Fluid Transients in Heterogeneous Fluid-Saturated Porous Geomaterials

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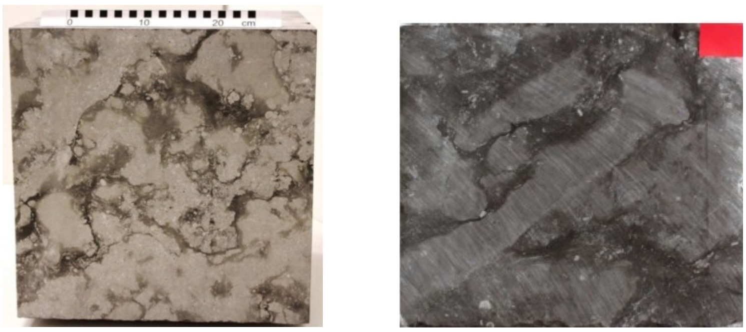
Abstract

This paper presents an approach for modelling fluid transients within a heterogeneous fluid-saturated porous medium, through the consideration of the effective parameters that can describe heterogeneity. This enables the application of the classical theory of piezo-conduction to the study of fluid transients in a porous medium where the permeability exhibits heterogeneity. The paper describes recent research that lead to the development of a measure for the effective permeability of a porous medium with hydraulic heterogeneity and presents the basic approach for examining the poroelasticity problem for a fluid-saturated medium with physical, hydraulic and elastic heterogeneity that can exhibit direct correlations. A computational procedure can be applied to validate the approach that represents the effective elasticity properties through mathematical relationships developed in the literature.

Keywords: Heterogeneous geomaterials, porous media, effective fluid transients, hydraulic heterogeneity, the piezo-conduction equation.

1. Introduction

Naturally occurring geologic materials are porous and they display heterogeneity at various scales. Heterogeneity can result from a variety of influences that range from depositional effects, reactive flows that can cause either dissolution or precipitation of minerals to stress induced alterations of the microstructure, similar to micro-cracking and damage. Determining the mechanical and physical properties of such naturally occurring heterogeneous geomaterials presents a considerable challenge to geoscientists and engineers, particularly as the hydro-mechanical processes that are of importance to geo-environmental and water resources applications, including, geologic disposal of contaminants, geologic sequestration of CO_2, energy resources recovery and geothermal energy and groundwater extraction, are largely controlled by the effective properties of the heterogeneous material. The effective properties in turn depend on the properties of the heterogeneities and their spatial distributions. In a majority of geologic settings the inhomogeneity is visible. An example of such a heterogeneous geomaterial is the Lindsay Limestone found in Southern Ontario, Canada, which is proposed as a host rock formation for the construction of a Deep Ground Repository for storing low and intermediate level nuclear waste (Selvadurai et al., 2011). The Lindsay Limestone is an argillaceous limestone, containing either nodular quartzitic limestone inclusions within a clayey or grey limestone or a quartzitic limestone containing veins of clayey grey limestone (Jenner and Selvadurai, 2011; Hekimi and Selvadurai, 2011 (Figure 1)).



(a) (b)

Figure 1. The Lindsay Limestone.

(a) A 406 mm cuboidal block (b) a 120 mm detail

The volume fractions of the species can vary but image analysis of photographic records indicate that samples used in the experimental research contained roughly equal proportions of the pure quartzitic limestone to the clayey grey limestone. An extensive research program has been initiated by the Nuclear Waste Management Organization, ON, to characterize the geomechanical properties of the heterogeneous limestone; these properties control the short term construction of the repository as well as the long term efficiency of radionuclide migration. The choice of the clayey limestone formation is dictated largely by its low permeability and its potential self-healing characteristics that can be attributed to the clay fraction in the grey component. The bulk permeability of the Lindsay Limestone can vary between 10^{-20} m^2 to 10^{-22} m^2, which requires transient techniques for its measurement. A second example of a heterogeneous geomaterial is Indiana Limestone; the permeability was recently investigated by Selvadurai and Selvadurai (2010, 2011a) using a permeameter specifically designed for measuring the near surface permeability of a 508 mm cuboidal block (Selvadurai, 2010a). In its general appearance the cuboidal region displays no overt signs of heterogeneity (Figure 2) but a detailed examination using steady state surface permeability measurements reveals a porous medium with a spatial variability in the permeability (Figure 3). The surface permeability was extrapolated to the interior of the cuboidal block using a kriging procedure.

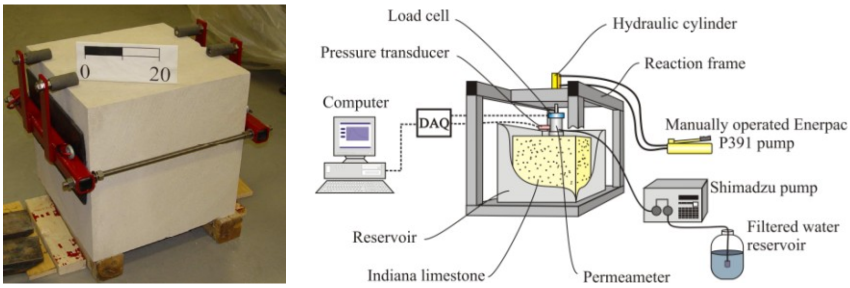


Figure 2. Cuboid of Indiana Limestone and the experimental configuration (Selvadurai, 2010a)

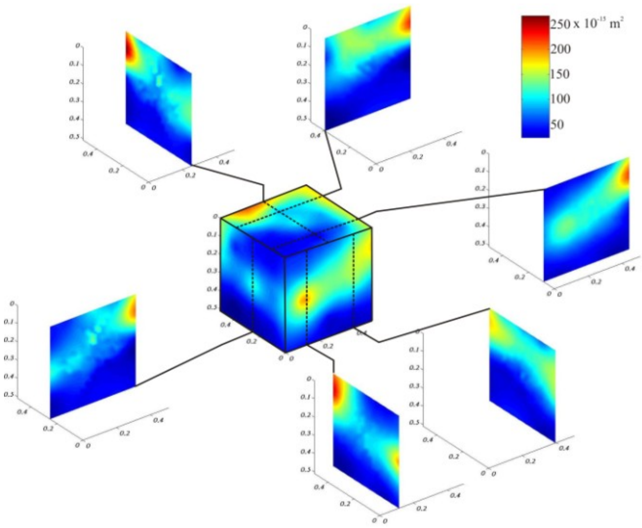


Figure 3. Permeability heterogeneity in the Indiana Limestone (Selvadurai and Selvadurai, 2010)

2. Geomaterial Heterogeneity and Fluid Transients

Experimental, mathematical and computational modelling conducted by Selvadurai and Selvadurai (2010) indicate that the permeability of the Indiana Limestone within the cuboidal block varies between 11×10^{-15} m^2 and 250×10^{-15} m^2, which represents a wide variation in the permeability. As discussed by Harr (1962), permeability of a geologic medium is usually the geomechanical property with the largest coefficient of variation, which can be attributed to spatial heterogeneity of the porous medium. The studies by Selvadurai and Selvadurai (2010) also point to the fact that although the permeability can exhibit spatial variability, it is possible to characterize the "effective permeability" for the heterogeneous porous medium by introducing suitable measures. The Wiener Bounds (Selvadurai and Selvadurai, 2010) represent the extreme limits for the effective permeability and several other representations, including those given by Landau and Lifshitz (1960), Matheron (1967), Journel et al. (1986), King (1987) and Dagan (1993), give useful and relatively close estimates of the effective permeability for the heterogeneous porous medium. Selvadurai and Selvadurai (2010) propose the following estimate for the effective permeability of a region with heterogeneity in the permeability: *Consider a heterogeneous porous medium with a spatial distribution of point-wise isotropic permeability that corresponds to a lognormal distribution. If hydraulically constrained one-dimensional permeability through this region, measured in* n *directions, is denoted by* K_n*, then the effective permeability of the porous medium of the region is given by*

$$K_{eff}^{SS} = \sqrt[n]{K_1 K_2 K_3 ......K_n} \tag{1}$$

Computational research by Selvadurai and Selvadurai (2011b) convincingly proves that this assertion is valid for any type of flow patterns initiated within the porous medium. Guided by this research, it is natural to enquire whether fluid transients in heterogeneous porous media could be examined in a similar way, where effective parameters are assigned to variables encountered in the formulation of tests such as the hydraulic pulse test. For example, the transient decay of the reduced Bernoulli potential $\Phi(\mathbf{x},t)$ (which consists of only the pressure and datum potentials) in the fluid-saturated pore space of a homogeneous geomaterial is governed by the piezo-conduction equation

$$\left(\frac{K}{\mu(n^* C_w + C_{eff})}\right)\nabla^2\Phi(\mathbf{x},t) = \frac{\partial\Phi(\mathbf{x},t)}{\partial t} \tag{2}$$

where K is the permeability, n^* is the porosity, μ is the fluid viscosity, C_w is the compressibility of the pore fluid, C_{eff} is the bulk compressibility of the porous skeleton, $\mathbf{x}$ is a position vector, t is the time variable and ∇^2 is Laplace's operator. The piezo-conduction equation can be solved for an appropriate experimental configuration where the decay of the fluid potential within a confined fluid region in contact with a surface of the saturated medium is used to estimate the permeability of the porous medium. In the instance when a hydraulic pulse test is conducted in a fluid-saturated porous medium with heterogeneity in the physical, mechanical and fluid transport characteristics, the partial differential equation governing the transient fluid transport problem can be formulated in relation to the effective values for the permeability $(\tilde{K})$, the compressibility of the porous skeleton $(\tilde{C}_{eff})$ and porosity $(\tilde{n}^*)$. It should be noted that such a reduction is applicable provided the spatial variations in the properties of the porous medium are small. Consider a hydraulic pulse test that is conducted in a *fluid-saturated heterogeneous porous domain*, where the effective values corresponding to the heterogeneities are used in the formulation. The partial differential equation governing the time-dependent fluid pressure in the porous medium is given by (Selvadurai, 2000, 2009; Selvadurai et al., 2005)

$$\left(\frac{\tilde{K}}{\mu(\tilde{n}^* C_w + \tilde{C}_{eff})}\right)\frac{\partial^2 \Phi(z,t)}{\partial z^2} = \frac{\partial \Phi(z,t)}{\partial t} \tag{3}$$

subject to the boundary and regularity conditions

$$\Phi_a \left(\frac{\partial \Phi}{\partial z}\right)_{z=0} = \left(\frac{\partial \Phi}{\partial t}\right)_{z=0} \quad ; \quad \Phi(\infty, t) \to 0 \tag{4}$$

and the initial condition

$$\Phi(z,0) = 0 \tag{5}$$

In (4), $\Phi_a = (A\tilde{K} / \mu V_w C_w)$, where A is the cross-sectional area of the sample and V_w is the volume of the fluid that is in contact with the surface area A and subjected to a potential of the form

$$\Phi(0,t) = \Phi_0\, \delta(t) \tag{6}$$

where $\delta(t)$ is the Dirac delta function. The initial boundary value problem defined by (3) to (6) can be solved using Laplace transform techniques and the result of interest to the hydraulic pulse test, namely the time-dependent decay of the fluid potential in the pressurized cavity region, is given by

$$\frac{\Phi(t)}{\Phi_0} = \exp(\Omega^2 t)\,\mathrm{Erfc}(\sqrt{\Omega^2 t}) \tag{7}$$

where $\mathrm{Erfc}(x)$ is the complementary error function defined by

$$\mathrm{Erfc}(x) = \frac{2}{\sqrt{\pi}} \int_0^{\infty} \exp(-\xi^2)\, d\xi \tag{8}$$

and

$$\Omega^2 = \left(\frac{A^2 \tilde{K}(\tilde{n} C_w + \tilde{C}_{sk})}{\mu V_w^2 C_w^2} \right) \tag{9}$$

The result (7) can be used to examine the accuracy of the representation of the properties $K(\mathbf{x})$, $E(\mathbf{x})$, $\nu(\mathbf{x})$ and $n(\mathbf{x})$ through their corresponding effective representations $\tilde{K}$, $\tilde{E}$, $\tilde{\nu}$ and $\tilde{n}^*$.

3. Computational Modelling of Transient Flow

We now examine, computationally, the transient fluid flow problem described previously within the context of its application to a typical heterogeneous porous medium similar to the Indiana Limestone examined by Selvadurai and Selvadurai (2010). Hydraulic pulse tests are specifically conducted on geomaterials of relatively low permeability and the Indiana Limestone is considered to be a porous medium where the permeability can be determined from steady state tests rather than transient pulse tests. The examination of pulse tests conducted on Indiana Limestone is sufficient for the purposes of demonstrating the reliability of the representation of the effective properties of a heterogeneous porous medium by appeal to measures such as the geometric mean. We consider the decay of a hydraulic pulse conducted in Indiana Limestone, with permeabilities derived from the steady state tests described previously but extended to cover 1000 sub-regions of homogeneous permeability conforming to a lognormal distribution (Figure 4). To computationally examine the effects of heterogeneity on the performance of a pulse test, we construct a synthetic heterogeneous medium where the upper and lower bounds for the variables are specified and variation in the properties relevant to the examination of hydraulic pulse tests (i.e. the permeability $K(\mathbf{x})$, the elastic modulus $E(\mathbf{x})$, the Poisson's ratio $\nu(\mathbf{x})$ and the porosity $n^{*}(\mathbf{x})$) are all assumed to correspond to lognormal distributions. In the domain of interest to the hydraulic pulse tests, each sub-region is assigned a property corresponding to a value that is randomly selected from the lognormal distribution. The selected distribution satisfies the *Kolmogorov-Smirnoff Test* (Massey Jr., 1951) and establishes the applicability of the lognormal distribution with a 95% confidence level. It is further assumed that the compressibility of the porous skeleton and the porosity correlates directly with the permeability. Experimental investigations conducted by Selvadurai and Głowacki (2008) and Selvadurai and Selvadurai (2010) indicate that the elastic modulus, Poisson's ratio and porosity of the Indiana Limestone can vary in the ranges of $E \in (23.3, 70.0)$ GPa, $\nu \in (0.207, 0.300)$ and $n^{*} \in (0.16, 0.30)$ respectively.

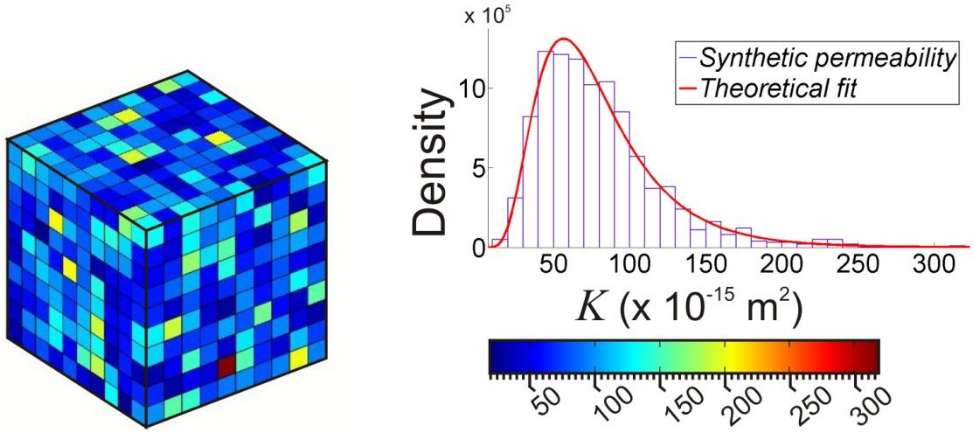


Figure 4. Cuboidal domain with hydraulic heterogeneity

To determine the effective compressibility of the cuboidal element with heterogeneity in the compressibility, we subject the cuboidal region to states of oedometric compression and pure shear (Davis and Selvadurai, 1996; Suvorov and Selvadurai, 2011), in order to determine

$$\tilde{E}_{oed} = \frac{\tilde{E}(1-\tilde{\nu})}{(1+\tilde{\nu})(1-2\tilde{\nu})} \quad ; \quad \tilde{G} = \frac{\tilde{E}}{2(1+\tilde{\nu})} \tag{10}$$

and these results yield $\tilde{E} = 32.06$ GPa and $\tilde{\nu} = 0.216$, which can be used to estimate the effective compressibility $\tilde{C}_{sk}$ of the heterogeneous porous skeleton from the relationship

$$C_{sk} = \frac{3(1-2\nu_{sk})}{E_{sk}} \tag{11}$$

Ideally, the effective properties of the heterogeneous porous skeleton should be estimated by subjecting the entire heterogeneous element, separately, to oedometric compression in three orthogonal directions and pure shear in three orthogonal planes. An alternative approach is to directly subject the porous medium with elastic heterogeneity to isotropic compression and to compute the effective compressibility of the medium by determining the overall volume change. Both these procedures give almost the same estimate for $\tilde{C}_{sk}$ of ~ 0.53 x 10^{-7} m^2 /kN. The effective porosity of the cuboidal porous element with porosity heterogeneity can be estimated in a straightforward manner using the arithmetic mean. It can be shown that the effective values of the parameters of interest to the analytical modelling of the response of hydraulic pulse tests can be evaluated as follows:

$$\tilde{K} = 73.96 \times 10^{-15}\ \mathrm{m}^2\ ; \quad \tilde{C}_{sk} = 0.53 \times 10^{-10}\ \mathrm{m}^2/\mathrm{N}\ ; \quad \tilde{n}^* = 0.196 \tag{12}$$

The hydraulic pulse test is modelled by finite elements and with properties assigned the spatial variations that conform to lognormal distributions and taking into consideration the correlations indicated previously. We consider the problem of a hydraulic pulse test conducted on a 508 mm cuboidal region of Indiana Limestone. The exterior lateral boundaries of the region are subjected to null Neumann boundary conditions for fluid flow. No other boundary conditions can be prescribed on the deformations since the governing partial differential equation strictly relates to the potential field $\Phi(z,t)$. In order to simulate the fluid reservoir of volume V_w that is subjected to the pressure pulse defined by (4), we consider a finite enclosure with cross sectional area A that is in contact with the porous medium; the boundaries are non-deformable and the permeability of the region is significantly larger than a plausible value for the rock that is being tested. This procedure was proposed by Selvadurai (2010b) and can successfully model pulse tests. At the start of the hydraulic pulse tests the fluid pressure in the reservoir region will be finite, whereas the fluid pressure in the porous domain will be zero. The discontinuity in the fluid pressures is accommodated for in the analytical solution (7), whereas numerical instabilities can occur in the computational simulations if suitable mesh refinement and time discretizations are not adopted. The computations should therefore consider adaptive algorithms in space and time to account for this requirement. In this research, however, extensive mesh refinement and small time steps are adopted to ensure stability of the solutions. The mesh refinement used in the study is illustrated in Figure 5. Both the "fluid" and the porous domains are discretized using Lagrangian (C_0 continuity) cuboidal elements. The computational approach is used to develop a solution to the problem of a one-dimensional hydraulic pulse test that is conducted on a heterogeneous porous medium where the relevant parameters exhibit lognormal variations. The computational solutions were developed using the multi-physics code COMSOL™.

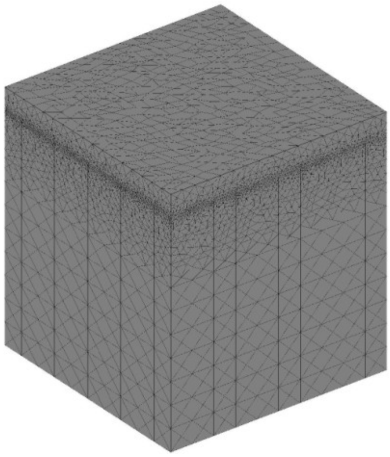


Figure 5. Finite element mesh for one-dimensional pulse tests (127,583 tetrahedral elements).

In addition to the parameters defined by (12) we note that

$$\begin{aligned} &A = 25.8064\ \mathrm{m}^2\ ;\ V_w = 13.1097\ \mathrm{m}^3 \\ &C_w = 4.541\times 10^{-7}\ \mathrm{m}^2/\mathrm{kN}\ ;\ \mu = 10^{-3}\ \mathrm{N\,s\,m}^{-2} \end{aligned} \tag{13}$$

From (12) and (13) we have

$$\Omega^2 = 0.197\ \mathrm{s}^{-1} \tag{14}$$

Figure 6 illustrates the pressure decay curve obtained for one-dimensional axial pulse tests conducted on the cuboidal sample of Indiana Limestone. Results are given for (i) one-dimensional hydraulic pulse tests conducted on a completely heterogeneous cuboidal porous domain where the *permeability*, *compressibility* and *porosity* of the medium exhibit *lognormal distributions*, and for (ii) one-dimensional hydraulic pulse tests conducted on a representative cuboidal element of the Indiana Limestone where the permeability is represented by its geometric mean and the compressibility and porosity are represented by their respective effective values derived through overall deformations of the heterogeneous domain and through the average value, respectively.

The results also illustrate that the flow velocity pattern within the cuboidal region is heterogeneous but largely one-dimensional. The magnitudes of the flow velocities tend to vary but the flow tubes are in general uniformly one-dimensional.

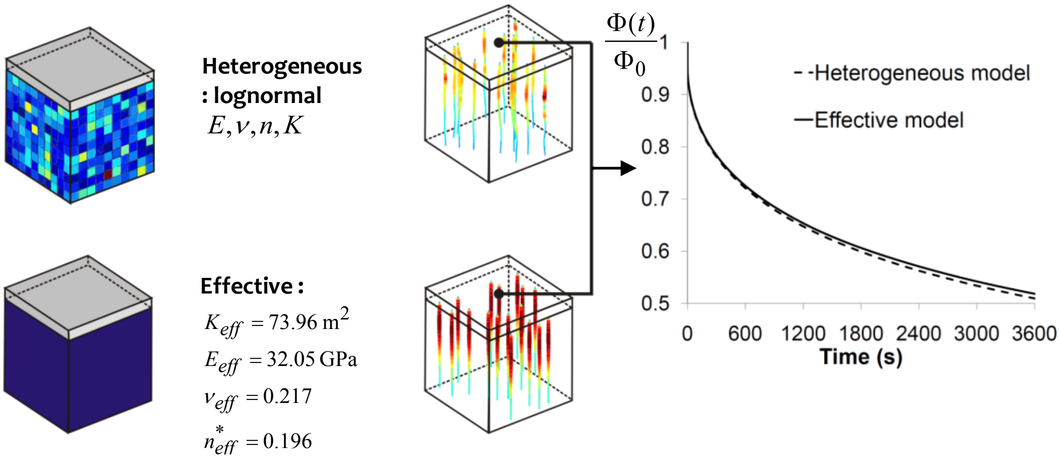


Figure 6. Transient decay of fluid potential in a hydraulic pulse test conducted on a cuboidal sample of heterogeneous Indiana Limestone

4. Concluding Remarks

Modelling fluid transport behaviour in naturally occurring heterogeneous porous media has to take into account the spatial variability of the permeability properties. In practical problems where the fluid flow is to be examined by appeal to a computational approach, there are two options; the first is to represent every part of the domain taking into account the exact spatial variability of the porous medium. This is manageable when the domain of fluid flow is not spatially extensive. For most regional ground water flow problems where the dimensions of the domain of interest are large, the computational modelling of the detailed permeability distribution is not a realistic option. Recourse must therefore be made to a simplified approach where the permeability can be represented through an equivalent measure. Laboratory research conducted on a sample of Indiana Limestone indicates that even at the scale of a cuboidal sample measuring 508 mm, the permeability can exhibit variability. The results of the research have shown that when the spatial permeability exhibits a *lognormal distribution*, the effective permeability can be accurately modelled by the *geometric mean*. This paper extends the concepts to include compressibility of the porous skeleton and porosity of the porous medium, which are necessary when hydraulic pulse tests are conducted to determine the effective permeability characteristics of a heterogeneous porous medium. It is shown that the geometric mean-based effective permeability, together with spatially averaged values of the compressibility and porosity, which also exhibit lognormal spatial variations, can be successfully used to examine hydraulic pulse tests. Results for the computational modelling of the pulse decay that uses the exact spatial variations of permeability, compressibility and porosity agree quite closely with the results for the pulse decay that uses the effective values for these parameters.

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