Optimization of Multipurpose Reservoir Operation Using Game Theory

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***Abstra**ct This paper introduces the reader to the concept of game theory and illustrates how it can be efficiently applied to optimize the allocation of water resources in multipurpose reservoirs. First part of the paper discusses the fact that life is full of conflicts, and these conflicts can be modelled as games. The second part discusses the mathematical modelling of a game. The basic characteristic of a game is that each player has a set of strategies and players do not know what strategy the other players are going to use. However, payoff to each player depends not only what he or she does, but also on the strategies chosen by other players. Final section of the paper develops a multipurpose reservoir operation as a game. In this game the two players are the reservoir operator and nature. Objective of the model is to optimize the benefit to the reservoir operator irrespective of what nature does..*

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1. INTRODUCTION

Life is full of conflict and competition. Some examples are cricket, war, political campaigns, advertising and marketing by business firms. Game theory is a mathematical theory that deals with conflict situations.

Water resources engineers in developed countries have been using various mathematical techniques to optimize the benefit from available water resource (Harboe 1997). Today they have ability to plan and operate water resources projects much more efficiently than what they did 10 years ago. For this purpose different mathematical models have been developed and relevant algorithms are used to optimize the benefits (Hall 1961).

The situation is different in developing countries. They have acquired the computational power to match those of developed countries. However, they are far behind in application of optimization techniques in planning and operating of water resource systems (Block 2011).

The situation is not different in Sri Lanka. Almost all organizations who are involved in managing water resources have acquired sophisticated computer equipment. This equipment is not used to optimize the benefits. They are primarily used to simulate the hydrologic processes and management of irrigated water.

1.1. Basic Characteristics of a Game

In mathematics a game is defined as a situation where two or more players are in conflict with each other. Players can be people or organizations. Basic properties of a game are listed below (Taha 2003).

- There are at least two players.
- Each player wants to win.
- The winner will get a payoff.
- Players compete with each other.
- Cooperation is not an advantage.
- Rules of the game are clearly defined and known to all players, in advance.
- Each player has a finite set of possible strategies.

- The outcome of the game depends on the strategies chosen by each player.

A game is played by a set of moves taken one at a time. This collection of moves is called a strategy. A player's optimal strategy is the one that gives maximum benefit to a player irrespective of what the other player does.

Game theory is the study of how players should rationally play games. Each player would like the game to end in an outcome which gives him as large payoff as possible. The four main components that interact with each other in a game are players, strategies, outcome and payoffs.

Games are defined by,

i) The number of players. When there are two players it is called a two person game. When there are n players it is called an n person game etc.

ii) The net winnings of the game. A zero sum game is where the net winnings is zero. For example in a two person zero sum game what one player wins the other looses.

iii) Fairness of the game. A game which is not biased toward any player is called a fair game.
A game in which a given player can always win by playing correctly is therefore called an unfair game.

5.1.1. Payoff table

A payoff table is a table showing payoff to a given player for different strategies of players(Hillier 2002). Sample pay off table for player A in a two person game is given below

Table 1: Payoff table for a two person game

		Player B		
		x_1	x_2	x_3
Player A	y_1	a_{11}	a_{12}	a_{13}
	y_2	a_{21}	a_{22}	a_{23}
	y_3	a_{31}	a_{32}	a_{33}

In this example player A has three strategies y_1, y_2 and y_3 while Player B has three strategies x_1, x_2 and x_3. According to this example payoff to player A is a_{13} if Player A plays strategy y_1 and Player B plays strategy x_3.

Let us consider a simple numerical example to illustrate the solution procedure.

Table 2: Pay off table for a game with pure strategy

		Player B		
		x_1	x_2	x_3
Player	y_1	3	2	4
A	y_2	6	9	5
	y_3	1	3	2

The methodology of obtaining the optimal strategy for the two players is explained below. The mathematical techniques involved are outside the scope of this paper. Interested reader can find them in (Hillier 2002)

For player A: Find the maximum entry in each column. Get the minimum of these maxima.
For Player B : Find the minimum entry in each row. Get the maximum of these minima.

Optimal strategy for A is minimax and that of B is maximin. In this case maximin is equal to minimax.
When minimax is equal to maximin there is a pure strategy. Optimal strategy of A is y_2 and that of B is x_3.

It was easy to solve this problem because there was a pure strategy. However, most of the problems that are encountered in real life do not have a pure strategy. They need to be solved using linear programming techniques (Saiseni 1969), which is explained later.

1.2. Application of Game Theory in Reservoir Operation

In United States the capacity of multipurpose reservoirs are generally large (Hall 1961). Most of their capacities are equivalent to about five times the annual inflow. Operators of these reservoirs are tempted to keep the reservoirs about half full most of the time. In the event of a drought there is sufficient water in the reservoir to meet the irrigation demand. On the other hand if a flood arrives there is sufficient capacity to accommodate the flood.

In developing countries, multipurpose reservoirs are relatively small. Their capacity is equivalent to about annual inflow. When the reservoir is half full the operator is in a dilemma. If he releases the water for other uses he might run out of water for irrigation. On the other hand if he retains the water there might not be sufficient capacity to accommodate a flood.

1.3 Similarities Between Multipurpose Reservoir Operation and a Game

Operation of a multipurpose reservoir has all the characteristics of a game. There are players. In this case two. One, the person who operates the reservoir and the other 'nature'. Let us look at the problem from the reservoir operators angle. He does not know what the nature is going to do next. However, payoff to him depends not only what he does but also what 'nature' does. For example let us assume that currently the reservoir is full and the operator decides to keep it full without releasing any water. If a flood arrives at this stage there could be a big damage. On the other hand if he decides to release as much water as possible, the damage will either be mitigated or eliminated.

Now that it is established that reservoir operation can be modelled as a two person game, theory of games can be used to optimize the benefits.

1.3.1. Preparation of a Payoff Matrix.

First step in the development of the model is to prepare a pay off matrix (Romp 2003). Let us prepare the payoff matrix for the reservoir operator. At each stage reservoir operator has several strategies and nature has several strategies.

In this paper a hypothetical reservoir is modelled to illustrate the solution procedure. For simplicity, the reservoir is assumed to have five different levels. At each level operator can take several decisions, in this case they are called strategies. Let us assume that a maximum of 2000 units of water can be released from the reservoir. If we assume that the water is released in blocks of 500 units of water, following are five possible strategies for the operator.

Strategy 1 – No release
Strategy 2 – Release 500 units
Strategy 3 – Release 1000 units
Strategy 4 – Release 1500 units
Strategy 5 – Release 2000 units.

Similarly we can define a set of strategies for the nature. For simplicity let us limit the number of strategies to five. Each strategy represents the inflow to the reservoir. The range will be the expected minimum to the maximum inflow.

Next process is the preparation of a payoff matrix for each pair of strategies of the two players. Each release is associated with a net benefit to the operator which is the sum of benefits from all uses, in the case of a multipurpose reservoir. This benefit will have to be evaluated in monetary terms. For example if the inflow is less than release more volume is available for flood control which will increase the flood control benefits. On the other hand it will lower the water level of the reservoir which will reduce the recreational benefits. This will increase the risk of water shortage. The net benefit is the sum of all

benefits, some of which are negative.

To illustrate the computational procedure, a hypothetical pay off table given below was prepared. In this example five strategies are available for both players. x_1, x_2, x_3, x_4, and x_5 are the strategies available for the nature and y_1, y_2, y_3, y_4 and y_5 are the strategies available for the operator. The payoff table given below represents the payoff to the operator. Now the problem is to find out the best operational policy which will optimize the benefit to the operator.

Table 3: Pay off table for the Operator vs Nature Game

		Nature				
		x_1	x_2	x_3	x_4	x_5
	y_1	12	35	24	-13	18
Operator	y_2	21	-19	16	19	27
	y_3	20	17	31	18	22
	y_4	14	13	28	18	24
	y_5	19	21	22	16	26

Let X=(x_1,x_2,x_3,x4,x5) and Y=(y1,y2,y3,y4,y5) represents the optimal mixed strategies forthe operator and nature respectively. The probability that operator plays strategy i is x_i, and the probability that the nature plays j is y_j

The gain to operator at each play is a random variable α and the expected value E of a play to the operator is given in eq.(1)

$$E(\alpha; X, Y) = \sum_{ij} a_{ij} x_i y_j \qquad (1)$$

Where a_{ij} is the gain to operator when operator plays strategy i, and nature plays the strategy j.

Operator wishes to choose X so that regardless of the nature of Y, his expectation at each play exceeds some amount v. Operator wishes v to be as large as possible.

Analysing on similar lines, expected gain for 'nature' is,

$$E(-\alpha; X, Y) = \sum_{ij} -a_{ij} x_i y_j \qquad (2)$$

Nature will play so that the expected value of his gain exceeds some number v.

These conditions will yield the following equations.

$$12y_1 - 21y_2 + 20y_3 + 14y_4 + 19y_5 <= v \qquad (3)$$
$$35y_1 - 19y_2 + 17y_3 + 13y_4 + 21y_5 <= v \qquad (4)$$
$$24y_1 + 16y_2 + 31y_3 + 28y_4 + 22y_5 <= v \qquad (5)$$
$$-13y_1 + 19y_2 + 18y_3 + 18y_4 + 16y_5 <= v \qquad (6)$$
$$18y_1 + 27y_2 + 22y_3 + 24y_4 + 26y_5 <= v \qquad (7)$$
$$12x_1 + 35x_2 + 24x_3 - 13x_4 + 18x_5 >= v \qquad (8)$$
$$21x_1 - 19x_2 + 16x_3 - 19x_4 + 27x_5 >= v \qquad (9)$$
$$20x_1 + 17x_2 + 31x_3 + 18x_4 + 22x_5 >= v \qquad (10)$$
$$14x_1 + 13x_2 + 28x_3 + 18x_4 + 24x_5 >= v \qquad (11)$$
$$19x_1 + 21x_2 + 22x_3 + 16x_4 + 26x_5 >= v \qquad (12)$$

In addition we have two equations indicating that sum of all probabilities equal to 1.

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1 \qquad (13)$$
$$y_1 + y_2 + y_3 + y_4 + y_5 = 1 \qquad (14)$$

Solving these equations as a linear programming problem using Lingo software, yields the following result.

```
Global optimal solution found.
  Objective value:                          21.17647
  Infeasibilities:                          0.000000
  Total solver iterations:                        10


                  Variable           Value        Reduced Cost
                         V        21.17647            0.000000
                        Y1       0.6639469            0.000000
                        Y2       0.2009488            0.000000
                        Y3        0.000000            0.000000
                        Y4       0.1351044            0.000000
                        Y5        0.000000            0.000000
                        X1        0.000000            6.000000
                        X2        0.000000            5.235294
                        X3       0.5294118            0.000000
                        X4        0.000000            22.88235
                        X5       0.4705882            0.000000
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[X]= {0,0,0.53,0,0.47}
[Y]={0.66,0.20,0,.14,0}

V = 21.2

As such the operator should select his strategy according to probability distribution [Y]. When he does this he is guaranteed a payoff of 21.2, irrespective of what nature does.

2. CONCLUSION

In mathematics a game is defined as a situation where two people who are called players are in conflict. Each player has a set of strategies. Each one wants to win. The one who wins get a payoff. The outcome of a game depends not only what one player does, but also what the opponent does.

Reservoir operation has all the characteristics of a game. The two players are the reservoir operator and the nature. The operator does not know what nature is going to do in hydrological terms. Hence reservoir operation can be modelled as a game. Game theory can be used to optimise the benefits from reservoir operation irrespective of what the nature does.

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