INFLUENCE OF SLAB CONNECTION ON SLENDERNESS EFFECTS IN SLENDER RECTANGULAR RC BEAMS

Dr. K. Girija
Assistant Professor, College of Engineering Trivandrum, India
(email: girijak.cet@gmail.com)

Dr. Devdas Menon
Professor, Indian Institute of Technology Madras, Chennai, India
(email: dmenon@iitm.ac.in)

Abstract

Slender reinforced concrete (RC) beams with narrow rectangular sections and no integral slab connections are commonly encountered in pre-cast construction, fascia and roof elements in buildings. Long and slender concrete beams are also encountered in prestressed concrete applications, typically in bridge girders, and instability failure have sometimes been observed during erection. More commonly, in concrete buildings, tanks, etc., slabs are integrally connected to the beams. Slenderness effects are not likely to be of concern if such slabs are located in the flexural compression zone of the beams. However, if the beams are very slender, and the slabs are located near the flexural tension zone, then slenderness effects can still be of significance, and need to be reckoned with in design. Such cases are commonly encountered in balcony construction and long cantilever beams, where "T-beam" action is absent and the beams are designed as rectangular beams in practice.

This paper reports the results of experiments conducted to study the effect of slab connection on the flexural tension side of narrow rectangular beams. The test results establish that the presence of an integrally connected slab in the flexural tension zone enhances flexural stiffness, mainly in the vertical plane.

Keywords: beams, critical buckling moment, instability, reinforced concrete, slenderness.

1. Introduction

Behaviour of slender RC beams is different from that of normal proportioned beams. These beams are susceptible to slenderness effects. Highly slender beams are prone to sudden instability (lateral torsional buckling) mode of failure. Moderately slender beams are also susceptible to slenderness effects, inducing lateral deflections and twisting of the cross-section, as in steel beams. Hence, such beams may undergo failure at moment values less than the estimated flexural capacity (M_{uf}). Estimations of the critical buckling moment (M_{cr}) and the ratio, M_{uf}/M_{cr}, are necessary to have an accurate prediction of the behaviour and ultimate moment capacity M_u of the slender RC beam. However, at present, these slenderness effects are not accounted for in the prevailing concrete design codes (ACI 318 (2008), BS 8110(1997), IS 456 (2000), EC 2 (2003), AS 3600 (2001)) unlike steel design codes. Concrete design codes aim at avoiding instability failures due to slenderness in RC rectangular beams, by imposing slenderness limits in terms of the dimensions of the beam (span/breadth and breadth/depth ratios). These slenderness limits specified in RC beams have been traditionally aimed at ensuring that the instability mode of failure does not precede the flexural mode of failure. This is achieved by ensuring that the critical moment capacity (M_{cr}) of the beam section exceeds the ultimate moment capacity (M_{uf}) due to flexural failure. However, there are disparities among the codes with regard to the specified slenderness limits. Furthermore, recent studies have shown that these slenderness ratios, expressed solely in terms of dimensions of the beam, are inadequate measures of slenderness. Other design variables, viz., the percentages of longitudinal and transverse reinforcement and the grades of concrete and steel, also need to be accounted for.

The presence of slabs which provide lateral support to beams influences the behaviour of slender beams. In practice, the vast majority of slender RC beams belong to this category. They are typically encountered in balcony parapets, fascia projections, walls of storage tanks, etc. The presence of integrally connected slab on compression side of the slender beam will add to the lateral stiffness of the beam and is beneficial for the lateral stability of the beam. But this beneficial effect will not be there if the slab is on the flexural tension side.

1.1 Literature review

The lateral stability of RC beams was first studied by Marshall in 1948, as reported in Hansell and Winter (1959). Based on the experimental studies conducted, Hansel and Winter (1959) proved that the ratio *L/b* is not a true measure of slenderness as suggested in ACI code. In a subsequent study, Siev (1960) experimentally established the influence of reinforcement ratio on the behaviour of slender RC beams. Sant and Bletzacker (1961) conducted experiments on over reinforced beams and proposed analytical expression to calculate M_{cr} and showed the importance of *d/b* ratio on behaviour of slender RC beams. Massey (1967) considered the contribution of longitudinal reinforcement as well as stirrups, along with that of concrete, to torsional rigidity of the section. Massey and Walter (1969) applied the theory proposed by

Massey (1967) to simply supported RC beams subjected to central point loading. Revathi and Menon (2006, 2007) conducted experiments on slender rectangular RC beams and proposed an expression to estimate M_{cr}, which attempted to account for stiffness degradation due to cracking of concrete. They concluded that a wide range of slenderness limits are feasible for different sets of design variables. However, subsequent tests carried out by the authors of the present study brought out certain shortcomings in the formulation, and suggested significant improvements (Girija and Menon (2011)).

Several experimental and theoretical studies have been carried out in various countries on slender RC rectangular beams, to predict the failure moment and the mode of failure. However, the effect of connecting slab on the behaviour has not been sufficiently studied. Theoretical predictions are rendered difficult by the three-dimensional response of the slender beam and the reduction in flexural and torsional stiffness with incremental loading on account of cracking of concrete.

2. Experimental Investigation

Experimental investigation consists of testing of both rectangular beams and beams with slab connection on flexural tension side.

2.1 Test Specimens

A total of four specimens with two rectangular beams and two beams with integrally connected slab on the flexural tension side were chosen for the experimental study. The details of specimens are shown in figs 1 and 2. For all the four beams the span was maintained as 6.0 m and a cross section of 100 mm wide and 450 mm deep was taken to get more slender test specimens. The rectangular beams were numbered B1a and B2a and the corresponding beams with slab connection were numbered as B1b and B2a.

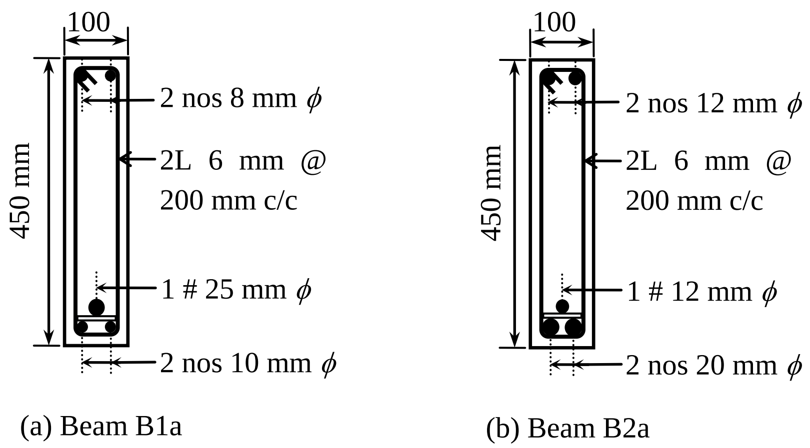

Figure 1: Reinforcement details of rectangular test beams

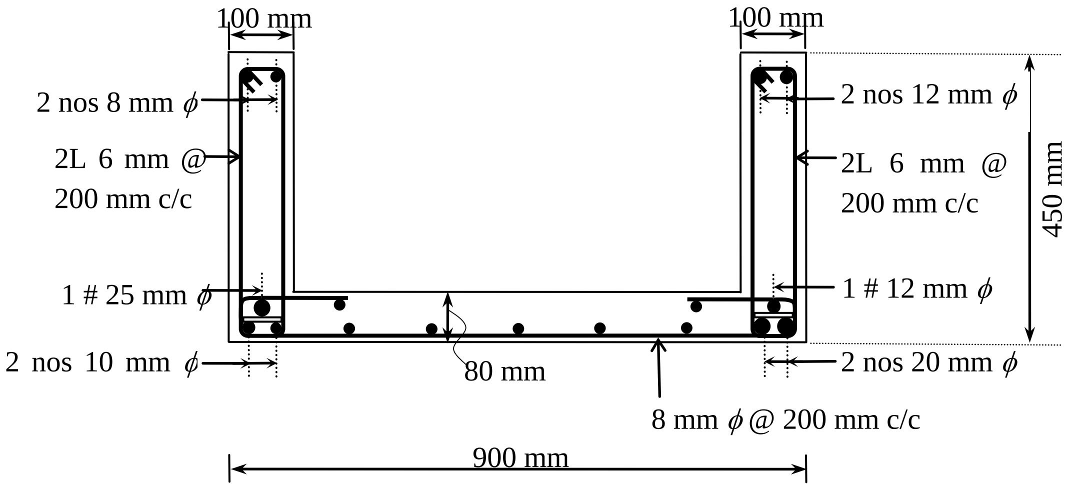

Figure 2: Reinforcement details of test beams with slab connection on flexural tension side

A mix proportion of 1: 2.3: 4.35 (cement: sand: coarse aggregate, by weight), with a water-cement ratio of 0.50 and a cement content of 330 kg/m^3 for getting a mean strength of 35 to 45 MPa was adopted. Ordinary Portland cement of 53 grade and high yield strength (Fe 500) rods of 8 mm to 25 mm diameter were used. Reinforcing steel with characteristic yield strength of 250 MPa was used for the stirrups.

Table 1 Average cube strength of test beams

Beam label	*B1a*	*B2a*	*B1b*	*B2b*
Average cube strength (MPa)	*35.5*	*35.5*	*32.8*	*32.8*

2.2 Specimen Instrumentation and Testing

The beams were placed in position along with loading and support assemblies. The experiment setup is shown in Fig. 3. The beams were loaded with two point loads at middle-third span points and with load steps such that failure would occur after 15 to 20 load steps. The loading scheme for test beams is shown in Fig. 4. Vertical as well as lateral deformations were recorded at mid-span and quarter-span locations. Dial gauges, with least count of 0.01 mm and 50 mm maximum deflection, were used for the measurement of the vertical deflections as well as lateral deflections at top and bottom locations. The beams were loaded to failure and the failure load and mode of failure were noted.

For testing beam B1b, the required support conditions were given for the test specimen and the beam B2b was supported throughout the span to get the simply supported end condition for the connecting slab. The observations of mid span and quarter span deflections in both vertical and horizontal directions were noted. After testing beam B1b, the position of support assemblies and loading arrangements were shifted to test the other beam, B2b.

Figure 3: Experiment Setup

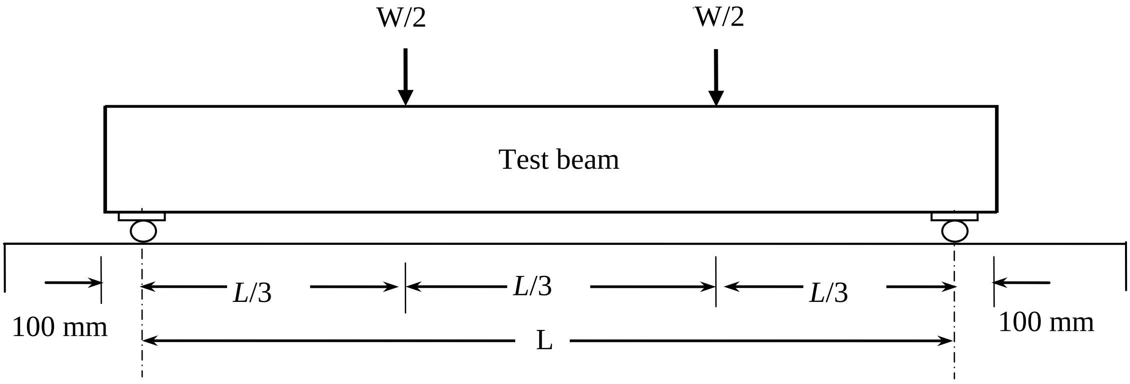

Figure 4: Load position for test beams

3. Theoretical formulation

Using the conventional theory of elastic stability (Timoshenko and Gere (1961)) it can be shown that, for a homogeneous elastic beam of length L, the critical buckling moment, M_{cr} can be expressed as:

$$M_{cr} = \frac{C_1 C_3}{C_2 L}\sqrt{EI_y GJ} \qquad (1)$$

where EI_y is the lateral flexural rigidity and GJ is the torsional rigidity. C_1 is a factor that accounts for type of loading, C_2 accounts for the end conditions of the beam and C_3 accounts for the location of the load with respect to the centroidal axis of the rectangular beam. This expression can be suitably modified to account for nonlinearity and cracking in reinforced concrete beams, by replacing the terms EI_y and GJ in Eq. 1 with the effective measures of lateral flexural and torsional rigidities, B_{eff} and K_{eff} respectively.

An expression for the critical buckling moment of a simply supported narrow rectangular beam subjected to point loads at middle third span points, can be generated from first principles by applying the principle of stationary potential energy (Girija K. (2011)) as:

$$M_{cr} = 1.09\frac{\pi}{L}\sqrt{B_{eff}K_{eff}}\left(\sqrt{1+\left(1.57\frac{\bar{a}}{L}\sqrt{\frac{B_{eff}}{K_{eff}}}\right)^2} - 1.57\frac{\bar{a}}{L}\sqrt{\frac{B_{eff}}{K_{eff}}}\right) \quad (2)$$

where $\bar{a}$ is the height of load above the centroid.

The moment curvature (M-φ) relationship of RC beams proposed by Bažant and Oh (1984) is adopted in the present study, to get the effective lateral flexural rigidity at any loading level. Corresponding to any point on the M-φ curve generated based on the model proposed by Bažant and Oh (1984), a measure of the effective flexural rigidity with respect to the major axis, ($EI_{eff,x}$), is obtainable as the ratio, M/φ. Using the value of secant modulus E, $I_{eff\ x}$ can be determined. An effective rectangular section with the same width b, but with a reduced depth d_{eqv} is assumed and an approximate expression for the effective flexural rigidity with respect to the minor axis is obtained as follows.

$$B_{eff} = EI_{eff,y} = E\left(\frac{d_{eqv}b^3}{12}\right) \quad (3)$$

The equation for effective torsional rigidity K_{eff}, of cracked concrete beam, proposed by Tavio and Teng (2004), is adopted in the present study.

$$K_{eff} = \frac{4\mu E_s A_o A_c}{p_o^{\,2}\left(\frac{1}{\rho_l + \rho_{tr}}\right)} \quad (4)$$

where μ is the rigidity multiplier taken as 1.5 as suggested by Tavio and Teng (2004), $\rho_l = A_l / A_c$ and $\rho_{tr} = (A_t p_1)/(A_c s)$ denote the ratios of reinforcement in the longitudinal and transverse directions respectively, E_s is elastic modulus of steel, A_l is total cross-sectional area of longitudinal steel along the periphery of the beam, A_t is cross sectional area of one leg of transverse stirrups, s is spacing of stirrups and A_o, A_c, p_o, p_1 are cross-sectional properties of the beam section.

The proposed expression for M_{cr} was validated (Girija (2011)) against all the reported test results on slender RC rectangular beams (which failed in sudden instability mode of failure). It was seen that the proposed formulation predicts M_{cr} more accurately than any of the existing theories, and with least dispersion (Girija and Menon (2011)).

4. Results and Discussion

Load deflection plots for test beams are furnished in Figs 4 and 5. Also, the test results are summarized in Table 2.

The estimated M_{cr} for the rectangular beams were less than the calculated flexural capacity of the beams and sudden instability mode of failure was observed as expected. The lateral deflection and twisting increased significantly with loading and cracks were found to open up on the convex side. The final instability failure occurred suddenly, with the lateral deflections increasing uncontrollably. The failure occurred at a reduced capacity as is evident from Table 2. Crushing of concrete on the concave side was also observed and the failure occurred at a moment value less than the calculated flexural capacity of the beam.

The final failure could not be achieved for beams B1b and B2b due to the lifting up of the simply supported slab and beam connected to the specimen before failure. The load-deflection behaviour observed is depicted here. Lateral deflection and twisting were observed for these beams also. Beams B1b and B2b were observed to carry more load than the corresponding rectangular beam.

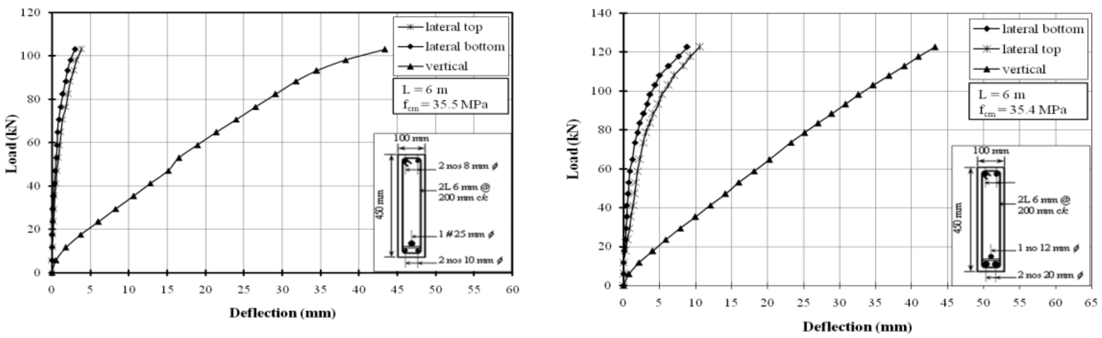

Figure 4: Mid-span deflection for beams B1a and B2a

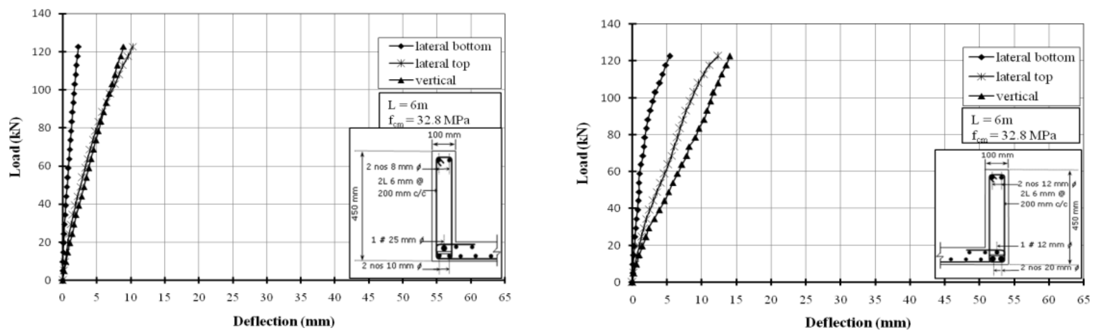

Figure 5: Mid-span deflection for beams B1b and B2b

Table 2 Test results: Failure moment and mode of failure

Beam Label	*Estimated critical buckling moment M_{cr} kNm*	*Calculated flexural failure moment M_{uf} kNm*	*Experimental failure moment M_{test} kNm*	*M_{test} / M_{uf}*	*Mode of Failure*
B1a	*112.1*	*126.5*	*106.5*	*0.842*	*Sudden lateral instability*
B2a	*134.7*	*145.1*	*133.1*	*0.917*	*Sudden lateral instability*

Comparison of the load deflection behaviour of the two sets of test beams showed that the presence of an integrally connected slab in the flexural tension zone enhances flexural stiffness, mainly in the vertical plane. The beneficial effect of increased stiffness in the lateral direction is offset by the increased tendency of the section to twist, due to the eccentricity in the lateral restraint. The tests conducted show that the vulnerability to buckling is not significantly reduced by the presence of a slab at the bottom; vertical deflections are much reduced, but lateral deflections at top increase. It is evident from the test results that the slab connection on the flexural tension zone of slender beams does not have significant influence on the slenderness of the beam.

5. Conclusions

1) The presence of an integrally connected slab in the flexural tension zone enhances flexural stiffness, mainly in the vertical plane.

2) The beneficial effect of increased stiffness in the lateral direction is offset by the increased tendency of the section to twist, due to the eccentricity in the lateral restraint.

3) The tests conducted show that the vulnerability to buckling is not significantly reduced by the presence of a slab at the bottom; vertical deflections are much reduced, but lateral deflections at top increase.

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