

APPLICATION OF CONTINUOUS WAVELET TRANSFORMATION METHOD IN ESTIMATION OF MODEL PROPERTIES

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Abstract

Continuous wavelet transform (CWT) has recently emerged as a promising tool for identification of modal properties such as natural frequencies, damping ratios and mode shapes through ambient excitation measurements of structures. This paper mainly discusses the capability of CWT method to identify the modal properties accurately using a practical application in a five storey reinforced concrete structure with masonry in-fills. Natural frequencies are identified by extracting windows paralleled to the frequency axis at wavelet ridges. Damping ratios are estimated using the wavelet-based logarithmic decrement method. They are further compared with the damping ratios obtained from the random decrement (RD) method. Furthermore, the mode shapes extracted using CWT method is compared with the results of the modal analysis.

Finally, a conclusion can be drawn that the CWT used for the output-only system identification of ambient excitation structures yields a good agreement with results of the RD method and the finite element models.

Keywords: Ambient excitation, continuous wavelet transform, natural frequencies, damping ratios

1. Introduction

Currently, ambient vibration measurements are commonly used in the assessment and structural health monitoring of civil engineering structures such as buildings, bridges and towers because the ambient vibration testing is cheap and fast, no excitation equipments required, no boundary condition simulations required and dynamic characteristics of the whole system can be estimated using a modal extraction technique. Furthermore, the extracted modal properties such as natural periods, mode shapes and damping ratios can also be used for verifying the design characteristics of a civil engineering structure and validating the numerical model that can be used to predict the response of a structure under an extreme loading condition.

Recently, a continuous wavelet transform (CWT) method is used for modal identification of civil engineering structures using only the output response as a time-frequency domain method. As the CWT method can decompose of a signal into time-frequency domain using a mother wavelet, multi degree of freedom (MDOF) systems can be handled directly. Furthermore, it can work as a band-pass filter and hence, this method can handle very noisy measurements (Staszewski (1997)). Another advantage of the CWT method is that the stationary assumption for an ambient vibration response is not required.

Therefore, this method has many advantages over the other methods in identifying modal properties using ambient vibration measurements. This method has been used successfully to extract the natural frequencies and the associated mode shapes through ambient vibration measurements of different types of civil engineering structures (Le and Tamura (2009), Meoa et al. (2006)). Regarding the damping estimation, the past studies by Staszewski (1997), Lamarque et al. (2000) and Ta and Lardiès (2006) have highlighted that damping ratios can be estimated adequately accurate through wavelet-based logarithmic decrement for lightly damped systems. However, their studies are limited to either the impulse response or free decay response of the structures.

The main objective of this study is to obtain the modal properties through ambient vibration measurements of low-rise buildings using CWT method. In order to estimation the damping ratio, this study uses two steps procedure. In the first step of the two step procedure, the free decay response of a structure is evaluated for the ambient vibration measurement using the random decrement (RD) method (Cole H.A., (1973)). In the second step, the free decay response is then decomposed into time-frequency domain using the CWT with the Morlet wavelet to estimate damping ratios. However, it is worth to note that in this study, a single step procedure is used to extract the natural frequencies and the mode shapes by decomposing of the ambient vibration measurement of a structure into the time-frequency domain using the CWT method.

2. Continuous Wavelet Transform Method

This section introduces a brief description on the theoretical background of the CWT method. However, authors strongly recommend to readers to refer the key papers to understand the theoretical background of the method. Fourier transformation (FT) transforms a given function $x(t)$ in the time domain into the frequency domain using a basic function $e^{j\omega t}$. This can be represented in the following form:

$$FT_{\mathbb{F}} = \int_{-\alpha}^{\alpha} x(t) e^{-j\omega t} dt \quad (1)$$

This linear transformation does not include any local time information of the function $x(t)$. Initially, Fourier transformation of the short time sliding window is used to overcome this limitation by decomposing of the function into frequency-time domain. However, this method, called the Short Time Fourier Transformation (STFT), suffers from time-frequency resolution limitation. Later, the CWT method is developed to obtain a better spectral decomposition as an alternative method to the STFT. The basic idea of the CWT is to find a function $\psi(t)$, which can generate a basic for the entire domain of the function $x(t)$, if the function $x(t)$ satisfies the condition that $x(t)$ decays to zero at $\pm\alpha$ as in the case of Fourier transform. The function $\psi(t)$ which displays fast decay in time domain and has the limited band width in frequency domain gives the local information of the function $x(t)$ both in time and frequency domains. It is not the case of Fourier transform where a global representation can only be obtained in the frequency domain. Such function $\psi(t)$ is also called as mother wavelet. Using a mother wavelet, the CWT method can be used to decompose of a function $x(t)$ into frequency-time domain as defined in the following form:

$$W_{(a,b)} = \frac{1}{\sqrt{a}} \int_{-\alpha}^{+\alpha} x(t) \psi^* \left(\frac{t-b}{a} \right) dt \quad (2)$$

where $\psi^*(t)$ and b are the complex conjugate of $\psi(t)$ and the parameter localizing the mother wavelet in the time domain, respectively and $W_{(a,b)}$ are the CWT coefficients that represent the measure of the similitude between the function $x(t)$ and the wavelet at the time b and the scale a . The complex Morlet wavelet is commonly used for the CWT method as a mother wavelet.

The damping ratio ζ of a system can be estimated from the slope of the straight line of the semi logarithmic plot of wavelet modulus cross section using Eq. (3).

$$\zeta = \frac{1}{2\pi m} \ln \left| \frac{W_{(a_0,b)}}{W_{(a_0,b+mT_0)}} \right| \quad (3)$$

The wavelet ridges are formed at an instantaneous frequency and time when the frequency of the response is equal to the frequency of the dilated mother wavelet at a time. Therefore, natural frequencies are evaluated from extracting a window parallel to the frequency axis of time-frequency plot at wavelet ridges where the CWT coefficients reach their maximum values.

Mode shapes are estimated through the wavelet transforms of output response at points k and the reference point as shown below:

$$\phi_i^k = \frac{W_{(a_i,b)}^k}{W_{(a_i,b)}^{ref}} \quad (4)$$

3. Practical Application in Low Rise Building

Picaso building is a six storey reinforced concrete structure including one under-ground storey. It is symmetric in plan and elevation. The floor plan is approximately rectangle with dimensions of 45m and 14.5m in length and width, respectively. In the transverse direction, the building has 2 bays and each bay is 7m wide while in the longitudinal direction, it has 16 bays and each bay is 2.6m wide. The height of each storey is 3.1m. The building consists of some interior and exterior in-fill walls.

Ambient excitations are measured at each floor level of the building and the ground during the field test by tri-axial seismometers (CMG-6TD). They are placed approximately at the centre of the each floor plan. All data is acquired at the frequency of 100Hz for the period of 1800 seconds. The recorded data is then pre-processed using a band pass filter (Chebyshev) at the specified frequency range from 2 to 15Hz to remove the non-zero mean noise and the uncorrelated to the structural response. Figure 1 illustrates the filtered velocity time histories at 4th storey level in East-West (E-W) direction and their fast Fourier transformation (FFT) plots. Furthermore, the displacement time histories are obtained by integrating the filtered velocity time histories in time domain using a trapezoid integration approach. They are shown in Figure 2(a).

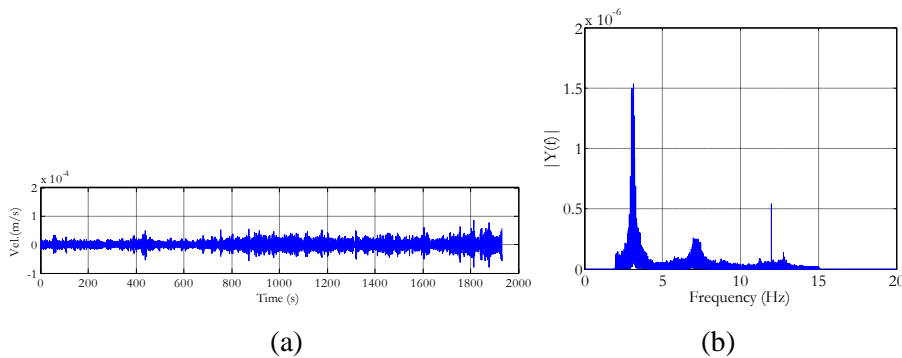


Figure 1: (a) Filtered velocity time history at 4th floor in E-W direction and (b) its fast Fourier transformation (FFT) plot.

Ambient vibration response of this structure is sensitive in the longitudinal direction (E-W direction). Therefore, model properties only in the longitudinal direction has been identified and discussed in this part of the study.

In order to estimate the damping ratios, two steps procedure is used. As the first step of the two steps procedure, RD signatures are evaluated from the displacement time histories at different

storey levels using the RD method. For this evaluation, the length of each RD signature is set to be 10 seconds (1000 samples). A level crossing triggering condition is used in this study with the optimal value of the triggering level. Figure 2(b) shows the resultant RD signatures of the displacement time histories at 4th storey level. It illustrates the damped free response clearly. To estimate the damping ratio associated with the first translational mode in longitudinal direction using Eq. 3, the semi-logarithmic plot is obtained by extracting a window parallel to time axis from each time-frequency plot at the frequency of 3.03 Hz. The damping ratio associated with second translational mode is estimated using the same approach but filtering out the first mode frequency content from the displacement time histories.

Furthermore, damping ratios are also estimated using only the RD signatures. However, only for this case, SDOF response data incorporated with any natural frequency is extracted from the integrated displacement histories by band-pass filtering before the RD method is used. It should be noted that RD signatures shown in Figure 2(b) do not represent the SDOF response data. They represent the MDOF response data used to estimate the damping ratios by two step procedure (RD+CWT). Table 1 compares the resultant damping ratios estimated using the two approaches. The maximum difference of 8.5% is observed at the 4th storey level.

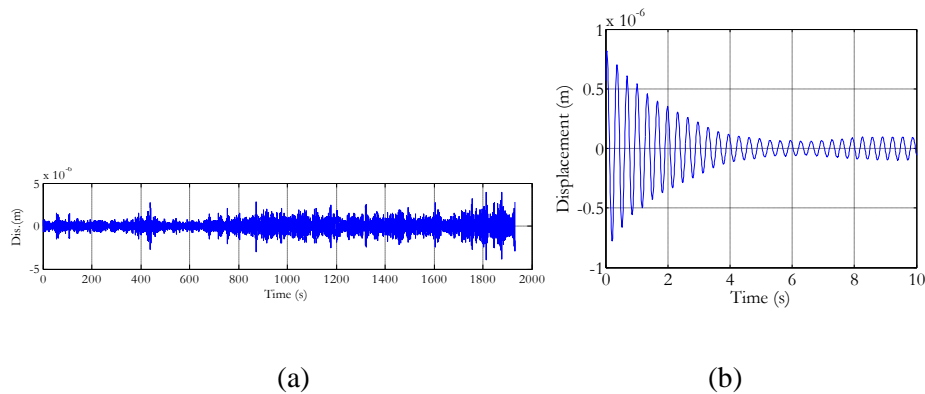


Figure 2: (a) Integrated displacement time history at 4th floor in E-W direction and (b) its RD signature plot.

The frequencies and mode shapes are extracted directly by decomposing the part of first 300 seconds of the displacement histories at each storey level into time-frequency domain using CWT with complex Morlet wavelet. Figure 3 shows the time-frequency plots obtained by decomposing of the 4th floor record. It indicates that the frequencies of the first, second and third translational modes in the longitudinal direction of the building can be identified clearly. Table 2 compares the frequencies estimated using CWT method and the modal analysis. It should note that the frequencies tabulated in Table 2 referring the CWT are the average periods obtained from the four records for each mode of the response.

Table 1: Comparison of damping ratios estimated using two approaches.

Floor No.	Mode 1			Mode 2		
	Damping (RD+CWT)	Damping (RD only)	Difference (%)	Damping (RD+CWT)	Damping (RD only)	Difference (%)
1	2.011	1.875	6.8	3.929	4.333	10.3
2	2.111	1.932	8.0	3.581	3.650	1.9
3	1.986	2.051	3.3	3.468	3.250	6.3
4	2.600	2.805	8.5	3.495	3.533	1.1

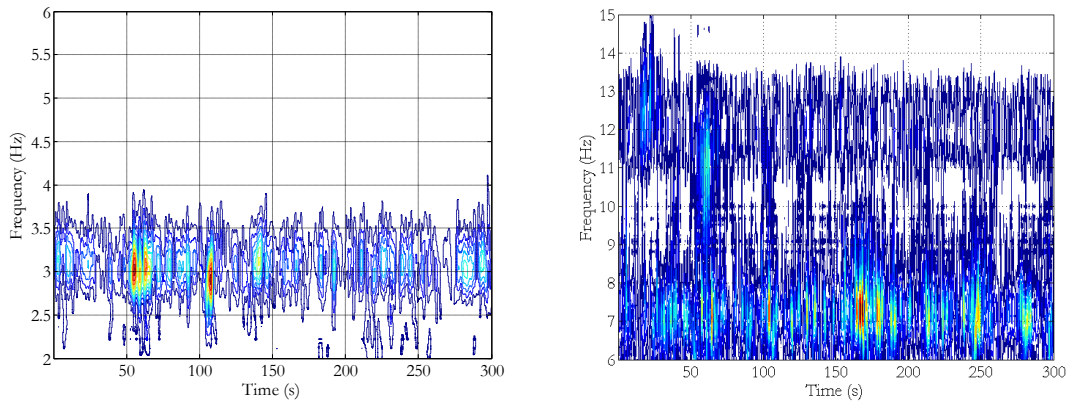


Figure 3: Time-frequency plot of modulus of CWT coefficients.

Figure 4 illustrates the comparison of the first mode shape extracted from CWT method and the model analysis. They are in good agreement. However, reasonably good estimation for the second mode shape could not be obtained using the approach used in this study.

Table 2: Comparison of frequencies extracted from CWT method and model analysis.

Mode	Period (s)- Numerical	Period (s)- CWT	Difference (%)
1	0.338	0.330	2.4
2	0.126	0.135	5.5
3	0.087	0.090	4.3

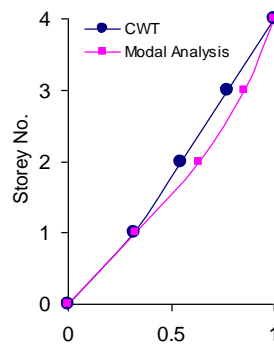


Figure 4: Comparison of mode shapes obtained from CWT and model analysis.

4. Conclusion

This study mainly investigates the capability of the continuous wavelet transformation (CWT) method using Morlet wavelet in estimation of model properties of low-rise buildings. For this purpose, a practical application is used. Based on the results of the application following conclusions can be drawn.

Based on the results of analysing ambient vibration measurements, recorded in the low-rise building, using the CWT method, dominant natural frequencies and the first translational mode shape can be extracted accurately compared to the results from model analysis. However, it is difficult to estimate the higher mode shapes using ambient vibration measurements with the approach used in this study. More investigation is required to establish a methodology to estimate the higher mode shapes accurately using CWT method. Furthermore, the damping ratios estimated using two steps procedure are well agreed with those obtained from the RD method.

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