Frequency Response Function Based Structural Damage Detection Using Artificial Neural Networks

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Abstract

Damage detection in structures has become increasingly important in recent years. While a number of damage detection and localization methods have been proposed, few attempts have been made to explore the structure damage with frequency response functions (FRFs). This paper illustrates the damage identification and condition assessment of a beam structure using a new frequency response functions (FRFs) based damage index and Artificial Neural Networks (ANNs). In practice, usage of all available FRF data as an input to artificial neural networks makes the training and convergence impossible. Therefore one of the data reduction techniques Principal Component Analysis (PCA) is introduced in the algorithm.

In the proposed procedure, a large set of FRFs are divided into sub-sets in order to find the damage indices for different frequency points of different damage scenarios. The basic idea of this method is to establish features of damaged structure using FRFs from different measurement points of different sub-sets of intact structure. Then using these features, damage indices of different damage cases of the structure are identified after reconstructing of available FRF data using PCA. The obtained damage indices corresponding to different damage locations and severities are introduced as input variable to developed artificial neural networks.

Finally, the effectiveness of the proposed method is illustrated and validated by using the finite element model of a beam structure. The illustrated results show that the PCA based damage index is suitable and effective for structural damage detection and condition assessment of building structures.

Keywords: Frequency Response Functions, Damage Detection, Damage Severity, Back Propagation neural network, Principal Component Analysis

1. Introduction

A large volume of research has been carried out in the past three decades in the field of vibration based damage identification, and many algorithms have been developed. Most of the work carried out so far use modal data such as natural frequencies and mode shapes for damage detection. In the early years of dynamic based damage detection, natural frequencies were especially popular as damage indicators as frequency measurements can be quickly conducted (Salawu, 1997). However, it is difficult to determine the damage location only by changes of modal frequencies in practice, since damage at different locations may produce the same amount of frequency change. Furthermore, in many cases natural frequencies tend to be insensitive to structural damage, especially for damage of lower severity (Chen et al., 1995). natural frequencies are heavily affected by environmental changes, such as temperature or humidity fluctuations (Kim et al., 2007) therefore it is a problem for field applications. Thus, a shift in natural frequencies along might not provide sufficient information for damage detection. Mode shapes and related functions or parameters are also frequently used as damage indicators for structural damage detection. Changes in mode shape measurements before and after damage are used either directly (Salawu et al., 1995) or indirectly as measures of mode shape curvatures (Pandey et al., 1991), modal strain energy changes (Stubbs et al., 1990) or dynamic flexibility (Pandey et al., 1994).

Among all sorts of methods developed for damage detection in structures, the use of frequency response functions (FRFs) seems very promising for damage detection in recent years as utilizing FRFs to form a damage indicator have several advantages. FRF require a small number of sensors and in situ measurements is straight forward (Fang et al., 2005) therefore among all the dynamic responses, the FRF is one of the easiest to obtain in real time. Measured FRF data are usually obtained from vibrational testing therefore these data provide information of structural dynamic behaviour. And also, FRF based damage detection techniques do not require experimental modal analysis therefore data is accurate as labours do not involve. Damage detection algorithms have been developed using either direct FRF measurements (Choudhury, 1996) or their derivatives such as FRF curvatures (Sampaio et al., 1999), FRF differences (Trendafilova et al., 2003) or compressed FRFs (Ni et al., 2006; Zang et al., 2001). However, FRF approach has many obstacles such as its large size and complexity of data. Furthermore, FRF are very sensitive to measurement noise and environmental fluctuations, which may lead to inaccurate damage identification and condition assessment.

This paper proposes a methodology to deal with FRFs data. FRFs from healthy and damaged beam structure and beam structure are used. Usage of all available FRF data as an input to artificial neural networks makes the training and convergence impossible. Therefore one of the data reduction techniques principal component analysis (PCA) is used to reduce the FRF data. The measured FRF is reconstructed using only a few PCs which are most significant and these PCs are used as input vector for ANNs instead of the raw FRF data. The method is validated using data of finite element models of beam structure. The results obtained are discussed in detail.

2. Theoretical background

Neural networks are used, for damage detection purposes, because of their ability to solve complex problems that are in deterministic and require a non-linear mapping between the input space and the output space, and their generalization capability that allows them to produce reasonable outputs from inputs not encountered during learning. Hence, in order to use the neural network for damage detection purposes the key would be to train the network with various damage signatures and their corresponding physical parameters.

There are different kinds of networks (Hagan et al., 1995) available, namely: feed forward back propagation network, radial-basis network, hamming network and self organizing maps. In this research study, feed forward neural networks were used. This is because it is evident from open literature (Fang, et al., 2005; Xu et al., 2001), that for inverse problems, in structural health monitoring, feed forward back propagation networks are the most effective and commonly used type of network. Artificial neural networks are trained using derived FRFs based damage indices following a novel algorithm.

Using FRFs data of the intact structure, matrix H= [h_{ij} (ω)] which has *m* rows of FRFs (*m* observations from different measurement locations); each with *n* frequency points is formed. In the present study, numbers of observations (*m*) is 7 and 10,000 frequency points are used for the beam structure. Each column of H is adjusted to have a zero mean by subtracting the mean of each column of FRFs of the intact structure and dividing each column by its standard deviation to get a unit variance to yield a response variation matrix $[\tilde{H}(\omega)_{mxn}]$ as follows.
The mean response of the *j*th column is given as:

$$\overline{H}_j = \frac{1}{m}\sum_{i=1}^{m} h_{ij}(\omega) \tag{1}$$

The corresponding standard deviation S_j can be defined as

$$S_j^2 = \frac{1}{m}\sum_{i=1}^{m} (h_{ij}(\omega) - \overline{H}_j)^2 \tag{2}$$

A typical element of the FRF matrix can now be replaced by:

$$\tilde{h}_{ij}(\omega) = \frac{h_{ij}(\omega) - \overline{H}_j}{S_j\sqrt{m}} \tag{3}$$

The correlation matrix can be defined as:

$$[C]_{nxn} = [\tilde{H}]^T_{nxm}[\tilde{H}]_{nxn} \tag{4}$$

By definition, the principal components are the eigenvalues and associated eigenvectors of the correlation matrix:

$$[C]\{\Psi_i\} = \lambda_i\{\Psi_i\} \tag{5}$$

where *i* is the principal component index.

The first principal component, i.e. the highest eigenvalue and its associated eigenvector, represents the direction and amount of maximum variability in the original data. The next principal component, which is orthogonal to the first component, represents the next most significant contribution from the original data, and so on. New FRFs for the detection stage is obtained and it is represented as new. This new complete observation $(H_{new})_{1xn}$ will be reconstructed based on $\tilde{H}_j$, S_j and $\{\Psi_i\}$ of the intact structure. In this algorithm FRFs of testing stage are not transferred to PCs. Element $(H_{new})_{1xj}$ of the FRF matrix $(H_{new})_{1xn}$ is transformed into

$$(\tilde{H}_{new})_{1j} = \frac{(H_{new})_{1j} - \overline{H}_j}{S_j} \tag{6}$$

The projection of the response variation matrix $(\tilde{H}_{new})_{1xn}$ on the n principal components derived from baseline FRFs of the intact structure is written as (Ni, et al., 2006)

$$(A)_{1xn} = (\tilde{H}_{new})_{1xn}(\Psi)_{nxn} \tag{7}$$

The projection matrix [A] and the eigenvector matrix [Ψ] can be partitioned into two sub-matrices with p principal components and (n-p) principal components. Setting those sub-matrices representing principal components (n-p) to zero, one obtained:

$$(\tilde{H}_R)_{1xn} = [A][\Psi]^T = [[A_{1xp} : [0_{1x(n-p)}]][[\Psi]_{nxp} : [0]_{nx(n-p)}]^T \approx [A]_{1xp}[\Psi^T_{pxn}] \tag{8}$$

Finally, the element $(\tilde{H}_R)_{1xj}$ of the reconstructed response variation matrix is used to obtain element $(H_R)_{1xj}$ FRF for the new observation,

$$(H_R)_{1xj} = S_j[\tilde{H}_R]_{1xj} + \overline{H}_j \tag{9}$$

In order to reduce the high dimensionality of FRF dataset, the complete FRF is divided into sub-observations with low dimensionality where each sub-observation contains r consecutive frequency points ($r<n$). To compare the new constructed signal $(H_{new})_R$ with the baseline signal H which is the mean response of the m observations of the intact structure, the damage index (DI) is defined as,

$$DI = (H_{new})_R / (H)_{baseline} \tag{10}$$

The integrated procedure for damage detection is summarized into the diagram in Figure 1.

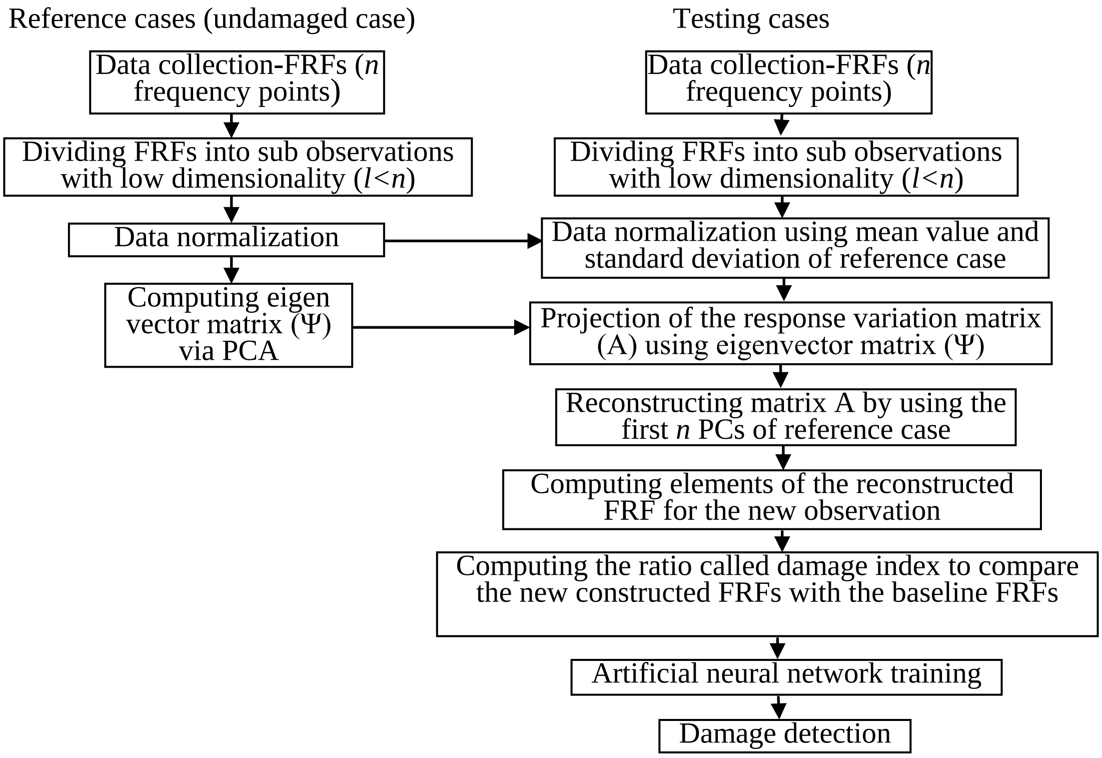


Figure 1 Damage detection diagram

3. Artificial Neural Networks

Neural network simulations appear to be a recent development. However, this field was established before the advent of computers. The artificial neural network (ANN) model is robust, fault tolerant and capable of pattern recognition and classification. They can model complex non-linear relationships and are robust in the presence of noise. ANN can also effectively deal with qualitative, uncertain, and incomplete information, making it highly promising for detecting structural damage. The feasibility of applying these networks to detect structural damage has received considerable attention (Kao & Hung, 2003).

Although many types of ANNs are used in practice, a back propagation (BP) type network, shown in Fig.2 is used for this study. The back propagation algorithm is capable of solving non linearity separable pattern classification problems; it was first popularized and widely used by Hagan et al. (1995). It consists of one output layer of neurons and one or more intermediate layers, referred to as hidden layers. The number of neurons in individual layers may be different, and they may each have a different transfer function. Training is usually done by iterative updating of weights, usually employing the negative gradient of a mean-square error function (Zweiri et al., 2003).

Back propagation neural networks are widely used by researchers. Elkordy et al.(1993) used back-propagation neural networks to identify damage in a five story building. Barai et al.(1995) examined vibration signature analysis of steel bridge structures in a nonconventional way using artificial neural networks. The back-propagation algorithm with multilayer perceptrons was used to train the network. The multilayer perceptron is very popular ANN architecture and has performed well in a variety of applications in several fields including a few structural engineering applications. Ortiz et al. (1997) presented a detailed study of the capabilities of artificial neural networks to detect damage in a cantilever structure. A three layer backpropagation neural network was trained and tested with deterministic and random information of the resonant frequencies of the damaged beam generated analytically, and it was also tested with experimentally measured resonant frequencies.

Wu et al. (1992) used back-propagation neural networks to extract and store the knowledge of the patterns in the response of the undamaged and the damaged structure. The results indicated that neural networks are capable of learning about the behavior of undamaged and damaged structure, damage localization and quantification from patterns in the frequency response of the structure. In Marwala and Hunt (1999), a new mapping topology called committee network to combine the information from FRF (Frequency Response Function) and modal data were proposed. Two back-propagation ANN models were used to predict the fault identity based on FRF and modal data respectively. Frequency energy calculated from FRF was used as the input for the first ANN model and modal properties for the second. The method was found to give errors that were generally lower than that given by the two existing approaches. Xu et al. (2005) presented a new robust two step algorithm for detecting the location and magnitude of damage. The damage extent was evaluated by a back-propagation neural network. The proposed algorithm was used to detect simulated damage in a simple finite element model of a slab and girder bridge. The damage extent was evaluated by a back-propagation neural network. The results showed that the proposed algorithm was quite effective in identifying the location and magnitude of damage, even in the presence of measurement errors in the input data. The results from many researchers have shown that neural networks can be trained to provide damage information in structures.

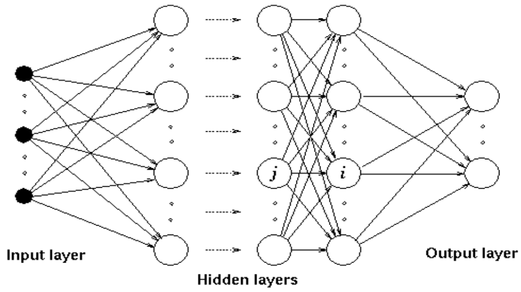


Figure 2. ***Architecture of back propagation (BP) neural network***

4. Methodology

4.1 Simulation of damage in a beam model

The objective of the finite element modeling of the beam structure is to create a model that represents the actual laboratory structure of Ulrike Dackermann (Dackermann, 2009) as close as possible. A beam structure was manufactured in the University of Technology Sydney (UTS) Metal Workshop to experimentally validate the proposed damage identification method. It is considered that the modulus of elasticity is $2*10^{11}$ N/m^2, the Poisson's ratio to 0.3 and the density to 7850 kg/m^3. The support conditions are set as pin-pin. For the numerical model, finite element models with different damage locations and damage sizes were created using commercial finite element (FE) analysis package ANSYS Classic and ANSYS Workbench.

28 damage cases of four locations and seven severities are created. The four damage locations are situated at 4/8th, 5/8th, 6/8th and 7/8th of the beam length (identified as "4", "5", "6" and "7") and the seven different damage severities correspond to a loss of the second moment of area, I, of 9.1%, 17.6%, 33.0%, 46.3%, 57.8%, 67.5% and 75.6% (termed as double extra light (XXL), extra light (XL), light (L), medium (M), extra medium (XM), severe (S) and extra severe (XS) respectively). The damage locations are illustrated in Figure 3. Frequency response functions are measured at locations "1" - "7" for twenty six damage cases.

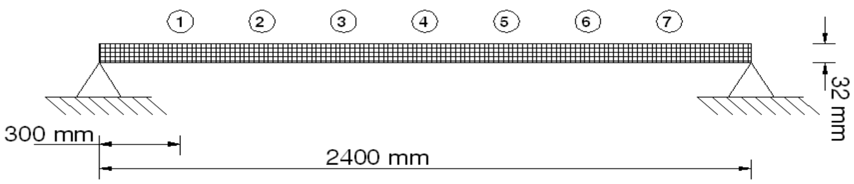
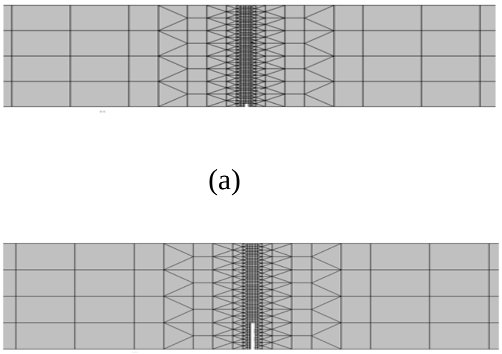


Figure 3 Frequency response functions measured in numerical beam model

Notch type damages, 1mm in length and 1mm, 2mm, 4mm, 6mm, 8mm, 10mm and 12mm in height are modelled. Damage is created as rectangular openings from the soffit of the beam. The meshed density is refined in the vicinity of the defect, as displayed in Figure 4.



(a) (b)

(c) (d)

Figure 4 Finite element modelling of damage with a width of 1mm and varying height of (a) 1mm, (b) 4mm, (c) 8mm and (d) 12mm.

A force of 800N, which is a typical impact force observed from experimental hammer excitation is applied at location "3". FRF summation functions from the beam structure with undamaged case and different damage cases are illustrated in Figure 5.

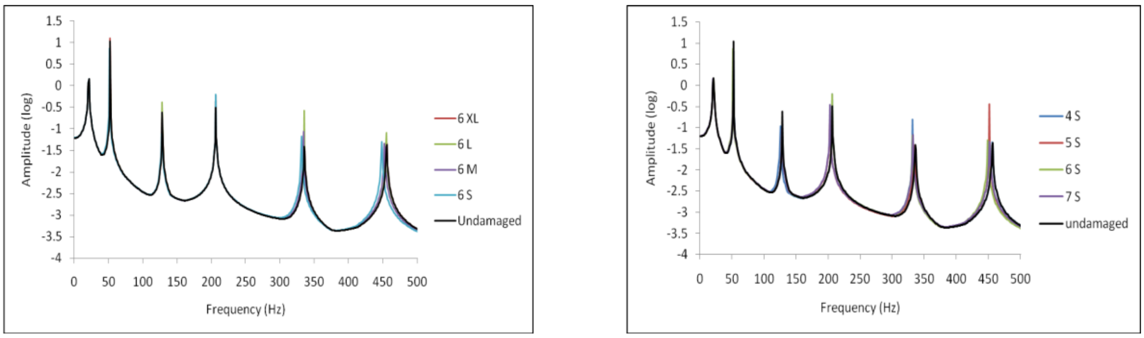
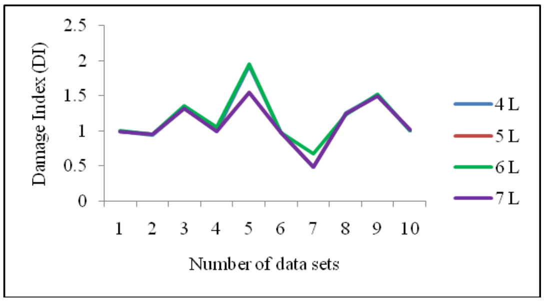


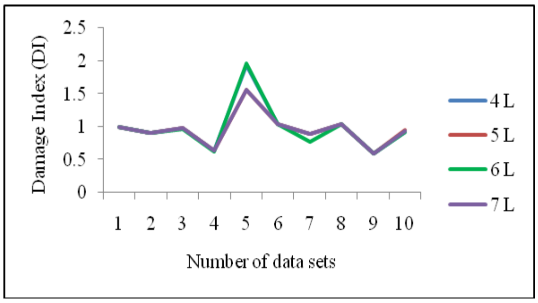
Figure 5 FRF summation functions from the numerical beam in the undamaged state and different damaged states (a) with defects at location '6' of severities extra-light (6XL), light (6L), medium (6M) and severe (6S) (b) with defects of severe extent at locations '4' to '7' (4S to 7S)

4.2 Artificial Neural Network Training

In this study, multi-layered feed-forward neural network architecture is implemented for structural damage detection. Such network consists of an input layer, one or more hidden layers, and an output layer (Fig. 2). The network is first trained using initial training data sets consisting of damage location and damage severity as target outputs and their corresponding FRFs as inputs.

Neural network must be put through a training cycle, for it to predict the severity and location of damages of beam structure. This training set should therefore encompass all damage sizes and locations. The input to the neural network is the damage indices of different damage signature, and the desired output (target) is the actual damage position and damage severity. In order to create a training set to train the network, a good number of finite element models have to be created. This is because various permutations and combinations of damage sizes and locations have to be modelled for the network to generalize accurately.
Figure 5 illustrates the damage indices obtained from measurements of location “1” and “2” using proposed algorithm when light damage is located at location “4”, location “5”, location “6” and location “7”.





(a) (b)

Figure 6 Damage index values obtained from different damage locations when FRF measurement location is at (a) location “1”and (b) location “2”.

For each data catergory two artificial neural networks are created estimating either the location (in length along the beam) or the severity (in loss of the second moment of area, I) of the damage cases. For the data catergory, the input samples are divided randomly into three sets as training, validation and testing in neural network training tool in MATLAB (2009a). The individual networks of the numerical beam data are designed with: one input layer of ten nodes, representing the damage index values of XXL, XL, L, M, XM, S and XS damage cases; two hidden layers of eight and four nodes, and output layer (denoted as a 10-8-4 network). Feedforward neural network is used as the network type and scaled conjugate gradient backpropagation algorithm is used as the training algorithm. The corresponding target output values for all damage cases for both network types are listed in Table 1.

Table 1 Neural network target output values

Network Output Damage Location (m)	*Damage Case*
1.2	*4XXL,4XL, 4L, 4M, 4XM, 4S, 4XS*
1.5	*5XXL,5XL, 5L, 5M, 5XM, 5S, 5XS*
1.8	*6XXL,6XL, 6L, 6M, 6XM, 6S, 6XS*
2.1	*7XXL,7XL, 7L, 7M, 7XM, 7S, 6XS*

Network Output Damage Severity (%)	*Damage Case*
9.1	*4XXL, 5XXL, 6XXL, 7XXL*
17.6	*4XL, 5XL, 6XL, 7XL*
33.0	*4L, 5L, 6L, 7L*
46.3	*4M, 5M, 6M, 7M*
57.8	*4XM, 5XM, 6XM, 7XM*
67.5	*4S, 5S, 6S, 7S*
75.6	*4XS, 5XS, 6XS, 7XS*

A total of eight individual networks are created; seven of these are trained with data obtained from the measurement locations '1' to '7', and one network is trained with data from summation FRFs which are obtained by adding up the FRF data from all individual FRF measurements. For all the cases first twelve PCs which contain 94% of original data set are used.

4.3 Performance measures of Artificial Neural Networks

Although there can be many performance measures for an ANN forecaster like the modelling time and training time, the ultimate and the most important measure of performance is the prediction accuracy of the output data. An accuracy measure is often defined in terms of the predicting error which is the difference between the actual (desired) and the predicted value. There are a number of measures of accuracy in the prediction literature (Zhang et al., 1998). The most frequently used are;

The mean absolute deviation (MAD) = $\dfrac{\sum e_t}{N}$

The sum of squared error (SSE) = $\sum[(e_t)]^2$

The mean squared error (MSE) = $\dfrac{\sum[(e_t)]^2}{N}$

The root mean squared error (RMSE) = $\sqrt{MSE}$

The mean absolute percentage error (MAPE) = $\dfrac{1}{n}\sum\left|\dfrac{e_t}{y_t}\right|(100)$

Where e_t is the individual prediction error; y_t is the actual value; and N is the number of error terms. In this study the mean squared error is used to predict the output error of the neural networks. Actual output values for identifying the structure damage location and damage quantification are shown in Table 1. Figure 7 illustrates the mean squared errors of outputs obtained from neural networks for all locations.

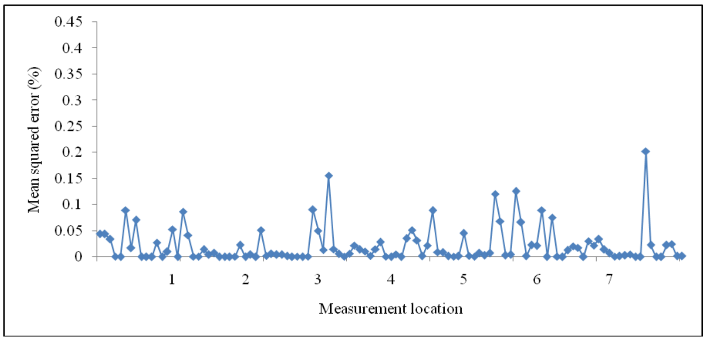


Figure 7 Mean Squared Error of neural network outputs of damage localization

The Table 2 lists results of seven individual networks (networks of 'location 1','location 2' etc), including training error, validation error, testing error and simulation error. From the network performance of Table 2, it is observed that the location predictions of the networks are corect for all damage cases. These impressive results clearly demonstrate the pattern recognition ability of neural networks. Specifications and the performance of the networks trained for damage quantification are listed in Table 3. It is noted from the data that the proposed FRF based damage identification scheme provides an accurate and robust identification method that requires only a small number of sensors.

As expected, among all trained networks, the best results for damage location are obtained with summation FRFs giving MSE of 0.021%, 0.036%, 0.058% and 0.569% for the training, validation, testing and simulation respectively while it is 0.365%, 0.407%, 0.261% and 0.323% for damage quantification. From the results it shows that the proposed algorithm performs well for damage quantification with identified damage patterns. It is further noticed that the difference in damage detection results from each individual network is very small (less than 1.8% for damage localization and 0.5% for damage quantification). This observation suggests that even with measurements from a single sensor, using the FRF based method, accurate and reliable damage identification can be obtained for two dimensional structures.

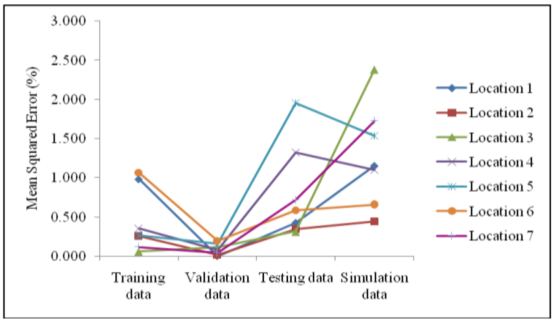
Table 2: Neural network performance (in mean squared error (MSE)) trained with data to identify damage location.

Measurement Location	*Training error (%)*	*Validation Error (%)*	*Testing Error (%)*	*Simulation Error (%)*
Location 1	0.980	0.050	0.426	1.152
Location 2	0.258	0.015	0.345	0.443
Location 3	0.059	0.109	0.313	2.383
Location 4	0.352	0.081	1.325	1.102
Location 5	0.265	0.156	1.956	1.540
Location 6	1.066	0.197	0.586	0.661
Location 7	0.118	0.043	0.718	1.728
Sum	0.021	0.036	0.058	0.569

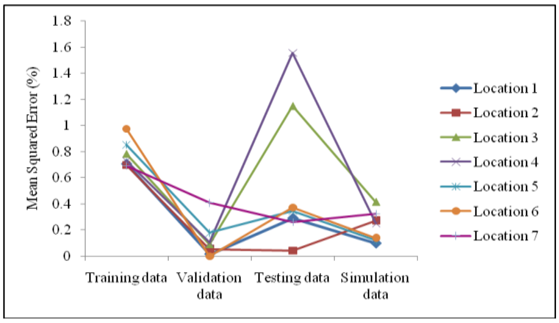
Table 3 Neural network performance (in mean squared error (MSE)) trained with data to identify damage severity.

Measurement Location	*Training error*	*Validation Error*	*Testing Error*	*Simulation Error*
Location 1	0.708	0.815	0.292	0.198
Location 2	0.697	0.555	0.143	0.273
Location 3	0.784	0.092	1.151	0.417
Location 4	0.728	0.104	1.557	0.253
Location 5	0.852	0.181	0.347	0.121
Location 6	0.977	0.536	0.373	0.142
Location 7	0.691	0.407	0.261	0.323
Sum	0.365	0.258	0.012	0.037

For the better view of results, training error, validation error, testing error and simulation error of neural networks trained for damage localization and severity estimation are plotted in Figure 8.



(a)



(b)

Figure 8. Neural network performance (MSE %) subdivided by measurement location with (a) damage location and (b) damage severity.

5. Conclusion

A novel algorithm based on FRFs and PCA is proposed for structural damage identification. In this study, damage patterns obtained from FRFs of different locations are fed to ANNs training. In order to reduce the dimensionality of the data set PCA is used. From this study it is noted that by projecting data onto the most important principal components, its size can greatly be reduced without significantly affecting the data.

A numerical example involving a beam structure is presented to demonstrate the feasibility of the proposed approach. It is seen that this method gives excellent location and severity predictions with MSE values less than 0.6% for summation FRFs. All individual networks trained for damage identification give precise results which confirm that FRFs measurements even from one sensor is enough for damage detection of two dimensional structures when using the proposed method. These outstanding results show the great potential of the proposed damage identification method.

References

Barai, S. V., Pandey, A.K. (1995). Vibration Signature Analysis Using Artificial Neural Networks. *Journal of computing in civil engineering, 9*(4), 259-265.
Chen, H. L., Spyrakos, C. C., & Venkatesh, G. (1995). Evaluating Structural Deterioration by Dynamic Response. *Journal of Structural Engineering, 121*(8), 1197-1204.
Choudhury, A. R. (1996). Damage detection in structures using measured frequency response function data, PhD thesis, Victoria University of Technology, Melbourne, VIC, Australia.
Elkordy, M. F., Chang, K. C., & Lee, G. C. (1993). Neural Networks Trained by Analytically Simulated Damage States. *Journal of computing in civil engineering, 7*(2), 130-145.
Fang, X., Luo, H., & Tang, J. (2005). Structural damage detection using neural network with learning rate improvement. *Computers & Structures, 83*(25-26), 2150-2161.

Hagan, M. T., Demuth, H. B., & H, B. M. (1995). Neural Network Design, 1st edn, PWS Publications, Boston.
Kim, J. T., Park, J. H., & Lee, B. J. (2007). Vibration-based damage monitoring in model plate-girder bridges under uncertain temperature conditions. *Engineering Structures, 29*(7), 1354-1365.
Ni, Y. Q., Zhou, X. T., & Ko, J. M. (2006). Experimental investigation of seismic damage identification using PCA-compressed frequency response functions and neural networks. *Journal of sound and vibration, 290*(1-2), 242-263.
Ortiz, J., Ferregut, C., & Osegueda, R. A. (1997). Damage detection from vibration measurements using neural network technology Artificial Neural for Civil Engineers. *Fundamentals and Applications ed N Kartam et al (New York: ASCE), Chapter 4*, 65-91.
Pandey, A. K., & Biswas, M. (1994). Damage Detection in Structures Using Changes in Flexibility. *Journal of Sound and Vibration, 169*(1), 3-17.
Pandey, A. K., Biswas, M., & Samman, M. M. (1991). Damage detection from changes in curvature mode shapes. *Journal of Sound and Vibration, 145*(2), 321-332.
Salawu, O. S. (1997). Detection of structural damage through changes in frequency: a review. *Engineering Structures, 19*(9), 718-723.
Salawu, O. S., & Williams, C. (1995). Bridge Assessment Using Forced-Vibration Testing. *Journal of Structural Engineering, 121*(2), 161-173.
Sampaio, R. P. C., Maia, N. M. M., & Silva, J. M. M. (1999). Damage detection using the frequency-response-function curvature method. *Journal of Sound and Vibration, 226*(5), 1029-1042.
Stubbs, N., & Osegueda, R. (1990). Global non-destructive damage evaluation in solids. *International Journal of Analytical and Experimental Modal Analysis, 5*(2), 67-79.
Trendafilova, I., & Heylen, W. (2003). Categorisation and pattern recognition methods for damage localisation from vibration measurements. *Mechanical Systems and Signal Processing, 17*(4), 825-836.
Wu, X., Ghaboussi, J., & Garrett, J. H. (1992). Use of neural networks in detection of structural damage. *Computers and structures, 42*(4), 649-659.
Xu, H. (2005). *Application of artificial neural networks in vibration-based damage identification.* Unpublished Ph.D., Carleton University (Canada), Canada.
Xu, Y. G., Liu, G. R., Wu, Z. P., & Huang, X. M. (2001). Adaptive multilayer perceptron networks for detection of cracks in anisotropic laminated plates. *International Journal of Solids and Structures, 38*(32-33), 5625-5645.
Zang, C., & Imregun, M. (2001). Structural damage detection using artificial neural networks and measured frf data reduced via principal component projection. *Journal of Sound and Vibration, 242*(5), 813-827.
Zhang, G., Eddy Patuwo, B., & Y. Hu, M. (1998). Forecasting with artificial neural networks:: The state of the art. *International Journal of Forecasting, 14*(1), 35-62.
Zweiri, Y. H., Whidborne, J. F., & Seneviratne, L. D. (2003). A three-term backpropagation algorithm. *Neurocomputing, 50*, 305-318.