DESIGN OF CONTINUOUS PRISMATIC PRESTRESSED CONCRETE SPINE BEAMS WITH VARIABLE PRESTRESSING FORCES

Eng W M D N Ranasinghe, Director Engineering, Western Province Engineering Organisation
Dr M T R Jayasinghe, Senior Lecturer, Dept of Civil Engineering, University of Moratuwa.

ABSTRACT: Design of continuous prismatic prestressed concrete continuous spine beams is a complex task, primarily due to the secondary moments that occur as a result of the prestressing effects. In prestressed concrete members, it is useful to select the smallest possible section while satisfying the criteria like the use of a particular construction technique, constructability and stress limits. The methods that can be employed for selecting the smallest section dimensions by considering the global and local bending, constructability, restrictions on depth etc. are highlighted. It is advantageous to minimise the total cable force ($\int Pds$) used for a continuous prestressed concrete prismatic spine beam with respect to economy and constructability. It is shown in this paper that the total cable force cannot be minimised by using a constant cable force throughout the length. For that, different cable forces could be used in span and support regions. It is also shown that for any given section, there is a maximum cable force that should not be exceeded over the supports. The selection of secondary moments to ensure that the cable profile zone resulting from the minimum cable forces will lie within the section is also highlighted. When the cable force changes, there will be point moments and point forces that will act at those sections. A straight forward method is presented for dealing with these when finding the cable profile that will satisfy the stress limits and also generate the secondary moment distribution already selected at the beginning of the design.

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Notation and Sign Convention

Tensile stresses are positive
Position within the beam is measured positive downward from the centroidal axis.
Sagging moments are positive.

A	Total cross-sectional area
c	Cover from edge of concrete to centre of tendon
d	Overall depth of beam
e	Tendon eccentricity
e_{max}	Maximum eccentricity at which tendon can be placed (y_2 - c)
e_{min}	Minimum eccentricity at which tendon can be placed (y_1 + c)
$e_{sn(i)}$	Eccentricity of new cable at i^{th} change point
f	Limiting stress condition
f_{cu}	Concrete cube strength
f_{cw}	Permissible compressive stress at the working load (-ve)
f_{tw}	Permissible maximum stress at the working load (+ve if tensile)
I	Second moment of area
M	Applied moment
$M_{a(act)}$	Maximum applied moment at the working load without secondary moments
$M_{b(act)}$	Maximum applied moment at the working load without secondary moments
M_a	Maximum applied moment at the working load with secondary moments
M_b	Maximum applied moment at the working load with secondary moments
M_2	Secondary moment
$(M_2)j$	Secondary moment at internal support j
P	Prestressing force in cable at transfer
P_B	Cable force corresponding to point B of Magnel diagram
P_i	Forces acting on i^{th} system
$P_{n(i)}$	Cable force in the new cable at the i^{th} cable force change point
$P_{r(i)}$	Cable force in the running cable at i^{th} cable force change point
R	Loss ratio

R_{ik} Reactions due to loading on beams
y_1 Position of top fibre (-ve)
y_2 Position of bottom fibre (+ve)
Z_1 Elastic section modulus of top fibre (I/y_1) (-ve)
Z_2 Elastic section modulus of bottom fibre (I/y_2) (+ve)
$\alpha_{n(i)}$ Inclination of the anchor for new cable
$\alpha_{r(i)}$ Inclination of the running cable at force change point I
β_j Distribution coefficient for M_2
$\beta_{ll(i)}$ Inclination of lower bound of cable profile at left hand side of i^{th}cabel force change point
$\beta_{ur(i)}$ Inclination of the upper bound of the cable profile at right hand side of the i^{th} cable force change point
Δ_i Displacements of the i^{th} system

INTRODUCTION

Box girder bridges are often used in the span range of 30-200 m (Swann, 1972). They are quite popular due to the inherent torsional rigidity that gives good load sharing characteristics. A box girder consists of single or multiple cells with cantilever overhangs on either side at the top flange. Since these structures are generally constructed as continuous over the supports, the design process becomes a complicated task. This is due to the secondary moments that are induced as a result of prestressing effects. Such secondary moments can alter the bending moments due to dead and live loads thus leading to many iterations in the design calculations.

A design method that can be followed as a logically evolving process by the design engineer to produce an optimum design was presented by Ranasinghe & Jayasinghe (1998). This method was presented for a constant prestressing force. In this paper, the above method was further refined to deal with variable prestressing forces. When the cable forces vary, there are additional constraints on the cable due to the location of anchor blocks. A straight forward method is presented which allows the designer to select the location of anchor blocks. It also gives a simple method to find an appropriate cable profile that generates the secondary moments selected earlier. With this rational design method, now it will be possible for any design engineer to produce continuous spine beam bridges which are very close to the optimum solutions that can be achieved with respect to the smallest cross section.

METHODOLOGY

The following methodology was used for the research work presented:

1. The design principles were developed to select minimum variable cable forces that would be needed for the continuous cable.

2. A design method was developed to select suitable secondary moments.

3. A straight forward design method was presented to find a suitable cable profile that will generate the above selected secondary moments and also fit within the limits on the cable profile.

THE DESIGN METHOD

The design method suggested can be presented as follows:

1. Selection of the smallest practically possible cross section.

2. Determination of minimum cable forces that can be used for the above section.

3. Selection of suitable secondary moments so that the smallest practically possible section can be used with the minimum cable forces.

4. Determination of the cable profile so that the secondary moments generated are quite close to those selected.

SELECTION OF SECTION DIMENSIONS

The selection of the smallest possible section based on construction aspects and other practical considerations is described in detail by Ranasinghe & Jayasinghe (1998). In order to clarify the design process, a design example is also presented along with the design method and principles. The box girder used for the design example is as follows. The box girder bridge consist of three spans of 40m, 50 m and 40 m. This bridge will carry a two lane highway of lane width 3.5 m and two walkways of width 1.5 m on either side. Thus the total top flange width is 10.0 m. It will be constructed using span by span construction technique. The section dimensions selected for the design example are presented in Table 1 highlighting the underlying principles. The section selected is shown in Figure 1.

The other useful design information are as follows:

1. Grade of concrete (f_{cu})= Grade 40 = 40 N/mm^2
2. Allowable stress in compression (f_{cw}) (BS 5400/4)= -16,000 kN/m^2
3. Allowable stress in tension (f_{tw}) (BS 5400/4)= 0.0 kN/mm^2
4. Allowable stress in compression used for calculations = -15,000 kN/m^2
5. Allowable stress in tension used for calculation = -1000 kN/m^2

It should be noted that the allowable stresses used for calculations have a certain allowance for any effects that cannot be taken into account accurately at the preliminary design stage.

As soon as the trial section is selected, it is necessary to check whether it can satisfy the minimum section required. This must ensure adequate behaviour at both the working loads and ultimate loads. Thus, a bending moment envelope should be developed taking account of dead, superimposed dead and live loads. For the bending moments due to dead loads, it is necessary to consider both as built and monolithic bending moment diagrams. As soon as the bridge is built, the dead load bending moments are given by the as-built bending moment diagram. However, this changes towards the monolithic (built at once) bending moment diagram due to long term creep defromations in concrete (Neville et al., 1981). Thus, a dead load bending moment envelope should be considered with as built and monolithic conditions as boundaries.

The resultant bending moment envelope for various loading conditions can be obtained by using a suitable method such as grillage analogy. For the design example, a shear flexible griallge, as shown in Figure 1, having two longitudinal beams representing the two webs and the associated flanges are used (Jaeger & Bakht, 1982). These are connected by using transverse members at 5.0 m intervals.

It is also considered that the bridge will be constructed by span by span construction technique with one span and 10m length of the next span being constructed in one operation. This can be illustrated as given in Figure 2. When the bending moment envelope is available, the global bending behaviour at working loads can be used to determine the section size required (lines 19 and 20 of the spread sheet).

Table 1: Dimensions selected for the box girder and the reasons for the selection

Dimension	Reason
Width of top flange = 10 m	To accommodate two lanes of width 3.5 m and two walkways of width 1.5 m.
Depth of the section = 2.1 m	A final depth of 2.1 which gives a span/depth ratio of 23.8 for the internal span
No of webs = 2	Two webs to minimise the number of webs (Podolny & Muller, 1982)
Width of webs = 0.35 m	A width of 0.35 m to anchoring of cables (Podolny & Muller, 1982)
Spacing of webs = 4.65 m	A centre to centre spacing of 5.0 m which gives a cantilever overhang of 2.325 m
Width of the bottom flange = 5.35 m	5.35 m with vertical webs
Thickness of bottom flange = 0.175 m	A thickness of 0.175 m which is the minimum to prevent horizontal cracks due to horizontal shear flow (Podolny & Muller, 1982)

In prestressed concrete beams, it is necessary to ensure that a Magnel diagram exist at all sections. Figure 3 shows a typical Magnel diagram for a beam subjected to a range of bending moments. Two pairs of lines originate from each of the Kern points ($e = -Z_1/A$ and $e = -Z_2/A$). One of each pair of lines relates to tensile stresses in the relevant extreme fibre, the other relates to compressive stresses in that fibre.

For the feasible region to exist, there must be a positive separation of the two bound lines emanating from each Kern points. This is equivalent to saying that the section must have elastic section moduli sufficiently large that the range of stresses caused by the range of applied moments are less than the range of stresses that the concrete can resist.

They can thus be expressed in the form:

$$Z_1 \leq \frac{M_b - M_a}{f_{tw} - f_{cw}} \quad (Z_1 \text{ is -ve}); \quad Z_2 \geq \frac{M_a - M_b}{f_{cw} - f_{tw}} \qquad \text{eq -1, eq - 2}$$

These expressions assume that the working load moments range govern the design, which is the case for the span by span construction technique considered in this paper. Therefore, the Z values required can be calculated as soon as the bending moment envelope is obtained. In a box girder, there are many ways of achieving the required Z values such as adjusting either the depth of the section or the thickness of the bottom flange or the width of the webs or the thickness of the top flange. The best option should give the required value of Z with the smallest cross sectional area. This is most likely to be the adjusting of the depth of the section. However, it is advisable to carry out a parametric study.

For the design example, the required value of Z_1 and Z_2 are -0.978 and 0.978 (eq. 1 and eq. 2), respectively at the most critical section, which is at the second internal support (lines 21 and 22 of the spread sheet). The minimum section selected with a depth of 2.0 m on the basis of guidelines explained in this paper gave Z_1 and Z_2 of -1.94 and 0.954. Thus, the initial section is not sufficient. A parametric study can be carried out as given in Table 2 in order to find the smallest possible section that gives the required section moduli. It can be seen that increasing the depth of the section to 2.1 m will give the smallest cross section while achieving the required Z values.

When the depth of the section is increased, there can be additional costs incurred by access roads resulting from more fill materials etc. If the depth is restricted and cannot be increased, then increasing the thickness of the bottom flange or the webs can be considered. In this particular example, depth is increased. Since the new section has a higher area than the initial section, the dead load bending moment envelope has to be re-evaluated. However, this is unlikely to affect the moment range drastically since it is primarily determined by the live loads.

Table 2: The results of parametric study for a spine beam having a top flange width of 10.0m and vertical webs.

Depth of section (m)	Thickness of a web (m)	Thickness of bottom flange (m)	Thickness of top flange (m)	Area of half section (m^2)	Z_1	Z_2
2.0	0.35	0.175	0.30	2.471	-1.940	0.953
2.0	0.35	0.200	0.30	2.525	-1.970	1.020
2.0	0.40	0.175	0.30	2.704	-1.945	0.982
2.0	0.35	0.175	0.35	2.540	-2.066	0.951
2.1	0.35	0.175	0.30	2.506	-2.079	1.025
2.1	0.35	0.200	0.30	2.560	-2.113	1.101
2.1	0.40	0.175	0.30	2.580	-2.085	1.050
2.1	0.35	0.175	0.35	2.738	-2.221	1.023

CRITERIA FOR THE SELECTION OF CABLE FORCES

Once the section is selected, an appropriate set of cable forces has to be selected. The cable forces can be either constant throughout the length of the beam or vary as appropriate. In both cases, it is advantageous to select the minimum cable force both in terms of cost and the need to accommodate all the ducts within the limited space available in the section.

When the cable forces are selected, whether constant or variable, it is possible to determine the line of thrust zone (e_p) along the beam. This also can be transformed to obtain the corresponding cable profile zone (e_s), when secondary moments (M_2) are known. The secondary moments should be selected so that the cable profile zone will lie within the section throughout the length. The relationship $e_s - e_p = \dfrac{M_2}{RP}$ can be used to find e_s when e_p, M_2 and P are known.

Any cable profile that fits within a line of thrust zone should be concordant; it should generate zero secondary moments. There is a unique condition for the existence of a concordant profile within a line of thrust zone (Burgoyne, 1987 (a)). This condition is valid irrespective of the type of cable forces; whether constant or variable. When a cable is fitted along the upper boundary of a line of thrust zone, it should generate hogging secondary moments. If a cable is fitted along the lower boundary of the line of thrust zone, it should generate sagging secondary moments. It is only then that a profile can exist that will generate zero secondary moments.

The minimum cable force corresponds to the case where the cable at the upper boundary of the line of thrust zone generates zero secondary moments over all supports. This can be achieved for practical cases only if variable cable forces are used. The use of a constant prestressing force is most likely to make the secondary moments zero only at one internal support.

Determination of secondary moments

It is possible to determine the secondary moments in a beam once the cable profile and the cable forces are known by using the generalised Clark Maxwell's theorem as described by Burgoyne (1987 a). This theorem states that when two force systems act on a linear elastic structure, the work done by the forces of first system on the displacements of second system is equal to the work done by the forces of the second system on the displacements of the first system ($\Sigma P_i \Delta_{ii} = \Sigma P_{ii} \Delta_i$).

The first system consist of a set of fictitious moment system and associated reactions as shown in Figure 4 with one system at each internal support. The reactions $R_{ik}^{'}$ are unknown, but need not be determined since they do not appear in the equations. The second system consist of the cable forces, corresponding secondary moments and associated reactions at the internal supports as shown in Figure 5. Due to cable forces and secondary moments, there is a curvature at each section given by $(\Sigma(\beta M_2)_j - RPe)/EI$. The support reactions resulting from cable forces are R_{jk}, but those need not be calculated since they appear at the supports where the displacements are zero in the first system. The application of the generalised Clark Maxwell's theorem results in Equation 3 from which $\left(M_2^{'}\right)_i$ can be canceled from the equation without loosing the generality, thus resulting in Equation 4.

$$\int \beta_i \left(M_2^{'}\right)_i \left(\sum_j \beta_j (M_2)_j - Pe \right) \frac{1}{EI} dx = 0 \qquad \text{for i = 2,3, ... , n-1} \qquad \text{eq-3}$$

$$\sum_j (M_2)_j \int \frac{\beta_i \beta_j}{EI} dx = \int \frac{\beta_i Pe}{EI} dx \qquad \text{for i = 2,3, ... , n-1} \qquad \text{eq-4}$$

The equations form a set of linear equations consisting of unknown secondary moments $(M_2)_j$. The cable profile, *e*, appears on the right hand side of the equations, and the coefficients of the $(M_2)_j$ terms are integrals of products of the β functions, many of which are zero since β_i is non zero only on either side of the support under consideration. The integration $\int \beta_i \beta_j dx$ results in simple expressions as indicated in the left hand side of the matrix notation given in equation 5 for a prismatic section. The integral $\int \frac{\beta_i RPe.dx}{EI}$ can be determined using numerical integration, preferably using the Simpson's rule, since the cable profile is known as a set of eccentricities with parabolic segments in between. In the spreadsheet given in Appendix A, the values of β factors and the Simpson's rule coefficients (s) have been indicated in lines 37, 38 and 39 of the spread sheet given in Appendix A. For each of the β coefficients, a row of $\beta.RP e_s.s$, have been prepared and then summed to give the totals shown at the end of the row. These values should be multiplied by h/3 as generally done in Simpson's rule summations where h is the interval considered for the cable eccentricities (5.0 m in this case). The matrix can now be inverted to obtain the unknown secondary moments.

$$\frac{1}{6}\begin{bmatrix} 2(L_1+L_2) & L_2 & 0 & - \\ L_2 & 2(L_2+L_3) & L_3 & - \\ 0 & L_3 & 2(L_3+L_4) & - \\ - & - & - & \end{bmatrix}\begin{bmatrix} (M_2)_2 \\ (M_2)_3 \\ (M_2)_4 \\ \end{bmatrix} = \begin{bmatrix} \int \frac{\beta_2 RPe}{EI} \\ \int \frac{\beta_3 RPe}{EI} \\ \int \frac{\beta_4 RPe}{EI} \\ - \end{bmatrix} \qquad \text{eq-5}$$

SELECTION OF MINIMUM CABLE FORCES

If the cable profile is concordant, the secondary moment at each support, (M_2), should be zero. It is both a necessary and sufficient condition for this that the terms in the right hand side of equation 4 are all separately zero. Thus, a cable profile is concordant if

$$\int \frac{\beta_i Pedx}{EI} = 0 \qquad \text{for all i.} \qquad \text{eq-6}$$

In order to determine the minimum cable forces, this condition can be used. When the cable is fitted along the upper boundary, the cable position is given by the following equations with respect to the prestressing force P_B shown on the Magnel diagram given in Figure 3 :

When $P \leq P_B$

$$e_{p-\min} = \frac{-Z_2}{A} - \frac{Z_2 f_{tw}}{RP} + \frac{M_b}{RP} \qquad \text{eq-7}$$

When $P \geq P_B$

$$e_{p-\min} = \frac{-Z_1}{A} - \frac{Z_1 f_{cw}}{RP} + \frac{M_a}{RP} \qquad \text{eq-8}$$

Hence, the choice of governing equations for $e_{p\text{-}min}$ depends only on the magnitude of the cable force in each region. It is shown later that there is no point in using a prestressing force greater than P_B. Hence, only equation 7 will determine the location of the cable at upper limits of e_p. It should be noted that these conditions are valid only if the secondary moments selected are large enough to ensure that the upper boundary of the cable profile zone will lie within the physical limits imposed by the dimensions (depth) of the concrete section.

The total cable force along a beam can be determined by $\int Pds$ where s is measured along the cable profile. In order to minimise the value of $\int Pds$, the following strategy can be adopted. It can be seen from equation 6 that the condition for zero secondary moment involve the function β which varies from zero to 1.0 and then to zero over a given support from either side. Therefore, it would be reasonable to suggest that the integral $\int \frac{\beta_i Pedx}{EI}$ is dominated by the value of P close to the support. This can be exploited in the following manner to find suitable minimum cable forces along the beam. It is considered that cable forces will be changed on either side of an internal support.

In order to minimise the total prestress, the cable forces over supports should be maximised and those in the span regions should be minimised. However, this should be done while satisfying the condition that a valid cable profile zone exist in the span regions. Thus, the minimum cable force that can ensure that there is a sufficient cable profile zone at the span critical section can be first found for each span region. Then, the corresponding prestressing forces should be found over the internal supports. If those values are less than P_B, then it is satisfactory. If the required prestressing cannot be provided with values below that given by P_B, then select a value close to P_B over the supports. Then adjust the prestressing forces in span regions until the condition for the existence of a concordant profile is satisfied. In either way, the cable forces selected will represent the least values that can be selected with the given number of cable force change points, thus leading to lowest for $\int Pds$.

This method of selecting cable forces is valid only if the upper boundary of the cable profile zone will lie within the physical limits imposed by the depth of the section. Otherwise the method explained by Jayasinghe (1992) has to be used.

In the design example, P_B = 37145 kN. The cable forces selected above first and second internal supports are 30,000 kN and 37000 kN respectively. At the second internal support, it was found that a cable force should be close to P_B. At the first internal support, the cable forces selected for the span regions on either side to allow a sufficient cable profile zone, allowed the use of a lower value than P_B of 30,000 kN. The corresponding cable forces in the span regions are 20,000 kN, 26,000 kN and 23,000 kN. The latter two values are not the lowest that can be used for those two spans, but have been appropriately increased to ensure the existence of a concordant profile. The secondary moments caused by these forces with the cable at upper limit of the line of thrust zone are -137.0 kNm and -6.0 kNm; these are sufficiently close to zero. This results in $\int Pds$ of 3,417,615. When this value is divided by the length of the beam, the corresponding average prestress can be determined. It is 3,417,615/130.0 = 26,289 kN. If a constant prestressing force is used, the minimum cable force that can be used is 31000 kN. This will result in $\int Pds$ value of 4,035,260. The corresponding average value is 31,040 kN. Thus, the use of variable cable forces can lead to a reduction of about 15% for the total prestressing force in this particular design example.

Importance of P_B for the existence of a cable profile

In order to show the importance of P_B, a closer look at the function, P x e, (cable force x eccentricity) can be taken. For the existence of a cable profile, $\int \frac{\beta_i Pedx}{EI}$ should be zero at each support. Since the cable is draped below the neutral axis in the span region, this function is positive. Thus, over the supports, this function should be as small as possible. The variation of P x e can be evaluated as follows:

When $P \leq P_B$

$$e_{p-\min} = \frac{-Z_2}{A} - \frac{Z_2 f_{tw}}{RP} + \frac{M_b}{RP} \qquad \text{eq. 9}$$

$$\frac{d(Pe_{p-\min})}{dP} = \frac{-Z_2}{A}; \text{ as P increases, } Pe_{p\text{-}min} \text{ decreases (since } Z_2 > 0)$$

When $P \geq P_B$

$$e_{p-\min} = \frac{-Z_1}{A} - \frac{Z_1 f_{cw}}{RP} + \frac{M_a}{RP} \qquad \text{eq. 10}$$

$$\frac{d(Pe_{p-\min})}{dP} = \frac{-Z_1}{A}; \text{ as P increases, } Pe_{p\text{-}min} \text{ increases (since } Z_1 < 0)$$

This means that there is no point in increasing the value of P beyond P_B, since the function increases. This is in contrary to the requirement. Thus, P_B, which is a unique value for a given prismatic section irrespective of the moments acting on it, can be considered as a maximum value when selecting the cable forces over the supports.

SELECTION OF SECONDARY MOMENTS

After selecting the minimum section on the basis of global bending behaviour, Magnel diagrams can be drawn along the beam. If the line of thrust zone given by Magnel diagram over critical sections (in this case the support sections) lies outside the physical limits imposed by the need for cover to the cable with the selected prestressing forces, there are two options available for the designer. He can either use the secondary moments to change the location of the Magnel diagram (lower it over the supports) so that it will lie below the physical limits. On the other hand, he can go for a larger section. The first option is better since the minimum section already selected can be used. The magnitude of the secondary moments can be selected as a percentage of the as built dead load moments. Any value between 0% and 90% could be feasible. A higher secondary moment will give a greater clearance over the supports for the cable profile zone and also ensure that the minimum cable force calculated as described above is valid.

Tests by Mattock et al. (1971) and Cohn & Frostig (1983) have shown that for all practical purposes, the beam acts as though secondary moments are present right up to failure. It is therefore valid to include secondary moments when determining the required ultimate moment capacity. This is particularly of importance when considering the resistance to hogging moments over the piers, since secondary moments are normally sagging, and they reduce the required ultimate moment capacity.

CABLE PROFILE FOR VARYING CABLE FORCES

If a constant prestressing force is used, it is quite straight forward to find a cable profile that will generate the assumed secondary moments. This can be achieved in the following way:

1. Find the line of thrust zone corresponding to the cable profile zone by using the relationship $e_p = e_s - M_2/(RP)$.

2. Fit a concordant profile into the line of thrust zone. Then transfer the concordant profile back into the cable profile zone using the relationship $e_s = e_p + M_2/RP$. This will not only fit within the bounds of the cable profile zone, but will also generate the assumed secondary moments.

Thus, the main task of finding the appropriate cable profile will be the determination of a concordant profile which fit within the bounds of a line of thrust zone; for this, use is made of an important property of concordant profiles. Any bending moment diagram due to a set of loads acting on the beam can be scaled to form a concordant profile. Hence the determination of a concordant profile is a matter of finding an appropriate distribution of 'notional loads', which give rise to a suitable bending moment. It is shown by Burgoyne (1987 b) that this process can be automated, if necessary, when a constant cable force is acting.

When the cable forces vary, there is an additional set of forces and moments that will act at the cable force change points. When a cable starts at a given point away from the centroid of a section with an inclination, it can give rise to a point force and a moment. When selecting the location of anchor blocks, the designer has to ensure that the centroid of the new cable and the running cable should lie within the bounds of the cable profile zone. Thus, it would be worthwhile to determine the limits on eccentricity for the new cables prior to selecting the actual locations. This method is explained later.

When the cable forces vary, the concordant profile should be a result of two sets of loads. Those are the 'notional loads' and a set of known forces and moments at sections that the cable forces change. Since it will be cumbersome to combine these two, there is an easy alternative. That is to modify the line of thrust zone using the bending moment diagram that results due to the known point forces and bending moments. Thus, the design process can be presented as follows:

1. Select the location of anchor blocks and the inclination of new cables.

2. Obtain a force x eccentricity (RP x e) zone by multiplying the limits of line of thrust zone by the cable force in each region.

3. Modify this by using the inverse of the bending moment diagram that results due to point forces and moments that act at the change over points.

4. Find a set of 'notional loads' that will give a bending moment diagram which fits within the modified force eccentricity zone.

5. Modify this bending moment diagram with the bending moment diagram due to point forces and moments. Transform this into the cable profile zone.

The resulting cable profile will not only satisfy the bounds on the cable profile, but also generate the assumed secondary moments. This process can be illustrated by using the design example.

Figure 6 shows the cable profile zone corresponding to the cable forces selected along with the actual cable profile selected. It can be seen that it is discontinuous at the places where the cable forces change. The corresponding force eccentricity zone is shown in Figure 7. This has discontinuities at cable force change points. Only the discontinuity at a chainage of 100 m is not allowing a continuous smooth cable. This means that only this point will need some adjustment for the moment.

The eccentricities selected and the corresponding vertical forces and point moments are given in Table 3. These will result in a bending moment diagram of the shape shown in Figure 8. Figure 9 shows the modified force eccentricity zone with the bending moment diagram resulting due to a set of notional loads. This can be transformed back into the actual cable profile as shown in Figure 6.

Table 4 gives a comparison of the assumed secondary moments and those generated. The loss ratio is 0.7.

Table 3: Vertical forces and point moments at cable force change points

Cable force change point	1	2	3	4
Chainage (m)	30	50	80	100
Force in new cable (kN)	10,000 x 0.7	4000 x 0.7	11,000 x 0.7	14,000 x 0.7
Eccentricity (m)	0	0	0	-0.2
Angle with horizontal	2^o	2^o	2^o	2^o
Point force (kN) ↑	244	97	268	341
Moment(anticlockwise +)	0	0	0	1960

Table 4: Secondary moments selected and those which actually occur

Location	Selected secondary moments (kNm)	Actual secondary moments due to the cable selected (kNm)
First internal support	7967	7976
Second internal support	7114	7136

FORCES AND MOMENTS AT CABLE FORCE CHANGE POINTS

The situation at a point where the cable force changes due to start of a new cable can be visualised as shown in Figure 10. If $P_{r(i)}$ is the force in the running cable, $P_{n(i)}$ is the force in the new cable, then on the left of i^{th} change point, the cable force is $P_{r(i)}$ and on the right, it is $P_{r(i)} + P_{n(i)}$. The centroid of total $P_{r(i)} + P_{n(i)}$ will represent the location of the resultant cable profile. This situation is equivalent to the application of a point moment equal to $P_{n(i)}\, e_{sn(i)}$ and a vertical force equal to $P_{n(i)} \sin\alpha_{n(i)}$ at the location where a new cable starts. The quantities $e_{sn(i)}$ (positive downwards) is the eccentricity of the new cable and $\alpha_{n(i)}$ is the inclination at which the new cable is started.

Limits on eccentricity

Since the cable forces have already been selected, the anchors cannot be placed at any eccentricity within the section. There are maximum and minimum eccentricities that are imposed by the shape that the cable profile can possibly take at the change points. These shapes can be visualised in terms of the force eccentricity zone and the bending moment diagram which fits into it as shown in Figures 11 and 12.

Figure 11 shows that the cable is placed at $e_{p\text{-}min}$ to the left of anchor point and $e_{p\text{-}max}$ to the right of it, which gives a maximum moment. Maximum moment that can act is at this point is $[P_{r(i)} + P_{n(i)}]e_{p\text{-}max} - P_{r(i)}\, e_{p\text{-}min}$. The corresponding maximum eccentricity at this point is $\{[P_{r(i)} + P_{n(i)}]e_{p\text{-}max} - P_{r(i)}\, e_{p\text{-}min}\}/P_{n(i)}$. The inclination of the cable is ignored.

Figure 12 shows that the cable is places at $e_{p\text{-}max}$ to the left of anchor point and $e_{p\text{-}min}$ to the right of it. This gives rise to a minimum moment. Minimum moment that can act at this point is $[P_{r(i)} + P_{n(i)}]e_{p\text{-}min} - P_{r(i)}\, e_{p\text{-}max}$. The corresponding maximum eccentricity at this point is $\{[P_{r(i)} + P_{n(i)}]e_{p\text{-}min} - P_{r(i)}\, e_{p\text{-}max}\}/P_{n(i)}$. The inclination of the cable is ignored. When the location of anchors are selected, the point moments should be in between these two minimum and maximum values.

CONCLUSIONS

A design method, that guides the designer with a logically evolving series of design decisions is presented. The main advantage of this design method is that the designer has the opportunity to select the section dimensions of the box girder by considering the governing criterion for each component such as top flange, webs and the bottom flange. A simple parametric study can be used to determine the smallest section that can be used.

It is shown that there is a unique criterion to satisfy the existence of a concordant profile within the line of thrust zone. This condition can be used to determine the minimum cable force that can be used. Then, the designer will be able to select suitable secondary moments that will ensure that the cable profile zone will lie inside the section throughout the length of the beam.

The use of minimum total cable forces for a prismatic beam is advantageous with respect to minimising the cost of construction. It is shown that when variable cable forces are used, the cable force over the support should not exceed a maximum value that can be derived for a given section. It is also necessary to ensure that the selected cable forces will ensure the existence of a cable profile zone at each cross section. A simple design method that can be easily followed by the designer to satisfy all the above conditions is presented. This allows the selection of a suitable set of cable forces that can lead to a minimum total cable force.

Once the cable forces are selected, a suitable cable profile should be found. This should not only satisfy the stress limits, but also should generate the secondary moments selected earlier inthe design process. When the cable forces vary, there are additional constraints due to point loads and moments due to cable anchorages. It is shown that the designer can modify the force eccentricity zone by using the effects of point loads and point moments acting at the cable force change points. Then a concordant cable profile can be fitted to this modified force eccentricity zone by using a suitable set of notional loads.

Thus, this design method allows the designer to use the smallest possible cross section with minimum prestressing force, while satisfying other criteria like constructability, long term behaviour and stress limits. The design method given for selecting the minimum cable force can be used for any section which is larger than the smallest given by the moment range. Therefore, this method can be used for generating a number of alternative solutions, for which the best can be selected on the basis of an overall cost analysis. This is of particular advantage in competitive bidding where the optimum designs are most likely to be successful. It is shown with a design example that all these calculations can be carried out using a suitable structural analysis program and a simple spreadsheet that can be prepared by the designer himself.

REFERENCES

BS 5400/2 (1978), Code of practice for Design of Concrete Bridge, British Standards Institution, BS 5400, Part 2, London.

BS 5400/4, (1984), Code of practice for Design of Concrete Bridge, British Standards Institution, BS 5400, Part 4, London.

Burgoyne, C.J. (1987 a), "Cable design for continuous prestressed concrete bridges", Proc. Instn. Civ. Engrs., Part 2, Vol 85, pp. 161-184.

Burgoyne, C.J. (1987 b), "Automated determination of concordant profiles", Proc. Instn. Civ. Engrs., Part 2, Vol 85, pp. 333-352.

Cohn, M. Z., Frostig, Y. (1983), Nonlinear analysis of continuous prestressed concrete beams, Procs, Int. Symp. on Nonlinearity and Continuity in Prestressed Concrete, Vol 2: Hyperstatic structures: Nonlinear analysis, 45-76, Waterloo, Ontario, Canada.

Gee, A.F. (1987), "Bridge winners and losers", The Structural Engineer, Vol 65A, No. 4, pp 141-145.

Jaeger, L.G. & Bakht, B. (1982), "The grillage analogy in bridge analysis", Canadian Journal of Civil Engineering, Vol 9, No. 2, June, pp 224-235.

Jayasinghe, M. T. R. (1992), "Rationalisation of prestressed concrete spine beam design philosophy for expert systems", Ph.D. Thesis, Department of Engineering, University of Cambridge, United Kingdom, p 234.

Mattock, A. H., Yamazoki, J & Kattula, B. T.(1971), "Comparative study of prestressed concrete beams, with and without bond", ACI Journal, 68/2, 116-125.

Neville, A.M., Dilger, W.H., Brooks, J.J. (1983), *Creep of plain and structural concrete*, Construction press, London.

Podolny, W. & Muller, J.M. (1982), *Construction and design of prestressed concrete segmental bridges*, John Wiley & Sons, New York.

Ranasinghe, W. M. D. N., Jayasinghe, M. T. R. (1998), "A design method for prismatic prestressed continuous box girder bridges", Proc. Research for Industry - 1998, University of Moratuwa, Sri Lanka, pp 52-68.

Swann, R.A. (1972), "A feature survey of concrete box spine-beam bridges", Technical Report 469, Cement and Concrete Association, London, 1972.

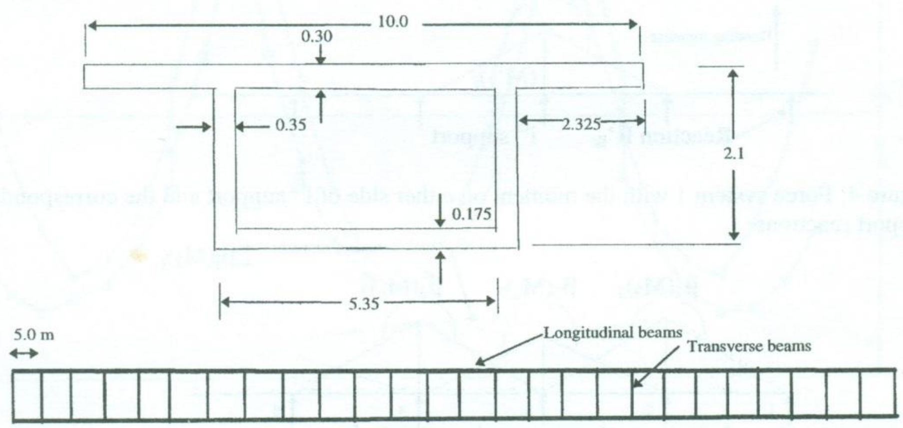


Figure 1: Cross section used for the design example and the grillage model used

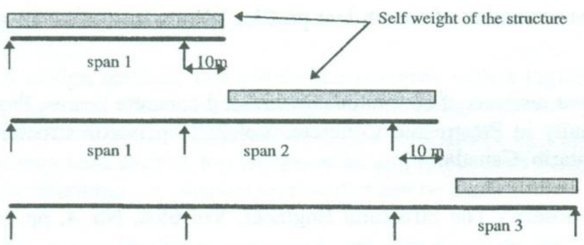


Figure 2: Loading due to self weight at as built condition

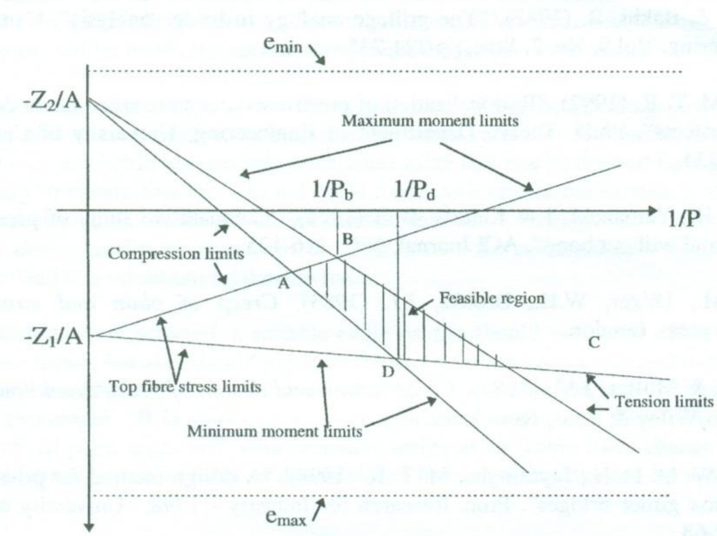


Figure 3: Magnel diagram showing governing conditions and the corresponding prestressing forces

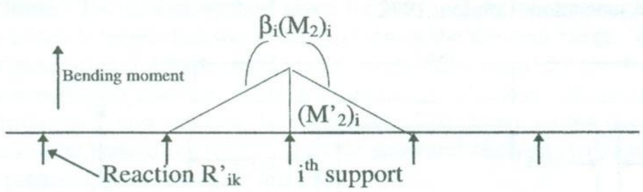


Figure 4: Force system 1 with the moment on either side of i^{th} support and the corresponding support reactions

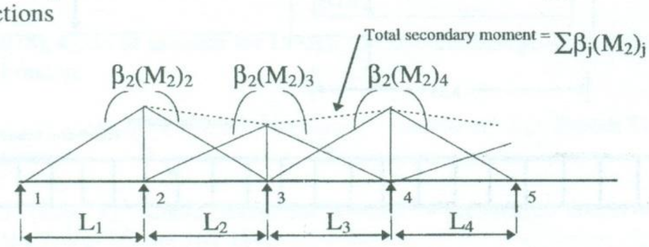


Figure 5: Force system 2 - Secondary moment at j^{th} support

Figure 6: Cable profile with varying prestressing forces and the upper and lower bounds of cable profile zone

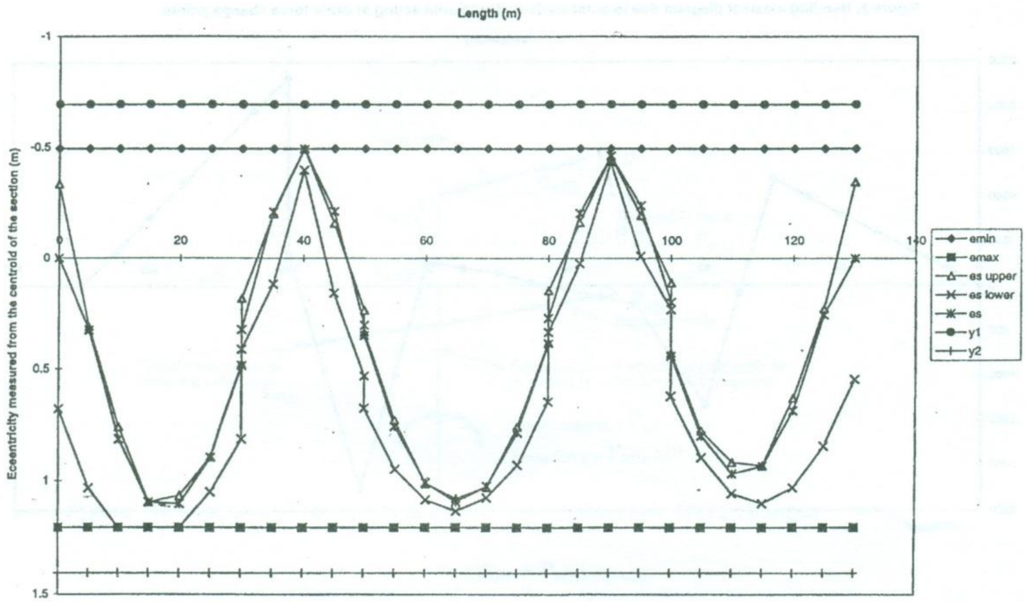


Figure 7: Force eccentricity zone due to force x eccentricity (ep)

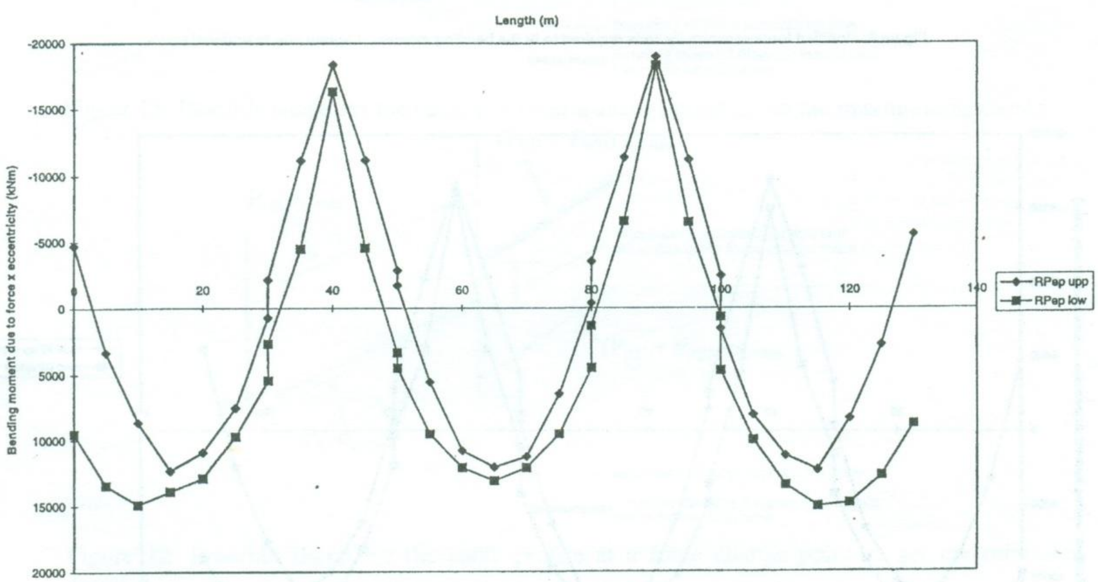


Figure 8: Bending moment diagram due to point loads and moments acting at cable force change points

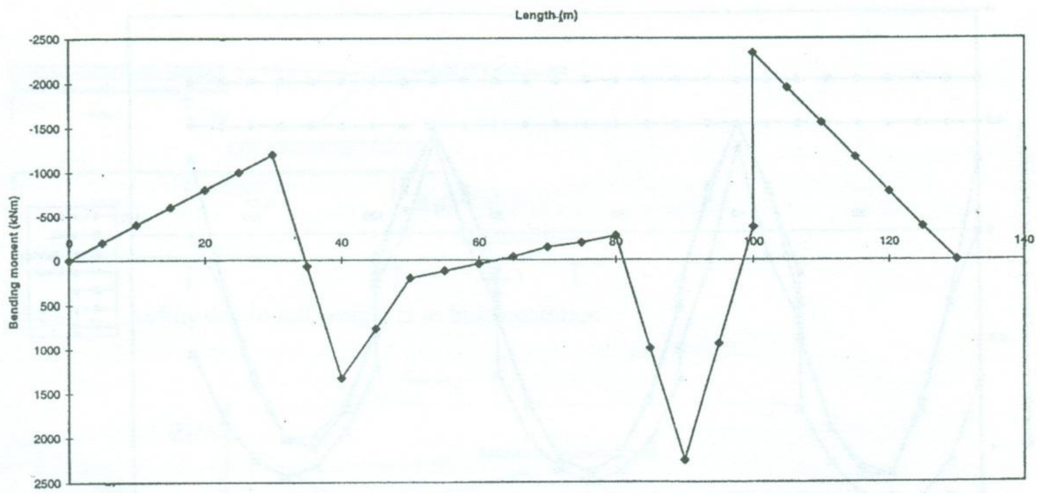
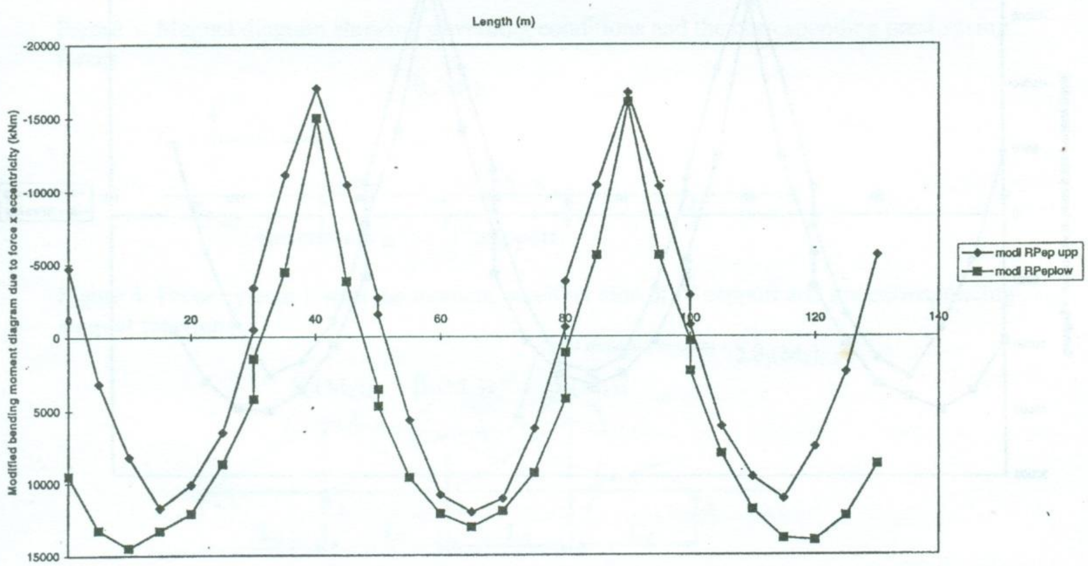


Figure 9: Modified force eccentricity zone required to fit the bending moment diagram due to notional loads



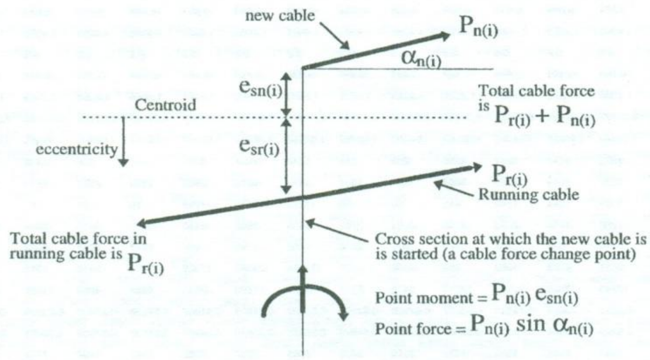


Figure 10: Forces and moments at a point where the cable force changes

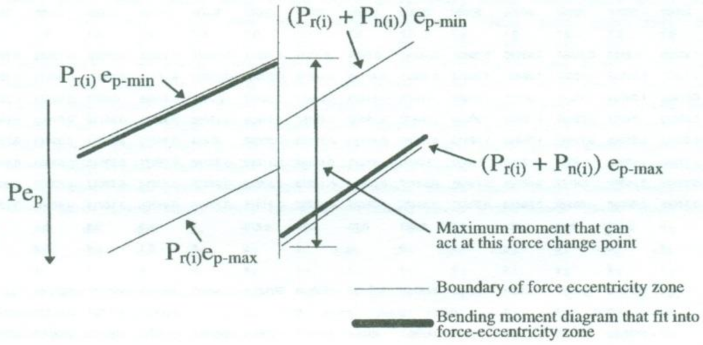


Figure 11: Possible shape for the cable profile at a change point to get the maximum moment

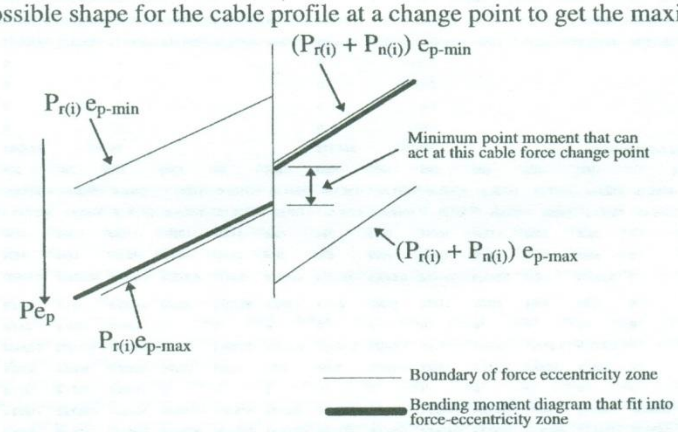


Figure 12: Possible shape for the cable profile at a force change point to get the minimum moment

1	chainage	0	5	10	15	20	25	30	30	35	40	45	50	50	55	60
2	fcw	-15000	-15000	-15000	-15000	-15000	-15000	-15000	-15000	-15000	-15000	-15000	-15000	-15000	-15000	-15000
3	ftw	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000
4	Ic	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44
5	Ac	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063
6	depth of sec	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1
7	y1	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694
8	y2	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065
9	cover	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
10	emin	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494
11	emax	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065
12	z1	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493
13	z2	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382
14	M monoli	0	3718	5933	6644	5851	2555	-245	-245	-5549	-12357	-5590	-326	-326	3432	5688
15	M as built	0	4018	6532	7543	7050	5053	1551	1551	-3453	-9959	-2966	2523	2523	6510	8992
16	M sup dead	0	853	1361	1524	1342	815	-56	-56	-1273	-2835	-1282	-75	-75	787	1305
17	M live min	0	-633	-1266	-1900	-2533	-3166	-3799	-3799	-4433	-7905	-4456	-3015	-3015	-2604	-2803
18	M live max	0	3183	5450	7962	7235	6349	3915	3915	1102	1125	607	2247	2247	4668	6920
19	Ma(act)	0	3938	6028	6268	4660	204	-4100	-4100	-11255	-23097	-11328	-3416	-3416	1615	4190
20	Mb(act)	0	8054	13343	17029	15627	12217	5410	5410	-3624	-11669	-3641	4695	4695	11965	17217
21	Z1 required	0	-0.294	-0.5225	-0.76864	-0.78336	-0.85807	-0.67929	-0.67929	-0.54507	-0.81629	-0.54907	-0.57936	-0.57936	-0.73929	-0.9305
22	Z2 required	0	0.294	0.5225	0.76864	0.78336	0.85807	0.67929	0.67929	0.54507	0.81629	0.54907	0.57936	0.57936	0.73929	0.9305
23	M2	0	995	1991	2987	3980	4975	5970	5970	6965	7967	7882	7797	7797	7712	7627
24	Ma	0	4933	8019	9255	8640	5179	1870	1870	-4290	-15130	-3446	4381	4381	9327	11817
25	Mb	0	9049	15334	20016	19607	17192	11380	11380	3341	-3702	4241	12492	12492	19677	24844
26	Pmax allow	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9
27	Ptrial	20000	20000	20000	20000	20000	20000	20000	30000	30000	30000	30000	30000	26000	26000	26000
28	R	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
29	es upper 1	-1.39525	-0.7489	-0.29997	0.03446	0.00525	-0.16725	-0.5824	-0.1123	-0.49511	-0.83049	-0.45225	-0.05935	-0.19585	0.19893	0.48283
30	as upper 2	-0.33537	0.31099	0.75992	1.09435	1.06513	0.89263	0.47749	0.18216	-0.20065	-0.53603	-0.15779	0.23511	0.33413	0.72891	1.01281
31	es lower 1	0.67968	1.03203	1.25246	1.34075	1.29682	1.0496	0.81325	0.81813	0.52479	0.0086	0.56498	0.9377	0.95459	1.22635	1.36316
32	es lower 2	0.68845	1.04081	1.26124	1.34952	1.30559	1.05838	0.82202	0.41185	0.11852	-0.39768	0.15871	0.53142	0.67602	0.94778	1.08459
33	es upper	-0.33537	0.31099	0.75992	1.09435	1.06513	0.89263	0.47749	0.18216	-0.20065	-0.494	-0.15779	0.23511	0.33413	0.72691	1.01281
34	es lower	0.67968	1.03203	1.2065	1.2065	1.2065	1.0496	0.81325	0.41185	0.11852	-0.39768	0.15871	0.53142	0.67602	0.94778	1.08459
35	ep upper	-0.33537	0.23992	0.6177	0.88099	0.78085	0.53727	0.05106	-0.10213	-0.53232	-0.87338	-0.53313	-0.13617	-0.09428	0.30517	0.59374
36	ep lower	0.67968	0.96096	1.06429	0.99314	0.92221	0.69425	0.38682	0.12756	-0.21315	-0.77706	-0.21663	0.16013	0.24762	0.52404	0.66553
37	beta1	0	0.125	0.25	0.375	0.5	0.625	0.75	0.75	0.875	1	0.9	0.8	0.8	0.7	0.6
38	beta2	0	0	0	0	0	0	0	0	0	0	0.1	0.2	0.2	0.3	0.4
39	s	1	4	2	4	2	4	1	1	4	2	4	1	1	4	2
40	beta1ep up	0	1679.42	4323.92	18500.8	10931.8	18804.6	536.137	-1608.48	-39125.2	-36682	-40304.3	-2287.71	-1372.67	15551.6	12967.4
41	beta2ep up	0	0	0	0	0	0	0	0	0	0	-4478.25	-571.927	-343.168	6664.99	8644.93
42	beta1ep lo	0	6726.73	7450	20856	12911	24298.7	4061.6	2009.11	-15666.6	-32636.4	-16377.1	2690.25	3605.29	26705.3	14535.1
43	beta2ep lo	0	0	0	0	0	0	0	0	0	0	-1819.67	672.563	901.322	11445.1	9690.09
44	RPep upp	-4695.15	3358.85	8647.85	12333.8	10931.8	7521.85	714.849	-2144.64	-11178.6	-18341	-11195.6	-2859.64	-1715.84	5554.16	10806.2
45	RPep low	9515.46	13453.5	14900	13904	12911	9719.46	5415.46	2378.82	-4476.18	-16318.2	-4549.18	3362.82	4506.61	9537.61	12112.6
46	min mom to be applied							0						0		
47	corres eccentricity							0						0		
48	Eccentricity selected							0						0		
49	corresponding moment							0						0		
50	corres verti force with 2 deg incli to horizontal							244.173						97.6691		
51	mom poi lo,mo	0	-200	-400	-600	-800	-1000	-1200	-1200	69	1330	770	201	201	122	43
52	modi RPep up	-4695.15	3158.85	8247.85	11733.8	10131.8	6521.85	-485.151	-3344.64	-11109.6	-17011	-10425.6	-2658.64	-1514.84	5676.16	10849.2
53	modi RPep lo	9515.46	13253.5	14500	13304	12111	8719.46	4215.46	1478.82	-4407.18	-14988.2	-3779.18	3563.82	4707.61	9659.61	12155.6
54	concord	0	3366	9008	11738	10643	6572	-386	-386	-11294	-16990	-11558	-1238	-1238	6206	10763
55	RPep actual	0	3566	9408	12338	11443	7572	814	814	-11363	-18320	-12328	-1439	-1439	6084	10720
56	epactual	0	0.25471	0.672	0.88129	0.81736	0.54086	0.05814	0.03876	-0.5411	-0.87238	-0.58705	-0.06852	-0.07907	0.33429	0.58901
57	beta1ep RPs	0	1783	4704	18507	11443	18930	610.5	610.5	-39770.5	-36640	-44380.8	-1151.2	-1151.2	17035.2	12864
58	beta2ep RPs	0	0	0	0	0	0	0	0	0	0	-4931.2	-287.8	-287.8	7300.8	8576
59	es actual	0	0.32579	0.81421	1.09464	1.10164	0.89621	0.48457	0.32305	-0.20943	-0.493	-0.21171	0.30276	0.34934	0.75802	1.00808
60	beta1es	0	2280.5	5699.5	22987.5	15423	31367.5	5088	5088	-15393	-20706	-16005.6	5086.4	5086.4	38628.8	22016.4
61	beta2es	0	0	0	0	0	0	0	0	0	0	-1778.4	1271.6	1271.6	16555.2	14677.6
62	ftw >	-5585.92	-1202.33	-1742.47	-1004.05	-1499.26	-1048.98	-1096.84	-3889.81	-819.924	-1882.61	106.021	-2387.59	-1270.4	-1517.52	-915.846
63	ftw >	-5585.92	-5765.21	-3956.95	-2660.52	-2316.89	-2034.96	-3217.65	-6010.61	-8430.94	-6076.64	-8860.83	-7426.08	-6308.9	-5107.89	-4114.6
64	fcw <	-5585.92	-7748.89	-7482.37	-7846.72	-7602.38	-7824.56	-7800.94	-10593.9	-12108.7	-11584.3	-12565.5	-11335.1	-10217.9	-10096	-10392.9
65	fcw <	-5585.92	-5222.58	-8887.29	-11514.7	-12211.1	-12782.5	-10385.6	-13178.6	-8273.4	-13044.8	-7402.15	-10309.9	-9192.71	-11626.7	-13639.8
66	ftw max	-5585.92	-1202.33	-1742.47	-1004.05	-1499.26	-1048.98	-1096.84	-3889.81	-819.924	-1882.61	106.021	-2387.59	-1270.4	-1517.52	-915.846
67	fcw min	-5585.92	-7748.89	-8887.29	-11514.7	-12211.1	-12782.5	-10385.6	-13178.6	-12108.7	-13044.8	-12565.5	-11335.1	-10217.9	-11626.7	-13639.8
68	total prestre		100212	100476	100157	100000	100084	100338		150848	150241	150237	150792		130434	130162
69																
70	M22 UPP=	-136.753		M22 LOWER =		4960.11										
71	M23 UPP=	-6.44172		M23 LOWER =		3561.86										
72																
73	M22 actual	7976.26		M22 selected		7967										
74	M23 actual	7135.96		M23 selected		7114										

65	70	75	80	80	85	90	95	100	100	105	110	115	120	125	130	
-15000	-15000	-15000	-15000	-15000	-15000	-15000	-15000	-15000	-15000	-15000	-15000	-15000	-15000	-15000	-15000	
-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	
1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	1.44	
2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	2.5063	
2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	
-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	-0.694	
1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	1.4065	
0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	
-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	-0.494	
1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	1.2065	
-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	-2.07493	
1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	1.02382	
6440	5688	3432	-326	-326	-5590	-12357	-5549	-245	-245	3555	5851	6644	5933	3718	0	
9971	9446	7418	3884	3884	-1153	-7691	-1502	3253	3253	6470	8183	8393	7099	4301	0	
1477	1305	787	-75	-75	-1282	-2835	-1273	-56	-56	815	1342	1524	1361	853	0	
-2737	-2803	-2604	-3015	-3015	-4456	-7905	-4433	-3799	-3799	-3166	-2533	-1900	-1266	-633	0	
7036	6929	-4668	2247	2247	607	1125	1102	3915	3915	6349	7235	7962	5450	3183	0	
5180	4190	1615	-3416	-3416	-11328	-23097	-11255	-4100	-4100	1204	4660	6268	6028	3938	0	
18484	17680	12873	6056	6056	-1828	-9401	-1673	7112	7112	13634	16760	17879	13910	8337	0	
-0.95029	-0.96357	-0.80414	-0.67657	-0.67657	-0.67857	-0.97829	-0.68443	-0.80086	-0.80086	-0.88786	-0.86429	-0.82936	-0.563	-0.31421	0	
0.95029	0.96357	0.80414	0.67657	0.67657	0.67857	0.97829	0.68443	0.80086	0.80086	0.88786	0.86429	0.82936	0.563	0.31421	0	
7542	7457	7372	7287	7287	7202	7114	6224	5335	5335	4446	3557	2667	1778	889	0	
12722	11647	8987	3871	3871	-4126	-15983	-5031	1235	1235	5650	8217	8935	7806	4827	0	
26026	25137	20245	13343	13343	5374	-2287	4551	12447	12447	18080	20317	20546	15688	9226	0	
37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	37144.9	
26000	26000	26000	26000	37000	37000	37000	37000	37000	23000	23000	23000	23000	23000	23000	23000	
0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	
0.54778	0.49893	0.23014	-0.14909	0.14136	-0.16632	-0.46211	-0.1981	0.10677	-0.33217	0.0177	0.15665	0.17087	-0.13087	-0.53223	-1.10528	
1.07776	1.02891	0.76012	0.38089	0.14621	-0.16148	-0.45727	-0.19325	0.11161	0.4282	0.77807	0.91702	0.93124	0.6295	0.22814	-0.34491	
1.41289	1.35382	1.20767	0.92657	0.89723	0.58847	0.13067	0.55352	0.79546	0.77572	1.04994	1.20938	1.25398	1.18385	0.99882	0.69901	
1.13432	1.07525	0.9291	0.648	0.33391	0.02514	-0.43266	-0.0098	0.23213	0.62208	0.8963	1.05574	1.10034	1.03021	0.84518	0.54537	
1.07776	1.02891	0.76012	0.38089	0.14621	-0.16148	-0.45727	-0.19325	0.11161	0.4282	0.77807	0.91702	0.93124	0.6295	0.22814	-0.34491	
1.13432	1.07525	0.9291	0.648	0.33391	0.02514	-0.43266	-0.0098	0.23213	0.62208	0.8963	1.05574	1.10034	1.03021	0.84518	0.54537	
0.66336	0.61918	0.35506	-0.0195	-0.13515	-0.43955	-0.73194	-0.43356	-0.09437	0.09683	0.50193	0.69609	0.76559	0.51907	0.17292	-0.34491	
0.71992	0.66553	0.52404	0.24762	0.05256	-0.25293	-0.70733	-0.25011	0.02615	0.29071	0.62015	0.83481	0.93469	0.91978	0.78997	0.54537	
0.5	0.4	0.3	0.2	0.2	0.1	0	0	0	0	0	0	0	0	0	0	
0.5	0.6	0.7	0.8	0.8	0.9	1	0.875	0.75	0.75	0.625	0.5	0.375	0.25	0.125	0	
4	2	4	1	1	4	2	4	1	1	4	2	4	2	4	1	
24146.3	9015.33	7754.59	-70.9683	-700.055	-4553.71	0	0	0	0	0	0	0	0	0	0	-2493.104
24146.3	13523	18094	-283.873	-2800.22	-40983.4	-37914.5	-39302.5	-1833.21	1169.25	20202.5	11207	18489	4178.5	1392	0	-799.5045
26205.2	9690.09	11445.1	901.322	272.235	-2620.33	0	0	0	0	0	0	0	0	0	0	107062.7
26205.2	14535.1	26705.3	3605.29	1088.94	-23583	-36639.6	-22672.4	507.883	3510.34	24961.1	13440.5	22572.7	7404.23	6359.23	0	88890.29
12073.2	11269.2	6462.16	-354.842	-3500.27	-11384.3	-18957.3	-11229.3	-2444.27	1559	8081	11207	12326	8357	2784	-5553	
13102.6	12112.6	9537.61	4506.61	1361.18	-6550.82	-18319.8	-6477.82	677.177	4680.46	9984.46	13440.5	15048.5	14808.5	12718.5	8780.46	
			0						881.826							
			0						-0.08998							
			0						-0.2							
			0						1960							
			268.59						341.842							
-35	-144	-193	-272	-272	993	2259	943	-372	-2333	-1944	-1555	-1166	-777	-380	0	
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13067.6	11968.6	9344.61	4234.61	1089.18	-5557.82	-16060.8	-5534.82	305.177	2347.46	8040.46	11885.5	13882.5	14031.5	12338.5	8780.46	
12320	11039	6721	-546	-546	-11452	-16770	-11422	-562	-562	6485	10471	11206	8504	2839	0	
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0.67885	0.61445	0.37989	-0.01505	-0.01058	-0.4805	-0.73471	-0.47741	-0.00734	0.11	0.52354	0.74696	0.76845	0.57646	0.19994	0	
24710	8946.4	8296.8	-54.8	-54.8	-4978	0	0	0	0	0	0	0	0	0	0	259.1
24710	13419.6	19359.2	-219.2	-219.2	-44802	-38058	-43277.5	-142.5	1328.25	21072.5	12026	18558	4640.5	1609.5	0	375.15
1.09324	1.02418	0.78495	0.38533	0.27077	-0.20243	-0.46004	-0.2371	0.19865	0.44137	0.79969	0.96789	0.9341	0.68689	0.25516	0	
39794	14912	17143.2	1402.6	1402.6	-2097.2	0	0	0	0	0	0	0	0	0	0	179204.6
39794	22368	40000.8	5610.4	5610.4	-18874.8	-23830	-21493.5	3858.75	5329.5	32187.5	15583	22558.5	5529.5	2054	0	168283.7
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130019	130012	130149	130415		165827	185245	185184	185701		115295	115065	115003	115140	115428	115150	3417615