DETERIORATION PREDICTION OF BRIDGE BY MARKOV CHAIN MODEL AND BAYESIAN THEORY

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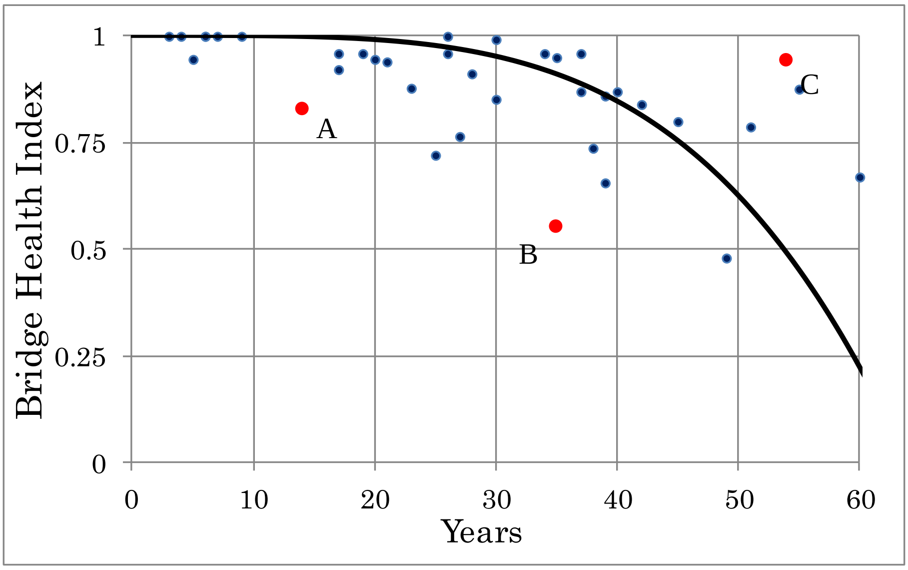
Abstract

This manuscript presents a bridge deterioration prediction method by using Markov chain model and Bayesian theory. Markov chain model works by defining discrete condition states and accumulating the probability of transition from one condition state to another over discrete time intervals. The probability of transition is generally expressed by the matrix. Though the previous studies have predicted the bridge deterioration by developing deterioration curves by using the Markov chain model, the predicted value will not be necessarily suitable for the measured value in the future. Therefore, this study demonstrates a method to predict deterioration progress as a prediction interval by taking account of the uncertainty by the Monte Carlo simulation. In addition, the method to update the prediction interval after the inspection is developed by Bayesian theory. This research was developed by using inspection results of existing bridges in Japan, and the proposed mechanism is convenient for bridge engineers to take rational decisions on the maintenance management plan of steel bridge infrastructures.

Keywords: Markov chain model, Bayesian theory, transition probability matrix, deterioration prediction interval

1. Introduction

Many bridges have been built since the 1960s in Japan, and recently the number of older bridges has increased. On the other hand, life cycle cost of bridge has been reduced due to the depression of Japanese economy. In order to use many bridges for a long time, efficient maintenance management is more important than new construction. Efficient maintenance management is developed to predict the deterioration of bridge. However, bridge inspection is conducted in every five years, but there are not many bridges with two check results for one bridge in present situation. Therefore, deterioration of bridge must be predicted with few inspection results though the accuracy is low. Some studies on deterioration prediction of bridge were conducted by several civil engineers to introduce methods for maintenance management of bridge infrastructures. For example, a deterioration prediction of bridge was derived as an



approximate curve of bridge condition (Figure 1).

Figure 1.Previous study of deterioration prediction

Though it is easy to calculate by this method, it is difficult to use them as deterioration prediction because of two problems. First, the theoretical line and the measured value are not necessarily the same. In particular, point A, B, C at Figure 1 are greatly away from the theoretical prediction line. Second, the deterioration progress of an individual bridge is uncertain in nature. If the deterioration of A bridge is predicted, the theoretical line has to be moved in parallel to coincide with this bridge. This prediction method is shown in Figure 2. But this curve does not satisfy the initial conditions, where the BHI (Bridge Health Index) is to be equal to 1.0 at just after the construction. When bridge age is 0 year old in this case, it is not necessarily 1.0 of BHI (Bridge Health Index). To improve these two problems, it is necessary to consider BHI dispersion. Therefore, the purpose of this research is to consider dispersion of measurement point. This research demonstrates how to express the probability distribution of transition rate by using Markov chain model. Although, many researchers have predicted the deterioration of a bridge as weighted average of the deterioration health condition, this study developed a prediction interval of bridge by using probability distribution of transition rate. The interval can consider the uncertainty in the deterioration. Furthermore, the deterioration of individual bridge will be predicted by Bayesian theory after new inspection results are obtained. This theory has the advantage of predicting the deterioration of bridge by inspection result of only one time.

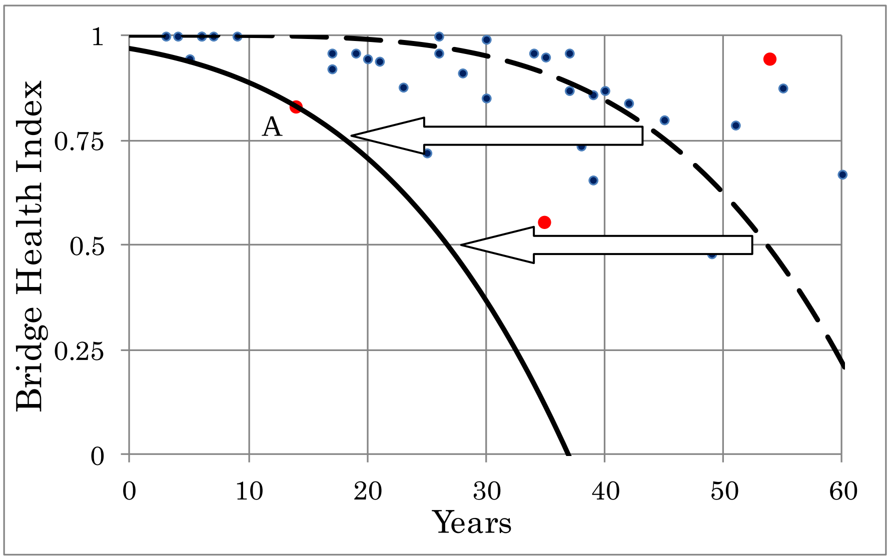


Figure 2.Previous study of deterioration prediction

2. Formulation of the Markov Chain Model

Bridge inspection frequency is every five years in Japan, and number of measurement data is few because the periodic check was started only about five years ago in almost all municipals. The Markov chain model has an advantage that can predict deterioration from the inspection result of one bridge. Characteristic of this method can calculate the decay of bridge as proportions. For example, the current inspection has been generally evaluated the damage of the bridge members as rank of ‘a’, ‘b’, ‘c’, ‘d’, and ‘e’. Rank ‘a’ has no damage, rank ‘e’ has the severe damage. Rank ‘a’ to ‘e’ are represented as proportions of which comes from the total of 100%. A bridge has some members including ‘road surface’, ‘floor slab’, ‘hinged bearing’, ‘bridge pier’ and so on. The inspection items of bridge members are ‘crack’, ‘reinforced steel bar exposure’, ‘waterleaks’ and so on. In this research, the health condition of a bridge parts obtained by the visual inspection is expressed as $R_i(i = a, b, c, d, e)$. $P_i(i = a, b, c, d)$ of the transition rate is the proportion that the health condition will change from n years to n+1 years like these ‘a’ → ‘b’, ‘b’ → ‘c’, ‘c’ → ‘d’, ‘d’ → ‘e’. This method is researched not to recover from present rank to better rank and not to deteriorate more than two ranks by one step in this paper. The relationship between percentage of n years and n + 1 years is represented by the following equation.

$$\begin{bmatrix} R_a(n+1) \\ R_b(n+1) \\ R_c(n+1) \\ R_d(n+1) \\ R_e(n+1) \end{bmatrix} = \begin{bmatrix} 1\text{-}P_a & 0 & 0 & 0 & 0 \\ P_a & 1\text{-}P_b & 0 & 0 & 0 \\ 0 & P_b & 1\text{-}P_c & 0 & 0 \\ 0 & 0 & P_c & 1\text{-}P_d & 0 \\ 0 & 0 & 0 & P_d & 1 \end{bmatrix} \begin{bmatrix} R_a(n) \\ R_b(n) \\ R_c(n) \\ R_d(n) \\ R_e(n) \end{bmatrix} \tag{1}$$

It was assumed that the bridge has no initial damage. So, damage rate of 1 year is expressed as $\{R_a, R_b, R_c, R_d, R_e\}$ = {1, 0, 0, 0, 0}. Therefore, the damage rate of n year is calculated by the following equation.

$$\begin{bmatrix} R_a(n) \\ R_b(n) \\ R_c(n) \\ R_d(n) \\ R_e(n) \end{bmatrix} = \begin{bmatrix} 1\text{-}P_a & 0 & 0 & 0 & 0 \\ P_a & 1\text{-}P_b & 0 & 0 & 0 \\ 0 & P_b & 1\text{-}P_c & 0 & 0 \\ 0 & 0 & P_c & 1\text{-}P_d & 0 \\ 0 & 0 & 0 & P_d & 1 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

$R_i(i = a, b, c, d, e)$ is represented by substituting the age of bridge for the variable n and calculating the right-hand side, which is the theoretical values of the damage rate after n years. The inspection results of rank 'a' to 'e' represent as $Q_i(i = a, b, c, d, e)$. To solve $P_i(i = a, b, c, d)$ has to equal $R_i(i = a, b, c, d, e)$ and $Q_i(i = a, b, c, d, e)$. However, this approach makes a discrepancy between $R_i(i = a, b, c, d, e)$ of the inspection results and $Q_i(i = a, b, c, d, e)$ of the theoretical values. As the number of variables and the formula doesn't coincide, exact numbers for $P_i(i = a, b, c, d)$ cannot be calculated. Because number of the variable number and the formula don't coincide, $P_i(i = a, b, c, d)$ of rigorous numbers cannot calculate. Therefore, $P_i(i = a, b, c, d)$ is obtained by calculating the following equation using the Nelder - Mead method.

$$Minimize\{\textstyle\sum_{i=a,\ b,\ c,\ d,\ e}[R_i(n) - Q_i(n)]^2\} \tag{3}$$

In reverse, the damage rate of n years is expressed as shown in Figure 3 to substitute $P_i(i = a, b, c, d)$ and number of n years for the right-hand side. For example, the bridge of forty years has each damage which is expressed as $\{R_a, R_b, R_c, R_d, R_e\}$ = {0.6, 0.2, 0.1, 0.05, 0.05} by the inspection results. According to Eq. (3), $\{P_a, P_b, P_c, P_d\}$ = {0.0127, 0.0363, 0.0545, 0.0739} is calculated.

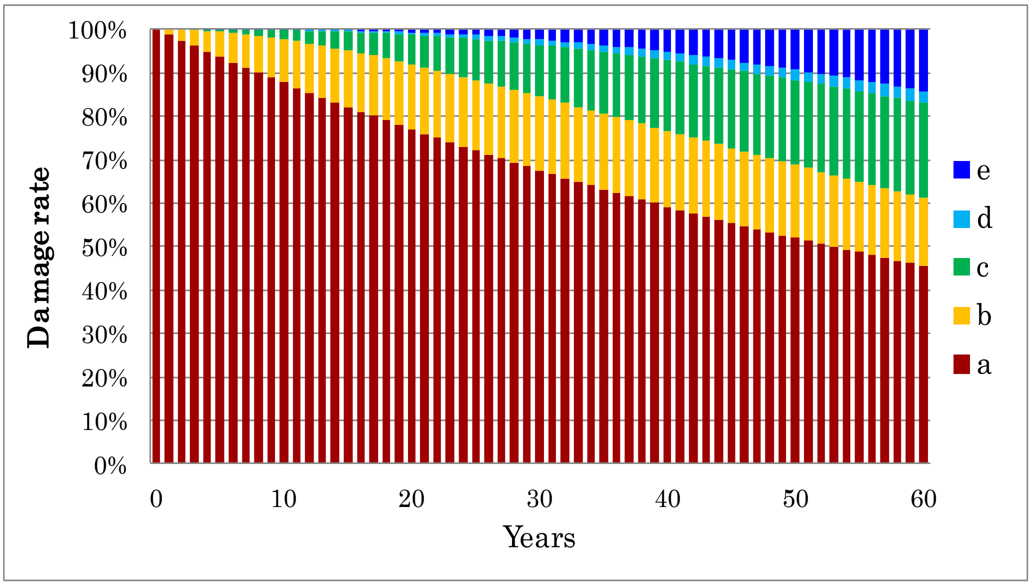


Figure 3.Damage rate by the number of years

In addition, BHI is calculated which is one of the most famous index to evaluate the health condition of bridge. BHI is the weighted average of the health condition which can be derived from the following equation. The result is shown as a curve in Figure 4.

$$\mathrm{BHI}=R_a(n)\times 1.0+R_b(n)\times 0.75+R_c(n)\times 0.5+R_d(n)\times 0.25+R_e(n)\times 0 \quad (4)$$

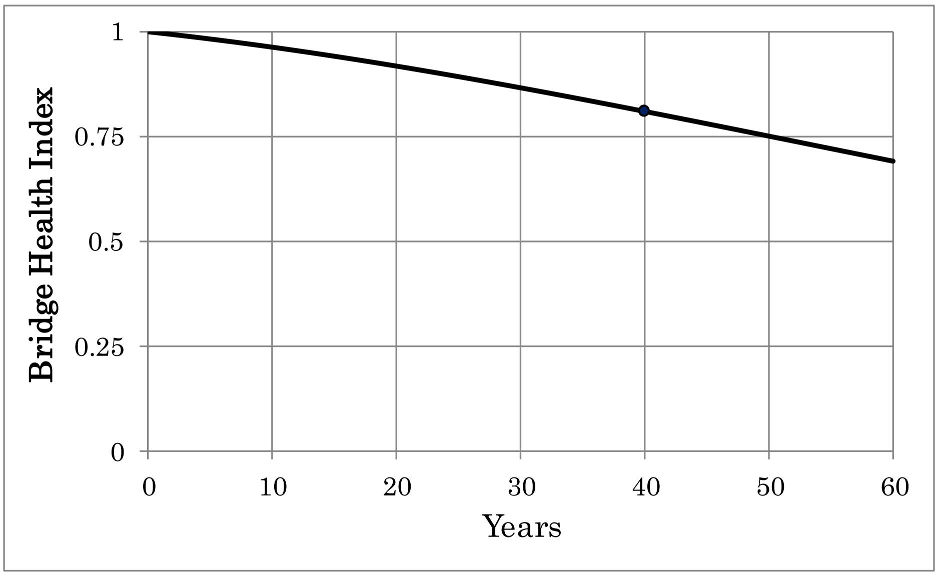


Figure 4.BHI by the number of years

3. Monte Carlo Simulation

This chapter demonstrates a method to form the deterioration interval from transition rate of probability distribution by using the Monte Carlo simulation. Markov chain model can calculate transition rate of all bridges of group A (Table 1) by Eq.(3). Group A has 36 PC (Prestressed Concrete) bridges which exist within 1000 m from the coast, constructed within 60 years and bridge length is from 5 m to 20 m. The bridge deterioration of the group is predicted as shown in Figure 1 in previous studies. The dispersion of inspection result cannot be expressed by deterministic estimation like an approximate curve method.

Table 1.Environment condition of group A

Coast distance	0~1000 m
Year	1~60 years
Length	5~20 m
Type	PC bridge

Therefore, this research estimates the dispersion as prediction interval by means of probability distribution. First, each of transition rates are calculated by Markov Chain Model in group A. And, the right side of Figure 5 are expressed the frequency as beta distribution as shown in Eq.(5). They are called 'Prior probability distribution' (abbreviate as 'Prior') and are expressed by beta distribution.

$$f(P_i;\alpha,\beta)=\frac{1}{B(\alpha,\beta)}P_i^{\alpha-1}(1-P_i)^{\beta-1}$$
$$(0\le x\le 1,\ 0<\alpha,\ 0<\beta,\ i=a\sim d) \tag{5}$$

The parameters of beta distribution show as α, β, and are calculated by Eq.(6). Mean μ, dispersion σ^2 are developed from each transition rate.

$$\alpha=\frac{\mu^2-\mu^3-\mu\sigma^2}{\sigma^2}$$
$$\beta=\frac{(\mu-1)(\mu^2-\mu+\sigma^2)}{\sigma^2} \tag{6}$$

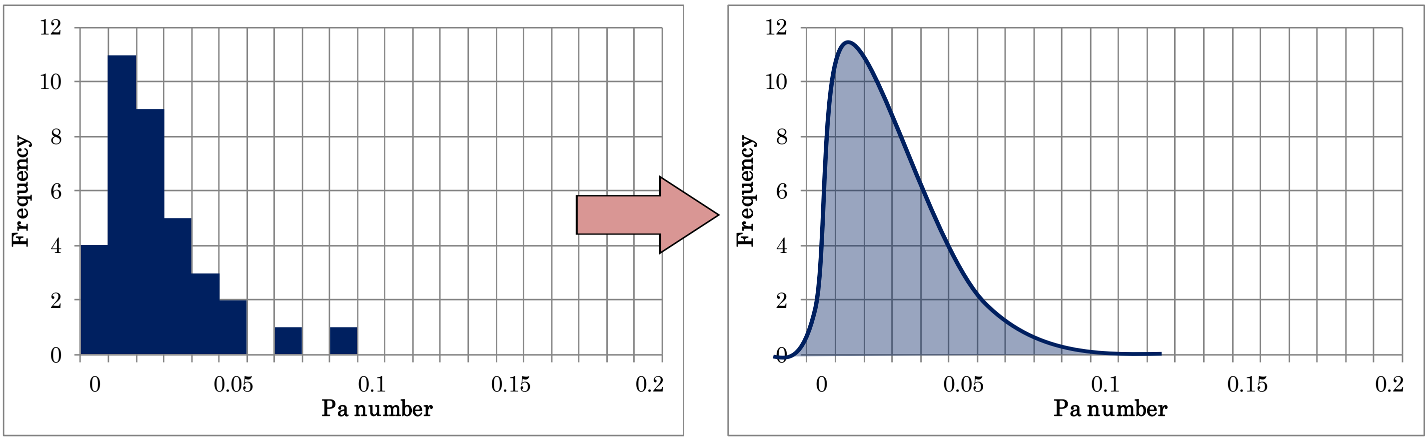


Figure 5.Change from histogram to probability distribution in the group

In order to generate a lot of generic bridges, many random numbers are generated from probability distribution. Moreover, deterioration curve of generic bridge is expressed by calculating Eq.(2) and (4). For example, 10^5 patterns of generic bridge are generated, and two-sided 95% prediction interval is developed as the line shown in Figure 6 by removing the upper and lower 2.5% of imaginary bridges. In this research, the upper line of interval is called 'Optimistic deterioration curve' and the lower line is called 'Pessimistic deterioration curve'. 'Optimistic deterioration curve' is the case of the bridge has no significant deterioration, and 'Pessimistic deterioration curve' is the case of the bridge has the most severe deterioration in this interval. Moreover, medium deterioration curve also can be developed by calculating one-side 50% of beta distribution. Therefore, Monte Carlo Simulation is able to compose of these patterns for predicting bridge deterioration. It is not difficult to plan and maintenance a lot of bridges by using these patterns.

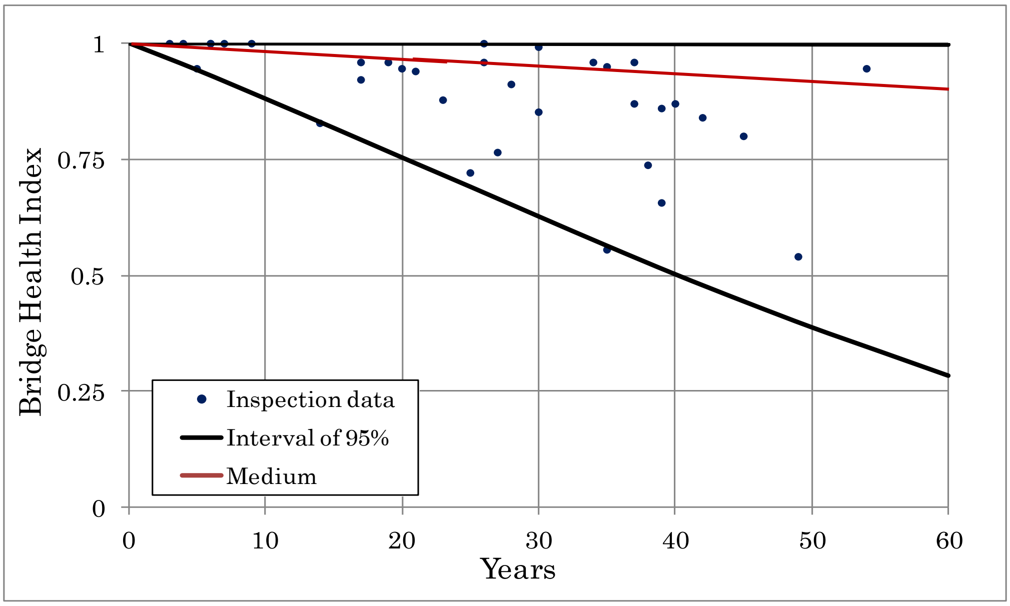


Figure 6.Prediction interval 95% of group A and Medium curve

4. Bayesian Theory

This chapter, a prediction of individual bridge is made by Bayesian theory. Bayesian theory has good characteristic that deterioration of individual bridge is composed by ‘Prior’, ‘Likelihood function’ and ‘Posterior probability distribution’ (abbreviate as ‘Posterior’). As described in the previous chapter, ‘Prior’ is the function that is expressed the characteristic deterioration of group as a probability distribution. ‘Likelihood function’ is expressed the characteristic deterioration of a bridge as a probability distribution. For example, when ‘Prior’ is expressed by beta distribution, ‘Likelihood function’ is also expressed by beta distribution. ‘Posterior’ is the function that is expressed to multiply ‘Prior’ by ‘Likelihood function’. Therefore, it has two characteristic deteriorations of ‘Prior’ and ‘Likelihood function’. It is expressed sample mean μ_0, universal dispersion $\sigma^2{}_0$, sample number n_μ, $n_{\sigma 2}$. μ of ‘Prior’ shows by a normal distribution as mean μ_0 dispersion $\sigma^2{}_0/\ n_\mu$, and σ^2 of ‘Prior’ shows by a reverse χ^2 distribution as $w(\sigma^2) \propto \chi^{-2}$ ($n_{\sigma 2}$ - 1, σ^2 ($n_{\sigma 2}$ + 1)). Therefore, ‘Prior’ is represented by following formula.

$$
\begin{aligned}
&w(\mu,\sigma^2) = w(\mu \mid \sigma^2) w(\sigma^2) \\
&\propto \frac{1}{\sqrt{\sigma^2 / n_{\mu 0}}} \exp\left| -\frac{(\mu-\mu_0)^2}{2\left(\sigma^2 / n_{\mu 0}\right)} \right| \cdot \chi^{-2}\left(n_{\sigma^2} - 1, \sigma_0^2\left(n_{\sigma^2} + 1\right)\right)
\end{aligned}
\tag{7}
$$

‘Likelihood function’ shows that beta distribution can be defined by μ, σ^2 and $P_{i(i=a\sim d)}$ of parameter. Therefore, ‘Likelihood function’ is represented by following formula.

$$
L\left(D \mid \mu,\sigma^2\right) = f(P_{i(i=a\sim d)};\mu,\sigma^2)
\tag{8}
$$

‘Posterior’ is composed to multiply by formula (5), (6), and following formula is expressed.

$$
w'\left(\mu,\sigma^2 \mid D\right) \propto w\left(\mu,\sigma^2\right) L\left(D \mid \mu,\sigma^2\right)
\tag{9}
$$

Moreover, a two-sided 95% prediction interval is developed by 'Posterior'. For example, bridge T and U are thirty years old, and BHI are 0.854 and 0.992. BHI of bridge U is higher than bridge T. And, the decay of the health of bridge U wasn't significant. Figure 7 shows the deterioration prediction interval of the group and both bridges T and U. BHI of bridge T is inclined to decrease than U. As a result, it is expected to show rapid progress of deterioration. In this way, individual bridge is expressed by considering individual transition rate. The prediction interval of each health condition can be used to work out maintenance needs and cost of repair more accurately.

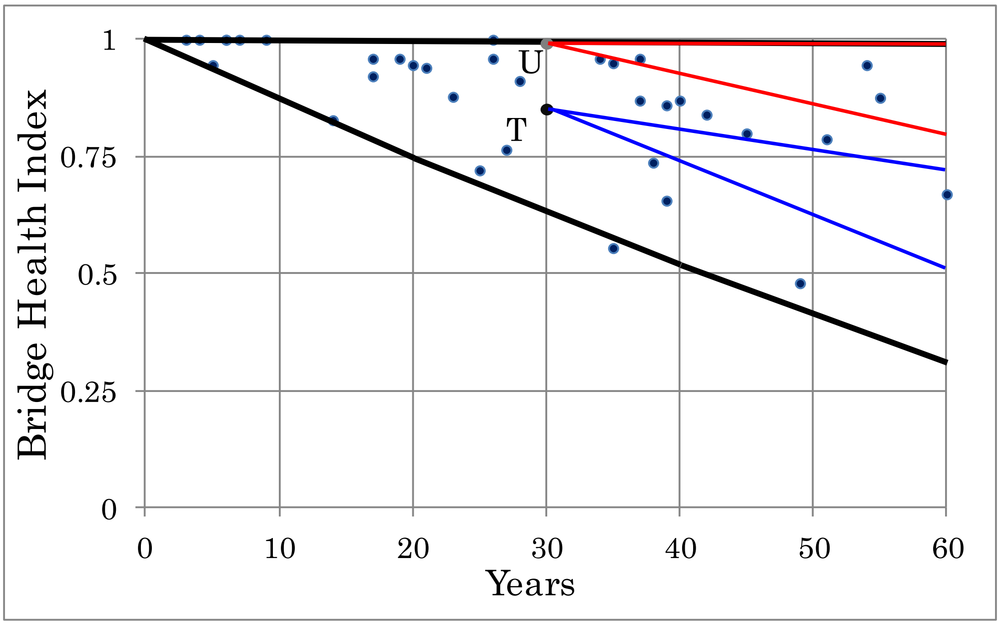


Figure 7.Deterioration interval of the group and both bridges T and U

5. Conclusions

1. This paper demonstrates a method to predict BHI by using deterioration interval in order to consider the uncertainty of the deterioration of bridges. This prediction method is not a curve but an interval. This provides a better insight into the estimation of maximum, average, and minimum maintenance costs to the bridge owners, which makes their decision making process more productive.
2. Moreover, it demonstrates a method to include characteristic deterioration of the bridge in deterioration interval of group by Bayesian theory. If condition and age of a bridge is known, this research can be used to predict the future deterioration of a bridge.
3. Prediction accuracy is better after using Bayesian theory, as it were, deterioration interval gets narrow. A lot of analyses of bridges are performed to get the deterioration more narrow. Furthermore, it is easy to develop for BMS (Bridge Management System) effectively.

Only little inspection data is available on bridge deterioration in Japan. It is expected that accuracy of this research can be improved by checking abundant detailed bridge parts in our future studies. Moreover, as it is assumed the bridge deterioration at age 10 and 40 to be different, bridge deterioration should not be calculated to keep using $P_i(i = a, b, c, d)$ of the transition rate as same distribution in the future.

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