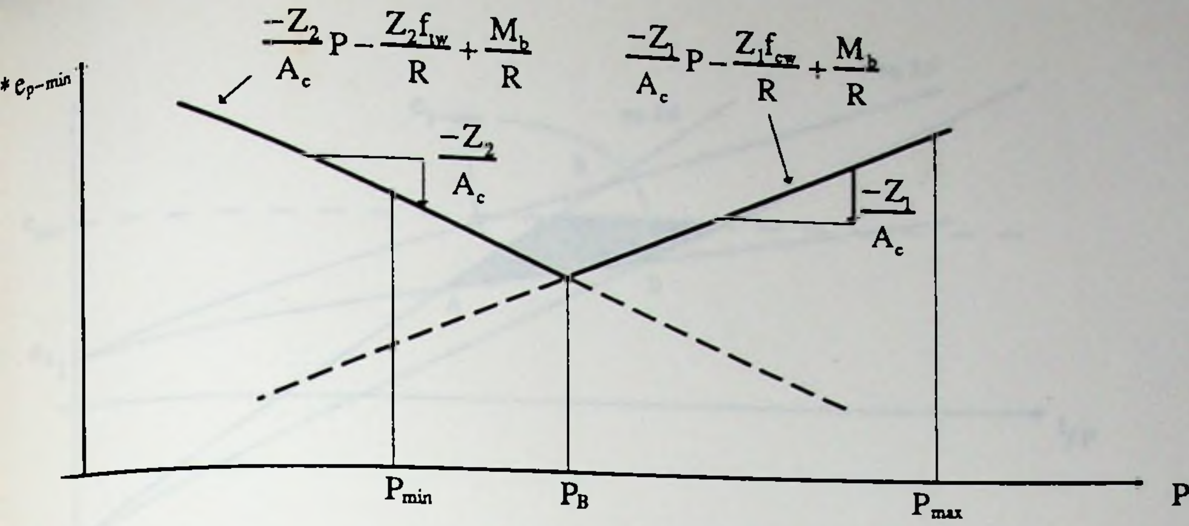

Figure 5.6: Pe_{p-min} versus P

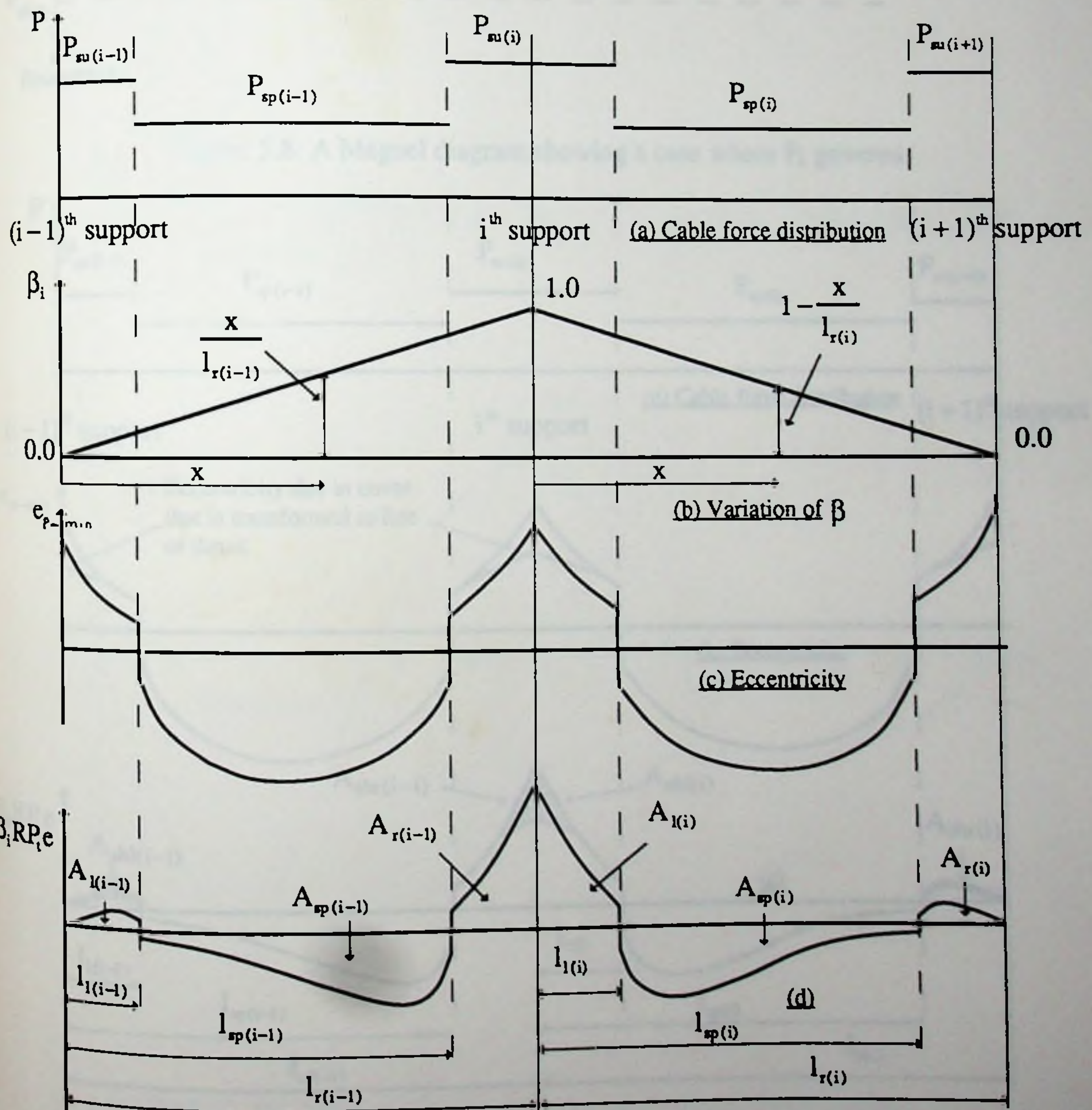

Figure 5.7: (a). The variation of cable forces in the span and support regions, (b). The variation of the function β_i over an internal support, (c). The variation of eccentricity corresponding to e_{p-min} and (d). Area covered by the function $\beta_i RP_t e$ over an internal span

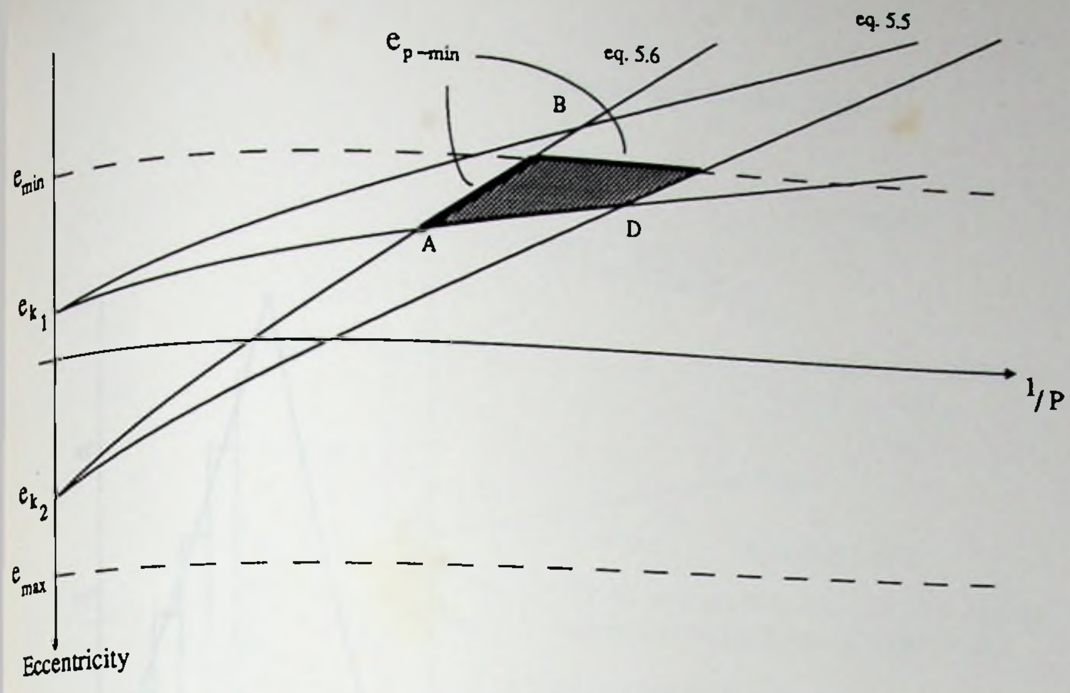

Figure 5.8: A Magnel diagram showing a case where P_4 governs

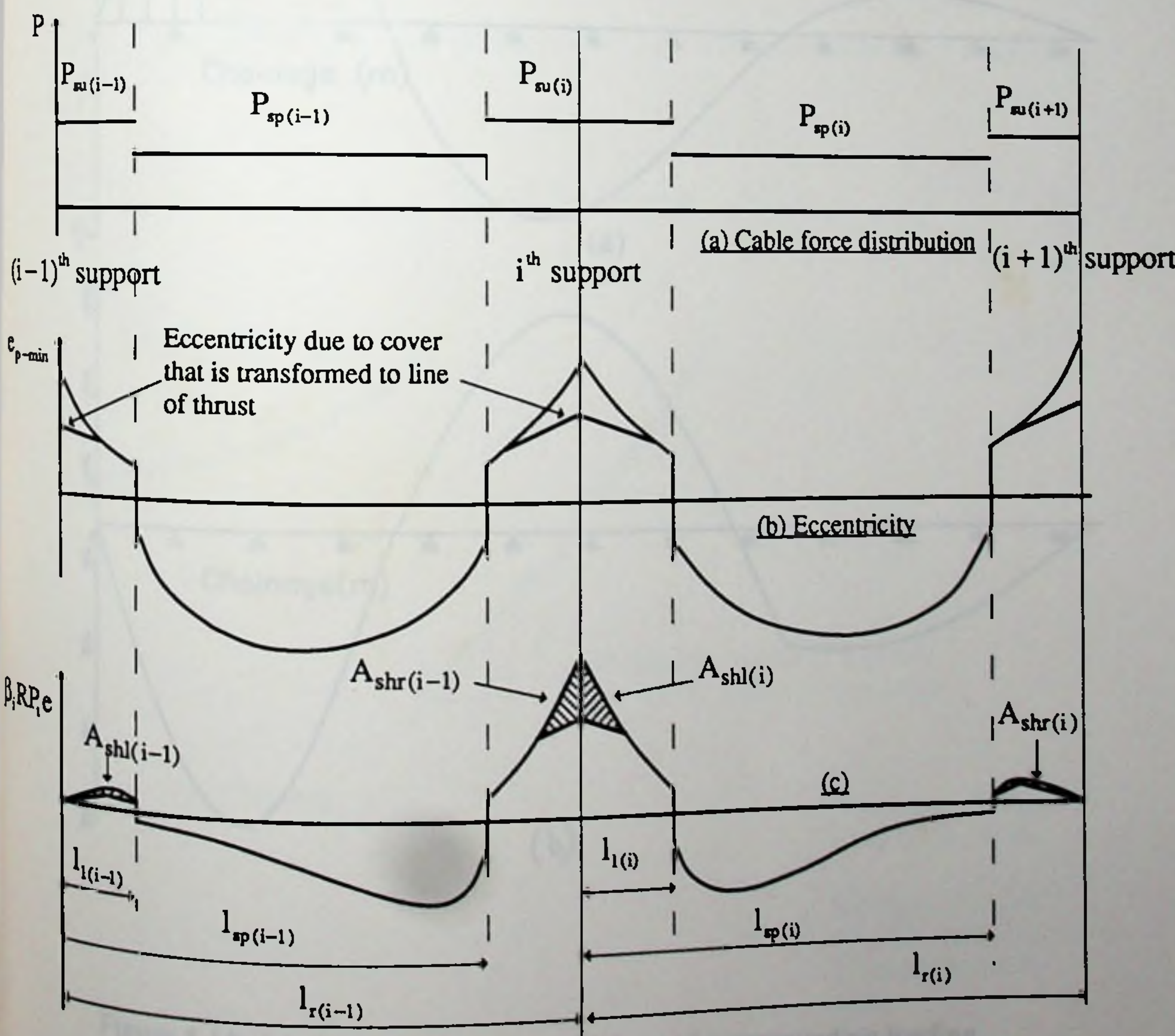

Figure 5.9: (a). The variation of cable forces in the span and support regions, (b). The variation of eccentricity corresponding to e_{p-min} with the additional limits imposed by the cover required for the prestressing cables, (c). Shaded area showing the additional constraint imposed on the function $\int \beta_i R P_i e dx$

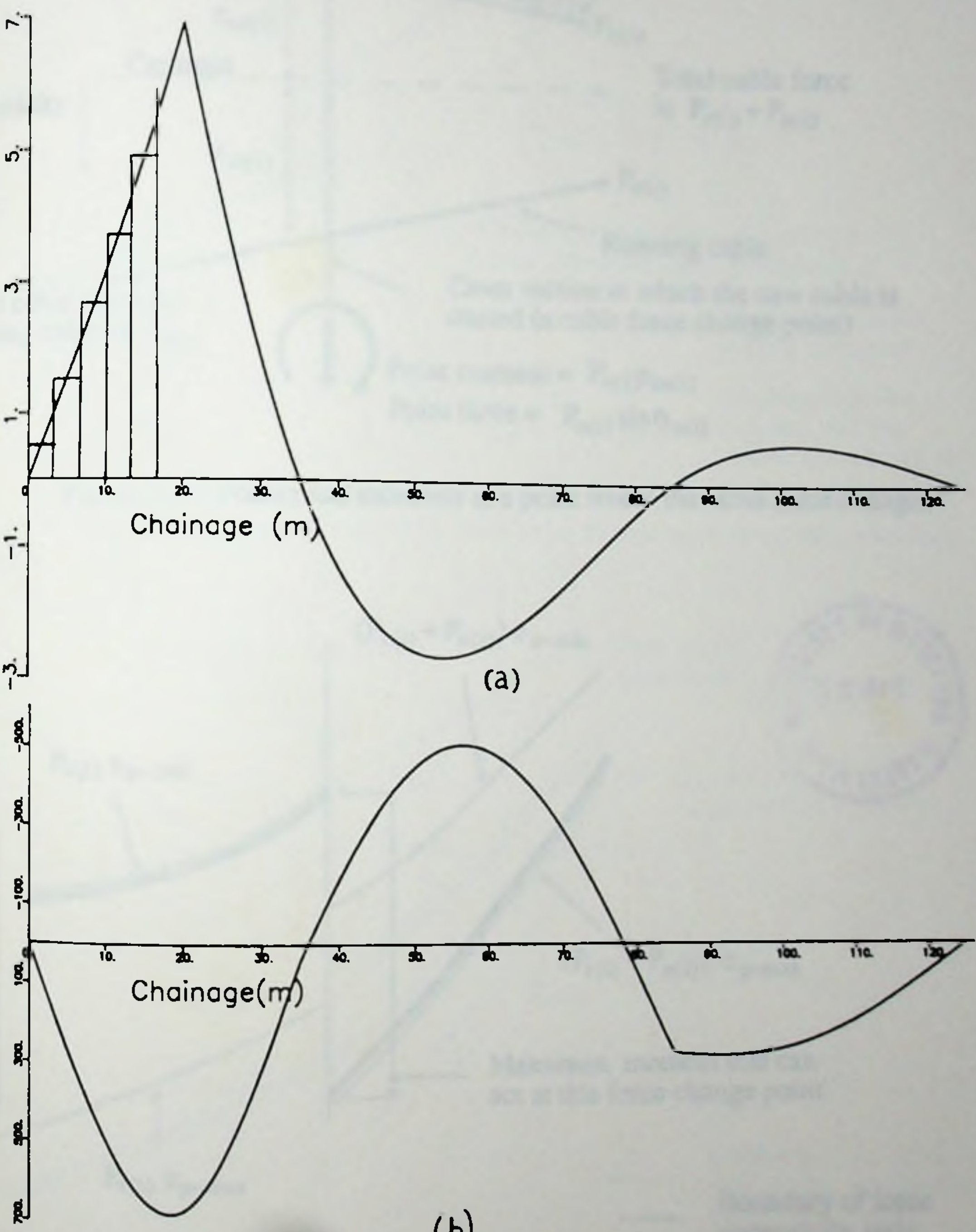

Figure 5.10: (a). Influence line at x=21.0m and corresponding loading.
(b). Shape of the bending moment diagram due to loads

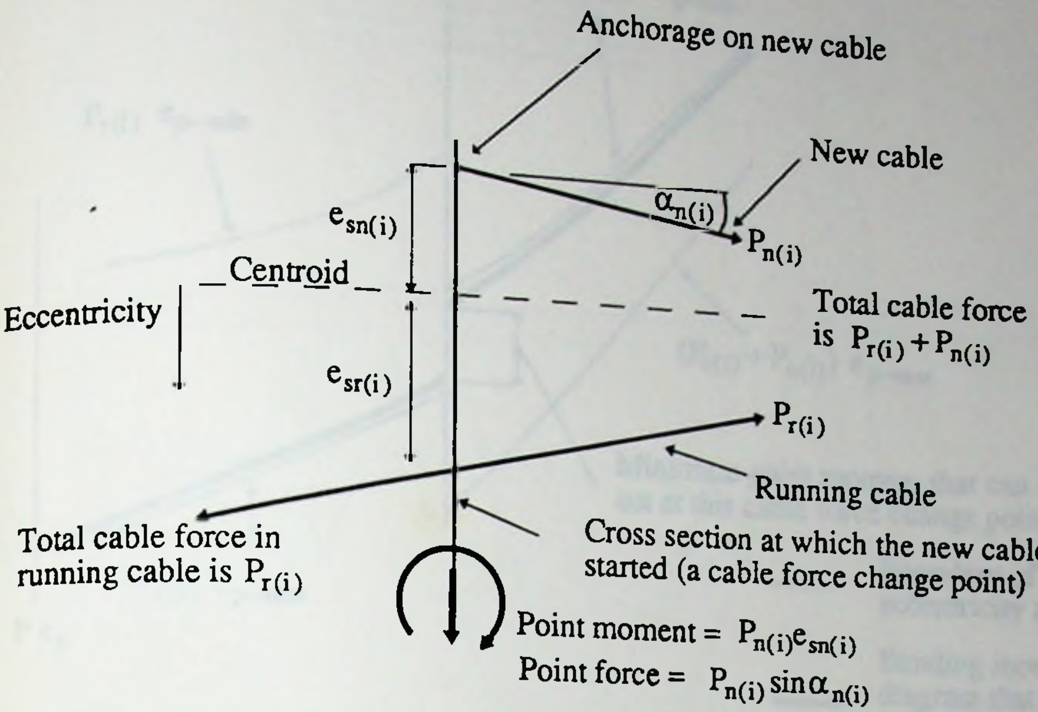

Figure 5.11: Forces and moments at a point where the cable force changes

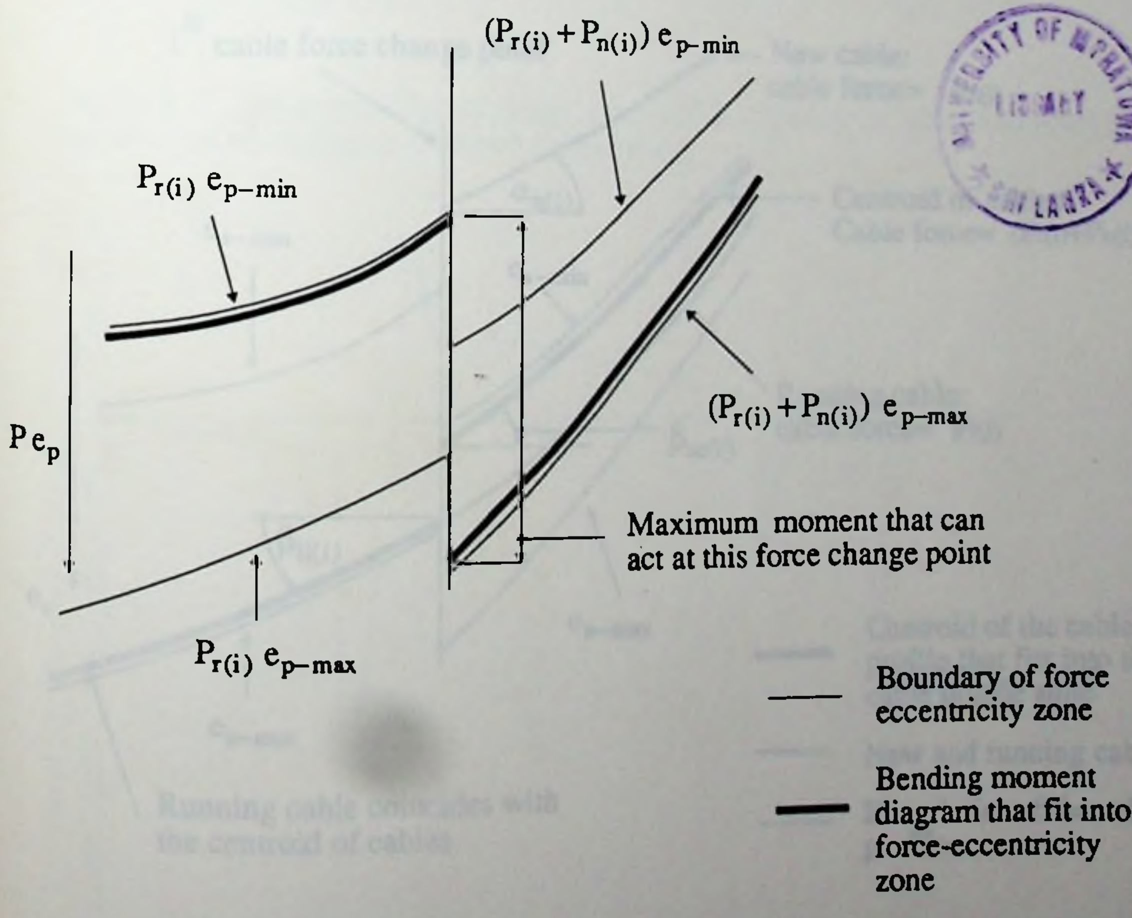

Figure 5.12: Possible shape for the cable profile at a change point to get the maximum moment

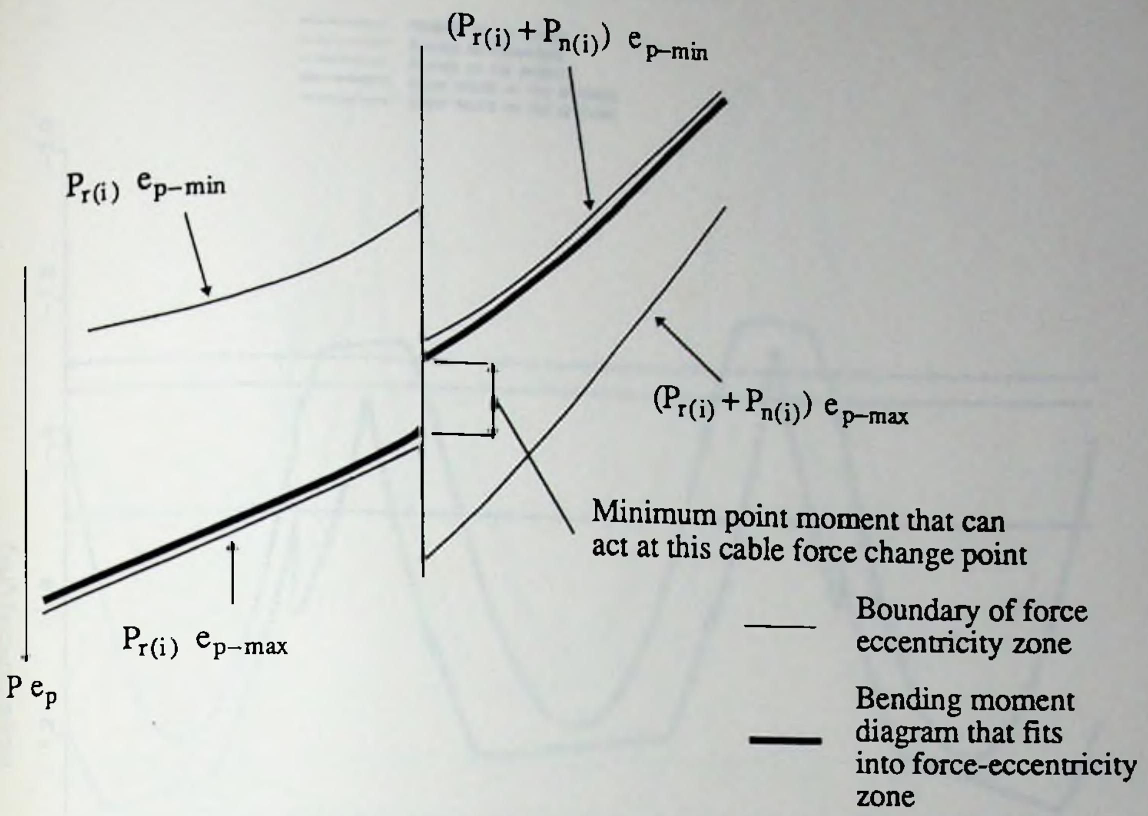

Figure 5.13: Possible shape for the cable profile at a change point to get the minimum moment

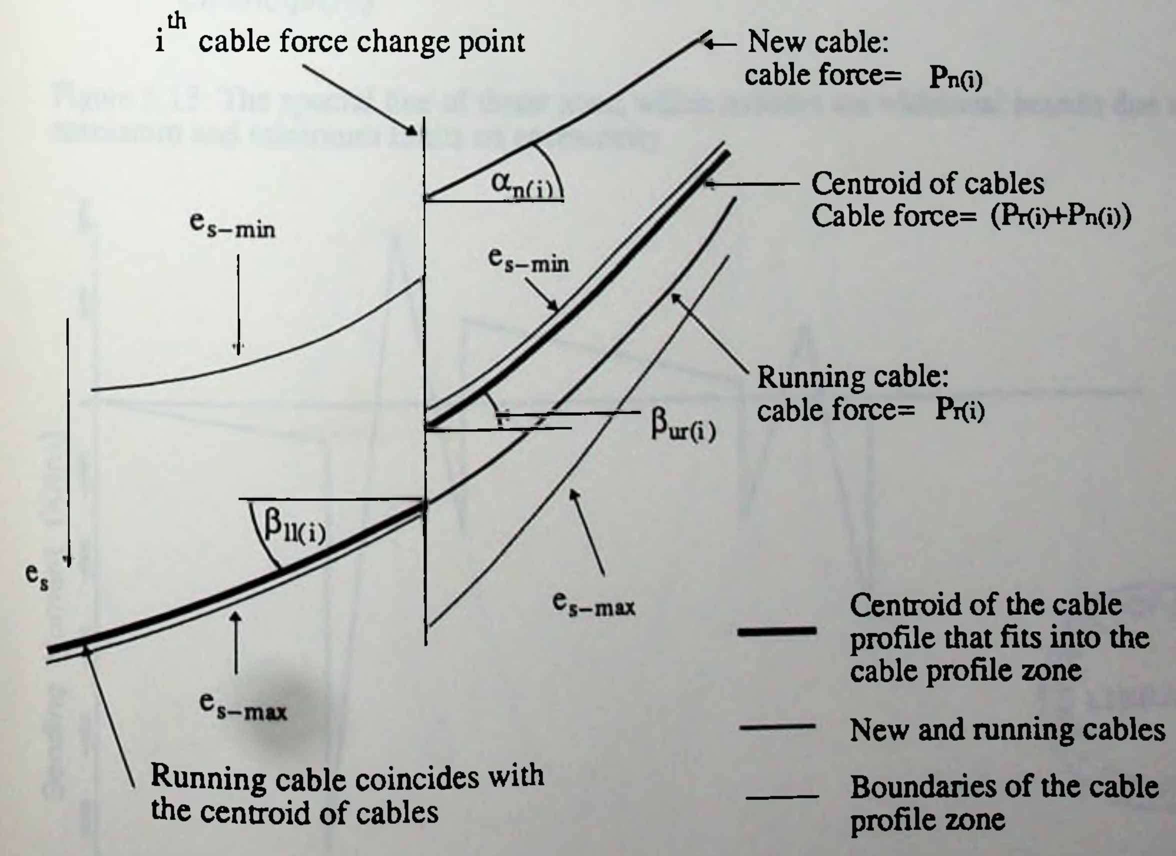

Figure 5.14: The inclination of the resultant cable placed at the limit of eccentricity at a section where the new cable is started (e_s, not e_p because dealing with angles which are distorted in the plot of e_p)

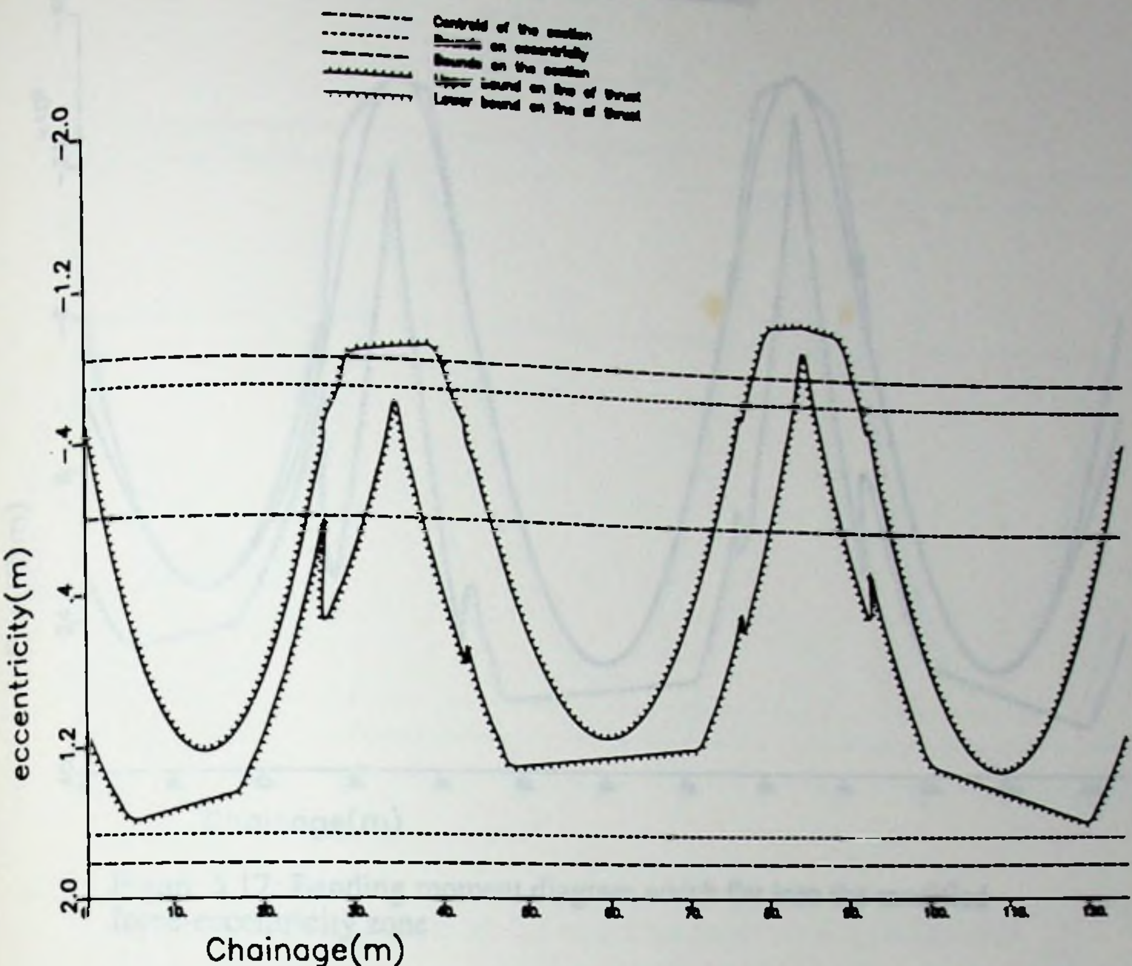

Figure 5.15: The special line of thrust zone, which includes the additional bounds due to maximum and minimum limits on eccentricity

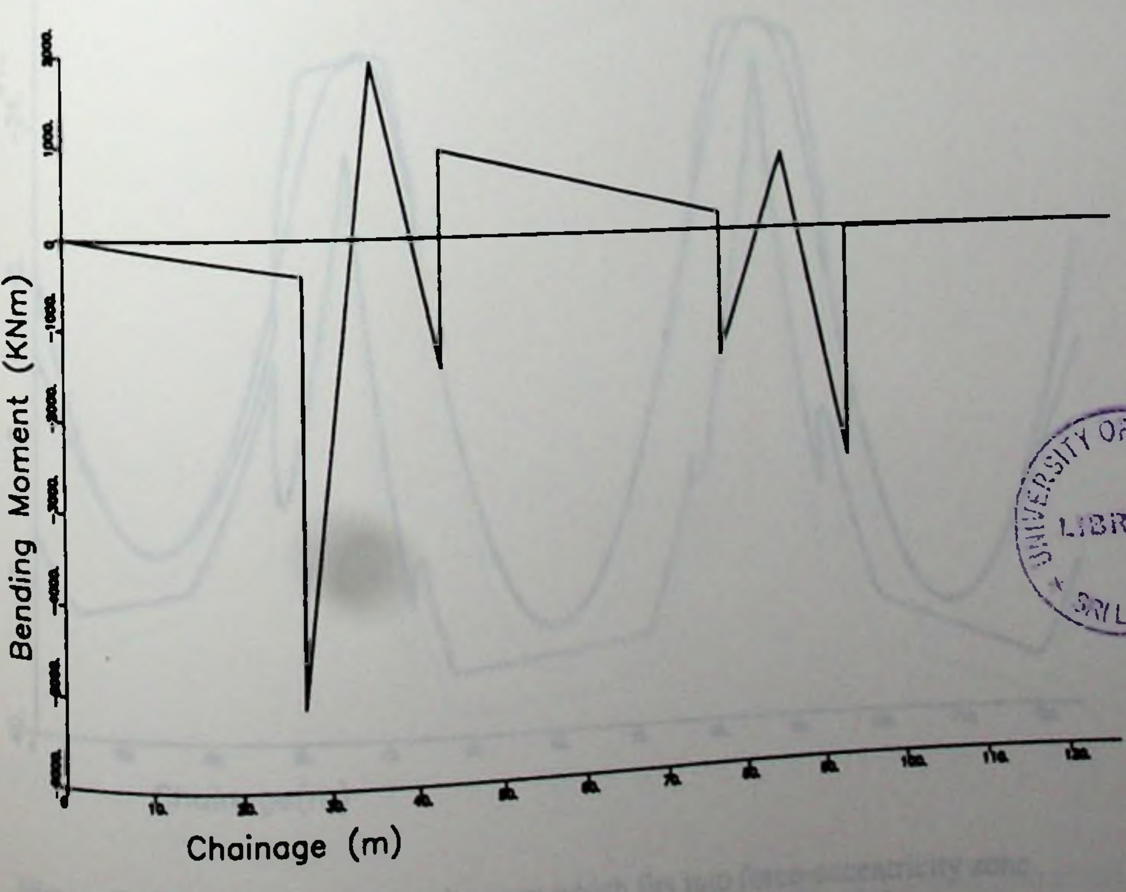

Figure 5.16: Bending moment due to point forces and point moments

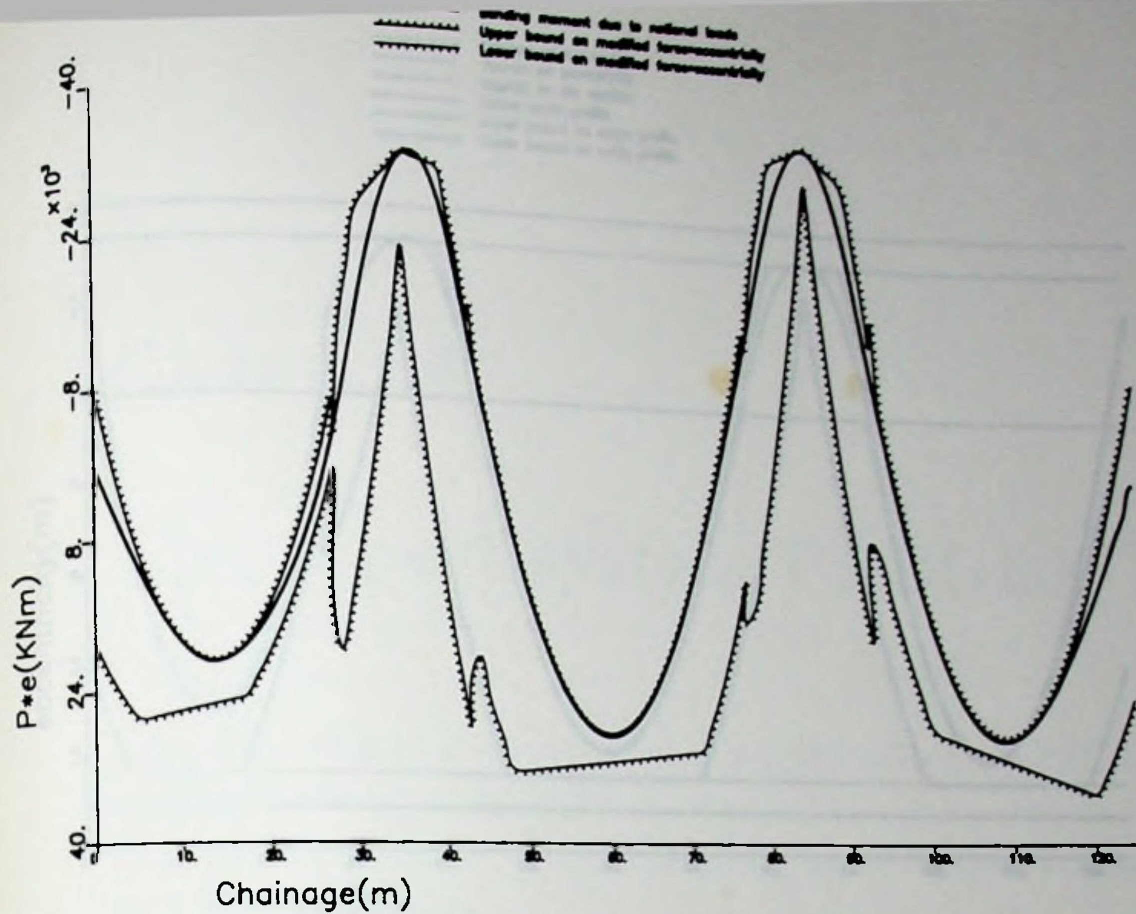

Figure 5.17: Bending moment diagram which fits into the modified force-eccentricity zone

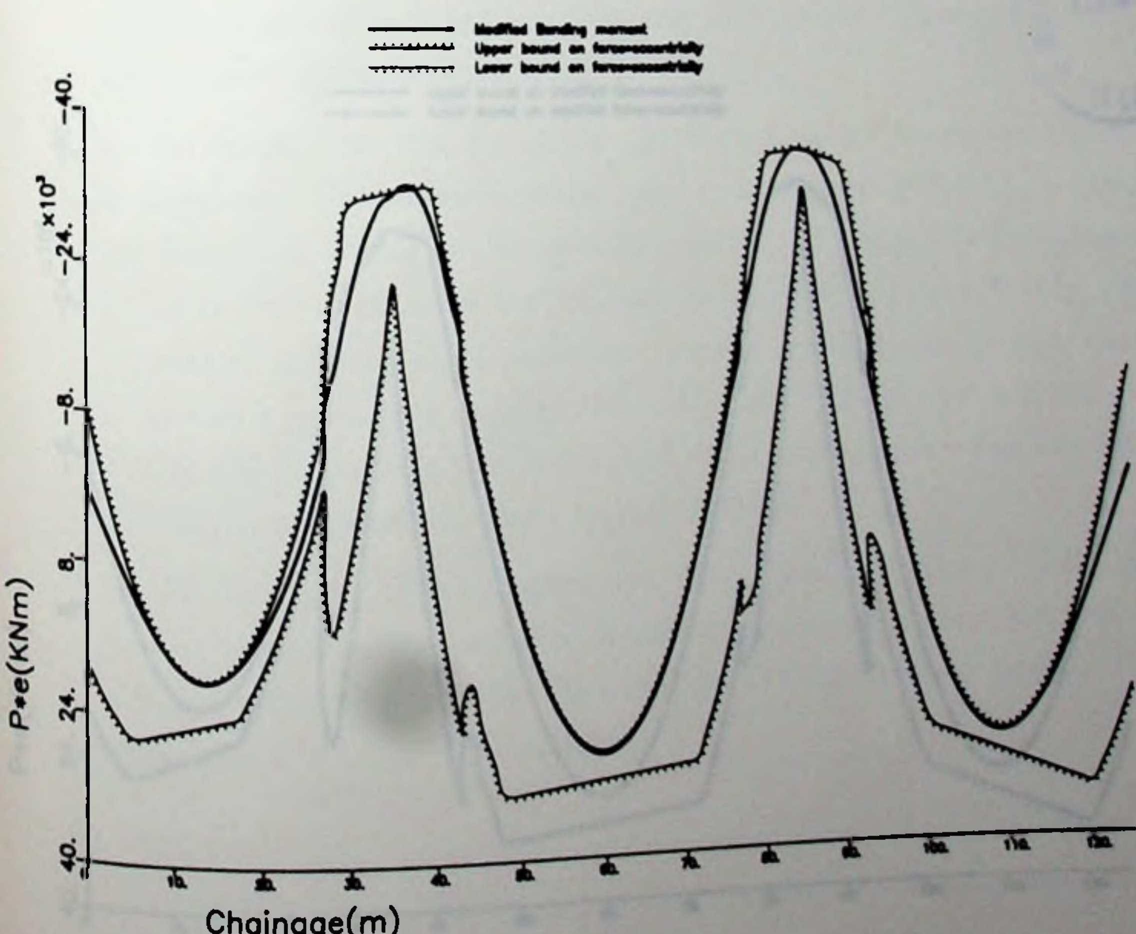

Figure 5.18: Bending moment diagram which fits into force-eccentricity zone

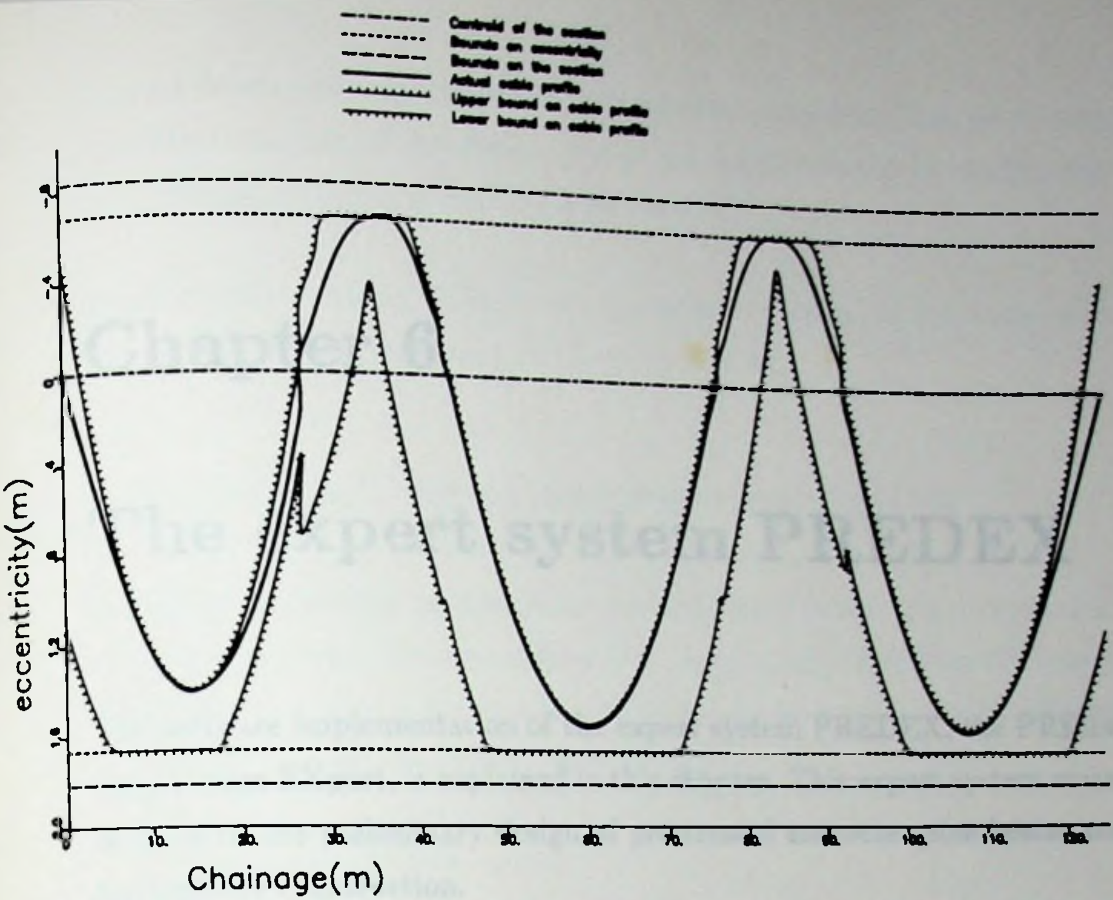

Figure 5.19: Actual cable profile with boundaries of cable profile zone

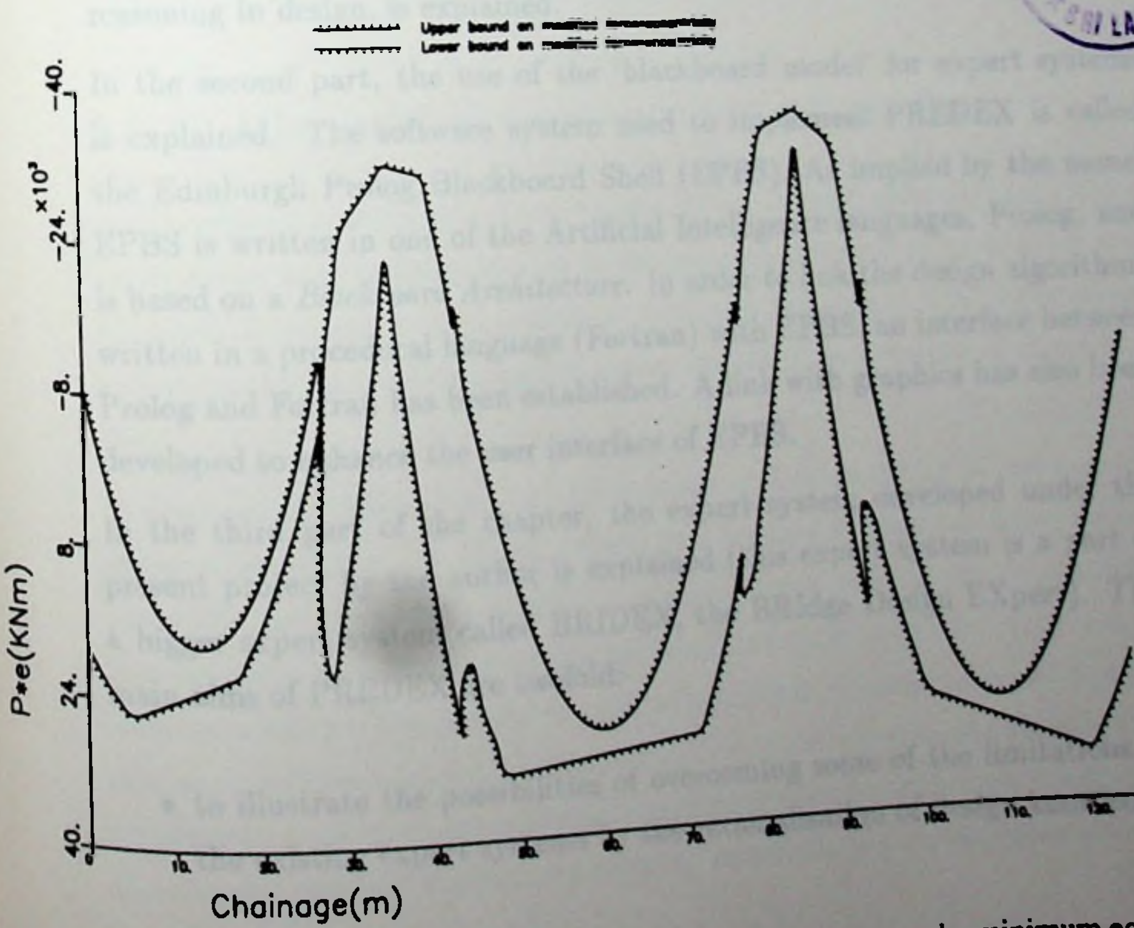

Figure 5.20: Modified force-eccentricity zone when the limits on the minimum eccentricity of anchor at first change point is violated

Chapter 6

The expert system PREDEX

The software implementation of the expert system PREDEX, the PREliminary Design EXpert, is explained in this chapter. This expert system mainly deals with the preliminary design of prestressed concrete spine beams used for highway construction.

This chapter is divided into four parts. In the first part, the background information pertaining to expert systems, such as the importance of proper modelling of engineering design as a hierarchy and the need for non-monotonic reasoning in design, is explained.

In the second part, the use of the 'blackboard model' for expert systems is explained. The software system used to implement PREDEX is called the Edinburgh Prolog Blackboard Shell (EPBS). As implied by the name, EPBS is written in one of the Artificial Intelligence languages, Prolog, and is based on a *Blackboard Architecture*. In order to link the design algorithms written in a procedural language (Fortran) with EPBS, an interface between Prolog and Fortran has been established. A link with graphics has also been developed to enhance the user interface of EPBS.

In the third part of the chapter, the expert system developed under the present project by the author is explained (this expert system is a part of a bigger expert system called BRIDEX, the BRIdge Design EXpert). The main aims of PREDEX are twofold:-

- to illustrate the possibilities of overcoming some of the limitations of the existing expert systems by the rationalisation of design knowledge

- to develop an expert system with knowledge modules that contain the least number of heuristics, which are supplemented by design algorithms, to tackle complex design tasks.

A considerable amount of the knowledge incorporated in the knowledge modules of PREDEX is explained in Chapters 4 and 5.

6.1 Modelling of engineering design

In engineering design, problems are generally partitioned into a hierarchy of sub-problems to handle the complexities. According to Engelmore & Morgan (1988), the way that a problem is partitioned makes a great deal of difference to the following aspects of an expert system:-

1. The clarity of the approach.
2. The speed with which the solutions are found.
3. The resources required.
4. The ability to solve the problem at all.

6.1.1 Hierarchical decomposition of problem

A hierarchical system is defined as one which is composed of inter-related subsystems; each of these in turn is hierarchical in structure until some lowest level of elementary subsystem is eventually attained. This hierarchical representation is used in complex design problems as a means of simplification.

In engineering design, two of the important characteristics of hierarchical systems highlighted by Simon (1969) can be found; they are *near decomposability* and *comprehensibility.* In design, the problems are decomposed into semi-independent sub-problems based on some property; this is *near decomposability.* The design of each sub-problem is undertaken with some degree of independence of the design of others; this is *comprehensibility.* This strategy is applicable since the sub-problems interact with each other predominantly through a selected property, unconstrained by the intricacies of the mechanisms which accomplish the solution to the sub-problems (Simon (1969)).

As an example, during the preliminary design stage of spine beams, the main goals are the selection of section dimensions, cable forces and cable profile. These tasks are not fully independent because there is a certain amount of interaction (if suitable dimensions are not selected for the section dimensions then it may be impossible to find a cable profile), but with suitably selected design parameters, each task can be carried out independently of the others.

Hierarchical decomposition is a very important concept for expert systems. This is because of the possibility of arranging the knowledge in modular form. The ability of knowledge modules to operate as self contained units is an indicator of a correct hierarchical decomposition.

One of the problems caused by the interaction of sub-problems is that solutions to one sub-problem may depend upon the decisions made in the others. One solution is to defer the decisions which bind the knowledge modules; this is called *the least commitment principle* (Sriram (1986)). In essence, this means postponing the making of these decisions until more information is available.

In design this may not be a trivial task. For some of the key design parameters, the expert system may be forced to make decisions even though sufficient information is not available. This may lead the design to be based on assumed values, which may lack any rational basis. An example is the selection of the cross sectional layout. This is one of the first decisions to be made in the design process for prestressed concrete spine beams. If no rational criteria are available, a section has to be assumed based on past experience. However this may have severe repercussions because most of the subsequent calculations are dependent on the chosen cross sectional layout.

6.1.2 Non-monotonic reasoning in design

In expert systems, preliminary design solutions are generated incrementally. Generally, it is not possible to start generating the solution at one end of the problem and working forward, which is known as *'monotonic reasoning'*. Instead, a number of knowledge modules arranged at different levels of the hierarchy may contribute to the solution, so the expert system must be able to cope with going backwards when required; a process known as *'non-monotonic reasoning'*.

Non-monotonic reasoning.has to be supported by maintaining consistency when new conclusions may invalidate some previous conclusions. The previous *'set of beliefs'* may undergo changes as new conclusions are drawn. Conversely, in monotonic reasoning, conclusions are constantly added on and there is no need to check on the consistency of the set of statements held by the system at any one time, provided the rules used are themselves consistant. However, in the real world, it is very difficult to obtain a single consistant set of rules which covers all eventualities. Therefore, non-monotonic reasoning is better suited to deal with situations that may arise in real domains.

The reasons why the expert systems for design need to support non-monotonic reasoning have been identified by Sham (1989b) as the following:

1. Decision making using incomplete information and a vague design brief.
2. Revision of design assumptions.
3. Detection of contradictions in the decisions.
4. Relaxation or intensification of design constraints.
5. Change of design intentions or requirements.

The proposed design techniques are developed with the intention of minimising the need for non-monotonic reasoning wherever possible because this represents a more logical representation of the problem. Two strategies are used:

1. The design parameters are determined so that subsequent calculations can be performed without invalidating the previous ones (*e.g.* the selection of cable forces: the limits on the cable forces are derived such that the existence of a legitimate cable profile is guaranteed (see Section 5.2)).
2. An allowance is made for constraints or structural behaviours quantitatively so that the subsequent design checks at the detailed design stage will succeed (*e.g.* making an allowance for long term creep (see Section 7.4)).

These strategies, however, are not able to address the need for non-monotonic reasoning arising from causes such as selecting improper values for reactant moments at the internal supports. If the designer finds that the selected section dimensions are not sufficient, then either the reactant moments at the internal supports or the section dimensions have to be changed. However, emphasis has been placed on keeping these changes of design parameters localised by identifying explicitly the possible iterative loops.

6.1.3 Truth Maintenance Systems

In expert systems, non-monotonic reasoning is supported by maintaining the consistency of the solution, a process known as *'truth maintenance'*.

When new information is provided to an automated problem solver of an expert system, the computational model may have to be revised. The Truth Maintenance System (TMS) is a problem solver subsystem for performing this function by maintaining the reasons for the beliefs (Doyle (1979)), which can be visualised as shown in Figure 6.1.

A truth maintenance system manipulates two data structures: *nodes*, which represent beliefs, and *justifications,* which represent reasons for beliefs *e.g.* D is true because A and B are true; D is the node and A and B are justifications. There are three fundamental actions performed by a TMS:

1. It can create a new *node*, to which the problem solving program using the TMS can attach the 'statement of beliefs'; either *true* or *false*. In the example given above the statement of beliefs for node D is 'true'.

2. It can add (or delete) a new *justification* for a *node*. In this case the *justifications* are A and B.

3. It can mark a *node* as a contradiction to represent the inconsistency of any statement of beliefs.

The software used for the implementation in the present project, the Edinburgh Prolog Blackboard Shell, is equipped with a TMS based on the algorithm of Doyle (1979), which makes use of the concepts of *dependency directed backtracking,* (DDB), developed by Stallman & Sussman (1977). In DDB, a database is built up which keeps a record of all deduced facts, their antecedent facts, as well as their *support justifications;* these records are known as dependency records. *Support justifications* are justifications for any assumptions made during the search for a solution. When the problem solver fails without producing a solution, it is able to backtrack using the DDB justifications rather than going through every step of the solution path. This results in much faster execution.

6.2 Blackboard model and systems

The blackboard architecture is a problem solving framework originally developed for the HEARSAY *II* speech understanding system. It treats problem solving as an incremental, opportunistic process of assembling a satisfactory configuration of solution elements. This process could be considered as similar to a group of people trying to solve a problem by writing their decisions on a common place, such as a real blackboard.

According to Hayes-Roth (1985), the blackboard architecture is based on three assumptions:

1. All solution elements generated during problem solving are recorded in a structured, global database called *'the blackboard'*.

2. These solution elements are recorded on the blackboard by independent processes called knowledge sources, which represent the individuals in the group discussion. Knowledge sources have a *condition* and *action* format. The condition describes situations in which the knowledge sources can contribute to the solution of the problem (the action).

   Each change to the blackboard constitutes an event that, in the presence of other specific information, can trigger one or more knowledge sources. Each such triggering produces a unique *knowledge source activation record* (KSAR). The KSAR represents a unique triggering of a particular knowledge source by a particular blackboard event. In the human example, when one member of the group puts something on the blackboard, the other members can say, *'But that is false because ......'*, or they can say, *'Using that information I can conclude that ......'*; each would be a separate KSAR.

Knowledge sources are independent, in that they do not invoke one another, and generally have no knowledge of each other's expertise, behaviour or existence. However, they are cooperative in that they contribute solution elements to a shared problem through the blackboard.

The blackboard architecture achieves simultaneous independence and cooperation among knowledge sources by permitting them to influence one another's problem solving behaviour only indirectly, by anonymously responding to and modifying information recorded on the blackboard.

3. In each problem solving cycle, several KSARs may compete to execute their actions. A monitor is thus needed to assess them and a scheduling mechanism determines which KSARs execute their actions, and in which order; this is like the chairman of the group intervening when more than one member is talking.

The components of a blackboard system can thus be summarised as follows (Nii (1986)).

1. **The knowledge modules:** The knowledge needed for solving the problem is partitioned into knowledge sources, which are kept separate and independent.
2. **The blackboard data structure:** The problem solving data are kept in a global database, the blackboard. Knowledge sources produce changes to the blackboard that lead incrementally to a solution. Communication and interaction among the knowledge sources take place solely through the blackboard. The data contained in the blackboard can be input data, partial solutions, alternative solutions or final solutions.
3. **Control:** There is no control component inherent in the blackboard model. Software implementations, however, provide the control structures, which can thus differ from one application to another.

The knowledge modules communicating through the blackboard can be visualised as shown in Figure 6.2.

6.2.1 Hierarchical representation of the blackboard

An important characteristic of the solution emerging on the blackboard is that it consists of small pieces of information, each associated with its supporting evidence. Hence, the database of the blackboard should be structured for fast retrieval of information and should also be able to hold every bit of information during the consultation process. Thus, it is convenient for the blackboard data structure to be arranged as a hierarchy; the information associated with one level could be the input to the knowledge sources at another level which in turn place new information on the same or other levels. For this project, it is logical to make the hierarchy of the blackboard the same as the decision making hierarchy of the design techniques proposed for the preliminary design.

6.2.2 Suitability of the blackboard model for PREDEX

The design technique proposed for spine beams is hybrid in nature; it uses the knowledge coded as heuristics and also derives part of the knowledge from design algorithms during a *consultation session*. Hence the solutions generated are evolutionary; they are not based fully on predefined knowledge included in knowledge modules. The blackboard model is ideal for evolving solutions since it can keep a record of the decisions made, and the knowledge derived during a consultation session.

6.2.3 Edinburgh Prolog Blackboard Shell

The Edinburgh Prolog Blackboard Shell (EPBS) has been developed by Jones *et al.* (1986) at the Artificial Intelligence Applications Institute (AIAI) of the University of Edinburgh. It is written in Prolog and is based on the blackboard model described in Section 6.2. It is a forward chaining rule-based system, which means that it works forward from what is already known to determine what can be derived from that information.

6.2.3.1 Entries in the EPBS

When an entry is posted on the blackboard, it takes the following form:

```
bb(Tag,Status,Index,Fact,Cf).
```

Tag is simply an identification number assigned to every entry by the EPBS.

Status is assigned to every entry to indicate its state at a particular moment. It will be one of the following:

- *in-* a new entry which is true.
- *amended-* an entry which has been changed; this changed entry is used to support the new 'in' entry.
- *inout-* is used when dealing with decisions to be taken when something is known to be false.

Every entry has one of these statuses depending on the reason for its presence on the blackboard.

Index represents a piece of information; *e.g.* width_of_the_beam_in_mm.

Fact is the current value of the parameter defined by *Index*; *e.g.* 300.

Cf is the certainty factor; 1 represents absolute certainty, and 0 complete uncertainty, with intermediate values representing partial truth. There could be a set of rules which would be used to calculate the certainty factors according to the principles of Bayesian logic, fuzzy logic, or other methods of dealing with imprecision (Forsyth (1984)). In the work presented in this thesis, Boolean logic is used, with *Cf* taking the values 0 or 1 only.

In addition to these, the **supports** of the entry are used to record the relationship between a new entry and others on the blackboard. This record is kept in a separate data structure as a collection of unit clauses of the following form:

```
supports(Tag1,Tag2)
```

This means *Tag2* is added to the blackboard because *Tag1* holds. These supports are used by the Truth Maintenance System to maintain consistency.

6.2.3.2 Syntax of rules of the EPBS

A typical rule of the EPBS has the following syntax.

`If`	*Condition*
`then`	*Goal*
`to`	*Effect*
`est`	*Est.*

The terms on the right are described below.

Condition

The *Condition* of a rule is a combination of tests, which finds the presence or absence of certain entries on the blackboard.

Goal

Goal is the action to be taken when the test for *Condition* is successful. Typically, the action taken may be the calculation of a certainty factor for an entry, the performing of a substantial manipulation, or the carrying out of a numerical computation. When no specific action is needed, the Prolog goal *'true'* can be used, since it always succeeds.

Effect

The *effect* of a rule can be one of the following:

- **add,** which adds an entry *Fact* on the blackboard under an *Index* with certainty factor *Cf*.
- **amend,** which amends an entry on the blackboard.
- **delete,** which deletes an entry on the blackboard.
- **action,** which allows the user to have an arbitrary Prolog goal as the conclusion of a rule; typically to print some result on the screen.

Est

Est is used to represent the *Estimate* of the usefulness of a rule. During each execution cycle, there may be more than one knowledge source activation record awaiting execution, thus some resolution of conflict is necessary to determine which should be executed first. In the default system provided by the EPBS, arithmetic expressions provided by the user are used as *Est*; the rule with the lowest *Est* is executed first. Hence, even for a knowledge base consisting of a number of knowledge modules, the rules in each knowledge module have to be assigned *Ests* so that they comply with the firing sequence of the complete knowledge base. Thus, the whole concept of modularity of knowledge modules is lost. In order to overcome this problem, the default is replaced by symbolic *Est* values; this is similar to the method adopted in INDEX (Kumar & Topping (1988a)).

The technique used for this is the following. Each *Est* parameter has been assigned a *name* and a *value*. The *name* is the same for all rules within each knowledge module, with the *value* defining the priority of rules within that module. The rule with the lowest *value* has the highest priority within a module. The scheduler uses the *name* to give an order of precedence to the knowledge modules before considering the *value*. For example consider the following two rules.

```
if      [sel_sec1,no_of_lanes(No_of_lanes),_ > 0]
then    ask_user1(width_of_lane,Width_of_lane,Cf)
to      add[sel_sec1,width_of_lane(Width_of_lane),Cf]
est     sel_sec(1500).
```

This rule states that:-

If the number of lanes are known
then ask the user what the width of each lane is
and add this information to the blackboard with a certainty factor of Cf.

This rule has an *Est* value of *sel_sec(1500)*.

```
if      notnow[sel_sec2,min_cable_force(loaded),_ > 0]
then    load_min_cable_force(Cf)
to      add[sel_sec2,min_cable_force(loaded),Cf]
est     cable_force(600).
```

This rule, which has an *Est* value of *cable_force(600)*, states that:-

If the procedural routine to find the minimum cable force is not loaded
then load it
and add *loaded* on the blackboard with a certainty factor of Cf.

In PREDEX, the sub-module for the selection of section dimensions *(sel_sec)* has precedence over the sub-module to select the cable forces *(cable_force)*. Therefore, *sel_sec(1500)* has precedence over *cable_force(600)*. It is also necessary to note that *sel_sec(100)* will have precedence over *cable_force(600)*. Within each sub-module, the order of precedence is determined by the numerical value.

6.2.3.3 Scheduler of the EPBS

The scheduler is the control mechanism which determines the sequence of execution of tasks. Subsequent to the addition of one or more entries to the blackboard, the rules that can possibly utilise this new information, have to be identified. These rules are placed on an agenda. Based on the *Est* values, the scheduler selects one pending task on the agenda for execution.

Detailed descriptions of the agenda and scheduler of the Edinburgh Prolog Blackboard Shell can be found in Chan & Johnson (1987). They have not been changed in the present implementation.

6.2.3.4 Front end facilities of the EPBS

The Edinburgh Prolog Blackboard Shell has a number of front end commands which are useful during the development stages of an expert system (Chan & Johnson (1987)). These can also be used to provide limited explanation facilities (Kumar & Topping (1987a)). The following are some of them:

1. **show this** - displays a summary of the current cycle (*i.e.* the knowledge source activation record which has just been carried out).
2. **show next** - displays a summary of the next cycle (*i.e.* the next knowledge source activation record that will be performed).
3. **show changes** - displays the entries that went into or out of the blackboard.

4. **show agenda** - displays the current agenda of rules available for execution.
5. **show supports** - displays the support relationships.
6. **show entries** - displays the entries on the blackboard.
7. **show causes** - displays the justification for the user entries.

6.2.3.5 Extensions to the EPBS

In order to enhance the capabilities of the Edinburgh Prolog Blackboard Shell, certain features, such as interfaces with procedural languages and graphics, have been developed in the course of this work.

Prolog, C and Fortran interface

The design algorithms developed for the spine beam design are written in Fortran. Edinburgh Prolog supports a direct interface with C (Hutchings (1986)), but does not provide any direct interface with Fortran; however, precompiled Fortran functions can be called from C. The interface between Prolog and Fortran is thus provided through C. Development of this interface is important for this project because it enhances the numerical processing capabilities of the EPBS substantially. This interface can be visualised as shown in Figure 6.3, where the C and Fortran routines share common storage.

In the present version of Edinburgh Prolog, only simple data types, such as integers, floating point numbers and character variables, can be passed between C and Prolog, but not arrays of these items. Special functions are provided within C to pass these simple elements into/out of Prolog clauses.

In the present implementation of the expert system, these functions have been used where only a small amount of data has to be transferred. When larger amounts of data are required, they are first written to a file by one language and then read by the other. Only the file name is passed directly between routines.

More detailed information about the Prolog-Fortran interface can be found in Kumar *et al.* (1987).

Graphical interface

According to Jain & Maher (1988), there are three main reasons for providing graphical facilities with expert systems:

1. Human beings assimilate information presented in graphical form more quickly and easily than if the same information is in text form (one picture is worth a thousand words).
2. Textual representation of information is usually more voluminous than the corresponding graphical representation.
3. While specifying design information, it is more convenient and natural for the user to enter it graphically where appropriate.

The graphical representation can be classified into two parts:

1. Static drawings- remain the same for all executions. These are usually used for providing input assistance.
2. Dynamic drawings- vary with the data produced by the expert system.

A dynamic interface is very important for presenting the output at each stage of the design process. In PREDEX, bending moment envelopes, permissible limits on the cable forces and cable profiles are presented graphically.

This graphics interface has been developed using a graphics package called *'GIPS'*, which has been developed by Burgoyne (1990b). This graphics package was designed to be called interactively as a subroutine from Fortran programs, and was found to be ideal for interfacing with the expert system. The data input is in the form of a text file, and hard copy output can be sent either to printers or plotters. Since the expert system uses the ***windowing*** facilities offered by *Xwindows* running under the UNIX operating system, the graphics window can be placed away from the text windows thus allowing access to both graphics and text at the same time.

6.3 The expert system

The BRIdge Design EXpert, BRIDEX, is an expert system intended to tackle the design of prestressed concrete spine beam bridges. Therefore, when it is completed, it should incorporate the design process at four main stages. These stages are the following:

1. Conceptual design.
2. Preliminary design.
3. Detailed design.
4. Design documentation.

Therefore, BRIDEX should consist of four main knowledge modules; a number of sub-modules may be associated with each of these. Of the main modules, that for the preliminary design (PREDEX) is developed in the present project by the author.

6.3.1 Features of knowledge module for preliminary design (PREDEX)

The knowledge module PREDEX incorporates the design technique proposed for prestressed concrete prismatic spine beams in Chapters 4 and 5, along with associated knowledge expressed as heuristics. The main goals of this development are to overcome some of the limitations of the existing expert systems outlined in Section 2.4.3, by using the knowledge gained through unravelling the underlying design principles. Some of the limitations addressed are the following:

- Inadequacies in reasoning knowledge are usually exemplified by heuristics; "we do *A* when *B,C,D,....* circumstances are true". A heuristic states *what* should be done, but fails to explain *why* the action is needed. This is not desirable. This problem is addressed in the expert system by trying to minimise the number of heuristics by using suitable design algorithms. These design algorithms are developed on the basis of explicitly identified structural behaviours and constraints which govern the key design parameters associated with each goal or sub-goal. These governing behaviours and constraints themselves answer the question *'why'* design parameters should have certain values.

When heuristics are used, wherever possible, some justification is attached to these rules, which in essence addresses the question *'why?'*.

- Inadequacies in domain factual knowledge are more serious, since they relate either to not including a governing condition, or to doing something at the wrong stage of the design process. The first error can lead to the expert system chasing a solution which is either false or non-existent, while the second can lead to the expert system fixing a parameter when it ought to be allowed to vary. Hence the expert system should address two problems; *'what'* should be done and *'when'* should it be done.

  1. *'what'* should be done in the process of selecting the design parameters is addressed in two phases. Firstly detailed studies are carried out into the design process to unravel the design principles that will govern the values for the key design parameters. Secondly design algorithms are developed to determine the corresponding governing limits to provide guide-lines for the designer (for example, the principles underlying the limits on the cable forces are unravelled and then design algorithms are developed to calculate these limits).
  2. *'When'* an action is to be taken is addressed in the expert system by explicitly identifying the design goals and the sub-goals and also the interactions between them. This enables the establishment of a goal structure which looks like a chronologically evolving series of tasks. The aim is to ensure that the user is able to visualise the design process as a logically consistent evolving process. This is explained in more detail in Section 6.3.2.

- Inadequacies in solution progression are associated with the way that expert systems work. This is addressed mainly with the facilities provided by the software as already explained in Section 6.2.3. However, this is supplemented by some of the features of the proposed design techniques.

  1. Database support - This is provided by the blackboard model since the blackboard is a global database.

2. Temporal reasoning - Once 'why' and 'when' a certain decision should be taken are known, it is possible to reason about the partial solutions at various stages of the design process.
3. Truth maintenance - The Edinburgh Prolog Blackboard Shell is equipped with a truth maintenance mechanism. However, this process is simplified by the proposed design technique since an effort has been made to minimise the backward loops in the design process.

Some of these aspects are discussed in the following sections with respect to the implementation of the expert system, in more detail.

6.3.2 Explicit representation of design goals

There are three main design goals at the preliminary design stage as shown in Figure 6.4. Each main goal consists of a number of sub-goals. The sub-goals for the selection of section dimensions are presented in Figure 6.5.

This is a hierarchical representation of the goals and sub-goals known as a part-of hierarchy in which the entity at the lower level is a part of that immediately above.

This type of hierarchical representation of goals is useful to understand the key design decisions. However, it is not sufficient as a representation of the way that these goals are achieved. Thus, a more comprehensive hierarchical decomposition should represent the constraints and structural behaviours which govern the design parameters and more importantly, when the decisions should be taken.

Hence, the hierarchical structure adopted for the selection of section dimensions in PREDEX is illustrated in Figure 6.6. The important features are the representation of *time* and the *interaction of key design parameters*. As an example, the thickness of the top flange depends on the web spacing. Thus, the web spacing acts as a constraint on the thickness of the top flange (the interaction between the key design parameters). It is also necessary determine the web spacing before determining the thickness of the top flange (when to take the decision).

6.3.3 Structure of PREDEX

The knowledge module PREDEX is mainly concerned with three main tasks, the selection of section dimensions, cable forces and cable profile. Hence, three sub-modules have been developed to achieve each of these tasks. A detailed description of each of these sub-modules is presented.

6.3.3.1 Sub-module for the selection of section dimensions

This is the starting point of the preliminary design process. Hence, there is a large amount of input data which is necessary for the calculations. According to MacLeod & Rafiq (1988), the design data processed in the expert system can be broadly divided into two categories.

- **Design parameters** - These are the parameters which will directly describe the object under consideration and are hence generally derived by the expert system. Examples are the thickness of the top flange, which is selected by the expert system, or the depth of the section, which is obtained from the user.
- **Design support parameters** - These are the parameters which are usually obtained either as input from the user or from a database. Design support parameters do not directly describe the object under consideration, but influence the parameters of the object. An example is the allowable compressive stress which can influence the required thickness of the bottom flange.

In PREDEX, the majority of design support parameters and some of the design parameters are obtained from the user.

Use of rules

In this sub-module, the section dimensions of an idealised section are selected using the rules and design algorithms. Of these rules, heuristics have been used only for the selection of the top flange thickness, because no rational design algorithm has been developed so far. This is an example of using heuristics instead of design algorithms because the required knowledge is not yet available. An example of such a heuristic is:

```
if     [sel_sec1,type_of_top_flange(transversely_reinforced),_>0]
and    [sel_sec1,web_spacing(Web_spacing),_>0]
and    holds (Web_spacing>3.0, Web_spacing=≤4.5)
then   true
to     add[sel_sec1,minimum_top_fl_thickness(0.25),1]
est    sel_sec1(800).
```

This rule states that:-

If the top flange is transversely reinforced
and the web spacing is known
and is between 3.0m and 4.5m
then proceed and take no specific action (the *goal* is *true*)
and add on the blackboard that the minimum thickness required for top flange is 0.25m.

This heuristic rule is based on the parametric study by Swann (1972), but the proposed thickness could be inadequate in some instances, because it does not give any regard to the loads acting on the section.

There are other rules in the same sub-module which are not heuristics, but are based on sound principles such as the provision of a sufficient thickness for construction. An example is the selection of web thickness.

```
if     [sel_sec1,type_of_construction(cast_insitu),_>0]
and    [sel_sec1,ducts_in_web(yes),_>0]
and    [sel_sec1,type_of_cables(draped_cables_with_anchors),_>0]
then   true
to     add[sel_sec1,thickness_of_each_web(0.35),1]
est    sel_sec1(900).
```

This rule states that:-

If the type of construction is cast *in situ*
and there are ducts in the webs
and there are draped cables with anchors
then proceed and take no specific action (the *goal* is *true*)

and add on the blackboard that the thickness required for each web is $0.35m$.

Use of design algorithms

As described in Section 4.3.6, a design algorithm has been developed for the determination of the thickness of the bottom flange. This algorithm has been integrated into the expert system. The thickness required for the bottom flange depends on the magnitudes of the bending moments acting at the critical sections, which in turn depend on the values selected for the reactant moments at internal supports. All the possible design scenarios have been identified in Section 4.3.6.1; the expert system has thus been written so that it supports all these different scenarios, which involve localised iterative cycles. The localised iterative cycles have been implemented using a facility provided in EPBS to amend previously known facts. An example of an interactive consultation session with the user is given below, where the selected bottom flange thickness is not sufficient (typewriter font is used to represent text on the screen):

```
Bottom flange thickness provided is   0.175m

Bottom flange thickness required at span sections
at chainage       15.00m is       0.032m
at chainage       60.00m is       0.069m
at chainage       108.00m is      0.058m

Bottom flange thickness required at support sections
at chainage       35.00m is       0.192m
at chainage       85.00m is       0.194m

Is the thickness of the bottom flange sufficient?

      1.  yes
      2.  no

Please enter one number followed by a full stop.
|:    2.
```

If the already selected bottom flange thickness is not satisfactory, then there are a few possible options as follows:

(a). If the bottom flange thickness is not sufficient only at support critical sections then there are two options:

1. Change the reactant moments thus changing the total bending moments so that the already selected bottom flange thickness will be satisfactory.

2. Change the bottom flange thickness in order to provide the sufficient bottom flange thickness.

(b). If the bottom flange thickness is not sufficient at span critical sections then

3. Reduce the reactant moments thus changing the total bending moments so that the span critical sections will not be critical any more.

4. Change the thickness of the top flange only if you want to stick to the selected reactant moment ratios.

Please enter one number followed by a full stop.

|: 1.

In this example the user has decided to use the first option, which means that the previously selected reactant moments should be changed. This is done by using a facility provided by the EPBS for amending an entry on the blackboard. The corresponding rule is as follows:

```
if      [sel_sec2,change_react_mom_ratio1(yes),_>0]
and     [sel_sec2,new_react_mom_ratio1(found),_>0]
and     @[sel_sec2,find_reactant_mom_ratios(found),_>0]
then    true
to      amend[sel_sec2,find_reactant_mom_ratios(found),1]
est     sel_sec2(1800).
```

This rule states that:-

If the reactant moment ratios should be changed

and the new reactant moment ratios are already found

then proceed and take no specific action (the *goal* is *true*)
and enter on the blackboard that a new set of reactant moments are known in place of the previous entry.

This rule will cause the expert system to work through another cycle with new reactant moments. The expert system will find the thickness of the bottom flange area required for the new bending moments at the critical sections. The same set of options will be available for the user this time as well. If the user chooses to modify the thickness of the flanges, then the expert system will go through a larger cycle, which includes the re-calculation of the bending moment as well.

When the expert system goes through iterative cycles, it would be irritating for the users if the system needs them to input the design support parameters over and over again. This is prevented by using another facility provided by the EPBS. An example is as follows:

```
if    notnow[sel_sec2,cover_to_tendon_at_top(Cov_to_tendon_at_top),_>0]
then  ask_user1(cover_to_tendon_at_top,Cov_to_tendon_at_top,Cf)
to    add[sel_sec2,cover_to_tendon_at_top(Cov_to_tendon_at_top),1]
est   sel_sec2(1680).
```

If the cover to tendon at the top fibre is not known
then ask the user what the cover to cable at the top fibre is
and add this information to the blackboard with a certainty factor 1.

This rule uses a special facility provided with the Edinburgh Prolog Blackboard Shell, *notnow*, to ensure that the cover to the tendon at the top fibre level is found from the user only if it is unknown (not present on the blackboard). This in essence means that the design support parameter is obtained from the user only once in any consultation session.

This sub-module is the largest of the sub-modules of PREDEX and consist of 72 rules.

6.3.3.2 Sub-module for the selection of the cable forces

As described in Section 5.2, there are a number of limits on the cable forces in a continuous beam. They are:-

1. The minimum (P_1 and P_2) and maximum limits imposed by the Magnel diagram.

2. The additional limits P_3 and P_4, which need to be explicitly satisfied by the designer.

The steps of the design technique are the following:

1. Select the cable forces in the span sections so that they satisfy the minimum and maximum limits imposed by the Magnel diagram.

2. Determine the tendon profile zone required to find P_3 and P_4.

3. Determine the values of P_3 and P_4 over each intermediate support.

4. Select the cable forces over the supports so that these values satisfy the limits due to P_3 and P_4, and also the minimum and maximum limits due to the Magnel diagram.

A number of these steps require the integration of design algorithms written in a procedural language to perform the required calculations. Hence, the majority of the rules included in this sub-module are linked with design algorithms. An example is the rule:

```
if     [cable_force,min_prestress_force(loaded),_>0]
and    [cable_force,find_pre_cable_zone(found),_>0]
and    [sel_sec1,minimum_compre_stress_at_working_load(M_c_s_w_l),_>0]
and    [sel_sec1,maximum_tensile_stress_at_working_load(M_t_s_w_l),_>0]
and    [sel_sec1,loss_ratio(L_r),_>0]
and    [sel_sec2,cover_to_cable_at_top(Cov_to_tendon_at_top),_>0]
and    [sel_sec2,cover_to_cable_at_bottom(Cov_to_tendon_at_bottom),_>0]
then   find_min_cable_force(M_c_s_w_l,M_t_s_w_l,L_r,
       Cov_to_tendon_at_top,Cov_to_tendon_at_bottom)
to     add[sel_sec2,find_min_cable_force(found),1]
est    cable_force(200).
```

This rule states that:-

If	the procedural routine to determine the minimum prestress force is loaded
and	the cable zone required to find the minimum prestressing forces is found
and	the minimum compressive stress at the working load is known
and	the maximum tensile stress at the working load is known
and	a value has been assumed for the loss ratio
and	the cover to the cables at the top fibre levels is known
and	the cover to the cables at the bottom fibre levels is known
then	find the minimum cable forces for above parameters
and	add an entry on the blackboard that the minimum cable forces are found.

This rule calls a procedural routine that calculates the limits P_3 and P_4 at each intermediate support.

The number of rules in this sub-module is 17 and most of them are used to control the procedural routines, which carry out the necessary calculations to guide the designer.

6.3.3.3 Sub-module for the selection of the cable profile

The main task of this knowledge module is to find the cable profile, which meets the requirements that:-

- It should generate the reactant moments that are the same as those initially assumed.
- It should lie within the allowable cable profile zone, thus assuring that all the allowable stress limits are satisfied.

In this sub-module also, the majority of the rules are linked to the procedural routines that find the cable profile. In this process the user has to determine the position and the inclination of the anchors. Since there are maximum and minimum limits on the eccentricity of the anchors, these limits are presented to the user as guidance.

Another example to illustrate the user interface and the type of guidance provided by the expert system is:-

```
The eccentricity of anchors is measured downwards from the
centroid of the section. Hence the following sign convention
should be adopted.
Eccentricity measured downwards in metres= + ve
Eccentricity measured upwards in metres= - ve
The angle should be measured from horizontal (---->) in the
clockwise sense and should be in degrees.
There are limits set on the eccentricity due to the
shape of the cable profile and the cover required for
cables. Hence, the position of the cable should be
selected so that these limits are not violated.

The maximum eccentricity due to cover is 1.664m
The minimum eccentricity due to cover is -.686m

Enter the eccentricity and the angle at which the
cable is started at the second force change point where
the maximum eccentricity due to the shape of the allowable
cable profile zone is .517
and the minimum eccentricity due to shape of the allowable
cable profile zone is -.227
> 0.3 357.0
```

The information relevant to all the points where the cable force is changed, is obtained similarly.

The automated routine for the determination of the cable profile is an example of the use of the analytical design method described in Section 2.4.2.2. It is the responsibility of the designer to set realistic objectives. In this case, the designer is guided by the expert system providing the bounds on the design parameters such as cable forces and the eccentricities for anchors; these bounds can be used by the designer to select values for design parameters such that a solution exists.

Once the cable profile is found, the results can be conveyed to the user by the graphical interface. These results include the bending moment envelopes, upper and lower bounds of the cable forces, the cable profile zone, the line of thrust zone, and the actual cable profile. The data used to produce these graphical outputs are stored in data files, so are readily accessible to the user. The graphical output is produced by the system at the request of the user.

This knowledge module consist of 15 rules which obtain all the details pertaining to the determination of the actual cable profile, including those for graphical interface.

6.3.3.4 Summary of sub-modules

The sub-modules presented illustrate some of the important achievements that are possible with the rationalisation of the design technique, before developing the expert system. Since all the possible solution paths have been explicitly identified including any localised iterative cycles, the software implementation has been straightforward; this has reduced the burden that would otherwise be placed on the truth maintenance system to maintain consistency.

Since the design algorithms can deal with a large number of possible cases (as an example, the algorithm for the selection of bottom flange thickness can deal with beams of different sizes), the number of rules that should be included in each sub-module is drastically reduced; this has a considerable effect in reducing the computation time needed to reach a solution. It is also possible to determine the required values for the design parameters accurately.

The most important advantage is that the preliminary solution reached satisfies the majority of the acceptance criteria, and hence stands a better chance of success at the subsequent detailed design stage.

6.3.4 User interface

The user interface provided in the expert system is not highly sophisticated since it is considered as secondary to the main objective of the project. However, it is capable of providing sufficient information to the user regarding the input required as well as the design process involved. An example is the following which provides guidance for the user when selecting the cable forces:

```
There are minimum bounds on the cable force in order to ensure
that a concordant cable profile exists. It can be shown that
the magnitudes of the cable forces over the support regions
have a dominant effect on the existence of the concordant
profile. Hence, the cable forces required over each support
are calculated. The cable forces selected should be larger
than these limits (P3 and P4) and should also be within the
limits set by the maximum and minimum cable force allowed
in each region.

Please wait, I am calculating the tendon profiles needed
for P3 and P4.


I have found the tendon profiles needed for P3 and P4.

Please wait, I am calculating the limits on cable force.

P3 at support 2 is     15011.98100 kN
P3 at support 3 is     16293.14800 kN
P4 at support 2 is     25244.61100 kN
P4 at support 3 is     22280.31200 kN
Please enter the cable forces at internal supports.

Enter the magnitude of the cable force at support 2.

The allowable cable force range due to the Magnel diagram is
22993.15 to 52585.78, P3 is 15011.98 and P4 is 25244.61 KN.
```

This information is provided to the user, who then enters the appropriate value for the cable force over support 2.

6.3.5 Explanation facilities

The Edinburgh Prolog Blackboard Shell has some front end facilities such as *show causes*, *show entries*. These facilities have been used for INDEX (Kumar (1989)) and QED (Sham (1989b)) to provide explanations on the line of reasoning. An example is the command *'show entries'* which can be used to find the entries posted on the blackboard. These entries form a summary of the decisions made in any consultation session.

```
222 in 1
sel_sec2
cover_to_cable_at_top(0.2)
```

This blackboard entry states that the cover required by the cables at the top fibres is $0.2m$, where 222 is the tag assigned by the EPBS, 1 is the certainty factor, sel_sec2 refer to the knowledge module to which this entry belongs.

```
221 in 1
sel_sec2
bottom_flange_area(loaded)
```


This blackboard entry states that the procedural routines to find the bottom flange area have been loaded.

```
207 amended 1
sel_sec2
find_reactant_mom_ratios(found)
```

This blackboard entry states that the reactant moments have been amended.

However, it is considered that although on-line explanation facilities on the line of reasoning are a primary requirement in expert systems, in PREDEX, the explanations provided by these front end facilities will be of little value to the user for the following reasons:

1. The design task undertaken is complicated and hence could not be explained with the aid of a few statements for a user who is not familiar with the design process.

2. Most of the important design parameters are derived using design algorithms which are based on sound design principles. Hence, although the reason for selecting a particular design algorithm or even the reason for various bounds on the design parameters can be explained using the user interface, it is very hard to explain some of the design principles which need a thorough understanding of the domain.

Therefore, it is suggested that a better solution for this type of expert system is to provide a hard copy of a manual which will explain the underlying design principles to the user as comprehensively as is in Chapters 4 and 5 of this thesis. This type of manual could then be used by the novice engineers to understand the design process properly and by near-experts to clarify any doubts.

However, a print out of justification for the selected design consisting of entries from the blackboard might be needed for a certifying authority or even for a team meeting.

6.3.6 Knowledge elicitation

A topic which has not been described so far is *knowledge elicitation* from the human expert on prestressed concrete design. In expert system jargon, knowledge elicitation means obtaining the information, which the human expert uses to solve problems, by means of various interviewing techniques; a detailed description of the interviewing techniques used to elicit knowledge from various bridge designers for the conceptual stage of bridge design process is given in Moore (1991).

As described by Chung & Kumar (1989), knowledge elicitation is one of the 'bottle necks' that hinders the development of versatile expert systems. The main reason is that in many instances, human experts find it difficult to explicitly state the knowledge that they possess and use, thus it takes a considerable time to acquire the knowledge required for the expert system. When expert systems were first introduced, it was considered necessary to employ a 'knowledge engineer', whose job was to help the human experts to structure their knowledge and to elicit and identify the relevant concepts.

From the beginning of this project, it was considered that although the expert structural design engineers would be able to give valuable information for the preliminary design stage, there may be a certain degree of subjectivity inherent in this information. Thus, no direct knowledge elicitation was carried out by interviewing expert designers. However, a certain amount of informal discussions were carried out to obtain information on the way that preliminary design is performed.

The majority of the knowledge included in PREDEX was therefore developed in this project by rationalising the preliminary design of prismatic prestressed concrete spine beams.

6.4 Conclusions

One important feature of PREDEX is the computational simplicity that can be achieved by using design algorithms instead of heuristics. These algorithms are capable of providing guidance for a variety of problems. The designer is able to experiment with the design algorithms to obtain a better understanding of the effects of various design parameters. For example, PREDEX can be used to understand the way that the depth of the section affects the required thickness of the bottom flange, or the effect of the reactant moments on the cable forces.

Another advantage of the use of design algorithms is that the computer is used mainly as a numerical processor during a consultation session. This approach helps to obtain the required solutions accurately, and also reduces the number of rules in each module; both contribute to improving the performance of the expert system with respect to execution times.

It can be concluded that expert systems can play an active role as an adviser in the design process. This is mainly due to the fact that in structural design, there are a number of decisions where the designer should use intuitive judgement. It is almost impossible to automatically take into account all of these decisions, and an attempt to do so may result in unnecessary complexity in the development stage of the expert system. An example is the determination of the required thickness of the bottom flange. In this stage, there are a number of possible options available for the designer, and the actual selection may be subject to a certain degree of personal preference. One designer may prefer to use reactant moments to minimise the required thickness of the bottom flange and whilst another may be willing to try a number of different options to select the most economical design. Thus, the task of the expert system should be to guide the designer in any of these possibilities, as is done by a true expert. However, it is open to others to extend PREDEX, by suggesting rules, either from heuristics or design algorithms, so that more decisions can be taken by the system.

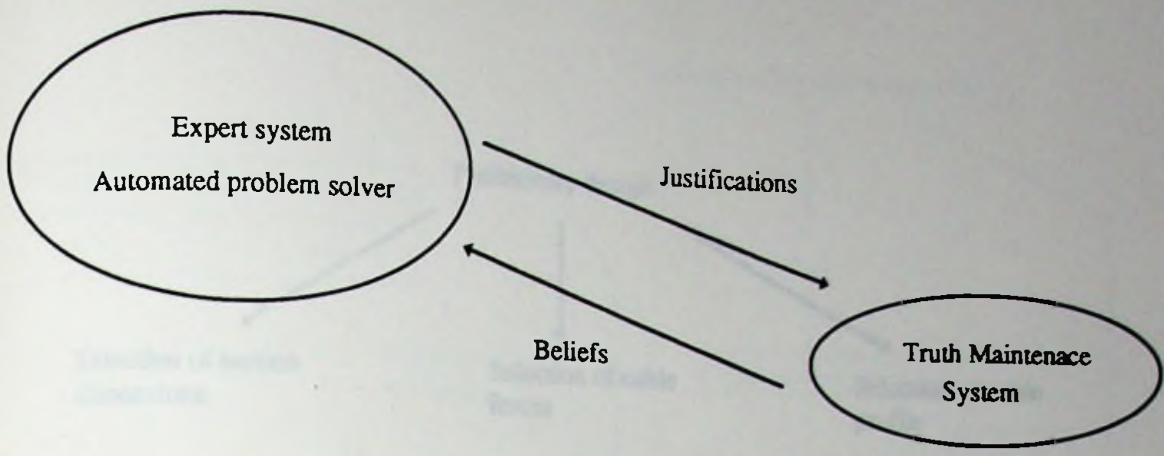

Figure 6.1: Coupling an expert system with a Thruth Maintenance System

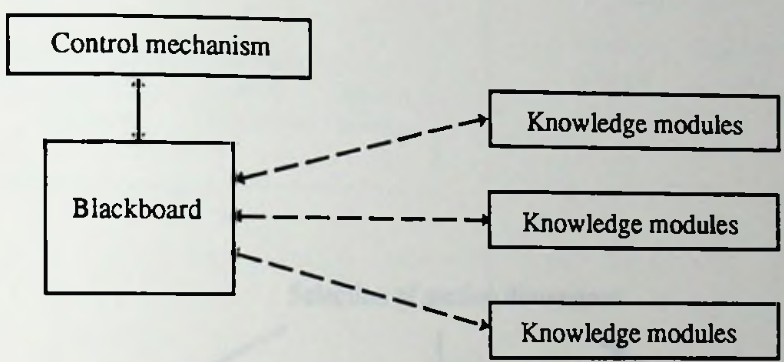

Figure 6.2: Knowledge modules communicating through the blackboard

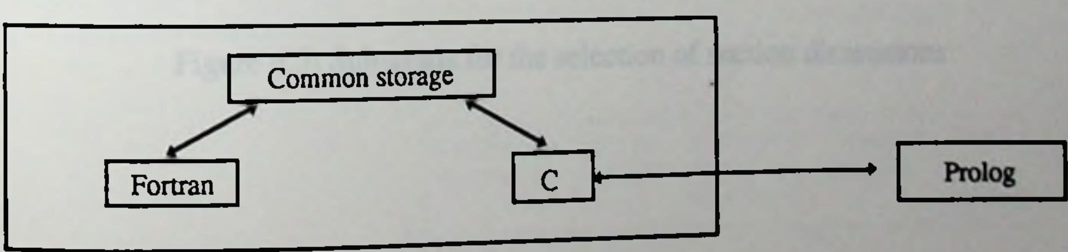

Figure 6.3: Prolog, C & Fortran interface

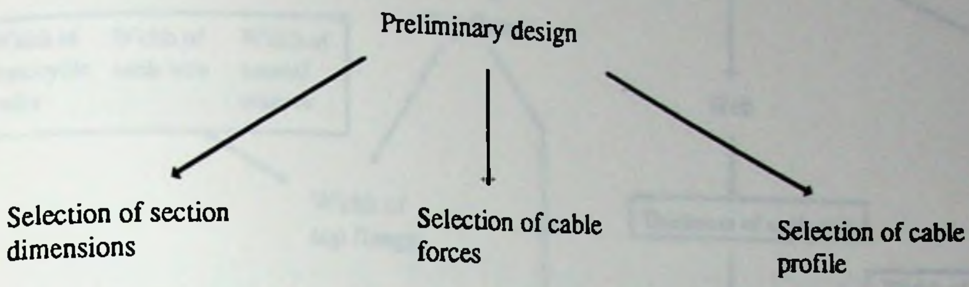

Figure 6.4: Main goals of the preliminary design process

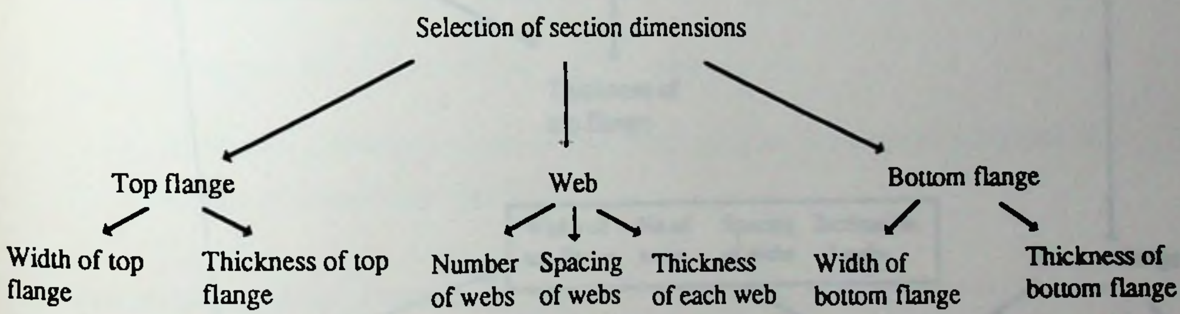

Figure 6.5: Sub-goals for the selection of section dimensions

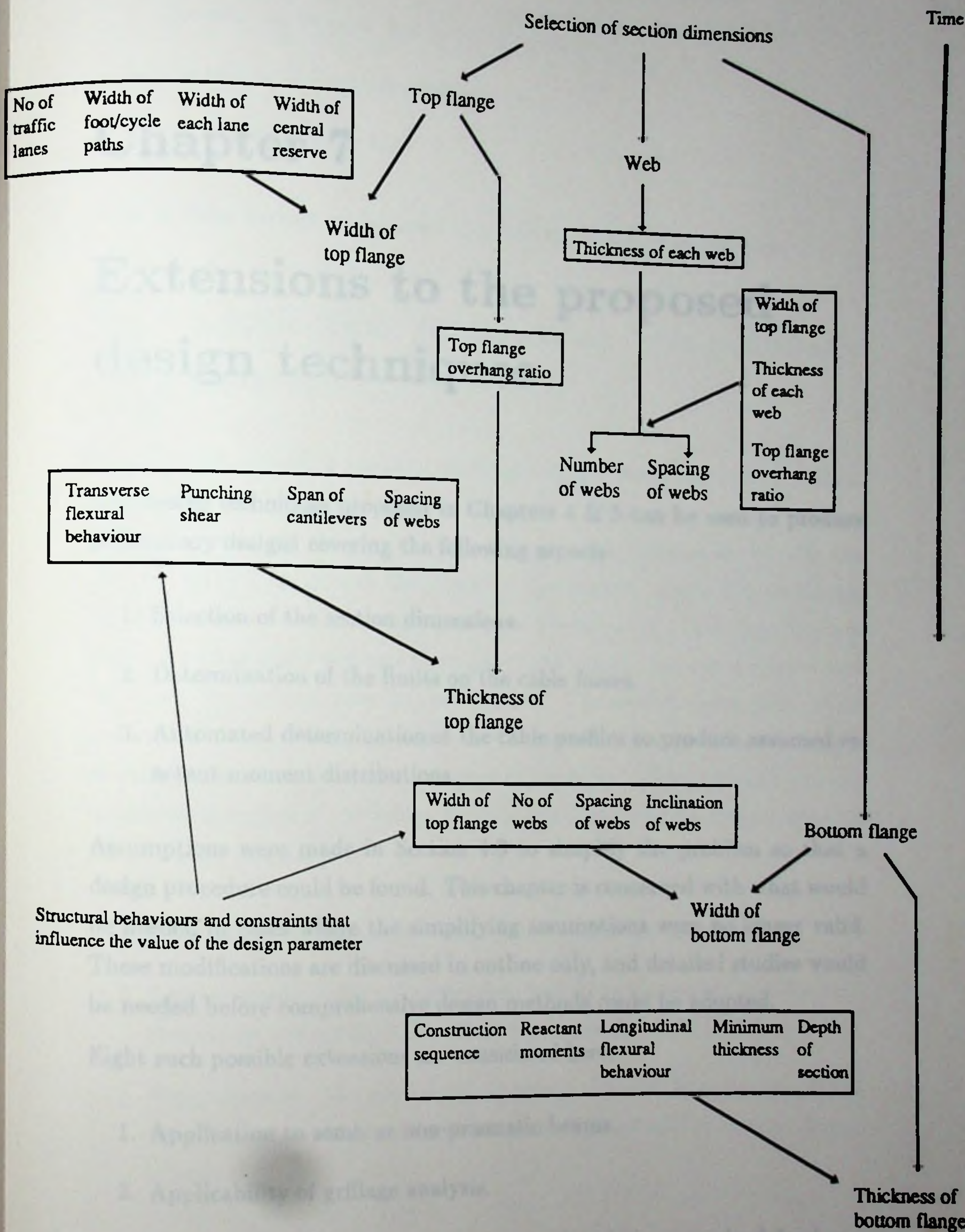

Figure 6.6: Hierarchical structure used for PREDEX

Chapter 7

Extensions to the proposed design techniques

The design techniques proposed in Chapters 4 & 5 can be used to produce preliminary designs covering the following aspects:

1. Selection of the section dimensions.

2. Determination of the limits on the cable forces.

3. Automated determination of the cable profiles to produce assumed reactant moment distributions.

Assumptions were made in Section 4.3 to simplify the problem so that a design procedure could be found. This chapter is concerned with what would be needed in cases where the simplifying assumptions were no longer valid. These modifications are discussed in outline only, and detailed studies would be needed before comprehensive design methods could be adopted.

Eight such possible extensions are considered here:

1. Application to semi- or non-prismatic beams.

2. Applicability of grillage analysis.

3. The effect of construction sequence and technique on dead load moments.

4. The effect of long term creep.

5. The effects of construction sequence on reactant moments.
6. The effect of stresses due to temperature.
7. The effects of foundation settlement.
8. The effects of shear lag.

Each of these aspects is discussed in detail in the following sections.

7.1 Application to semi- or non-prismatic beams

In spine beams used in highway construction, and especially in box girders, it is possible to identify four main parameters, which can describe the type of beam. These are the area of the top flange, the thickness of the web, the area of bottom flange and the depth of the section. According to Swann (1972), in almost all the bridges covered in his survey of 172 bridges, the top flange area remained constant throughout the length; the web thickness also remained fixed except close to the supports. The section parameters which are readily changed along the beam are the bottom flange area and the depth of the section; these determine the type of the beam. Thus, according to the longitudinal section of the bridge deck, bridge beams can be categorised into the following three types:

1. Prismatic – constant cross section throughout the length.
2. Semi-prismatic – the outside dimensions remain constant but the bottom flange and/or the webs thicken towards the supports.
3. Non-prismatic – the total depth increases towards the supports; this is usually accompanied by an increase in the web and/or bottom flange thickness.

The longitudinal section shapes of these bridge types are shown in Figure 7.1. The main difference between these types, for design, is that in prismatic beams the live load moment acting on the section is independent of the actual size of the section; thus the design is straightforward. In semi- or non-prismatic beams, the section stiffness along the beam can directly influence the moment distribution due to dead and live loads.

7.1.1 Application to semi-prismatic beams

7.1.1.1 Selection of section dimensions

In the proposed design technique, a design algorithm was developed for the determination of the bottom flange area, when top flange and web areas are known at each cross section. The determination of the bottom flange area is done in two steps. In the first step, the minimum bottom flange area required at a particular cross section is determined. In the second step, the bottom flange area that should be adopted in each region is determined.

For semi-prismatic beams, the bottom flange area is increased close to the support in order to resist the hogging bending moments. Generally, this increased bottom flange area is kept as localised as possible in order to simplify the construction. Since the hogging moments over the internal supports can be adjusted by using the reactant moments, the following design procedure can be suggested.

1. Assume the minimum bottom flange thickness and thus calculate the initial bending moment envelope.
2. Then select a reactant moment distribution and determine the bottom flange thickness required at a sufficient number of cross sections along the beam.
3. Adopt these new bottom flange thicknesses and then determine the new bending moment envelope. In this case, the increased flexural stiffness close to the supports will cause more dead and live load moments to be attracted to the support.
4. Then use reactant moments to adjust the new bending moments at internal supports close to the previous values. This will ensure that the previously selected bottom flange thicknesses are satisfactory.

This procedure is valid only if the span sections are not critical and hence the minimum thickness is sufficient. If this condition is not satisfied, then a different procedure is required. Hence, a detailed study is needed to identify the possible scenarios as carried out in Section 4.3.6.1 for a prismatic beam.

7.1.1.2 Determination of limits on the cable forces

If the section dimensions are selected as in the previous subsection, then each cross section will have a valid Magel diagram, from which a minimum bound on the cable force can be calculated. This minimum cable force satisfies the P_1 and P_2 limits (see Section 5.2). The cable forces selected must also ensure that a concordant cable profile exists within the limits of the line of thrust zone, which can be done by ensuring that P_3 and P_4 are satisfied (see Section 3.3.2). The condition for the concordancy remains $\int \frac{\beta P e dx}{EI} = 0$, but EI now varies. Since β is independent of the section modulus, the argument that the cable force over the supports has a dominant effect on satisfying this condition still holds. Therefore, the design procedure explained in Section 5.2 can be adopted for semi-prismatic sections as well, but with a new set of equations derived by considering the variable section stiffness.

7.1.1.3 Determination of the cable profile

In Section 5.3, an analytical design procedure has been suggested for the automated determination of the cable profile. This process uses the influence lines as guidance when selecting the notional loads. A modification needed to this design procedure is that the influence lines should be generated by taking account of the varying section stiffness along the beam. A check on convergence of the algorithm ought to be carried out.

7.1.2 Application to non-prismatic beams

In non-prismatic beams, there are two design parameters that vary along the length. These are the depth of the section and the bottom flange thickness. Since these beams are mainly used in long span bridges ($60m <$ span $< 150m$), it is important to minimise the dead weight of the structure. Hence, the cross section should be selected such that it corresponds to the smallest possible one that can be used at each position analysed. This can be achieved by obtaining the smallest possible thickness for the bottom flange.

In non-prismatic beams, there should be a sufficient minimum depth in the middle to permit access inside the box beam for the removal of formwork, tensioning of prestressing cables, checking of structures and, usually, the siting of services.

The longitudinal section required also depends on the construction technique, which is often the balanced cantilever method, where the beam acts as a cantilever until the final prestress is applied to make the beam continuous (Mathivat (1983)). Therefore, the forces acting during the construction may be more critical than those acting under the working load conditions, and so may govern many of the design parameters. The design is further complicated by long term redistribution of moments due to creep[1] because the bridge has to resist substantially different force systems in its life; the structure acts as a cantilevered beam during construction and as a continuous beam after construction thus giving rise to large trapped moments[2].

The study presented in this thesis is carried out for a structure constructed monolithically (*i.e.* no trapped moments) but with provision made for span-by-span construction by defining force change points near the support. Thus it is not possible to directly suggest any design procedures for non-prismatic beams for the selection of section dimensions, cable forces, or profile, without performing a detailed investigation into the implications of balanced cantilever construction. Nevertheless, the design technique proposed for the selection of the bottom flange thickness for a given moment can be usefully employed in finding the least weight structure, because it determines the minimum bottom flange thickness required at any cross section.

7.2 Applicability of grillage analysis

The design techniques described in Chapters 4 and 5 are based on the assumption that a spine beam bridge can be adequately modelled as a continuous beam. In practice, however, the most widely used analytical tool is the

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[1]see Section 7.4

[2]see Section 7.3

'grillage analogy method'. Spine beams are modelled as a grillage of beams because this is a better method of representing the numerous combinations of highway loads, such as partially loaded lanes and skew of supports, to obtain an approximate estimate for the torsional and shear effects (Jaeger & Bakht (1982)).

The application of the *grillage analogy method* for various types of bridge decks was described in detail by Hambly (1976). In essence, the spine beam is divided into longitudinal and transverse members to form a grid. The longitudinal members are *I-beams* which are selected to coincide with the webs of the section so that the centroid of each of them coincides with the principal axis of the deck; this means that the *I-beams* can be made identical, as illustrated in figure 7.2(a) and (b). The transverse members representing the top and bottom slabs are generally selected to be orthogonal to the longitudinal members.

If the spine beam has edge beams, the outer beams of the grillage selected as above will be stiffer than the inner beams, due to the larger top flange area. This problem is solved by introducing two small beams to represent the edge beams as shown in figure 7.2(c). Hence, the selection of a grillage of identical *I-beams* to represent the spine beam is not difficult except when the webs are steeply inclined. The procedures for the selection of design parameters with a grillage analysis are explained in the following subsection.

7.2.1 Application to right bridges

7.2.1.1 Selection of section dimensions

The section dimension that needs to be selected on the basis of grillage analysis is the bottom flange thickness (the web is checked for shear and torsion). Since there are a number of parallel beams, the maximum and minimum moments acting on all of them at a particular cross section will form the actual bending moment envelope. Once these moments are determined, the procedures explained in Section 4.3.6.1 and 7.1.1.1 can be used depending on whether they are prismatic or semi-prismatic.

7.2.1.2 Determination of the cable profile in each beam

If the cable forces and profile are identical in each I-beam of the grillage, then the beam deflections due to the prestress are the same and there will be no interaction of prestressing effects between the beams. It then follows that the technique based on virtual work (see Section 5.2.3.1) is applicable. The cable force and profile determined for one beam can be applied to the others.

However, it is possible that this method will overdesign some of the internal beams of a multicell spine beam if they are less heavily loaded. But if less prestress is applied to these internal beams, then the interaction between the beams must be considered using the grillage analysis and the equivalent load method.

The cable profile can also be determined as explained in Section 5.3, since the influence lines can be used to find the notional loads acting on each beam.

7.2.2 Application to skew bridges

In skew bridges, there may be a considerable amount of interaction between the torsional and flexural behaviour. According to Hambly and Pennels (1975), this interaction becomes dominant when the skew angle exceeds 20^0. The design techniques presented in this thesis have been developed on the assumption that the flexural behaviour plays a dominant role and the torsional behaviour plays only a secondary role. Therefore, the design technique can be adopted only for small skew angles ($< 20^0$).

A grillage layout for a skew bridge is illustrated in Figure 7.3. As can be seen, at any cross section taken perpendicular to the axis of the beam, the chainage of the centre beam is different from the outer beams, if the chainage is measured from the beginning of each grillage beam. This in essence means that the cable profile selected for the central beam cannot be adopted for others. This problem is more severe if the bridge is wide. Hence, a detailed study is needed for the following:

- to determine up to which skew angle the proposed design technique can be adopted, and

- to find alternative techniques for bridges which are either wide or have a large skew angle.

7.3 Effect of the construction technique and sequence

For the design of prestressed concrete bridge decks, a particular construction technique and sequence is generally assumed and built-in (Gee (1987)). A knowledge of construction technique is very important at the preliminary design stage since it can influence the governing criteria for design parameters considerably (for example see Section 7.1.2) and can also affect the values of design parameters.

The term *monolithic construction* is used to define the state where the structure is built in one operation because it can be considered as a datum for all the other conditions that exist in practice.

Any stresses existing in the structure at the completion of each phase of construction are locked in and carried forward to the next phase. The final condition at the completion of construction then becomes the initial self weight condition. The difference between the moments due to monolithic construction and the actual construction method are called the *trapped moments* (see Figure 7.4). By definition, these moments are zero at the ends and vary linearly between the supports.

This means that the bending moment envelope used for the determination of design parameters in the proposed design technique should be based on the dead load bending moments due to the initial self weight condition, and not on the monolithic bending moment envelope as described in Section 4.3.6.1. The dead load moment due to the initial self weight condition can be calculated if the construction technique and sequence are known. However, it needs to be considered at an early stage in the design process, and there may be complications for non-prismatic sections.

7.4 The effect of long-term creep

According to Neville et al. (1982), no creep-dependent moments are induced in a monolithically constructed continuous beam as long as the creep properties are the same throughout the beam and the support conditions are not changed. But nearly all concrete structures are built in stages, so there are trapped moments, and the creep properties also differ. All practical concrete structures, therefore, are subjected to a time-dependent redistribution of moments.

The analysis of the structure for long term creep is a complicated process, but it is well documented (Neville et al. (1982)). However, the most important task at the preliminary design stage is to make an adequate allowance for this effect so that a proper design check will succeed at the detailed stage.

The redistribution of moments due to creep causes the initial self weight condition to change towards that of monolithic construction. This invariably causes a loss of efficiency. This is illustrated in the Figure 7.5 which shows the initial and final dead load bending moments along the longitudinal axis of the Kylesku Bridge in Scotland (Nissen et al. (1985)).

This means that the bending moment envelope effective at the initial loading is different from that a long time after construction. Hence, the only possible way to take account of these different bending moment distributions is to consider the bending moment due to self-weight itself as an *envelope*, which will represent initial and final dead load moment diagrams.

One boundary of this envelope is already known. That is the dead load moment diagram due to the initial self weight condition including trapped moments. The other boundary is the dead load bending moment diagram due to self-weight a long time after construction. However, it is difficult to determine this because sufficient data is not available at the preliminary design stage, but it is known that this boundary is close to the bending moment due to monolithic construction (Gee (1987)) which should therefore be used. This envelope has to be added to that due to live load.

Therefore, the design should be based on a modified bending moment envelope as shown in Figure 7.6. Since the determination of the design parameters is based on the overall bending moment envelope, this increase in the moment range will be reflected in the section dimensions and the cable forces.

7.5 The effect of the construction sequence on the reactant moments

In the design techniques proposed in Chapters 4 and 5, the reactant moments are initially assumed and the cable forces and profiles are determined so as to generate the assumed reactant moments. However, the actual construction sequence affects the reactant moments generated by these cables.

Reactant moments are due to curvatures imposed by the prestressing cables. If the time dependent change in the prestressing forces are neglected, these curvatures can be visualised as those caused by a set of *permanent loads* acting from the cable onto the structure. These loads can be considered as analogous to the dead weight of the structure, which is also affected by the construction sequence. If this analogy stands, the initial reactant moments should also change towards those that would occur in monolithic construction under the effects of creep. Thus the reactant moments should not be selected as a single bending moment diagram but should be selected as a bending moment envelope. This is illustrated in Figure 7.7.

The determination of the boundaries of this bending moment envelope is not straightforward. One boundary can be approximated as the assumed reactant moment distribution. However, it is impossible to determine the other boundary of the reactant moment envelope until the cable forces and the profile are known. Hence, one remedy would be to assume the other boundary to be a percentage of the assumed values (*e.g.* 80% of the actual reactant moments (see Figure 7.7)). This is clearly an example of a situation where the designer must get some feedback from the design before selecting precise values, so the procedural design method must be used. However, the task of the designer is simplified by isolating the parameter which gives rise to this trial and error procedure so that the feedback from a failed design will provide accurate guidance for redesign.

7.6 The effect of temperature

When a cross section is subjected to a differential temperature distribution (see Figure 7.8), there is a linearly varying strain distribution $(\epsilon_0+\psi z)$, where ϵ_0 is the direct strain and ψ is the curvature. There is also a self equilibrating stress distribution f_{temp} which is given by Equation 7.1 (see Figure 7.9).

$$f_{temp} = E_z(\epsilon_0 + \psi\, z - \alpha_z t_z) \tag{7.1}$$

The calculation of ϵ_0 and ψ are explained in detail in Clark (1983). The following two equations are applicable for an uncracked section for which the coefficient of thermal expansion of concrete is considered as constant α_c.

$$\epsilon_0 = \frac{\alpha_c}{A_c I}\left(I \int_0^d b_z t_z dz - A_c \bar{z} \int_0^d b_z t_z (z - \bar{z}) dz \right) \tag{7.2}$$

$$\psi = \frac{\alpha_c}{I} \int_0^d b_z t_z (z - \bar{z}) dz \tag{7.3}$$

The distance z is measured from the bottom fibre and $\bar{z}$ is the distance to the centroid of the section from the bottom of the section.

If only one cross section of the structure is restrained longitudinally, ϵ_0 just causes unrestrained expansion and contraction.

The curvature ψ gets restrained at supports if the structure is statically indeterminate. Thus this curvature induces moments which are of the same form as reactant moments (see Section 7.6.1). There will be maximum and minimum effects due to temperature, so they can be regarded as another moment envelope that will modify the total moment envelope.

The stress f_{temp} is applied at any section independent of all the other loads. The permissible stresses that are used to determine the design parameters can be modified to take account of f_{temp}, which can be calculated relatively easily from equation 7.1, 7.2 and 7.3.

7.6.1 Calculation of continuity moment due to temperature effects

In Section 5.2.3.1, a method is described to calculate the reactant moments due to the curvature caused by the prestressing effects. The differential temperature distribution also causes the same type of curvature, so Equation 5.3 can be used to determine the continuity moment due to temperature, M_t. In this case, the curvature ψ is equivalent to the curvature Pe_s/EI caused by prestressing effects. Thus the following equation can be obtained.

$$\sum_j (M_t)_j \int \frac{\beta_i \beta_j \, dx}{EI} = \int \psi \, dx \quad for\ i = 2, 3, \ldots, n-1 \qquad (7.4)$$

This equation can be used for all types of cross sections, but can be simplified for a prismatic section, where EI is constant. The integrals can be performed analytically, which yields the following result. The secondary temperature moment at each intermediate support can then be obtained.:

$$\frac{1}{6}\begin{pmatrix} 2(l_1+l_2) & l_2 & 0 & 0 & \cdots \\ l_2 & 2(l_2+l_3) & l_3 & 0 & \cdots \\ 0 & l_3 & 2(l_3+l_4) & l_4 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}\begin{pmatrix} (M_t)_2 \\ (M_t)_3 \\ (M_t)_4 \\ \vdots \end{pmatrix} = \frac{EI\psi}{2}\begin{pmatrix} l_1+l_2 \\ l_2+l_3 \\ l_3+l_4 \\ \vdots \end{pmatrix}$$

The unknown moments are $(M_t)_j$. Since the curvature due to temperature can be either positive or negative, these continuity moments can be represented as an envelope.

7.6.2 Application of temperature effects to the proposed design technique

The temperature effects are of two types:

1. the stresses induced as a direct result of differential temperature distribution, which are represented as f_{temp} on the figure 7.9.
2. the continuity moments M_t due to temperature, that are caused in continuous beams as derived above.

The design techniques proposed in Chapters 4 and 5 to determine the section dimensions, cable forces and profile use the permissible concrete stresses in compression and tension to derive values for design parameters. Therefore, the stresses induced in each cross section due to temperature can be taken into account by modifying the permissible concrete stresses. The actual amount of modification can be calculated only when the section dimensions are known. However, a simple study of typical cross sections should allow a single value (about $2.0N/mm^2$) to be used for preliminary design. Alternatively, the actual temperature stresses can be calculated at the start of each iteration performed to determine the thickness of the bottom flange (see example in Section 7.9).

The bending moments due to continuity can be used to modify the overall bending moment envelope used for design. Since the bottom flange area required at the critical sections depend on the allowable concrete stresses and the bending moments acting on the section, these effects may alter the required area of the bottom flange and also the cable forces.

7.7 The effect of foundation settlements

Settlement of foundations leads to a variation of the reactions at supports due to dead load. Since allowance has to be made for any combination of relative settlements, this leads to another contribution to the envelope of moments. For prismatic beams the required calculations can be done before starting the design. There may be complications in semi- and non-prismatic beams due to varying stiffnesses.

7.8 Effect of shear lag

The assumption that plane sections remain plane is valid only when the deformations due to shear strains are negligible. In order to use simple beam theory for the analysis of beams with wide flanges, the flanges are attributed an *'effective flange width'* (see Figure 7.10).

Continuous bridge decks have significant variations in effective flange width along their length. This is partly due to the different distance between the points of contraflexure in span- and support-regions, and partly due to different dominating loads that are distributed over spans and concentrated at supports.

According to Clark (1983), within the limitations of the *effective width concept,* the recommendations given in BS 5400 Part 5 can be used to determine the *effective flange widths* of concrete bridges.

A design technique was described in Section 4.3.6.1 to determine the required thickness of the bottom flange, given the actual widths of the top and bottom flanges. If shear lag effects should be considered, these actual widths can be replaced by effective widths. However, shear lag needs more detail consideration before incorporating it into the proposed design technique because:-

- shear lag affects stress due to applied loads,
- shear lag *does not* affect axial stress due to prestress, but
- shear lag *does* affect stresses due to the curvatures caused by the prestressing effects.

7.9 The design example

This example illustrates the effects of trapped moments and temperature on a continuous beam. The beam considered is as in Section 4.5, so the same section dimensions and permissible stresses are applicable. The reactant moment ratios (RMR) are selected as 0.25 and 0.4 at the second and third supports. When the effects of temperature and trapped moments were ignored, the minimum thickness of the bottom flange was sufficient with these RMR.

Allowance for temperature is made as follows. The section is initially assumed as designed in Section 4.5. The differential temperature distributions shown in Figure 7.11 are taken from BS5400/2 and the section analysed as described in Section 7.6 to get the curvature due to temperature (ψ) and self-equilibrating stresses (f_{temp}, see Figure 7.11). The curvature gives rise to a range of temperature moments $(M_t)_j$ (as described in Section 7.6.1), which is given in Table 7.1; thus they affect both M_a and M_b. The self-equilibrating stresses vary between a maximum tension of $1.6N/mm^2$ and a compression of $2.63 N/mm^2$, so the stress limits used to determine the cross section are adjusted by these amounts.

The trapped moments are determined by assuming that the beam will freely cantilever $8.0m$ from each internal support, during construction. The resulting trapped moments are sagging along the length of the beam, and the values at the internal supports are also given in Table 7.1. The trapped moments will dissipate with time due to creep, so it is assumed that in the long term they will reduce to zero. Thus the sagging trapped moments increase the value of the maximum moment (M_b), but do not alter the value of the minimum moment (M_a).

Position	1^{st} span	2^{nd} supp.	2^{nd} span	3^{rd} supp.	3^{rd} span
Chainage	15.0	35.0	61.0	85.0	107.0
RMR		0.25		0.4	
Maximum $(M_t)_j$ (kNm)		4007.1		4100.3	
Minimum $(M_t)_j$ (kNm)		-294.0		-300.3	
Trapped moment (kNm)		3112.5		15489.7	
M_b (kNm)	35324.8	-20521.1	66253.5	-8200.2	53770.7
M_a (kNm)	11402.5	-51984.6	27973.1	-52673.7	21193.1
t_b required (m)	0.027	0.229	0.119	0.235	0.082

Table 7.1: The selection of the bottom flange thickness with trapped moments and temperature effects: Step 1

As can be seen from the Table 7.1, the minimum thickness of the bottom flange ($0.175m$) is not sufficient. Thus a second iteration is carried out by selecting the bottom flange thickness to be $0.25m$.

Since the section is different, the trapped moments and temperature effects should be different. However, a detailed analysis has revealed that the increase of the bottom flange thickness has only a small effect on the self-equilibrating temperature stresses, thus the same allowance is used for this step. The new trapped moments and $(M_t)_j$ values are shown in Table 7.2 which shows that the new thickness is satisfactory.

This example illustrates that when trapped moments and temperature are considered at the preliminary design stage, a different section may result. It should be noted that the temperature effects are considered with the dead

Position	1^{st} span	2^{nd} supp.	2^{nd} span	3^{rd} supp.	3^{rd} span
Chainage	15.0	35.0	61.0	85.0	107.0
RMR		0.25		0.4	
Maximum $(M_t)_j$ (kNm)		4007.1		4100.3	
Minimum $(M_t)_j$ (kNm)		-50.1		-52.3	
Trapped moment (kNm)		3304.5		16454.4	
M_b (kNm)	36329.4	-21737.4	68675.3	-8523.3	55644.3
M_a (kNm)	12428.5	-53149.8	30036.3	-53713.5	21945.2
t_b required (m)	0.027	0.240	0.124	0.245	0.089

Table 7.2: The selection of the bottom flange thickness with trapped moments and temperature effects: Step 2

and live load moments determined for load combination 1 (as defined in BS 5400/2) in this illustrative example.

More precise calculations of temperature and creep effects will still be needed at the detailed design stage, but these can now be carried out in the expectation that the structure will be satisfactory.

7.10 Discussion

The design techniques explained here to take account of the assumptions made in Chapter 4 should produce better preliminary solutions. However, many of them need rigorous validation and feedback from experts in industry. Until then, it would be premature to develop rules based on these techniques since some aspects such as applicability of grillage analysis, creep and shear lag may be subject to considerable debate. Another point that should be emphasised is that there can be a considerable amount of interaction between these aspects, such as taking account of the effects of shear lag with a grillage analysis.

Most of the work presented in this chapter is aimed at making an adequate allowance at the preliminary design stage for effects that are too complicated to be taken into account precisely. It is essential to make a realistic allowance for these effects because any over-estimate will cause over-design which will not be economically competitive. This is particularly true in the case of the effects due to long term creep because this tends to modify a number of effects such as the reactant moments, trapped moments and support settlements.

The method suggested to overcome this problem is to consider an envelope of moments, which is superimposed on the overall bending moments. However, some of the boundaries suggested are clearly over-estimates because no other alternative is possible. In this type of situation, the *machine experimentation* idea (see section 2.3.2.4) that Adeli & Balasubramanyam (1988a) have used in BTEXPERT can be useful in obtaining rules that can be incorporated in the expert system. In machine experimentation, the expert system is used to obtain a large number of solutions in order to derive rules. These rules can then be used to provide advice to the user.

The expert system described in this thesis is not capable of taking into account most of the aspects covered in this chapter. Nevertheless, an example is presented to illustrate the way that the design parameters are affected when some of these effects are considered.

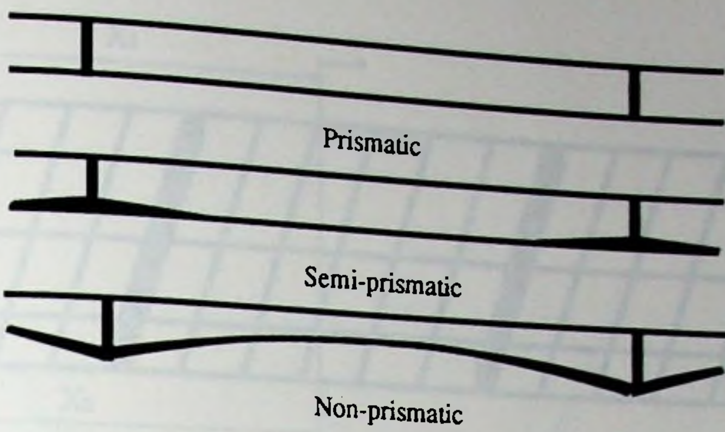


Figure 7.1: Longitudinal sections of different bridge types

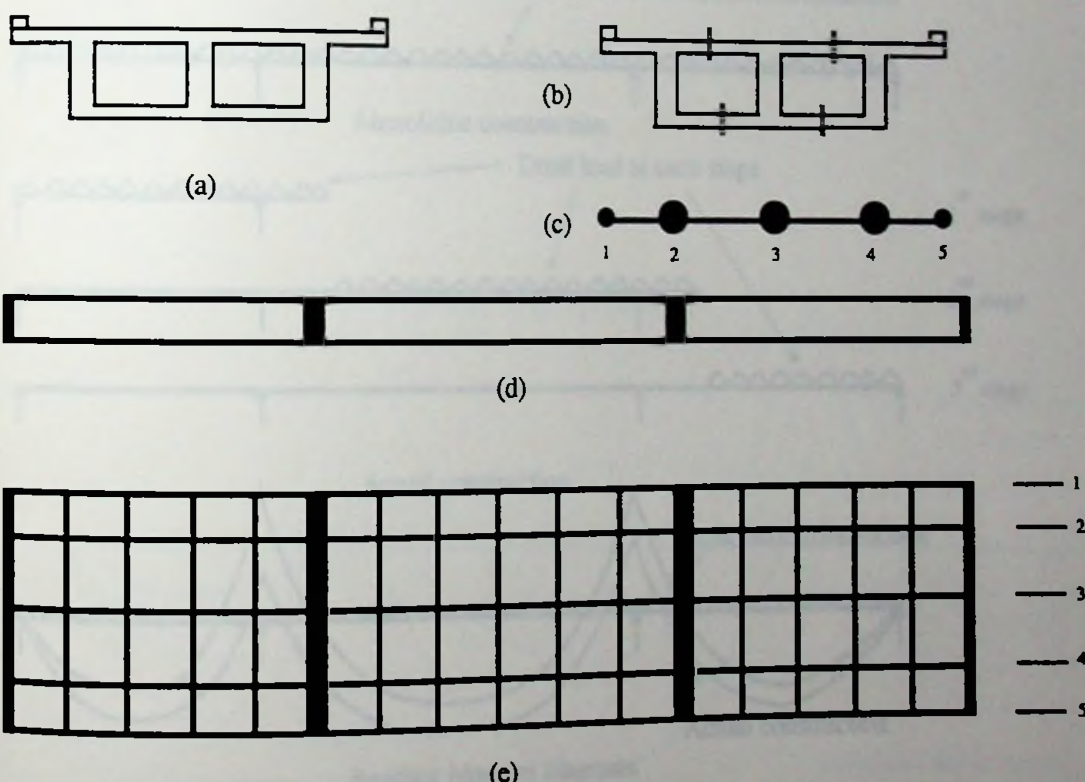


Figure 7.2: Grillage layout for a three span right, prismatic twin-cell concrete spine beam deck. (a) Deck section. (b) Grillage beams. (c) Grillage section. (d) Deck longitudinal section (e) Grillage mesh

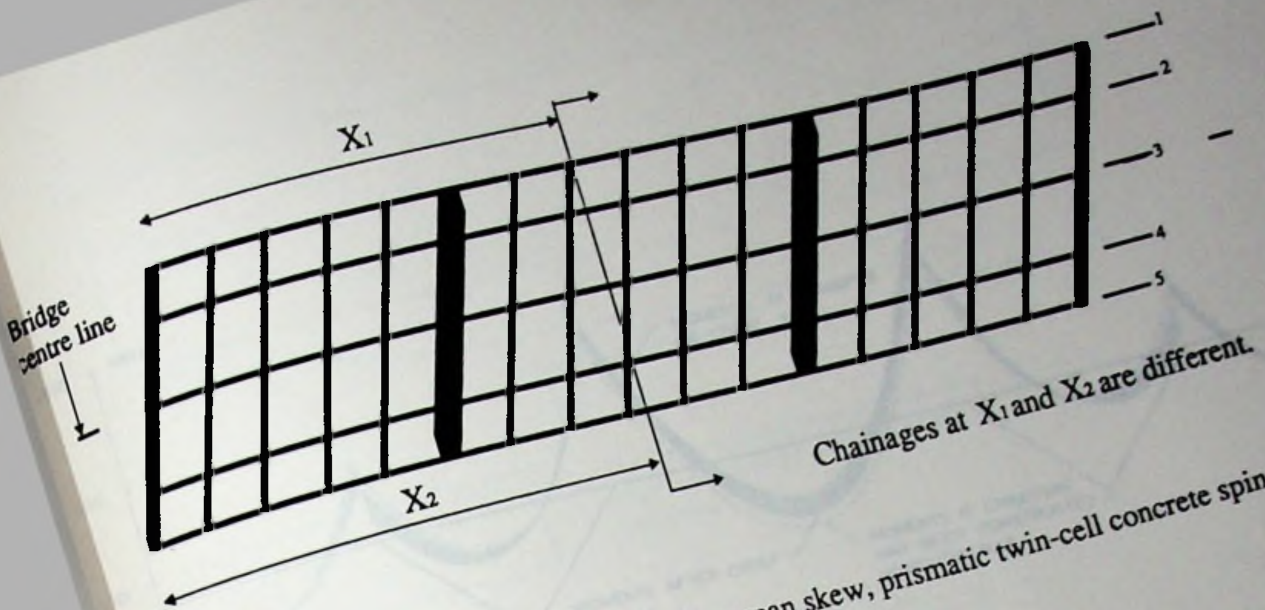


Figure 7.3: Grillage layout for a three span skew, prismatic twin-cell concrete spine beam deck

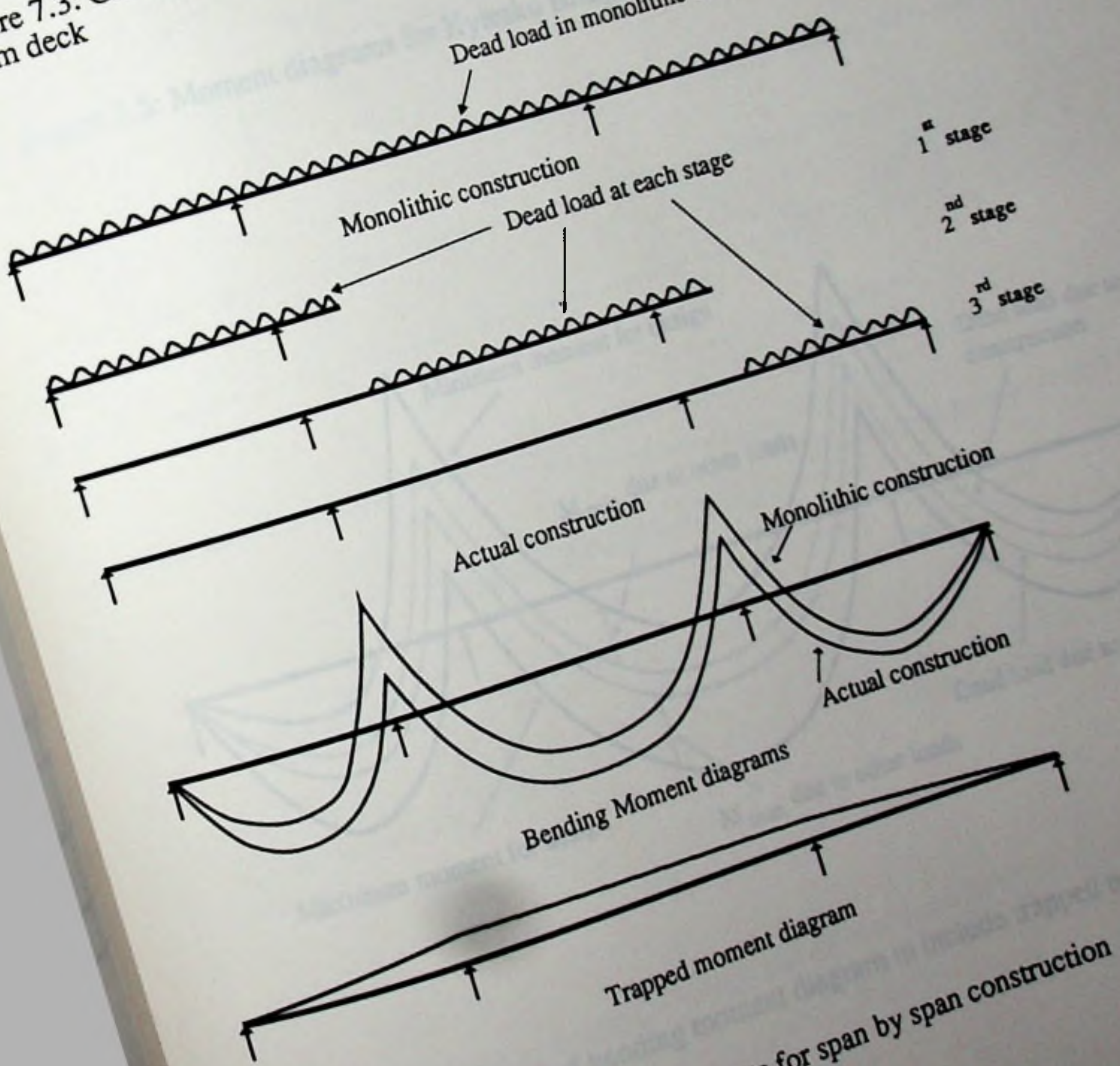


Figure 7.4: Trapped moments for span by span construction

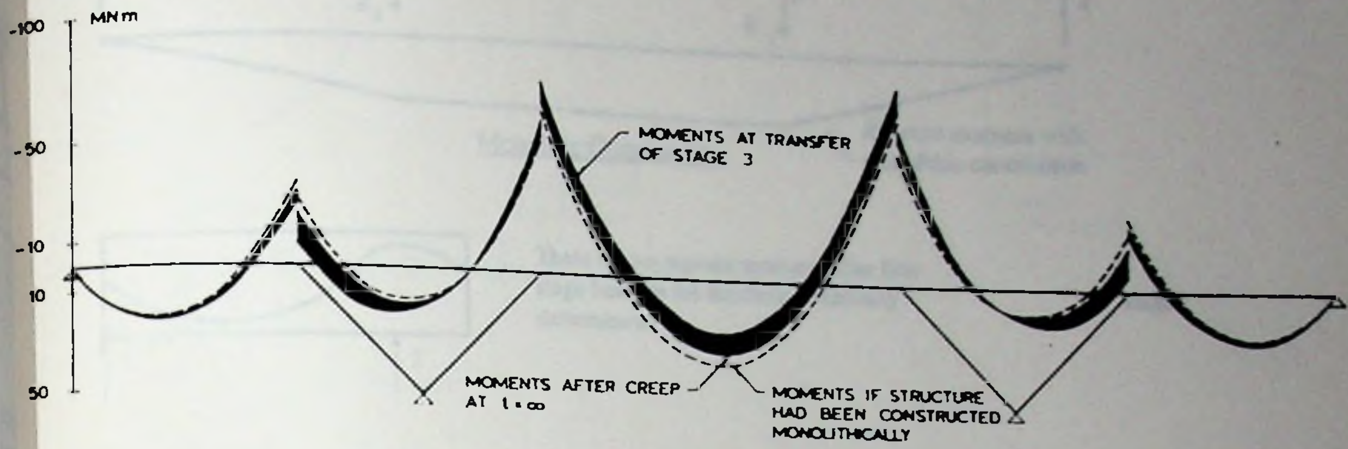


Figure 7.5: Moment diagrams for Kylesku Bridge (taken from Nissen et al. (1985)

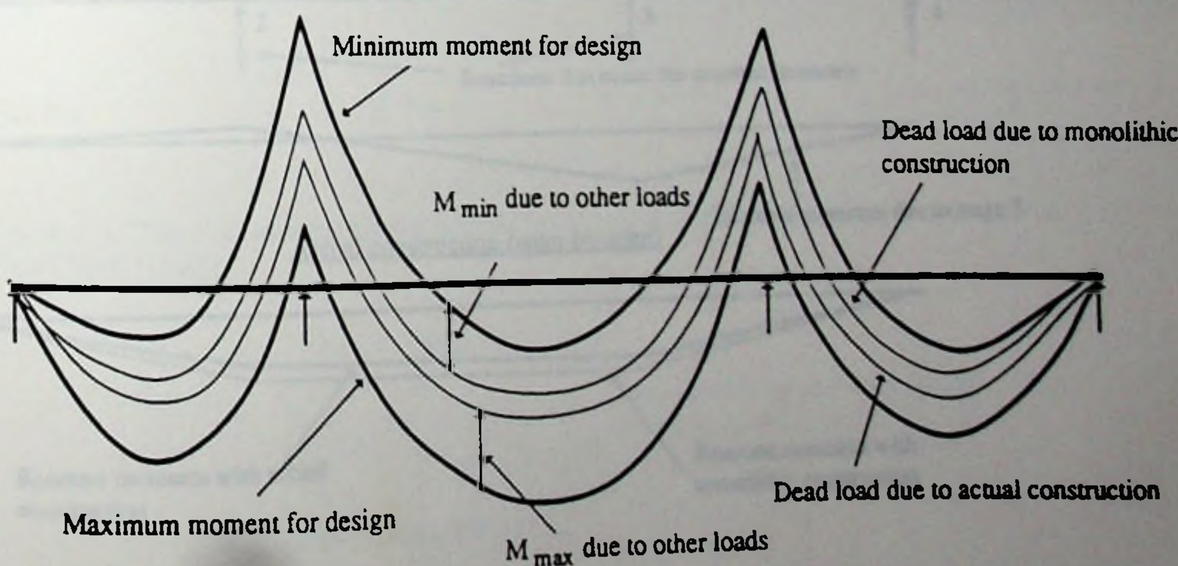


Figure 7.6: Modified bending moment diagram to include trapped moments

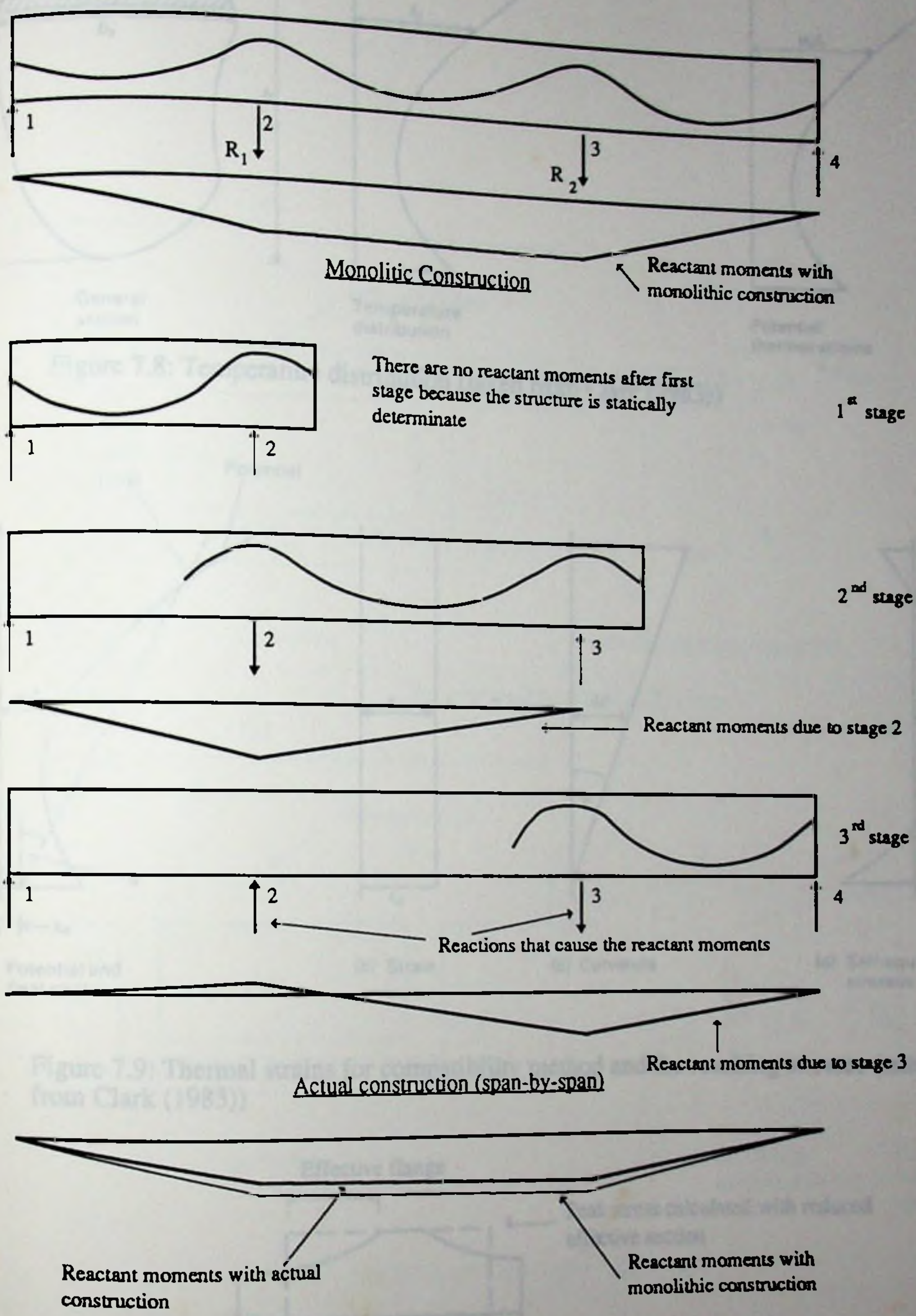


Figure 7.7: Effect of construction sequence on reactant moments

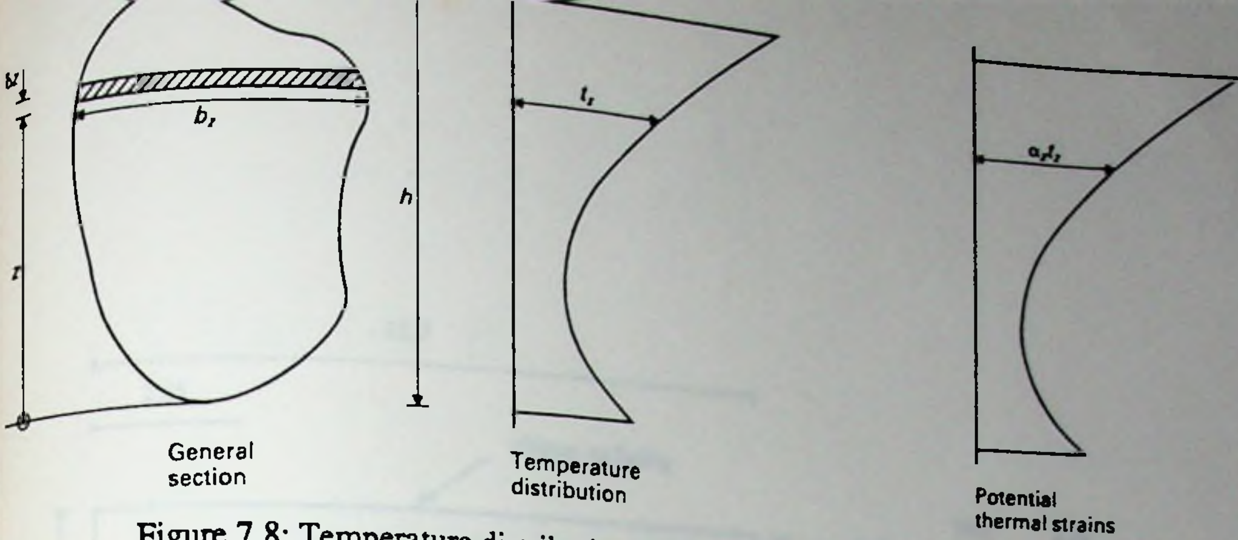


Figure 7.8: Temperature distribution (taken from Clark (1983))

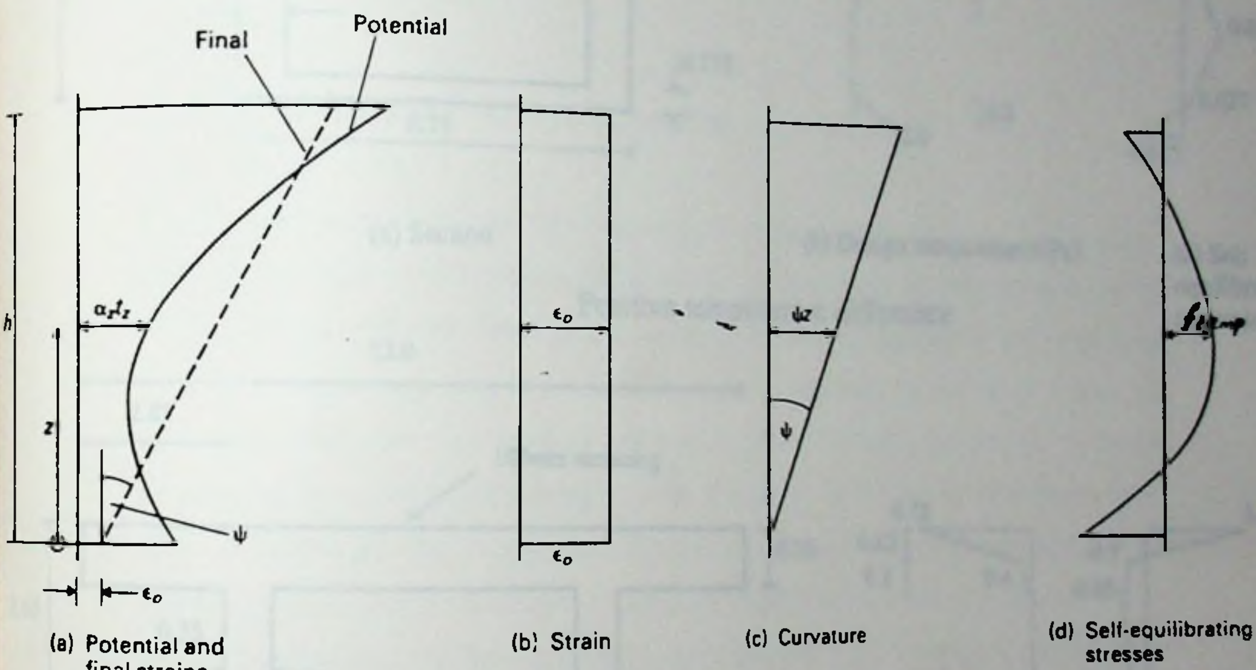


Figure 7.9: Thermal strains for compatibility method and the resulting stresses (taken from Clark (1983))

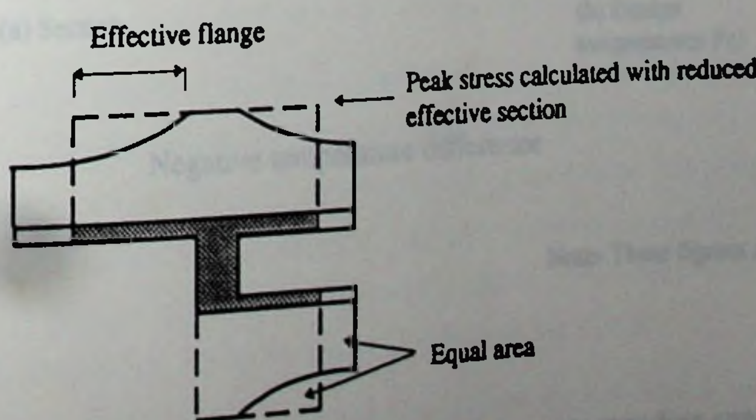
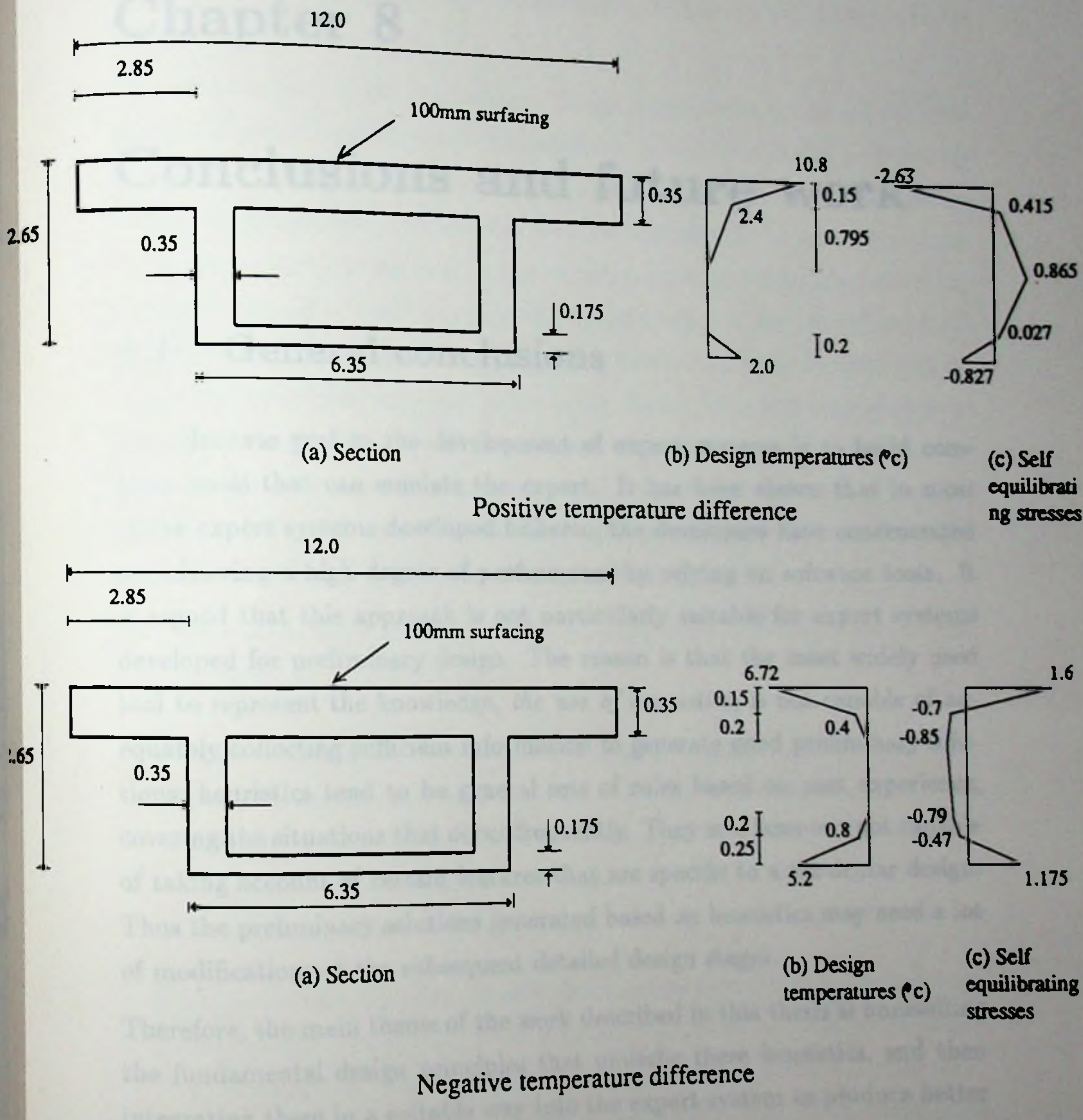


Figure 7.10: Shear lag



Note- These figures are not to scale

Figure 7.11: Differential temperature distributions and the corresponding stresses

Chapter 8

Conclusions and future work

8.1 General conclusions

The ultimate goal in the development of expert systems is to build computer tools that can emulate the expert. It has been shown that in most of the expert systems developed hitherto, the developers have concentrated on achieving a high degree of performance by relying on software tools. It is argued that this approach is not particularly suitable for expert systems developed for preliminary design. The reason is that the most widely used tool to represent the knowledge, *the use of heuristics,* is not capable of adequately collecting sufficient information to generate good preliminary solutions; heuristics tend to be general sets of rules based on past experience, covering the situations that occur frequently. They are, however, not capable of taking account of certain features that are specific to a particular design. Thus the preliminary solutions generated based on heuristics may need a lot of modifications at the subsequent detailed design stages.

Therefore, the main theme of the work described in this thesis is unravelling the fundamental design principles that underlie these heuristics, and then integrating them in a suitable way into the expert system to produce better solutions. It is emphasised that in order to produce versatile expert systems, it is necessary to extensively use the desirable features of the computer; the most important capability being its numerical processing power.

It is also emphasised that good preliminary solutions should have a special character in that they should be able to satisfy the majority of the acceptance criteria at the detailed design stage. This means that the majority of the constraints and structural behaviours that govern the key design parameters should be explicitly identified and taken into account in the preliminary solutions. If there are some behaviours that are too complicated to be calculated at the preliminary design stage, an adequate allowance should be made for them.

An understanding of the fundamental design principles is combined with the numerical processing power of the computer to produce better preliminary solutions. This essentially means that the rationale behind the heuristics should be unravelled, and should then be included in design algorithms. These algorithms can be used in the expert system to replace the heuristics. Heuristics should be used only if design algorithms are either not yet available or impossible to develop. This approach has been adopted for the preliminary design of prestressed concrete spine beam design with a certain degree of success; the main achievements are the development of design techniques for the selection of section dimensions, cable forces and cable profile. These are the main design parameters that are needed at the end of the preliminary design stage for the selection of economical solutions.

The main advantages of using the design algorithms to produce the preliminary solutions are the following:

1. Each design algorithm can deal with a large number of similar problems, thus the number of heuristics that would otherwise be needed to cover each and every possibility can be reduced considerably; this has the advantage of producing expert systems which are faster in execution and more accurate in their answers.

2. Once the rationale behind each decision has been properly understood, it is possible to identify situations where an incorrectly selected design parameter may lead to a dead end. An example is the determination of bounds on cable forces described in Section 5.2. If an insufficient cable force is selected, then the expert system will be searching for a solution that does not exist.

3. The majority of the structural behaviours that may govern the key design parameters are explicitly identified and expressed mathematically for use in the design algorithms. Thus, it is possible to make allowances

for certain structural behaviours that are too complicated for accurate determination, sometimes by making modifications to the constraints. An example is dealing with the long term redistribution of moments due to creep as explained in Section 7.4. This approach is particularly useful at the preliminary design stage, because making a sufficient allowance at the initial stage enhances the possibility that the design will succeed at the detailed design stage. This also allows the designer to consider the design process as a logically evolving series of tasks.

4. When making allowances for structural behaviours that are too complicated to be calculated by modifying constraints, it is inevitable that a certain degree of over-design may result. However, a certain degree of refinement may be made by incorporating rules which could provide some guidance. For example, if only 80% of the trapped moments are redistributed in normal cases, then this information can be used to achieve some cost saving. However, these rules are not readily available. These must be developed: machine experimentation explained in Section 2.3.2.4 is one of the options available. The expert systems developed as described in this thesis will allow machine experimentation, because a large number of accurate designs can be created in a short time.

Another important aspect of the work covered in this thesis is on the design process of prestressed concrete spine beams. It is argued that a computer tool, which reaches a solution as a result of a large number of iterations, will not emulate the expert. Therefore, it is emphasised that an identification of the actual nature of the design process is very important. Since, the design of prestressed concrete spine beams is a complex task, it is not possible to suggest good solutions in one step. Any preliminary solution should go through an evolutionary process. Hence, the method that should be adopted for the overall design is the procedural design method. This becomes a complex process when there are a large number of iterative loops, especially when some of these possible loops are not known beforehand. In order to overcome this problem, a detailed study has been carried out to explicitly identify *when* to make the decisions and also where iterative loops are needed, and *what* are the possible redesign decisions available. This in essence means the adoption of redesign at the preliminary design stage with precise redesign knowledge.

It is also emphasised that for each decision it is necessary to determine whether the design parameter can be calculated or can be based on experience. If the design parameter can be calculated, then a design algorithm should be developed, which allows the analytical design method to be adopted. If the design parameter needs experience, which is based on sound principles, heuristics can be used, so the experimental search design method is suitable. If the design parameter is not amenable for either method, then the procedural design technique should be used. Even then all the possible redesign decisions should be identified explicitly. This approach has the following advantages:

1. Determining *when* to make a decision is very useful in developing rule based expert systems, because the sequence in which the decisions will be made is predetermined and included in the expert system. Also, making a decision at an incorrect position in the design process may have a number of repercussions, because the decision may either not be based on sufficient information if decided too early, or may not provide sufficient information for some other decisions if decided too late. In either case, it may lead to unnecessary redesign cycles.

2. Identifying all the possible iterative loops in the design process is very important at the implementation stage of the expert system because a special provision can be made for these loops, if necessary. It is also possible to include sufficient description of the possible redesign steps, so that the user will be guided when performing the redesign.

When expert systems were initially introduced, they were viewed as computer programs which will replace the design engineer by automating the design tasks. However, this has not happened so far and is unlikely in the future. A better approach is to consider expert systems as computer tools which will complement the role played by the non-expert in the design process, by providing guidance where necessary.

When the decision that has to be taken is straightforward, it is possible for the expert system to use rules to make decisions. The user, however, should be able to override the decisions of the expert system, if necessary. Examples are the selection of the width of the top flange and the required number of webs.

There are certain decisions which are too complicated for the expert system, such as the case where there is more than one feasible solution. In these instances, the expert system can provide useful advice to guide the design engineer, but the design engineer should take over the decision making. An example is the selection of the thickness of the bottom flange. The expert system can calculate the required minimum thickness but the designer should decide whether to accept it or to get a different minimum value by changing the other design parameters. A special feature of the incorporation of design algorithms is that the designer can be guided by suggesting precise bounds on the design parameters.

Therefore, it can be concluded that the work described in this thesis fulfils two requirements. Firstly, the rationale behind the decisions made by the expert designer is unravelled; this understanding can be used for expert systems and also possibly for use in teaching future engineers about rational design techniques. Secondly, the expert system will act as a true adviser to the user, by assisting in the decision making process.

8.2 Suggestions for further research and development

The work covered on the design of prestressed concrete spine beams is primarily for prismatic box girder beams constructed monolithically with some provision for span-by-span construction. There are several practical techniques used for the construction of spine beams, such as balanced cantilever, progressive launching and incremental launching. Since a construction technique and sequence should be assumed and built into the prestressed concrete beam design, similar studies should be carried out for other construction techniques. It is also necessary make a distinction between *in situ* and pre-cast segmental construction.

Past experience has shown that the most frequently changed section design parameters in spine beams are the depth and the thickness of the bottom flange. The depth of the beams, which determines the type of beam, is an important design parameter; this is often governed by requirements such as clearance for navigation, and when not restricted could be used to obtain alternative solutions. In the case of prismatic beams, the overall depth of the beam can be varied to obtain alternative solutions. In the case of non-prismatic beams, the depth along the length of the beam can also be varied to produce alternative solutions. For the determination of the required bottom flange thickness, a design technique has been developed for the selection of the feasible minimum thickness, when the moments acting on the section are known. Because the construction technique, the construction sequence, and the type of bridge determine the governing moments, the criteria developed for the selection of section dimension should be extended for other construction techniques and types of bridges.

For the selection of cable forces, the techniques suggested are based on certain governing conditions like the existence of a valid cable profile zone and the existence of a concordant cable profile. Detailed studies are needed to verify whether these conditions are the governing ones for the various construction techniques; the same exercise should be carried out for the method presented for the selection of the cable profile. Therefore, the important aspect is to carry out detailed studies in order to adopt the known design principles and also to unravel hitherto unknown principles. These design techniques expressed as design algorithms will be the core of more versatile expert systems for prestressed concrete spine beam design in the future.

Another type of spine beam that has not been considered in this study is the double-T beam. Double-T beams do not have a separate bottom flange area, and cannot resist high hogging moments over the internal supports. Therefore, the bending moments acting on the section should be redistributed using the reactant moments. With the design techniques explained in Chapters 4 and 5, this is not a difficult task, because the designer has full control over the reactant moments. However, the section dimensions to be selected at the preliminary design stage are different from the box girder sections. The dimension which is most likely to be dependent on the longitudinal flexural behaviour is the thickness of the webs, and the required values are likely to be higher than the minimum required for construction. Thus, new design algorithms should be developed. When concentrated loads act on the structure, it is also likely that larger moments will be transferred onto the individual spines; this is due to the higher flexibility of the structure. Therefore, a detailed study is needed to cover these aspects.

The design principles covered in this thesis for spine beam designs are primarily for highway construction. Another important application of spine beams is for railway bridges. Beams used for railway bridges are often prismatic channel beams. Thus, the design principles applicable for these will be different from those needed for highway bridges. A detailed study into railway beams to rationalise the design philosophy would be useful in extending the scope of the expert system for design of spine beams.

Appendix A

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Appendix B

The governing equations and their graphical representation

B.1 Stress limit criteria

If a prestressed concrete section is subjected to a prestressing force, P, acting at an eccentricity e, and a sagging moment M, then the possible stress conditions can be illustrated as shown in figure B.1. The sign convention used is as follows.

- Tensile stresses acting on concrete are positive.
- Sagging bending moments are positive.
- Distances measured downwards from the centroid are positive. Thus, the section modulus of bottom fibre is positive and the section modulus of top fibre is negative.

The following expressions can be written for the stresses at the top and bottom fibres.

$$f_c \leq -\frac{P}{A_c} - \frac{Pe}{Z_1} + \frac{M}{Z_1} \leq f_t \tag{B.1}$$

$$f_c \leq -\frac{P}{A_c} - \frac{Pe}{Z_2} + \frac{M}{Z_2} \leq f_t \tag{B.2}$$

If the minimum working load moment M_a and the maximum working moment M_b acting on a particular section are considered as the governing conditions, then four inequalities can be derived in order to satisfy the allowable stresses. If the moment at transfer is M_t then another four governing inequalities can also be derived.

These conditions can be rearranged to give inequalities on the permissible eccentricities, as functions of the applied moments, the section properties and the prestressing force. The actual prestressing force at working load conditions is RP_t where P_t is the prestressing force at transfer and R is the loss ratio.

$$e \geq -\frac{Z_1}{A_c} - \frac{Z_1 f_{ct}}{P_t} + \frac{M_t}{P_t} \tag{B.3}$$

$$e \geq -\frac{Z_1}{A_c} - \frac{Z_1 f_{cw}}{R P_t} + \frac{M_b}{R P_t} \tag{B.4}$$

$$e \geq -\frac{Z_2}{A_c} - \frac{Z_2 f_{tt}}{P_t} + \frac{M_t}{P_t} \tag{B.5}$$

$$e \geq -\frac{Z_2}{A_c} - \frac{Z_2 f_{tw}}{R P_t} + \frac{M_b}{R P_t} \tag{B.6}$$

$$e \leq -\frac{Z_1}{A_c} - \frac{Z_1 f_{tt}}{P_t} + \frac{M_t}{P_t} \tag{B.7}$$

$$e \leq -\frac{Z_1}{A_c} - \frac{Z_1 f_{tw}}{R P_t} + \frac{M_a}{R P_t} \tag{B.8}$$

$$e \leq -\frac{Z_2}{A_c} - \frac{Z_2 f_{ct}}{P_t} + \frac{M_t}{P_t} \tag{B.9}$$

$$e \leq -\frac{Z_2}{A_c} - \frac{Z_2 f_{cw}}{R P_t} + \frac{M_a}{R P_t} \tag{B.10}$$

These inequalities form straight bound lines on a plot of $1/P_t$ versus e (a Magnel diagram). By considering various combinations of these conditions, it is possible to derive three bounds on each elastic section modulus, to ensure that a valid combination of cable force and eccentricity can be found (see Section 4.1.3):

$$Z_1 \leq -\frac{M_a - R\,M_t}{R\,f_{ct} - f_{tw}} \tag{B.11}$$

$$Z_1 \leq \frac{M_b - R\,M_t}{f_{cw} - R\,f_{tt}} \tag{B.12}$$

$$Z_1 \leq \frac{M_b - M_a}{f_{cw} - f_{tw}} \tag{B.13}$$

$$Z_2 \geq -\frac{M_b - R\,M_t}{R\,f_{ct} - f_{tw}} \tag{B.14}$$

$$Z_2 \geq \frac{M_a - R\,M_t}{f_{cw} - R\,f_{tt}} \tag{B.15}$$

$$Z_2 \geq \frac{M_a - M_b}{f_{cw} - f_{tw}} \tag{B.16}$$

By considering all combinations of top and bottom fibre stress limits, the following limits on the prestressing force can be derived.

$$P_t \geq -\frac{A_c\,f_{tw}}{R} + \frac{A_c}{R(Z_2 - Z_1)}(M_b - M_a) \tag{B.17}$$

$$P_t \geq \frac{A_c}{R(Z_2 - Z_1)}\left[(R\,f_{tt}Z_1 - f_{tw}\,Z_2) + (M_b - R\,M_t)\right] \tag{B.18}$$

$$P_t \geq \frac{A_c}{R(Z_2 - Z_1)}\left[(R\,f_{tw}Z_1 - R\,f_{tt}\,Z_2) + (R\,M_t - M_a)\right] \tag{B.19}$$

and

$$P_t \leq -\frac{A_c\,f_{cw}}{R} - \frac{A_c}{R(Z_2 - Z_1)}(M_b - M_a) \tag{B.20}$$

$$P_t \leq \frac{A_c}{R(Z_2 - Z_1)}\left[(R\,f_{ct}Z_1 - f_{cw}\,Z_2) - (R\,M_t - M_a)\right] \tag{B.21}$$

$$P_t \leq \frac{A_c}{R(Z_2 - Z_1)}\left[(f_{cw}Z_1 - R\,f_{ct}\,Z_2) - (M_b - R\,M_t)\right] \tag{B.22}$$

Of these ineqalities, those associated with the working load moments M_a and M_b have been extensively used in this thesis.

B.2 The Magnel diagram

The Magnel diagram is the graphical representation of the stress inequalities (Equations B.3 - B.10) applicable to prestressed concrete sections. It is drawn by plotting the reciprocal of the prestressing force against the eccentricity, so that all the inequalities become straight lines. All these lines intersect the eccentricity axis either at $-Z_1/A_c$ or $-Z_2/A_c$. These are the kern points of the section; if a resultant force acts within the limits set by the kern points, then no tension will develop within the section due to that force. These kern points are denoted by e_{k_2} and e_{k_1}, respectively.

The basic requirement for the existence of a valid Magnel diagram is the satisfaction of equations B.11 to B.16.

There are additional limits on the Magnel diagram due to the limits imposed on eccentricity by the minimum cover required for the prestressing cables. These limits are denoted as e_{min} and e_{max}, which correspond to minimum and maximum limits on the eccentricity.

Although there are eight lines that can form the Magnel diagram (see Figure B.2, there will be only four lines which will govern the feasible region in the Magnel diagram. In this thesis, the design techniques have been suggested as though the stress limits corresponding to the working load conditions govern, but this is not always the case. Thus, the method should be extended to cover the transfer conditions, if used in practical applications.

In Figure B.2, the feasible region is shown to be additionally restricted by the eccentricity limit due to cover so the area enclosed by ABEF is the region of valid combinations of cable force and eccentricity.

Figure B.3 shows a simplified Magnel diagram for a section governed by working load conditions only. The following points can be observed about this Magnel diagram.

- The points C and A correspond to the prestressing cable forces given by equations B.17 and B.20, respectively. They are the minimum and maximum cable forces allowed by the Magnel diagram, under working load conditions.
- The limit imposed by the concrete cover required for steel cables intro-

duces another minimum limit on the cable force. This correspond to P_E.

- The cable forces corresponding to points B & D are dependent only on the section properties and the allowable stresses, for working load conditions. Hence, they are fixed for a particular cross section layout. They are given by the following equations:

$$P_B = \frac{Z_1 f_{cw} - Z_2 f_{tw}}{R(Z_2 - Z_1)/A_c} \tag{B.23}$$

$$P_D = \frac{Z_1 f_{tw} - Z_2 f_{cw}}{R(Z_2 - Z_1)/A_c} \tag{B.24}$$

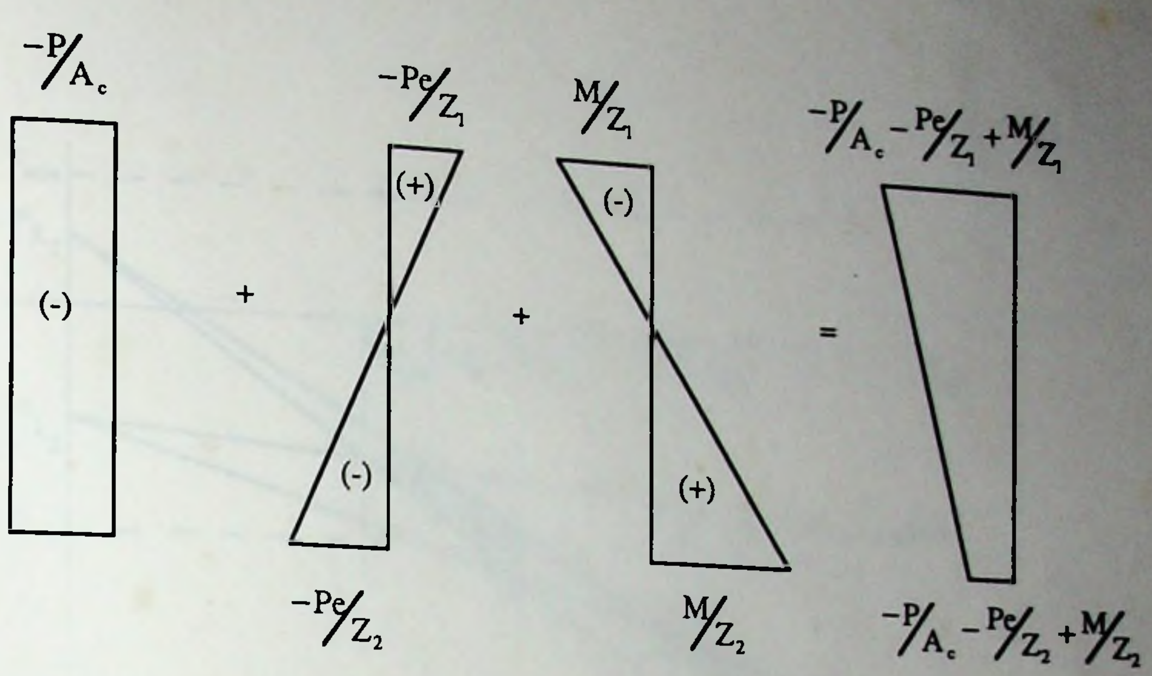

Figure B.1: Stresses due to prestress and moment

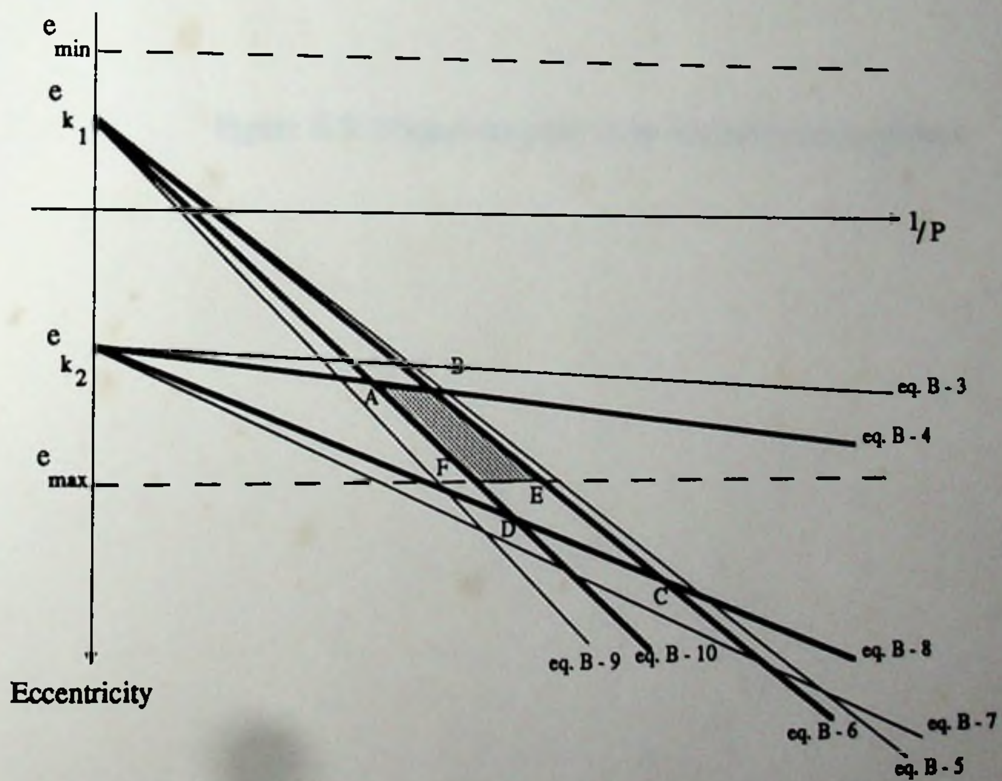

Figure B.2: A typical Magnel diagram

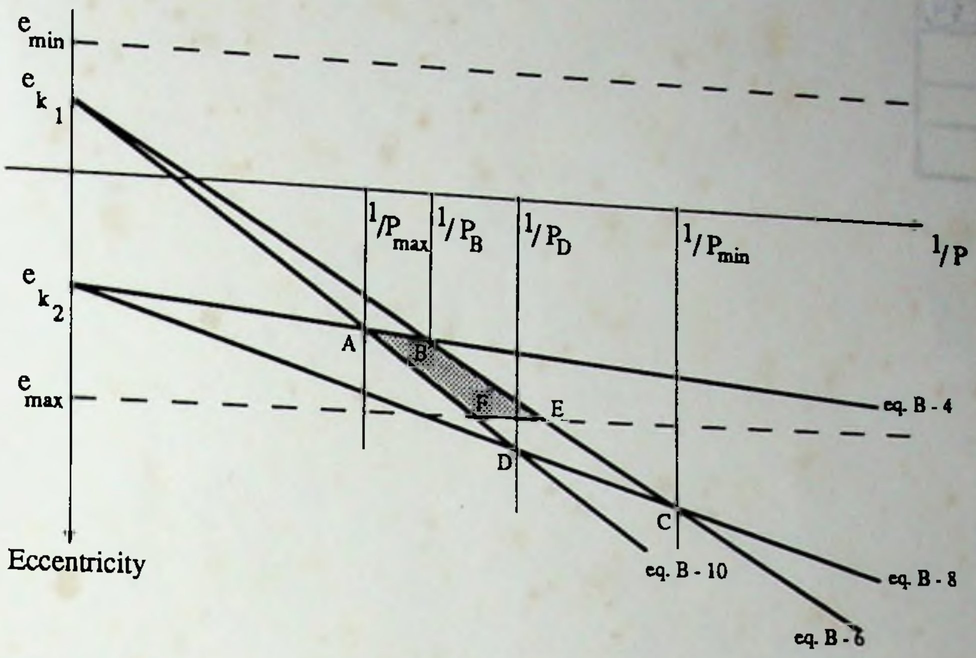

Figure B.3: Magnel diagram under working load conditions