FITTING GENERALIZED LAMBDA DISTRIBUTION TO SOLAR RADIATION IN COLOMBO, SRI LANKA

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Abstract

There is a need to look at the country's potential to develop solar power as a key power resource. As increasing the importance of solar radiation it is essential to identify a proper statistical distribution, to get an idea about the availability of solar radiation .The daily solar radiation data (MJ/m^2) collected by the Meteorological department from 1997 to 2007 at a station in Colombo at latitude 6 54 30' and longitude 79 51 30' were used for the analysis.

The daily solar radiation for monthly periods in Colombo tends to have a non-normal distribution and hence there is a need to identify a distribution which will cover different curve shapes. Thus, this study aims at fitting the Generalized Lambda Distribution (GLD) to daily solar radiation for each month. The parameters were estimated using the maximum likelihood method. The quality of the fits were assessed using histograms, qqplots, and resample KS (Kolmogorov-Smirnov) test. The fitted distribution was compared with the distributions in the literature namely the Weibull, Logistic, Normal and Lognormal using root mean square error.

The above mentioned goodness of fit test and the plots show that the GLD fits the data well. When compared with the distributions in the literature GLD has the minimum root mean square error. All the months tends to have negatively skewed distributions. The daily solar radiation totals from February to September show strong negative skewness and almost same distribution pattern. Further the months January, October, November and December show the least skewness. The mean solar radiation for the period 1997 to 2007 is 16 MJ/m^2. By taking16 MJ/m^2 as the threshold value probabilities were calculated above this value. The chance that the daily solar radiation above 16 MJ/m^2 is more than 87% for the months from February to September and for the rest of the months there is no chance of having solar radiation above this value. Since three fourths of the months have solar radiation more than 87% of the average value, a substantial potential exists in the city of Colombo for harnessing solar energy.

Key words: Solar Radiation, Generalized Lambda Distribution, Solar Energy, Four parameter distribution

1. Introduction

Due to the geo-chemical conditions, Sri Lanka is rich with several forms of renewable energy sources. Solar radiation is one of the cheap, sustainable, more environment friendly sources of power. Sri Lanka is just above the equator between 5^0 55' N and 9^0 55' N and between the eastern longitude 79^0 42' and 81^0 52'. Therefore, for a country like Sri Lanka solar radiation is a very good alternative renewable energy source where the sunshine is available in abundance.The most common uses of solar energy in Sri Lanka include drying, heating and electricity generation. There is a need to look at country's potential to develop solar power as a key power resource. To develop solar-based technology the knowledge of the quantity and quality of solar radiation in a given location is very much important. Thus it is essential to identify a proper statistical distribution to get an idea about the availability of solar radiation in different locations in the country and also the time periods of high solar radiation.

2. Materials and Methods

The daily solar radiation data collected by the Meteorological department for the period January- 1997 to December-2007 at a station in Colombo at latitude 6 54 30' and longitude 79 51 30' were used for the analysis. The unit of measurements is Mega joules per meter square (MJ/m^2). The daily solar radiation totals for monthly periods in Colombo tends to have non-normal distribution and hence need to identify a distribution which will cover different curve shapes. Thus this study aims at fitting the probability distribution introduced by Ramberg; namely the Generalized Lambda Distribution (Lakhany.A & Mausser.H, 2000) to daily solar radiation for each month. This is a four parameter distribution which can be used to fit a wide variety of curve shapes.

The Generalized Lambda Distribution originally proposed by Ramberg and Schmeiser (RS GLD) which is a four parameter distribution, is defined by its inverse distribution function:

$$F^{-1}(u) = \lambda_1 + \left(\left(u^{\lambda_3} - (1-u)^{\lambda_4}\right)/\lambda_2\right);\ 0 \le u \le 1$$

where λ_1 , λ_2 , λ_3 , and λ_4 are respectively the location , scale , and shape parameters of generalized lambda distribution GLD(λ_1 , λ_2 , λ_3 , λ_4) (Steve Su, 2007). In this study GLDEX (1.0.4.1.) and gld (1.8.4.) packagers in R software were used to fit the Generalized Lambda Distribution to empirical data and the parameters were estimated using the maximum likelihood method (Steve Su, 2007; Steve Su, 2010). It also provides the comparison of the mean, variance, skewness, and kurtosis between data and the fitted distribution. Then the quality of the fits were assessed by using histograms, qqplots, and resample KS (Kolmogorov-Smirnov) test (Steve Su, 2007). The quantile plots provide whether the data that the GLD distribution appears to give an adequate fit. The resample KS (Kolmogorov-Smirnov) test assesses the similarity between fitted distribution and actual data by sampling a proportion of the data and fitted distribution and calculating the KS test p-value. This process is then repeated many times, and the number of times the p-value is not significant is recorded. Further the frequency distributions of solar radiations are investigated using four kinds of probability density functions. That is the Weibull, Logistic,Normal and Lognormal. To evaluate the performance of the probability functions the root mean square error (RMSE) is considered as judgment criteria.(Tian Pau Chang,2010). The smaller the errors the better the fit is.

3. Results and Discussion

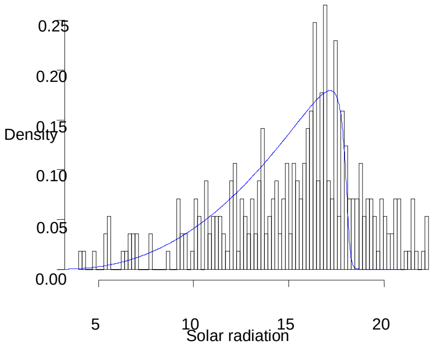
The Generalized Lambda Distribution was fitted to the daily solar radiation totals for each month. The estimates of the parameters and the skewness of the fitted distributions are given in Table 1.

Table1: Parameter estimates and the skewness of the distribution for daily solar radiation

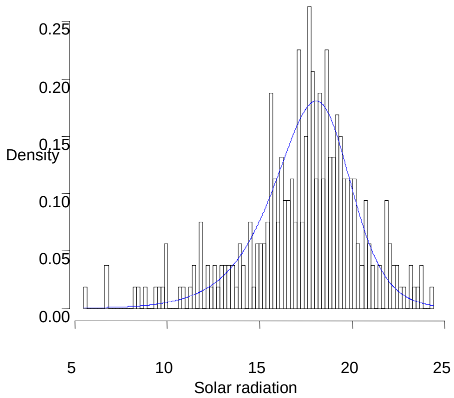
Month	*GLD* (λ_1 , λ_2 , λ_3 , λ_4)				*Skewness of the distribution*
	λ_1	λ_2	λ_3	λ_4	
January	*15.416*	*7.685*	*-0.874*	*3.204*	*-0.729*
February	*17.604*	*5.480*	*-0.390*	*3.468*	*-0.836*
March	*18.704*	*1.850*	*-0.377*	*3.225*	*-1.900*
April	*18.796*	*4.990*	*-0.614*	*3.914*	*-1.643*
May	*17.156*	*6.540*	*-0.439*	*2.941*	*-0.980*
June	*16.770*	*7.643*	*-0.901*	*3.523*	*-0.876*
July	*17.081*	*7.118*	*-0.754*	*4.071*	*-1.109*
August	*16.668*	*9.856*	*-0.902*	*3.272*	*-0.910*
September	*17.735*	*8.368*	*-1.514*	*10.278*	*-1.124*
October	*15.524*	*21.810*	*-0.730*	*3.150*	*-0.730*
November	*15.215*	*7.564*	*-0.616*	*3.231*	*-0.665*
December	*14.828*	*6.728*	*-0.683*	*3.227*	*-0.653*

Considering the each month separately it can be seen that the negative coefficient of skewness ranging from -1.900 to -0.653. These results demonstrate that all the months tend to have negatively skewed distributions. The daily solar radiation totals from February to September show strong negative skewness and almost same distribution pattern. Further the months January, October, November and December show the less skewness and have almost same distribution pattern. The behavior of the distribution for each month is shown in figure 1.

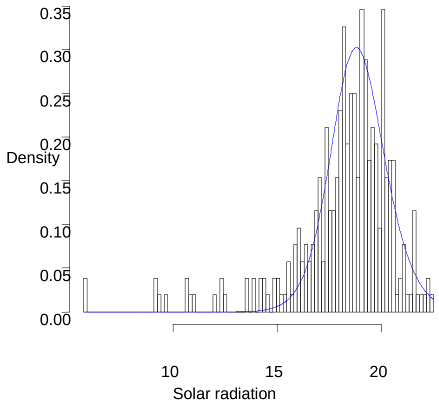
January



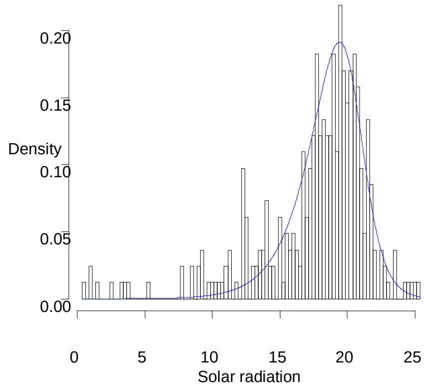
February



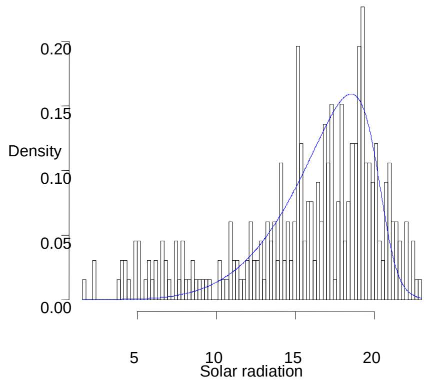
March



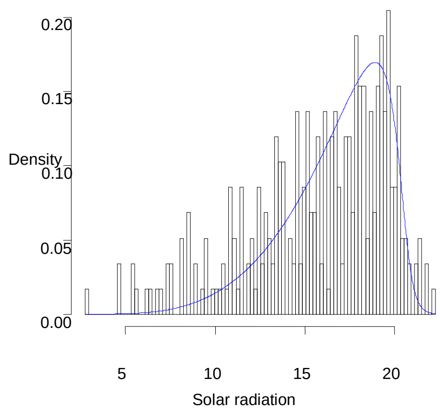
April



May



June



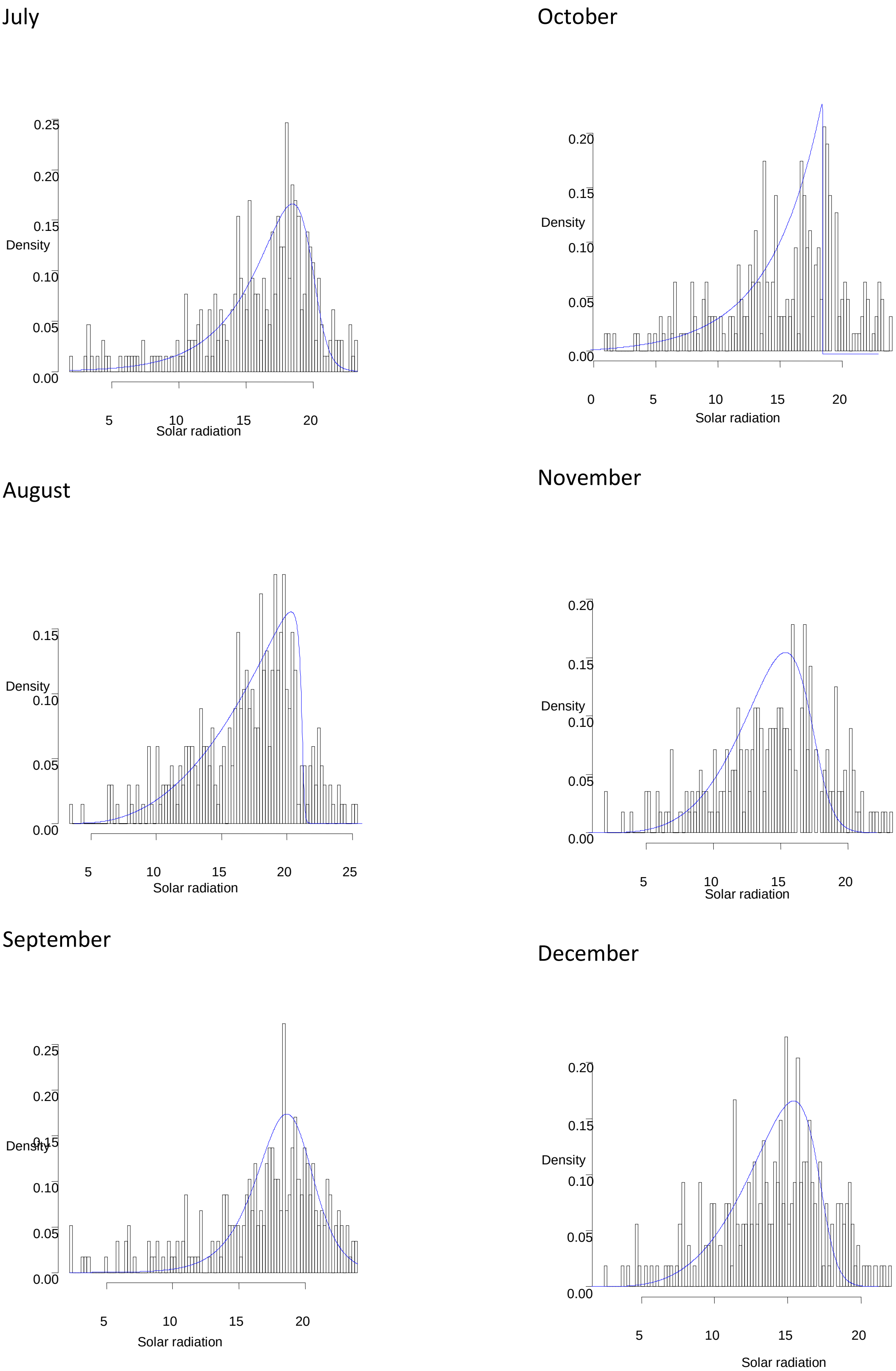


Figure 1: Frequency histograms of solar radiation and the density functions

The months from February to September show almost same distribution pattern. When considering the corresponding quantile plots for the above mentioned months, the data and the fitted distribution coincide we can say that the data fits well with the distribution. One example is given in figure2.

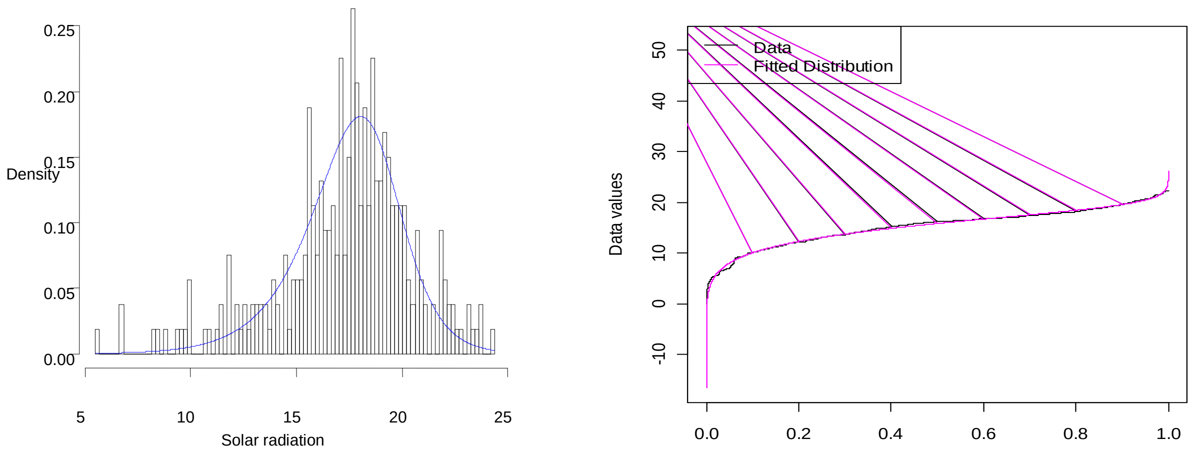


Figure2: Empirical and GLD fitted frequencies and Quantile plot for GLD fits on the solar radiation for the month February

Further the months January, October, November and December show almost same distribution pattern. The corresponding quantile plot for month January is given in figure3.

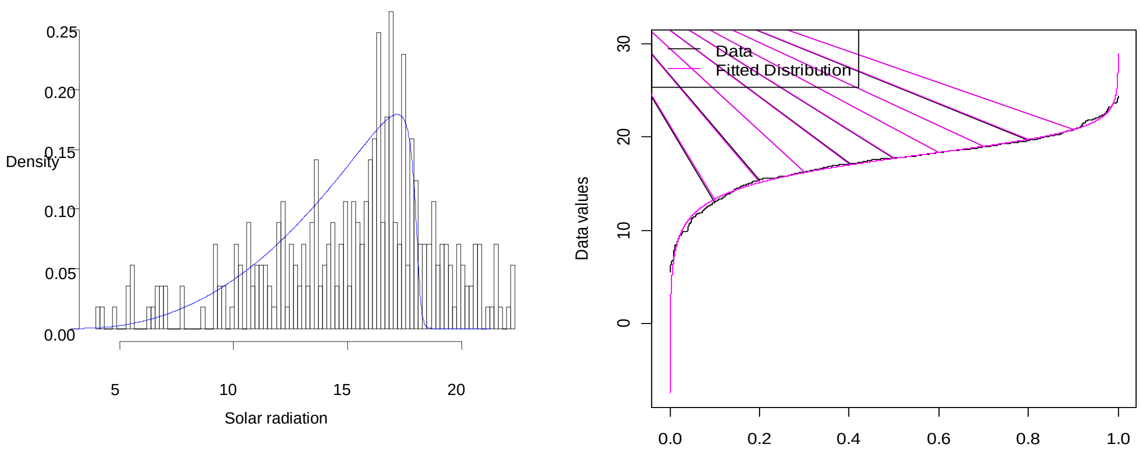


Figure3: Empirical and GLD fitted frequencies and Quantile plot for GLD fits on the solar radiation for the month January

The KS resample test gives 904 for the fitted distribution in figure 2 and 823 for the fitted distribution in figure 3. Thus more than 90% of the time for the distribution in figure 2 and more than 82% of the time for the distribution in figure 3, the KS test indicates that there is no difference between the fitted distribution and the empirical data.

Further, the fitted GLD was compared using four kinds of probability density functions in the literature, i.e. the Weibull distribution, Logistic distribution, Normal distribution and Lognormal distribution using root mean square error. The results are given in table 2.

Table2: The root mean square errors (RMSE)

	RMSE				
	Weibull	*Normal*	*Lognormal*	*Logistic*	*GLD*
January	*5.632*	*5.714*	*5.645*	*5.665*	*4.334*
February	*4.632*	*4.529*	*5.061*	*4.545*	*3.197*
March	*3.636*	*3.724*	*4.283*	*3.807*	*2.956*
April	*5.858*	*5.724*	*8.922*	*5.851*	*4.288*
May	*6.339*	*6.579*	*6.743*	*8.781*	*4.886*
June	*5.142*	*5.389*	*5.752*	*5.152*	*4.233*
July	*5.616*	*5.902*	*8.409*	*5.669*	*4.460*
August	*5.284*	*5.302*	*5.884*	*5.330*	*4.144*
September	*6.249*	*6.388*	*8.310*	*6.424*	*3.157*
October	*6.795*	*6.542*	*8.659*	*6.624*	*4.817*
November	*5.803*	*5.650*	*6.654*	*5.588*	*4.093*
December	*5.506*	*5.760*	*6.751*	*5.270*	*3.809*

The minimum root mean square error is given by the fitted GLD which confirms that the best distribution for daily solar radiation in Colombo is the GLD.

Further, the mean solar radiation for the period 1997 to 2007 is 16 MJ/m^2. A daily solar radiation value of 16 MJ/m^2 is considered as the threshold value. By fitting the probability density function and the distribution function, the probabilities of receiving solar radiation above the threshold value in each month at Colombo was calculated. It is given in Table 3.

Table3: Probability of daily solar radiation of greater than 16MJ/m²

Month	*January*	*February*	*March*	*April*	*May*	*June*
P(solar radiation>16)	*0.000*	*0.978*	*0.941*	*0.975*	*0.965*	*0.872*

Month	*July*	*August*	*September*	*October*	*November*	*December*
P(solar radiation>16)	*0.922*	*0.885*	*0.874*	*0.000*	*0.000*	*0.000*

It can be seen that the chance of getting solar radiation above 16 MJ/m^2 is more than 87% for the months from February to September and for the rest of the months there is no chance of having solar radiation above this value.

4. Conclusion

Since GLD is a four parameter distribution it fits the data more accurately. The high daily solar radiation totals can be seen from month February to September. Thus there is substantial potential exists in the city of Colombo for harnessing solar energy during these months. Further by taking any threshold value the probabilities that the solar radiation above the selected threshold value could be calculated.

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