EXTRACTION OF STRUT AND TIE MODEL FROM 3D SOLID ELEMENT MESH ANALYSIS

[1]Dammika A.J., [2]Naveed Anwar

[1]"Mahanuwara Gedara" Pallegama, Hungama. Sri Lanka.
dammikaaj@gmail.com M Eng, Structural Engineering, Asian Institute of Technology, Thailand

[2]School of Engineering and Technology, Asian Institute of Technology,
P.O Box 4,Klong Luang, Pathumthani,12120, Thailand.
nanwar@ait.ac.th Associate Director, ACECOMS

ABSTRACT

Strut-and-tie model (STM) method is a lower bound method based on the theory of plasticity, which can be used especially for the design of structural concrete members in D- Region. An approach for automatically finding an appropriate STM for structural concrete members modeled and analyzed by using solid element mesh is introduced in this study. The finite element analysis can be performed in any FEA computer program that has the solid element meshing capability. The solid element principle stress trajectories of the concrete member are obtained and struts and ties are extracted based on the direction cosines and magnitude and graphically displayed. The algorithm includes two main important features: (1) to extract and display an appropriate STM from the output of FEA; and (2) to refine, analyze and design the extracted appropriate STM. As a sample application, concrete pile cap configuration is used in demonstrating the capability of the proposed method in finding an appropriate strut-and-tie model and compared with the previous theoretical and experimental studies that deal with STM for the purpose of verification of the results.

Keywords: Strut and Tie Model, Stress Trajectories, Solid Element, Principal Stress, Finite Element Analysis

1. Introduction

The strut and tie model (shortened as STM) approach which is originated from truss analogy concept has become more rational to use as a tool for designing of structure's disturbed or discontinuity regions (D-region) of concrete structures. So, it fulfills the void in many design codes and guidelines by providing reasonable approach to design structure's D–regions instead of using rule of thumbs and experience to design in such structural components. The structural member is idealized as a truss by introducing uniaxial compressive struts and tension ties in this model which represent the actual load transfer mechanism of the particular member under applied loads and given support conditions. In the concept of truss analogy, truss mechanism is defined for fully cracked reinforced concrete section and concrete is considered to be as no more tensile strength. The truss action is produced by diagonally cracked web concrete struts while longitudinal and transverse reinforcement are act as horizontal and vertical ties. The locations where struts and ties intersect are called as nodes and these nodal zones represent biaxial or tri axial stress fields depending on two dimensional or three dimensional truss configurations of particular member. This particular study is focused on three dimensional strut and tie model which cannot be idealized from two dimensional strut and tie model as usual practice for many three dimensional structural components.

According to the experience of many researchers, there is no single or unique STM is available for any particular structural component. So, engineering judgments are helpful in this occasion to find out correct truss model for particular member which is defined based on stress trajectories obtained from elastic analysis or load path approach as described by Schlaich (1987).

Since STM becomes rational design approach for designing structure D-regions in which no sufficient guide lines are provided in many of design code of practices. Many of the structures including joints, brackets, openings, corbels, deep beams, pile caps contains D-regions in which geometric or static or both discontinuities are available. In fact proper design guide lines are necessary to overcome problems and failures concentrated on those D-regions due to empirical design provisions and detailing practices. So, finding of accurate truss idealization for such structural components is essential for economical and safest design practices. Unfortunately finding of necessary STM for structural members has to undergo so many barriers although that is the most suitable approach

available to identify the load transfer mechanism of such components. As understood from the literature, available tools to find STM are iterative and more time consuming. Although FEM approach provide clear picture of stress distribution of any structural configurations, in many instants, even imagination of STM for a particular situation is difficult as their complexities in stress distribution in many structures D-regions. On the other hand difficulties occur in confirming the adequacy of identified STMs and selecting better model from available many options. So, it is necessary to pay attention on those unresolved problems associated with STM design approach.
Obtaining of elastic stress distribution for complex structural configurations can be easily achieved by employing finite element method (FEM). FEM is the available most powerful tool which is more popular among designers to analyze complex structural configurations to find out its response for applied loading conditions. For the present study, FEM is the basic tool which is going to be used to find the hidden truss mechanism in three dimensional structural components according to its elastic stress distribution. In this study, it is proposed to use three dimensional solid element mesh analysis of a pile cap in order to extract its strut and tie mechanism use in load transfer.

2. Material and Methods

History

The concept of truss analogy in structural design spreads over hundreds years of history (Ritter, 1899; Mörsch, 1902)(Schlaich (1987).According to the past records, although certain part of the structures are designed with almost required accuracy, some other parts are designed using rule of thumb or judgment based on past experience. In that case most of the researchers are contribute their effort to apply concept of truss analogy to find solutions for designing of such irregular reinforced concrete members.
As a further development of the approach, the application of strut and tie model for design of structure D-regions as well as for B regions are present by Schlaich (1987) in well described manner. After that the concept of strut and tie approach becomes rational in D-region design and is included in many design codes as guide line to design of such deep and irregular members.
In the strut and tie model proposed for reinforced concrete structures, loads are carried through set of compressive fields (strut) and these are interconnected by tension ties which is usually reinforcement bars, pre-stressed tendons or concrete stress fields. These compression strut and tension ties are interconnected at nodes. Once a suitable truss model is identified, the forces of the strut and ties are calculated satisfying equilibrium between applied loads and inner forces in order to sizing of them. Especially this method implies that the structure is designed according to lower bound theorem of the theory of plasticity.
As it can propose many strut and tie models for a structural member, it is necessary to find out optimized model for particular member. Since loads try to use the path with the least forces and deformations, the models with the least and shortest ties are best because of reinforced ties are much more deformable than concrete struts. This simple criterion is formulated as follows by Schlaich. (1987)

$$\sum F_i l_i \varepsilon_{mi} = Minimum$$

Where

$$F_i = force\ in\ strut\ or\ tie\ i$$
$$\varepsilon_{mi} = mean\ strain\ of\ member\ i$$
$$l_i = length\ of\ member\ i$$

There are basically three types of struts and ties are identified. Those are Concrete strut in compression (C_c), Concrete ties in tension without reinforcement (T_c) and Ties in tension with reinforcement (T_s). The nodes can recognize as CCC-node, CCT- node, CTT-node or TTT-node depending on the combination of above mentioned strut and ties. The principle remains same for the nodes which combine more than three strut and ties too.
As mentioned earlier T_s is one dimensional element between two nodes which is essentially longitudinal reinforcement bars or pre-stressing tendons as well as stirrups. In the case of longitudinal reinforcement adequate anchorages need be provided in order to avoid brittle anchorage failures at load below the ultimate capacity. Whereas C_c and T_c are two dimensional or three dimensional stress fields which spread between two adjacent nodes.

The successes full usage of strut and tie model is depends on understanding of basic member behavior and engineering judgment and it is basically a design tool. The process of developing a strut and tie model for particular member is an iterative and graphical procedure and it can be done using few approaches as identified. This can be done either by using stress trajectories based on Elastic Analysis or load path approach or by using standard models.
The strut and tie model is based on the cracked section and it gives lower bound capacities according to the lower bound theorem of plasticity which is the theoretical basis of truss analogy. It is assumed that crushing of concrete (struts and nodes) does not occur prior to the yielding of reinforcement (ties or stirrups) for this is to be true.

Finite Element Analysis

In the case of finding appropriate STM for a structural concrete member, as it is understood from the literature, identification of the flow of internal forces that is stress trajectories of that particular member is the main focus. In that case, FEM is the available more reliable tool which can be used to analyze any complex structural components by meshing it in to small elements. Nowadays it is more popular among engineers because of its compatibility with modern digital personal computers even for analysis of much complex problems.
As it is clear from the literature, FEM is used by many researchers as their tool to find the stress trajectories of structural components. As a similar approach, in proposed study, Finding of stress trajectories of a pile cap is going to be done employing linear elastic finite element analysis using solid element mesh with aid of digital computer.

Computational Approaches for Developing Necessary Strut and Tie Models

As experienced by many researchers and designers, the traditional methods which are used to find STM for particular structural concrete member is much time consuming and most of the time it would be an iterative procedure based on designer's intuition and previous experience. Also it is a difficult task for designers to find correct strut and tie model for members with complicated geometry and loading conditions. So, many researchers focused on their studies to find out necessary STM for structural concrete members using computational approaches. Most of such approaches are related to the automatic generation of STM using computer.
As a computational approach, Liang et al.,(2000) proposed a performance based evolutionary topology optimization method for automatic generation of optimal strut-and-tie models for reinforced concrete structures based on evolutionary structural optimization method (ESO).. In that approach, the element virtual strain energy is calculated for element removal, while a performance index is used to monitor the evolutionary optimization process. In this method, the load transfer mechanism of the structural member is gradually characterized from remaining elements of the discritized concrete member after systematic removal of elements that have the least contribution to the stiffness.
Furthermore, Liang et al. (2001) again introduced the method for automatic generation of STM for prestressed concrete beams by using the performance based optimization (PBO) technique.. In PBO, the performance objective of topology optimization is the minimizing of weight of the continuum structure while maintaining deformations within acceptable limits.
Ali and White (2001) also introduced a computer aided approach for designing of D-regions of concrete structures using optimization approach to define the topology of the equivalent truss structure. In their optimization algorithm, two new features are included in order to avoid the generation of impractical reinforcement layout and to account stress redistribution.
The basic idea of the ESO method is also used by Kwak (2006) to determine more rational strut-and-tie models. In their method, the ESO method using truss elements are effectively used in finding the best strut-and-tie models in RC structures. Brick element composed of six truss elements is used as a basic structural element unit in order to prevent the structural instability that may occur during the evolutionary optimization process. Systematic removal of each brick element that has the least virtual strain energy is used to find the optimal load transfer mechanism of an RC structure, which is equivalent to the optimal topology of the strut-and-tie model. The optimization criterion of minimizing the total elastic strain energy of the structure is applied in the method.
All of the attempts discussed from the literature are focused on two dimensional STM. Liang-Jenq leu et al. (2006) discussed about STM methodology for three dimensional RC structures and they proposed refined ESO method (RESO) to develop STM in three dimensional space. This method also

utilizes the finite element model with given loading and support conditions and optimum topology is found from removal of ineffective materials which is determined from strain energy density of each element.

Although many attempt have been taken place in finding of necessary strut and tie model for D-regions of structural concrete members, most of the time they were end up with inefficient conclusions. On the other hand, the usage of finite element output is not completely used by any researcher although they get the help of FEM. In many cases only ESO method is and its modifications are governed the finding of necessary STM. As described by Schlaich et al. (1987) stress trajectories are straight forward to use to find STM. Although stress trajectories are used to identify STM, it is not easy to use for any cases especially when structure is too complicated. On the other hand it is not an easy task to identify the relevant STM through stress trajectories in three dimensional stress states as far as internal element stress trajectories cannot be visualized.

As a new approach, considering the FE analysis results, modified space truss model is proposed by Kanok-Nukulchai et al. (1996) especially for pile caps which bare even larger size of columns and any number of piles under pile cap. This modified truss model gives more realistic results taking in to account of column size and its location, pile stiffness and dimensions of the pile cap. In this model, the column axial loads and moments are assumed to be transferred to the pile cap at the corners of the equivalent rectangular column section which is bounded my main reinforcement cage. These column nodes is used to apply the equivalent loads determined from axial loads, moments and shear forces act at column base which is main advantage over simple truss models proposed by previous researches.

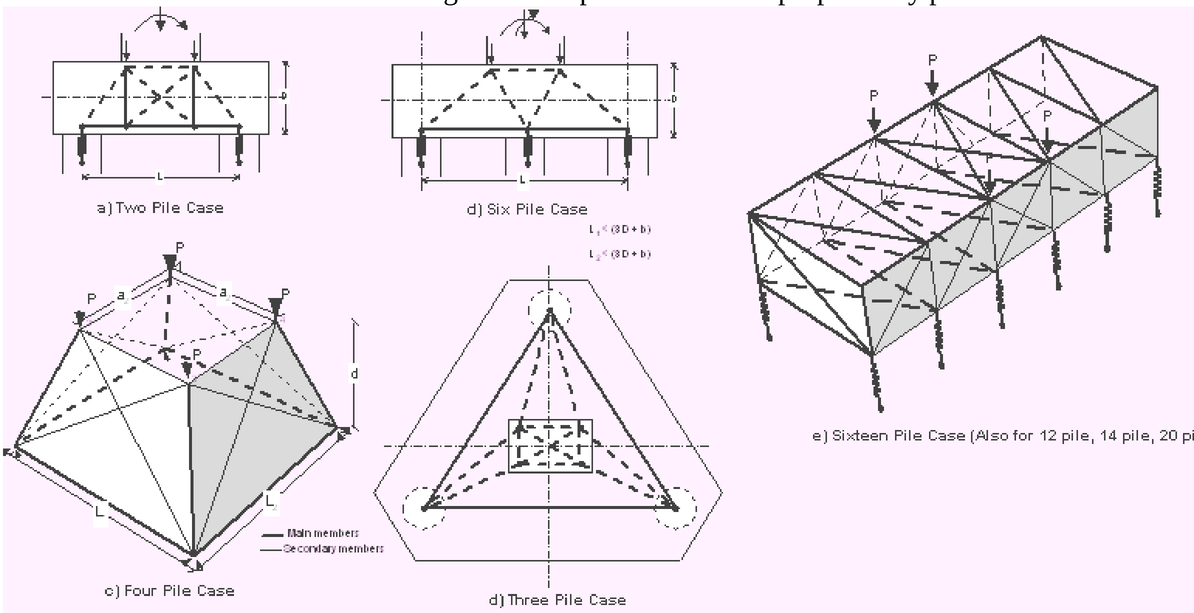


Figure 2.1: *Modified 3- D Truss Model for Pile Caps (Nukulchai & Anwar, 1996)*

With full usage of finite element mesh analysis results, Jason (2008) prepared the computer programming platform to extract STM in two dimensional structural members using stress trajectories obtained from shell element mesh finite element analysis. His study is limited to shell element mesh of shear walls and deep beams. As an extension of that research, present study is focused on to develop computer programming platform to extract STM for three dimensional structural components using 3D solid mesh FE analysis. Pile cap is the three dimensional structural D-region concrete member focused on this study which is analyzed using solid element mesh in order to extract necessary strut and tie model.

Present Methodology

The methodology followed by this study is mainly comprised of two major steps; Linear elastic analysis of selected structural component and developing of Visual Basic programming platform to extract the STM. Steps of the research methodology are adapted in schematic diagram below.

FINITE ELEMENT ANALYSIS OF STRUCTURAL MEMBERS

(1) Linear elastic analysis of selected structural members using solid element mesh in SAP 2000 V12

(2) Obtain the following data from FEA output results
- (i) Solid element stresses at corner joints
- (ii) Direction cosines of principal stresses at solid element corner joints
- (iii) Solid element corner joints and centroid coordinates
- (iv) Solid element joint connectivity
- (v) Solid Element Properties

STM EXTRACTION PROCESS IN VISUAL BASIC PROGRAMINGPLATFORM

(3) Import and store data obtained from step 2

(4) Calculations
- (i) Calculation of average principal stresses at solid elements centroids and corner joints.
- (ii) Calculation of direction cosines of average principal stresses in each element

(5) Groupings
- (i) Solid element grouping with nearly equal principal stress directions
- (ii) Solid element grouping with nearly equal principal stress values which having nearly equal direction for strut and tie layout

(6) Display
- (i) Averaged principal stress vectors of the structural member
- (ii) Primary strut and tie layout

(7) Refining of primary strut and tie model based on
- (i) Limit of stress magnitude(stresses less than specified percentage of maximum stress available are ignored)
- (ii) Number of divisions in direction cosines variation
- (iii) Number of divisions in stress range variation in particular direction
- (iv) Size of the strut or tie

Refined strut and tie layout – Figures/Captions

3. Computational Aspects

Solid Element Stress Vectors Grouping with Nearly Equal Directions

After the averaged maximum absolute principal stress of each nodes and element centroids are extracted together with their direction cosines, the model becomes system of points which represent the solid element joints and solid element centroids with having a principal stress vector at each point. The next step is the grouping of stress vectors with nearly equal directions. Separate module is written to do the screening of all principal stress vectors according to their direction cosines. Since the directions of each and every stress vector are specified through three direction cosines, screening will follow one after another direction cosine values which range from -1 to +1. For simplicity, direction cosine values with respect to each global axis are divided in to number of equal groups which are ranged from -1 to +1. Once this step of screening is completed, the output is stress vectors and those are stored in separate cells which represent particular direction. The cells having no stress vectors are eliminated.

Solid Element Grouping with Nearly Equal Principal Stresses Magnitudes

Once the stress fields which are having nearly equal directions are grouped together, the next step is to identify the stress fields which are having nearly equal magnitude from previously screened nearly equal direction groups and grouped them together. The number of stress value groups belongs to one directional set can vary from one to any reasonable value in order to extract better strut and tie layout. Once the above two steps are completed, the output is separate stress paths of having nearly same magnitude and direction. Each of these stress vector groups represent either strut or tie member depending on sign of the stress. In the computer algorithm, all of these screened stress vectors stored

in three dimensional data structure array and these data is used for graphical presentation of strut and tie model and it is called primary extracted strut and tie layout.

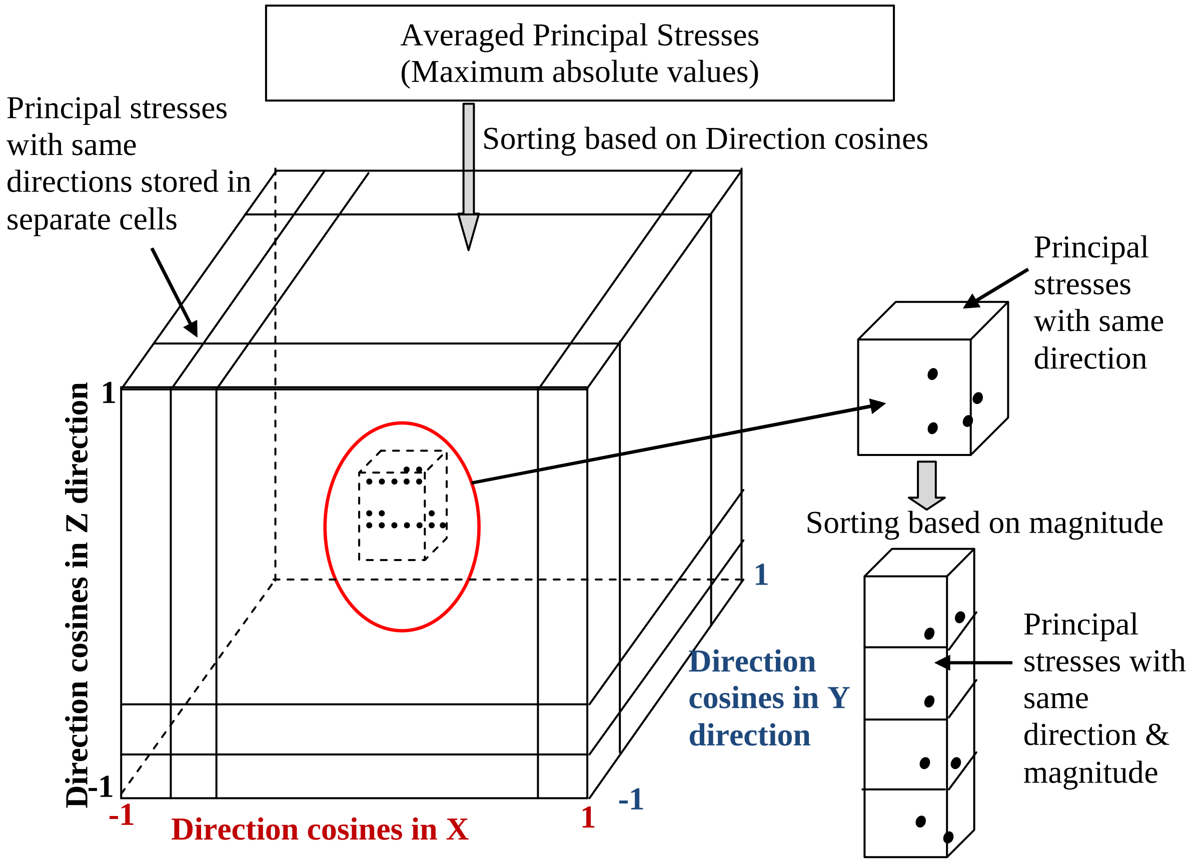


Figure 3.1: *Graphical Representation of Grouping Process*

Definition and Graphical Representation of Primary Strut and Tie Members

Once the grouping of stress vectors completed, all the coordinates of the stress vector points which follows the same strut or tie member is known. The next step involved in STM extraction is identifying of those struts or ties by sizing them. The first step involved in identification of particular strut or tie is the aligning of all the points belongs to that particular stress fields with z (vertical) axis and bring it in to axis origin. Then all the stress vectors are directed in to vertical direction and scattered along the vertical axis. Then plan area of those scattered points is divided in to grid introducing sufficient grid spacing in which each cell in the grid represent the cross section area of the strut or tie. Then, the stress field is divided in to vertical tubes having rectangular cross section based on above grid spacing. The stress points belongs to separate rectangular tubes are identified as strut or tie according to sign of their stress. In here, tubes with no stress vectors can be eliminated and length of strut or tie determined based on lower and upper stress points belongs to particular stress tube. The axis of the tube is considered as axis of the strut or tie and those axes are again rotate back to their original positions in order to locate the strut or tie in their respective positions. This procedure is followed for all stress fields sorted out in order to extract the primary strut and ties layout. In the graphical representation, those strut or ties are displayed by their center lines.

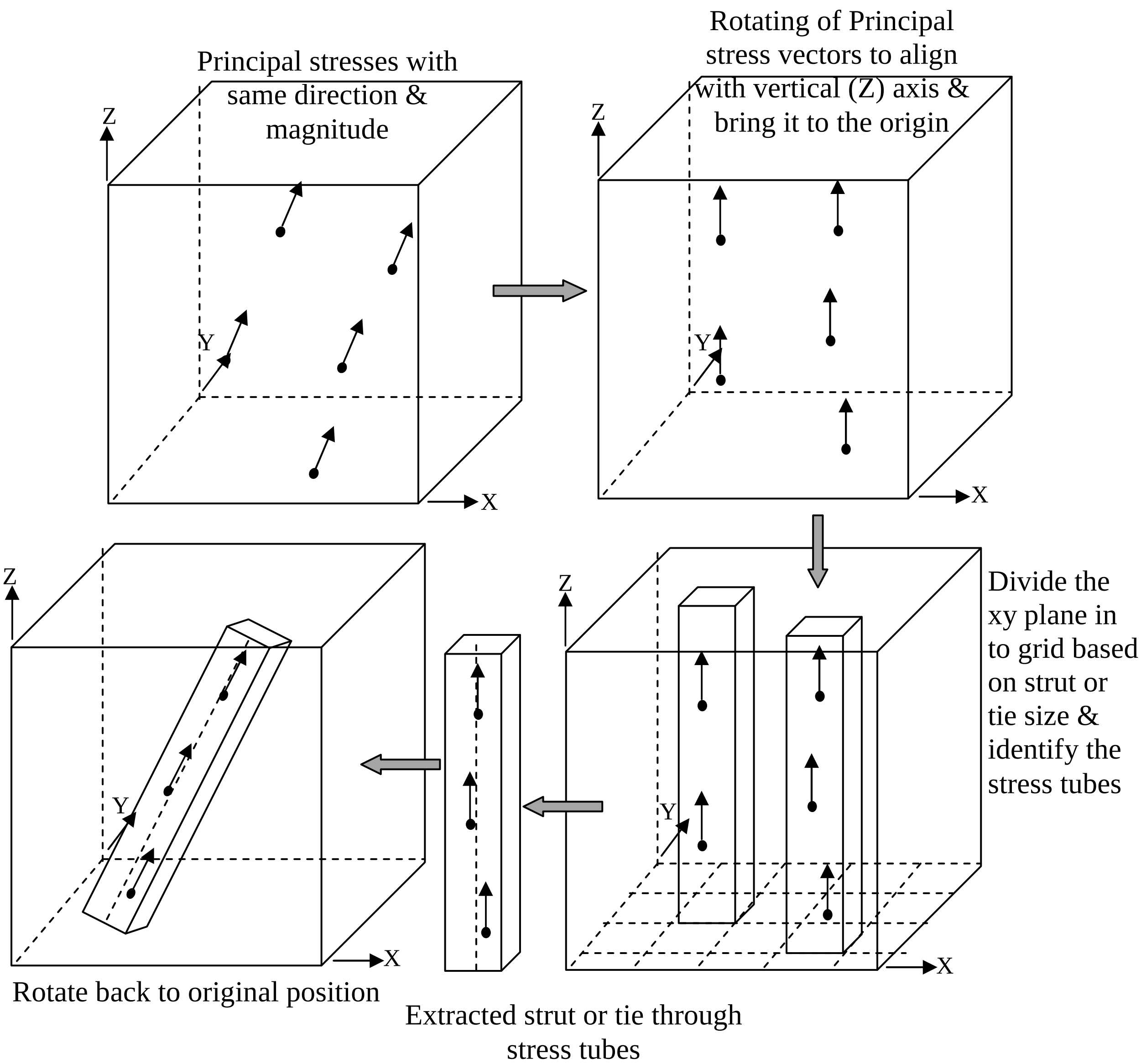


Figure 3.2: *Graphical Representation of Strut and Tie Extraction Process*

Refining of Primary Strut and Tie Layout

Once the primary struts and tie layout is extracted, the refinement of it can be achieved by varying the governing parameters involved in extraction process for better strut and tie layout. There are four parameters introduced in extraction algorithm;

(i) Stress limit tolerance
(ii) Number of divisions in direction cosines
(iii) Number of divisions in stress range
(iv) Strut or tie size

Once this refinement is done, the output is refined strut and tie layout and it is displayed in graphically in program output.

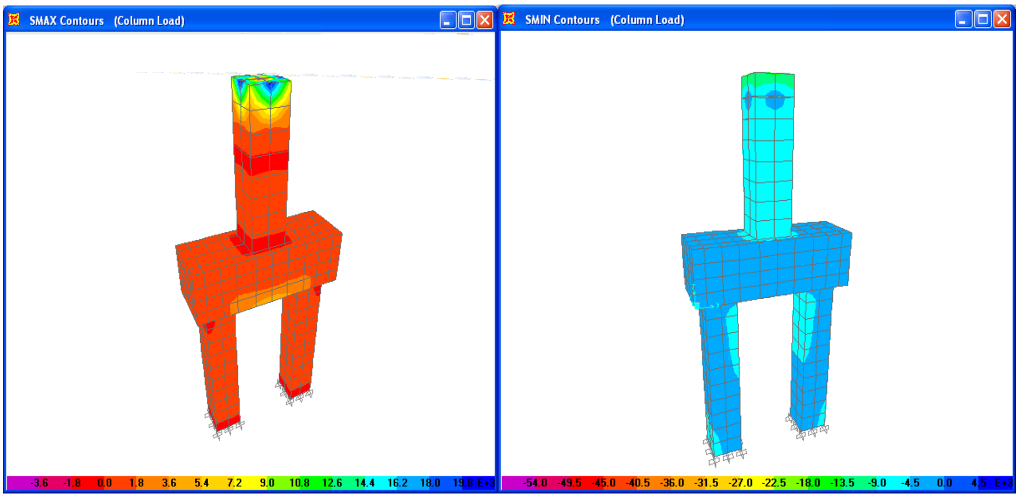
4. Results and Discussion

General

Based on present implementation, the major output coming from the process is appeared in two forms in which graphical interface and text file format. Four graphical views can be viewed through the graphical interface. It includes view of the of stress trajectories of the structural member, view of stresses in directed slots, view of compact directed stress field and view of strut and ties. View of strut and tie is the major output concern in present study. The data text files output coming from the process include details of directed stresses, details of directed stress groups and details of strut and ties. The details of strut and ties include direction cosines and coordinates of the extracted strut and ties together with their stress values and it is used to model strut and ties in SAP 2000. The output results of nine pile cap configurations and three pier configurations modeled and analyzed in SAP 2000 with solid element mesh are used to verify the present implementation and some of the results are adapted in here.

Two Piles-Pile Cap

Two piles-pile cap taken from usual construction practice modeled in SAP 2000 and analyzed with the application of point load at the column center and output data files used in STM extraction process.



(a)Principal Compressive Stress (b)Principal Tensile Stress

Figure 4.1: ***Principal Compressive Stress & Tensile Stress Contours of Two Piles-Pile Cap***

Although bottom tensile zone of the pile cap is cleared from the stress contours, compressive stress flow is not cleared. This is because of usage of solid element for modeling of piles and column. Since column is modeled using solid element, the load coming from column spreads at the top of the pile cap. Normally column and pile caps are modeled using frame elements as it is better to represent axial load transferring members.

The refining configurations of direction cosine divisions=4, Stress range divisions=4, Strut size=0.5 and Stresses > 6% of Maximum stress are used in extracting the above strut and tie layout. Although many strut and tie members still present in the layout, the basic expectation of triangular shaped strut and tie layout is achieved.

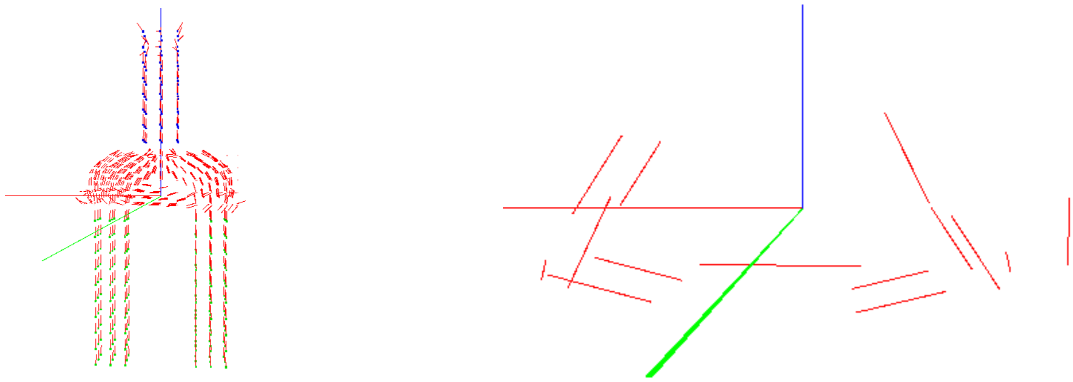
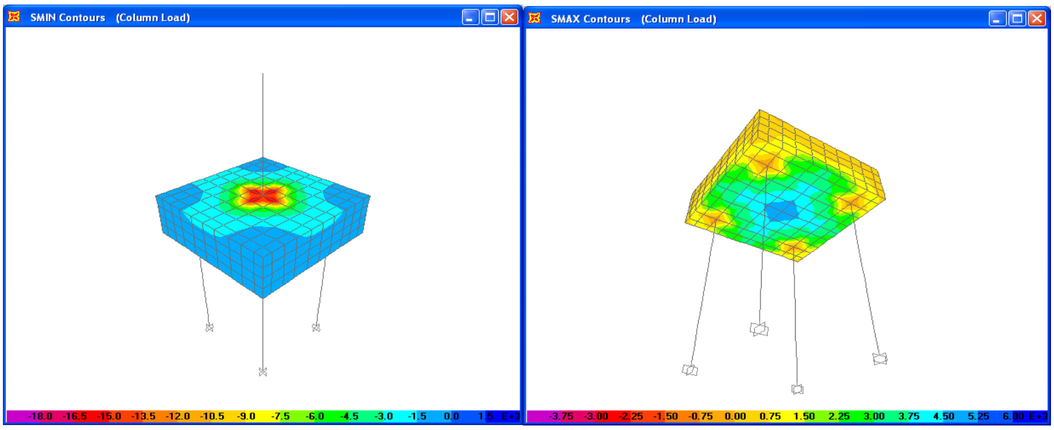


Figure 4.2: ***Stress Trajectories & Refined Strut and Tie Layout from Program Output***

Four Piles-Pile Cap

Four piles –pile cap configurations of having span to depth ratio varying from one to four are considered in the strut and tie extraction. Initially results from all the pile caps modeled from solid element including piles and column are used to test the extraction process but it doesn't show better results. Then, piles and columns in all models are replaced by frame elements to overcome that problem.

The commonly used pile cap configuration of having span/depth is equal to two is considered as first four piles pile cap case.



(a)Principal Tensile Stress (b)Principal Compressive Stress

Figure 4.3: ***Principal Tensile & Compressive Stress Contours of Four Piles-Pile Cap***

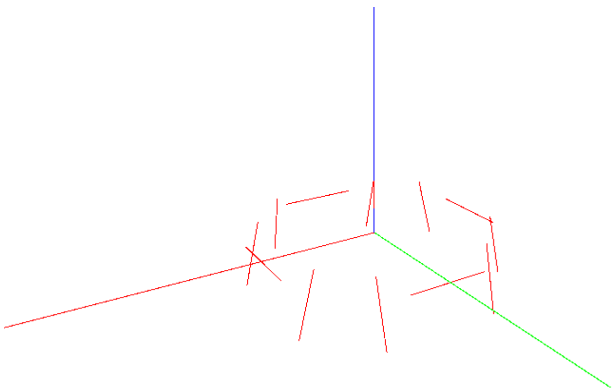


Figure 4.4: ***Refined Strut and Tie Layout for Four Piles Pile Cap (Span/Depth=2)***

The refining configurations of direction cosine divisions=4, Stress range division=2, Strut size=1.2 and Stresses >11% of Maximum stress are used in extracting this strut and tie layout. As it is shown in above figure two inclined struts can be seen in each corner of the pile cap while horizontal tie members lie in between each corner. Although the inclinations of struts are not sufficient to intersect within the body of the pile cap, it can be idealized as a shape of pyramid when each corner struts are represented by a single strut. According to Adebar et al., (1990), the shape of the simple 3D strut and tie model for four piles pile cap is pyramid shaped as shown in Figure 4.8.

Almost similar to pyramid shape four inclined strut layout is clearly shown in program output for the case of four piles pile cap with span/depth is equal to one. This implies that formation of struts possible in rather deep members compared with shallow members.

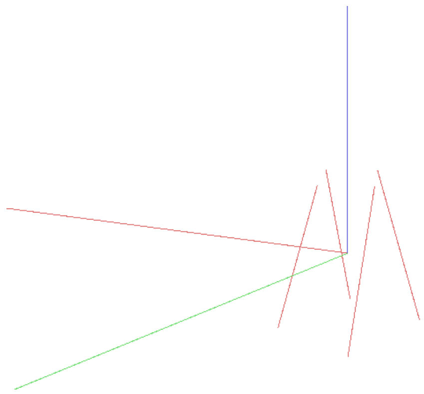


Figure 4.5: ***Refined Strut and Tie Layout for Four Piles Pile Cap (Span/Depth=1)***

The above shown strut configuration is extracted through the use of refining criteria of Direction cosine division=3, Stress range division=2, Strut size=1.5 and considering stresses greater than 15% of maximum stress present in the system.

All other pile cap configurations with four piles considered are rather shallow members compared to the above two cases. So, only tension ties at the bottom of the pile cap are cleared in extracted layout as bending is governing action in such members.

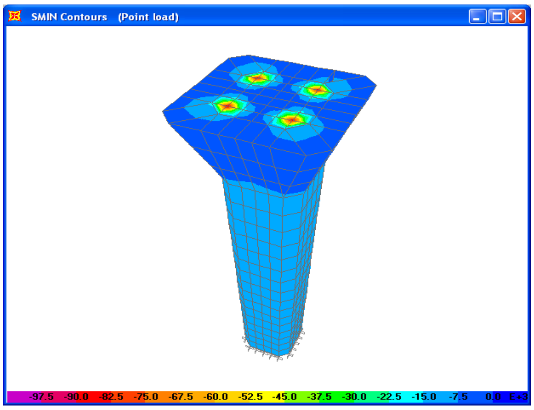
Pier Configurations

The use of strut and tie model for designing of D-region members is the present practice as their stress distribution is complex. Apart from the pile cap configurations, the validity of the results coming out from proposed strut and tie extraction algorithm is checked against such irregular and complex pier configurations used in many of the elevated structures built in the vicinity of the city of Bangkok. Three of such pier configurations are considered in this study. The first one is the common pier configuration used in elevated railway track in Bangkok. The second case is the pier with relatively thick pier head spread over large area. These types of structures are common in most of the interchanges and other locations of elevated highway structures. The third case considered is pier with curved pier head which is commonly used in U-turn bridge structures.

Common Pier Configuration used in Elevated Railway Track in Bangkok

Toller pier with tapered head is modeled by solid elements and fixed support condition is used at the bottom. The load coming from both side spans are applied as point loads acting on pier head as similar to the real situation and results coming from linear static analysis is used as input for the extraction algorithm.

The program output gives the strut and ties layout clearly showing four inclined struts at pier head region and four vertical struts at stem region as shown in following figure. This strut layout is extracted with refining criteria of Direction cosine division=3, Stress range division=1, Strut size=1 and neglecting stresses less than 3% of maximum stresses present in the system. The strut layout modeled in SAP 2000 by removing the small strut and tie element present within the body of the stem is also shown in other figure and it is evident that the program output extract the struts that show the path of the compressive stress flow of the structural member.



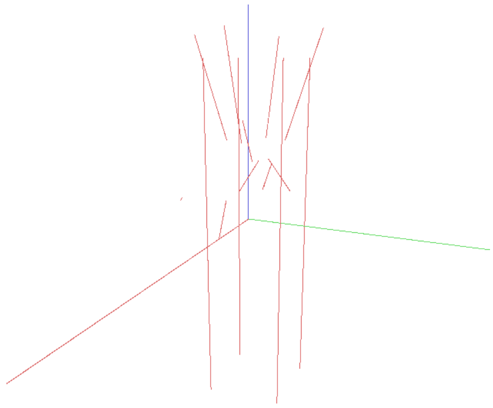
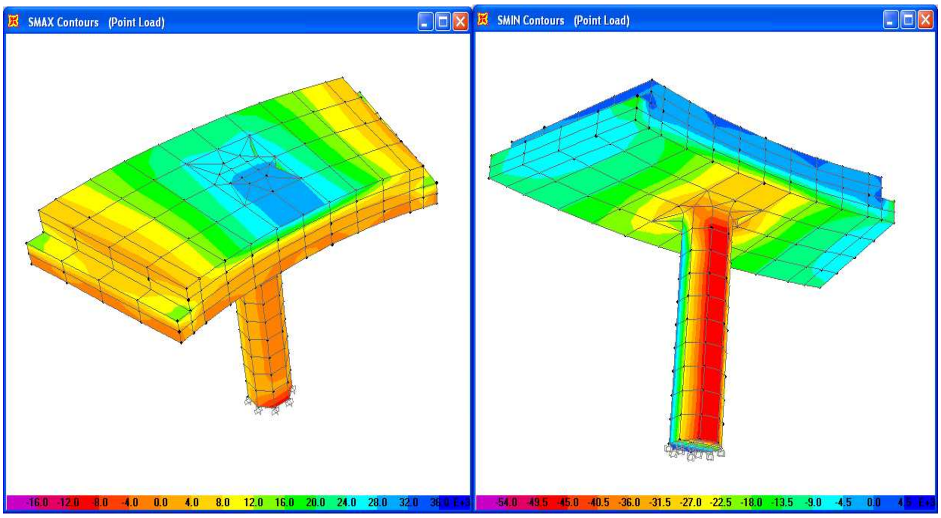


Figure 4.6: ***Principal Compressive Stress Contours of Pier Head under Point Loading & Extracted Strut and Tie Layout***

Pier with Curved Pier Head

Similarly as previous pier configurations, curved head pier is modeled with solid element and bridge girder loads coming from both sides spans of the pier are applied as point loads on relevant locations. A linear static analysis result of the model is used in extraction of possible strut and tie layout for the pier head configuration. Pictures of solid element model, principal stress contours due to girder loads and extracted strut and tie layout are shown in following figures.



(a)Principal Tensile Stress (b)Principal Compressive Stress

Figure 4.7: ***Principal Tensile Stress & Compressive Stress Contours of Curved Pier Head***

Tension ties on top of the pier head and vertical compressive struts at stem of the pier can be seen from the extracted strut and tie layout which is match with principal stress contours of the pier. But compressive struts cannot be clearly seen at the bottom of the pier head. The above strut and tie layout is extracted through the use of refining criteria of Direction cosine division=4, Stress range division=4, Strut size=1 and neglecting stresses less than 5% of maximum stress present in the system.

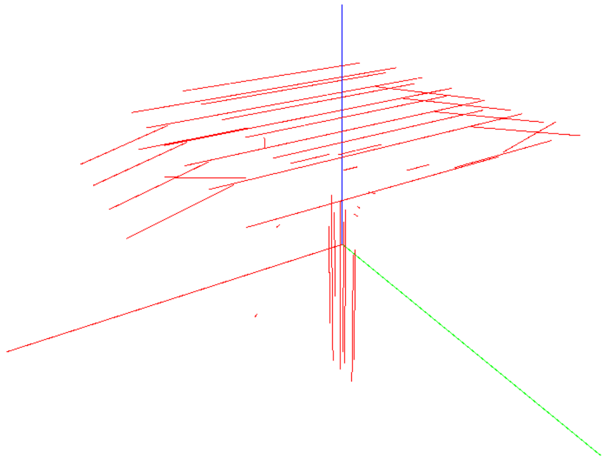


Figure 4.8: ***Strut and Tie Layout for Pier with Curved Pier Head***

5. Conclusions

Based on results and experienced gained through the present study, following conclusions are drawn.

(i) Solid element mesh analysis can display the internal stress flow of three dimensional structural members and initial strut and tie layout can be visualized through it.
(ii) The proposed method can extract the possible strut and tie member layout that match with internal stress flow of three dimensional disturbed region members.
(iii) Further modifications are required to improve the quality of the results.

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