COMBINED HIGH AND LOW CYCLE FATIGUE MODEL FOR PREDICTION OF STEEL BRIDGE LIVES

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ABSTRACT

A new fatigue model is presented to predict life of steel bridges for combined high and low cycle fatigue. It consists of a modified strain-life curve and a new strain based damage index. The damage variable is based on a modified von Mises equivalent strain to account for effects of loading non-proportionality and strain path orientation in multiaxial stress state. The proposed model was verified with experimental test results of two materials, available in the literature. Then, the proposed model was applied to a wrought iron railway bridge to estimate the fatigue life due to usual traffic and earthquake loadings. The obtained results confirm the importance and effectiveness of the proposed model over commonly used Miner's rule based life prediction of steel bridges.

Keywords: High cycle fatigue, Low cycle fatigue, Steel bridges, Life prediction, Earthquake loading.

1. Introduction

Bridges are generally subjected to high cycle fatigue (HCF) due to low amplitude loading by usual traffic during their service life. However, they may be subjected to low cycle fatigue (LCF) due to high amplitude loadings such as earthquake loadings. The combined damage of HCF and LCF may be a reason for a much reduced life (Kondo and Okuya 2007).

Most of fatigue life estimation of bridges is concentrated on multiaxial high cycle fatigue. There is almost no literature considering the combined damage of HCF and LCF of bridges. In other fields such as aircraft engineering, von Mises equivalent strain and Coffin-Manson strain-life curve are used with the Miner's rule as the general method to estimate the life for combined damage of HCF and LCF (Suresh 1998). However, von Mises equivalent strain cannot capture the effects due to non-proportional loading and orientation of strain path (Borodii and Strizhalo 2000). The Miner's rule is the simplest and the most widely used fatigue life prediction technique. However under many variable amplitude loading conditions, Miner's rule based life predictions have been found to be unreliable since it cannot capture loading sequence effect (Siriwardane et al. 2008).

These reasons raise the question about accuracy of the Miner's rule based life estimation for combined damage of HCF and LCF in bridges which are generally subjected variable amplitude

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loading and multiaxial stress state. Therefore, it is necessary to have a different model, which is based on commonly available material properties, to estimate more accurately the life for combined damage of HCF and LCF due to variable amplitude loading.

The objective of this paper is to propose a new model to accurately estimate the fatigue life (crack initiation life) when a bridge is subjected to combined damage of HCF and LCF. Initially, the proposed combined HCF and LCF is presented. Then, verification of the model is discussed. Finally, the proposed model is applied to an existing railway bridge to estimate fatigue life.

2. Proposed fatigue model

This section proposes the new fatigue model to estimate life of steel structures. Initially, the details relevant to proposed damage variable, modified strain-life fatigue curve are discussed. Then, the proposed damage indicator is explained.

2.1. Damage variable

The damage variable for combined HCF and LCF is given as (Borodii and Strizhalo 2000),

$$\varepsilon_{eq} = (1+\alpha\phi)(1+kSin\varphi)\varepsilon_{VM} \tag{1}$$

where ε_{eq} is the equivalent strain amplitude, α is the material parameter for loading non-proportionality, ϕ is the cycle non-proportionality parameter, k is the material parameter for strain path orientation, φ is the angle from the principal direction to the applied strain path and ε_{VM} is the von Mises strain as given,

$$\varepsilon_{VM} = \frac{1}{(\sqrt{2}\times(1+\nu))}\left[(\varepsilon_{xx}-\varepsilon_{yy})^2+(\varepsilon_{yy}-\varepsilon_{zz})^2+(\varepsilon_{zz}-\varepsilon_{xx})^2+\frac{3}{2}\times(\gamma_{xy}^2+\gamma_{yz}^2+\gamma_{zx}^2)\right]^{1/2} \tag{2}$$

where ν is the Poisson’s ratio. ε and γ are the axial and shear strain amplitudes in respective planes.

2.2. Strain-life curve

It is necessary to modify the strain-life fatigue curve in HCF regime in order to consider the combined damage of HCF and LCF. The proposed curve consists of two parts as shown in Figure 1. The first part of the curve describes fatigue life of plastic strain cycles which usually affect LCF. To describe this part, Coffin-Manson strain-life curve is utilized as shown below.

$$\varepsilon_{eq} = \frac{\sigma_f^{'}}{E}(2N)^b + \varepsilon_f^{'}(2N)^c \tag{3}$$

where ε_{eq} is the equivalent strain amplitude, N is the number of cycles to failure, $\sigma_f^{'}$ is the fatigue strength coefficient, b is the fatigue strength exponent, $\varepsilon_f^{'}$ is the fatigue ductility coefficient, c is the fatigue ductility exponent and E is the elastic modulus of the material.

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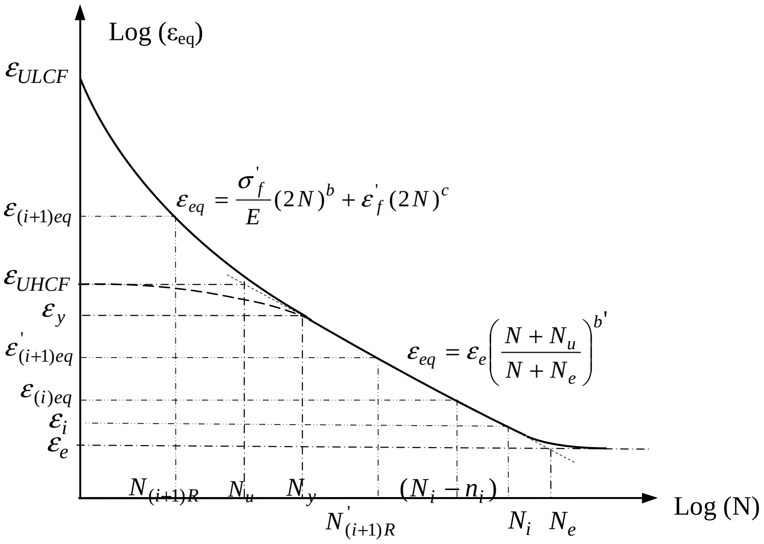

Figure 1: Schematic representation of the proposed strain-life curve

The ultimate strain of low cycle fatigue $(\varepsilon)_{ULCF}$ which is the equivalent strain amplitude corresponding to failure in half reversal (a quarter of a cycle) is obtained from Eq. (3) as,

$$(\varepsilon)_{ULCF} = \varepsilon'_f \tag{4}$$

The second part of the curve describes the fatigue life of elastic strain cycles which usually affects HCF. This part of curve represents hypothetical fully known curve. The shape of the curve is obtained by directly transforming the previous fully known stress-life curve (Siriwardane et al. 2008) to elastic strain-life curve as shown below.

$$\varepsilon_{eq} = \varepsilon_e\left(\frac{N+N_u}{N+N_e}\right)^{b'} \tag{5}$$

where ε_e is the strain amplitude of the fatigue limit, N_e is the corresponding number of cycles to failure. The ε_y and N_y are the yield strain and the corresponding number of cycles to failure. The b' is the slope of the finite life region of the curve. The $(\varepsilon)_{UHCF}$ is the ultimate strain of HCF which is the elastic strain amplitude corresponding to half reversal (a quarter of a cycle) is expressed as,

$$(\varepsilon)_{UHCF} = \left(\frac{\sigma_u}{E}\right) \tag{6}$$

where σ_u is the ultimate tensile strength of the material. The N_u is the number cycles corresponding to the intersection of the tangent line of the finite life region and the horizontal asymptote of the ultimate elastic strain amplitude $(\varepsilon)_{UHCF}$ as shown in Figure 1.

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2.3. Damage indicator

The proposed damage indicator considers combined damage of HCF and LCF due to variable amplitude loading. Suppose a component is subjected to a certain equivalent strain amplitude $(\varepsilon)_i$ of n_i number of cycles at load level *i*. N_i is the fatigue life (number of cycles to failure) corresponding to $(\varepsilon)_i$ (Figure 1). Therefore, the reduced life at the load level *i* is obtained as $(N_i{-}n_i)$. The damage equivalent strain $(\varepsilon)_{(i)eq}$ (Figure 1), corresponding to the failure life $(N_i{-}n_i)$ is defined as i^{th} level damage equivalent strain. Then, the new damage indicator, D_i is stated as,

$$D_i = \frac{(\varepsilon)_{(i)eq} - (\varepsilon)_i}{(\varepsilon)_u - (\varepsilon)_i}$$

(7)

where the $(\varepsilon)_u$ is

$$(\varepsilon)_u = \begin{cases} \varepsilon_{ULCF} & (\varepsilon)_i \geq \varepsilon_y \\ \varepsilon_{UHCF} & (\varepsilon)_i < \varepsilon_y \end{cases} \tag{8}$$

At the end of i^{th} loading level $(\varepsilon)_{i+1}$, damage D_i has been accumulated (occurred) due to the effect of loading cycles, the damage is transformed to load level *i+1* as below.

$$D_i = \frac{(\varepsilon)'_{(i+1)eq} - (\varepsilon)_{i+1}}{(\varepsilon)_u - (\varepsilon)_{i+1}}$$

(9)

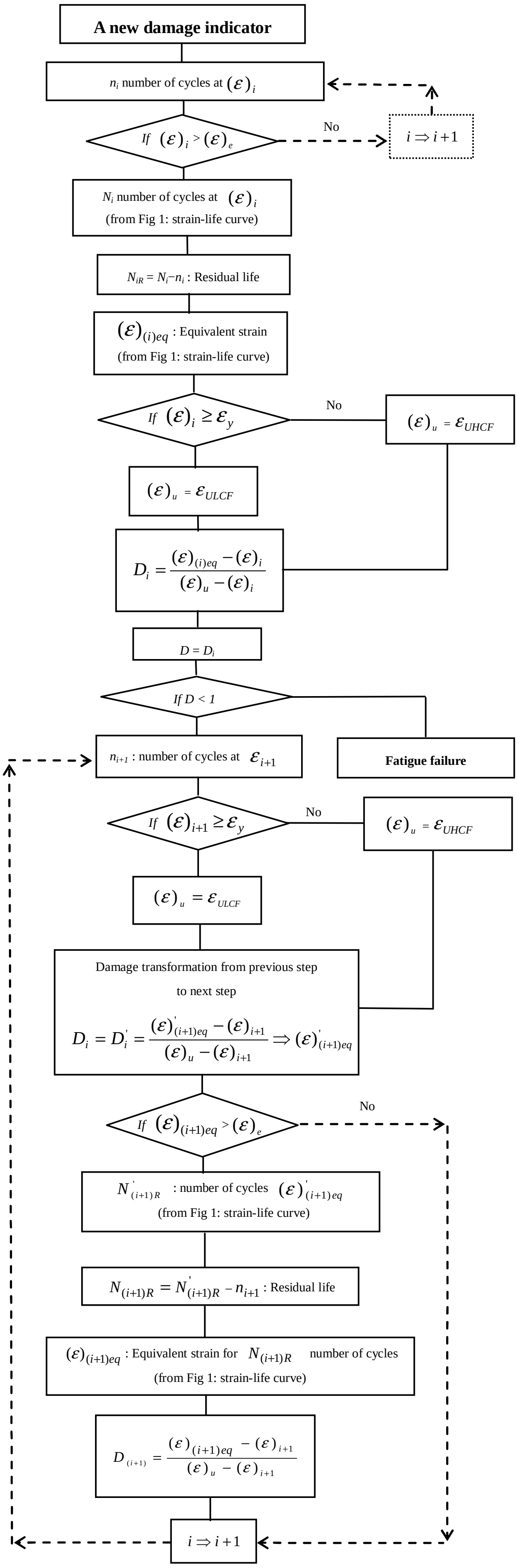
and $(\varepsilon)_u$ is expressed,

$$(\varepsilon)_u = \begin{cases} \varepsilon_{ULCF} & (\varepsilon)_{i+1} \geq \varepsilon_y \\ \varepsilon_{UHCF} & (\varepsilon)_{i+1} < \varepsilon_y \end{cases} \tag{10}$$

Then, $(\varepsilon)'_{(i+1)eq}$ is the damage equivalent strain at loading level *i+1* and it is calculated as,

$$(\varepsilon)'_{(i+1)eq} = D_i[(\varepsilon)_u - (\varepsilon)_{i+1}] + (\varepsilon)_{i+1}$$

(11)



The corresponding equivalent number of cycles to failure $N'_{(i+1)R}$ is obtained from the strain-life curve as

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shown in Figure 1. The $(\varepsilon)_{i+1}$ is the strain at the level *i+1* and supposing that it is subjected to $n_{(i+1)}$ number of cycles, then the corresponding residual life at load level *i+1*, $N_{(i+1)R}$ is calculated as,

$$N_{(i+1)R} = N'_{(i+1)R} - n_{(i+1)}$$
(12)

Therefore, strain, $(\varepsilon)_{(i+1)eq}$ which corresponds to $N_{(I+1)R}$ at load level *i+1*, is obtained from the strain-life curve as shown in Figure 1. Then the cumulative damage at the end of load level *i+1* is defined as,

$$D_{(i+1)} = \frac{(\varepsilon)_{(i+1)eq} - (\varepsilon)_{i+1}}{(\varepsilon)_u - (\varepsilon)_{i+1}}$$
(13)

This is carried out until D_i is equal to 1. Flow chart of the damage indicator is given Figure 2.

3. Verification of the proposed model

This section explains the verification of the proposed fatigue model by comparing fatigue test results available in the literature. Two experimental test results were used: S45C steel and Haynes 188.

3.1. Verification for S45C Steel

Fatigue tests performed by Chen et al. 2006 were used to verify the proposed fatigue model. Axial (A) and torsional (T) testing were performed in HCF and LCF regimes. The parameter, *k*, is estimated as 0.15 from the constant amplitude fatigue tests (Kim et al 1999). Then, fatigue lives of the proposed model and Miner's rule based previous model are estimated as given Table 1.

Table 1 Experimental summary and predicted fatigue lives of S45C steel

Test	First load level		Second load level		Experimental life (n_1+n_2)	Predicted life	
	Strain amplitude	No of cycles (n_1)	Strain amplitude	No of cycles (n_2)		Previous model	Proposed model
AT4	0.0047	1250	0.0035	119638	120888	202891	117808
AT5	0.0047	2500	0.0035	116826	119326	184654	75357
AT6	0.0019	25000	0.0097	4547	29547	35112	36241
AT7	0.0019	50000	0.0097	6411	56411	58280	60547
AT8	0.0019	75000	0.0097	6019	81019	81418	82633
TA1	0.0105	1250	0.0022	44879	46129	97492	57322
TA2	0.0105	2500	0.0022	35756	38256	84697	47253
TA3	0.0105	3750	0.0022	21598	25348	71901	34153
TA4	0.00495	25000	0.00464	4091	29091	33598	34346
TA5	0.00495	50000	0.00464	3281	53281	56284	61352
TA6	0.00495	75000	0.00464	2327	77327	75142	77950

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The percentage variations of the predictions are determined with the experimental results. The previous model gives a percentage variation of 23.9 % while the proposed model gives a value of 6.2 %. Therefore, the proposed model based fatigue lives are more accurate than previous model predictions.

3.2. Verification for Haynes 188

Fatigue test performed by Kalluri and Bonacuse 2002 were used verify the proposed fatigue model. Axial (A) and torsional (T) testings have been performed in different sequences (AA, AT, TT and TA). The parameter, *k* was estimated as 0.17 from constant amplitude tests given (Kalluri and Bonacuse 1999). Experimental results were compared with the predicted lives of the proposed fatigue model. In addition, the previous model used with the Miner's rule was also used in this case. The obtained comparisons are given in Table 2.

Table 2 Experimental summary and predicted fatigue lives of Haynes 188

Test	First load level		Second load level		Experimental life (n_1+n_2)	Predicted life	
	Strain amplitude	No of cycles (n_1)	Strain amplitude	No of cycles (n_2)		Previous model	Proposed method
AA1	0.0033	3926	0.0101	789	4715	4365	4413
AA2	0.0033	7851	0.0101	758	8609	8249	8337
AA3	0.0033	15702	0.0101	659	16361	15977	16147
AA4	0.0033	23553	0.0102	815	24368	23709	23931
TT1	0.0060	5857	0.0173	1250	7107	7276	7414
TT2	0.0060	11714	0.0175	1100	12814	12923	13189
TT3	0.0060	23427	0.0173	1343	24270	24316	24832
TT4	0.0059	35141	0.0173	1467	36608	35677	36219
TT5	0.0060	40998	0.0175	1294	42292	41348	41812
AT1	0.0035	3926	0.0174	1189	5115	5084	5345
AT2	0.0035	7851	0.0173	1218	9069	8660	9093
AT3	0.0033	15702	0.0172	930	16632	16058	16600
AT4	0.0033	23553	0.0173	1253	24806	23885	24185
TA1	0.0061	5857	0.0101	560	6417	6316	6367
TA2	0.0060	11714	0.0101	494	12208	12133	12216
TA3	0.0059	23427	0.0100	459	23886	23740	23907
TA4	0.0059	35141	0.0102	427	35568	35322	35588

The percentage variations of previous model predictions with experimental results were estimated as 0.74 % while the proposed model has a percentage variation of 0.62 %. Therefore, the predicted fatigue lives by the proposed fatigue model are more accurate than previous model predictions.

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4. Case study: fatigue life estimation of a bridge member

The proposed model was applied to a wrought iron railway bridge member to estimate the fatigue life due to traffic and earthquake loadings. The selected bridge is situated near Colombo in Sri Lanka and one of its members was selected for life estimation. The evaluations are especially based on secondary stresses and strains, which are generated around the riveted connection of the member due to stress concentration effect of primary stresses caused by usual traffic and earthquake loadings. The selected member is shown in Figure 3 (a) and (b).

Considered member

(a) (b)

Figure 3: Views of (a) the bridge; (b) considered member

The combined damage of HCF and LCF is evaluated considering all six rivets are active while all the riveted locations have no clamping force. The clamping force is generally defined as the compressive force in the plates which is induced by the residual tensile force in the rivet. Since this study assumes that the riveted locations have no clamping force (value of clamping force is zero), the connected members are considered to subject to the biaxial stress state. Therefore, a critical member without rivets can be considered to analyze the biaxial state of stress of a 2D finite element analysis. The nine node isoperimetric shell elements were used for the FE analysis.

Earthquake was considered to occur at different times in the bridge life as shown in Table 3. It is assumed that usual traffic load is followed after the earthquake. The fatigue life of the member was estimated using approaches: (1) proposed model; (2) previous model (Coffin-Manson curve with the Miner's rule). The obtained results are given in Table 3. The results indicate that combined damage of HCF and LCF causes an appreciable reduction of bridge life. For the proposed model, percentage reduction of life is the highest when the earthquake occurs at 50 years. If the earthquake amplitude is increased, the maximum percentage reduction occurs before 50 years. For the previous model, the reduction of service life is constant irrespective of time of earthquake occurrence since Miner's rule cannot capture the loading sequence effect. Comparison of fatigue life reveals that the proposed model predictions differ from the previous model predictions. This verifies that the proposed strain-life curve with new damage indicator better represent the combined HCF and LCF behaviour than Coffin-Manson relationship with Miner's rule.

Table 3 Fatigue life of the member for different earthquake occurrences

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Time of earthquake* (years)	Previous model (Miner's rule)		Proposed model	
	Fatigue life (years)	Percentage reduction of life (%)	Fatigue life (years)	Percentage reduction of life (%)
10	127.7	5.0	130.9	19.6
50	127.7	5.0	109.6	32.7
75	127.7	5.0	116.3	28.6
100	127.7	5.0	130.5	19.9
No earthquake	134.5		162.8	

*After construction

The differences of case study results confirm the importance of accurate combined HCF and LCF model to estimate the fatigue life of existing steel bridges.

5. Conclusions

A new model for combined damage of HCF and LCF was proposed to estimate life of steel bridge. A verification of the proposed model was conducted by comparing the predicted lives with experimental lives of two materials. It was shown that the proposed fatigue model gives an accurate fatigue life for combined damage of HCF and LCF where detailed stress histories are known. The proposed fatigue model was applied to estimate the fatigue life of a wrought iron railway bridge Case study realized the importance of consideration of the earthquake induced LCF damage in addition to HCF damage due to usual traffic loading in steel bridges. The importance and effectiveness of accurate prediction of combined damage of HCF and LCF was also confirmed.

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