Evaluation of minimum shear reinforcement in reinforced concrete beams

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Abstract

Failures in reinforced concrete (RC) structures under shear loads are proved to be brittle with the presence of web reinforcement.

The minimum web reinforcement specified by many codes of practice is intended to maintain adequate reserve strength and ductility after the diagonal cracking and to control widening of diagonal cracking. However, the expressions for estimating the minimum shear reinforcement in codes of practice are based on the experimental data base observed on small beam depth made of normal strength concrete (NSC). Hence, these provisions need to be reassessed and altered for large size members made of high strength concrete (HSC). Further, there has been no consensus on the amount of shear reinforcement to be provided by different codes of practice, as they differ significantly in respect of HSC members. In this paper, many factors influencing the minimum shear reinforcement in RC beams has been incorporated. An expression has been proposed incorporating a wide range of concrete strengths. A comparison of the minimum shear reinforcement predicted by the proposed expression has been made with the codes of practice.

Key words: Shear reinforcement, reinforced concrete beam, ductility, high strength concrete.

1. Introduction

It is believed that the web reinforcement in the form of vertical stirrups does not affect the diagonal cracking strength significantly. However, the presence of shear reinforcement tends to alter the behaviour of reinforced concrete (RC) beams and enhances the shear capacity by improving the shear transfer mechanisms through better dowel action, restraining crack propagation, minimising bond splitting failure and enhancing the contribution of uncracked concrete in the compression zone. In RC structures, provision of minimum shear reinforcement is mandated when the factored shear force exceeds one-half the design shear strength of concrete. The intension of providing minimum shear reinforcement in RC elements by the codes of practice is (i). to prevent sudden brittle failure as soon as reaching the first diagonal cracking load, (ii). to control widening of cracks at service loads and (iii). also to impart adequate ductility before failure. Several national codes of practice such as ACI 318, Canadian code and AASHTO specify the minimum shear reinforcement varying with the compressive strength of concrete, while it is varying with the yield strength of shear reinforcement only as per IS and BS codes of practice.

2. Literature review

The review of literature on the minimum shear reinforcement is reported. Lin and Lee (2001) reported that the increase in the quantity of tension reinforcement increases the shear strength but decreases the ductility of the beams. Further an increase in the percentage of compression reinforcement and the strength of concrete improves the ductility of the beams effectively. Beams with high strength shear reinforcement exhibit the same crack control ability as compared to the beams with normal strength shear reinforcement. In another study, Lin and Lee (2003) concluded that the factors affecting ductility are shear span-to-depth (a/d) ratio, spacing of stirrups and strength of shear reinforcement. Also it was observed that by increasing the quantity and the strength of shear reinforcement does not show apparent influence on the diagonal cracking strength of beams. However, increasing the strength of concrete and strength of shear reinforcement increases the ultimate strength, while the decrease in the shear-span-depth ratio and the spacing of stirrups increases the ultimate strength. In beams with low a/d ratio, the effectiveness of shear reinforcement is much low.

Xie et al. (1994) carried out experimental study on ductility of reinforced concrete beams in shear with and without web reinforcement, in which the following variables were varied; strength of concrete from 40 MPa to 109 MPa, shear span-to-depth ratio from 1.0 to 4.0 and the percentage shear reinforcement from 0.0 to 0.784. In this study, the post-peak response was quantified through shear ductility. Based on the comparison of tests on two large size beams by Johnson and Ramirez (1989) and the tests on reduced size beams, Frosch (2000) reported that the size of the beam did not affect the post cracking behaviour or the shear strength provided by the stirrups. However, from the evaluation of test results and previous studies, Johnson and Ramirez (1989) demonstrated that the overall reserve strength beyond diagonal tension cracking diminished with the increase in compressive strength, f'_c for the beams designed with the current provisions of minimum shear reinforcement. This situation would be more critical for beams with large a/d ratios at small percentage of longitudinal reinforcement. Yoon et al. (1996) concluded that for HSC members the crack spacing is a function of spacing of longitudinal and transverse reinforcement. The spacing of shear cracks increases as the size of the member increases. Hence the Canadian Standards Association (CSA) (1984) underestimates the shear strength of large size members at failure.

Roller and Russell (1990) reported that the minimum shear reinforcement specified in the codes of practice needs to be increased as the strength of concrete increases. The equation proposed by Yoon et al. (1996) was revaluated by Ozcebe et al. (1999) and stated that the amount of shear reinforcement could be 20% smaller than that of the minimum shear reinforcement required as per the provisions of ACI code (1999). Experimental investigations by Angelakos (2001) showed that the minimum amount of reinforcement in the form of stirrups provided by ACI code (1999) exhibited inadequate safety margins.

3. Code provisions for minimum shear reinforcement

The provisions for minimum shear reinforcement by various codes of practice are shown in Table 1. The variation of the shear reinforcement index ($r*f_y$) with compressive strength of concrete is shown in figure 1 ($r = A_{sv}/bS_v$). It demonstrates that the minimum shear reinforcement is a function of the compressive strength of concrete as per ACI (1999), AASHTO (2000) and CSA (1984) codes. It is the function of the strength of shear reinforcement alone as per BS (1985) and IS (2000) codes. Also, the difference is much large for HSC beams compared to NSC beams.

Table 1. Provisions for Minimum Shear Reinforcement by various Codes of Practice

Sl. No	*Code*	*Equation*	*Equation*
1	***ACI (1999)***	$\frac{A_{sv}}{bS_v} \geq \frac{\sqrt{f_c}}{16f_y} \geq \frac{0.33}{f_y}$	*(1)*
2	***AASHTO [2000]***	$\frac{A_{sv}}{bS_v} \geq \frac{\sqrt{f_c}}{12f_y}$	*(2)*
3	***CSA(1984)***	$\frac{A_{sv}}{bS_v} \geq \frac{\sqrt{f_c}}{16.67f_y}$	*(3)*
4	***IS 456(2000)***	$\frac{A_{sv}}{bS_v} \geq \frac{0.4}{0.87f_y}$	*(4)*
4	***BS 8110 (1985)***	$\frac{A_{sv}}{bS_v} \geq \frac{0.4}{0.87f_y}$	*(4)*

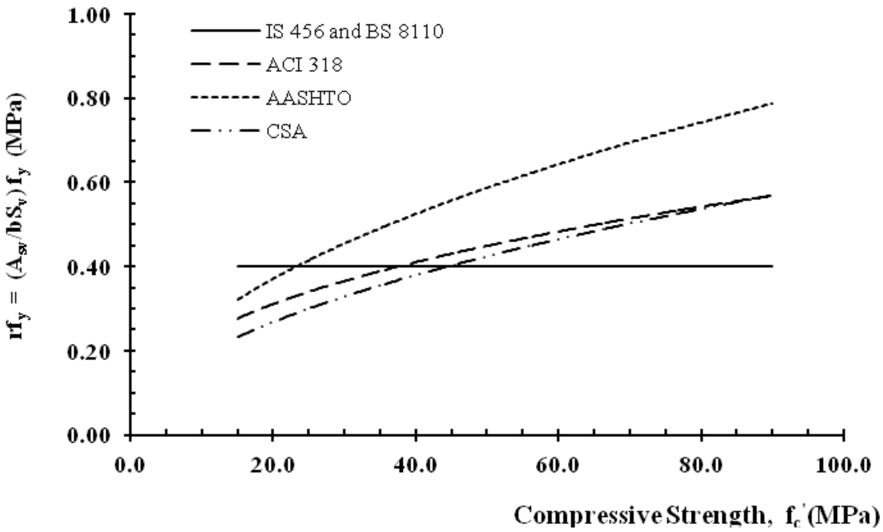

Figure 1. Shear reinforcement index vs. Compressive strength of concrete.

4. Estimation of minimum shear reinforcement

The minimum shear reinforcement specified by the codes of practice is intended to ensure that the strength of a member after cracking exceeds the load at which the diagonal cracking occurs or in other words for a beam with the given geometry and materials the minimum shear reinforcement is necessary to increase the shearing strength of the beam to a value 'V' greater than that of the cracking strength, $V_{cr.}$ The studies carried out by Johnson and Ramirez (1989) showed that the reserve strength decreases with the increase in compressive strength of concrete and that it would be more critical with increase in the shear span-to-depth ratio and decrease in the quantity of the longitudinal reinforcement. Therefore, the minimum shear reinforcement must be a function of the shear-span-to-depth ratio and the longitudinal reinforcement apart from the compressive strength of concrete.

$$V_c + V_s \geq V_{cr}$$

$$V_{cr}\ (\text{Injaganeri, 2007}) = \left(v_u \frac{\sqrt[3]{a/d}}{2\rho^{1/6}} \right) bd \tag{5}$$

$$\text{Taking } Vc = \left(\frac{0.17\sqrt{f_c^{'}}}{\gamma} \right) bd \tag{6}$$

$$v_u = 0.5\sqrt{f_c^{'}} \tag{7}$$

$$\text{and } V_s = A_{sv} f_y \left(\frac{d}{s_v} \right) \tag{8}$$

$$\therefore \left(\frac{A_{sv}}{bs_v} \right) f_y = \sqrt{f_c^{'}} \left(0.25 \frac{\sqrt[3]{a/d}}{\rho^{1/6}} - 0.22 \right) \tag{9}$$

Where a = shear span, a/d = shear span-to-depth ratio, A_s = area of tension reinforcement, A_{sv} = Area of shear reinforcement, b = breadth of beam, d = effective depth in mm, $f_c^{'}$ = cylindrical compressive strength in MPa, f_y = yield strength of longitudinal steel, f_{vs} = yield strength of shear reinforcement, ρ_l = longitudinal reinforcement ratio, r = A_{sv} / bS_v, S_v = Spacing of stirrups, v_{cr} = diagonal cracking strength, v_u = ultimate shear strength, V_c = shear capacity of concrete, V_s = shear strength of reinforcement, V_{cr} = cracking shear strength, and V_{ul} = factored shear force at critical section

Using the relation obtained between the diagonal and the ultimate shear strength by Injaganeri (2007) and the condition $(V_c + V_s) > V_{cr}$, an expression for the minimum shear reinforcement has been established. The minimum shear reinforcement is a function of the compressive strength of concrete, the shear span-to-depth ratio and the percentage of the flexural reinforcement as shown in Eq. 9. Figure 2 demonstrates the variation of the minimum shear reinforcement as per Eq. 9 with the compressive strength of concrete.

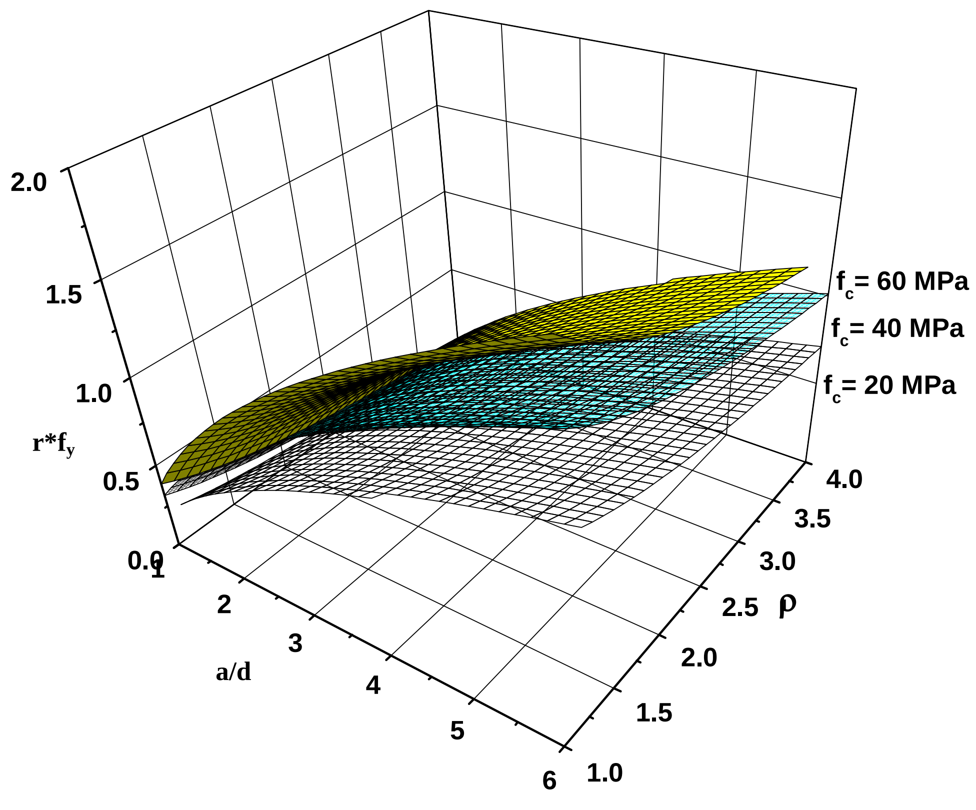

Figure 2. Minimum Shear Reinforcement with a/d ratio and longitudinal reinforcement, ρ.

5. Results and discussion

The minimum shear reinforcement as per the ACI and IS codes demands that the ultimate shear strength be equal to the shear strength of concrete 0.48 MPa and 0.4 MPa respectively for $f_c^{'}$ = 60 MPa. However, the provisions for shear reinforcement by the codes of practice are independent of the shear span-to-depth ratio and the percentage of longitudinal reinforcement. Figure 2 shows the variation of shear reinforcement with a/d ratio and ρ_l for different strengths of concrete. It can be observed that the web reinforcement decreases with the increase in ρ_l and increases with increase in the a/d ratio. The codes of practice specify the minimum shear reinforcement to ensure minimum ductility apart from ensuring the shear strength greater than its diagonal cracking strength. It is to note that the addition of even a small quantity of shear reinforcement increases the ductility of a member. However, the minimum ductility needs to be defined for that member.

In order to study the minimum shear reinforcement, six geometrically similar reinforced concrete beams of two different sizes fabricated with high strength concrete (HSC) were tested under three-point loading using a displacement control testing system of 500 kN capacity. The spacing of stirrups was altered to study the variation of ductility and compare the minimum shear reinforcement estimated from the provisions of the codes of practice. Details of the specimens are shown in Table 2.

Table 2. Geometric and Material Properties with Reinforcement Details

Beam	*b*	*d*	l_e	*a/d*	f'_c	A_{st}	A_{sv}	$r*f_y$	f_y	f_{vs}
H20-0.4	*150*	*161*	*966*	*3.0*	*45.7*	*3 #16*	*#3 @100 c/c*	*0.4*	*521*	*447*
H20-0.6	*150*	*161*	*966*	*3.0*	*40.1*	*3 #16*	*#4 @90 c/c*	*0.6*	*521*	*400*
H20-0.8	*150*	*161*	*966*	*3.0*	*37.6*	*3 #16*	*#4 @70 c/c*	*0.8*	*521*	*400*
H60-0.4	*150*	*536*	*3216*	*3.0*	*45.7*	*6 #20*	*#5 @300 c/c*	*0.4*	*595*	*479*
H60-0.6	*150*	*536*	*3216*	*3.0*	*40.1*	*6 #20*	*#6 @267 c/c*	*0.6*	*595*	*425*
H60-0.8	*150*	*536*	*3216*	*3.0*	*41.7*	*6 #20*	*#6 @200 c/c*	*0.8*	*595*	*425*

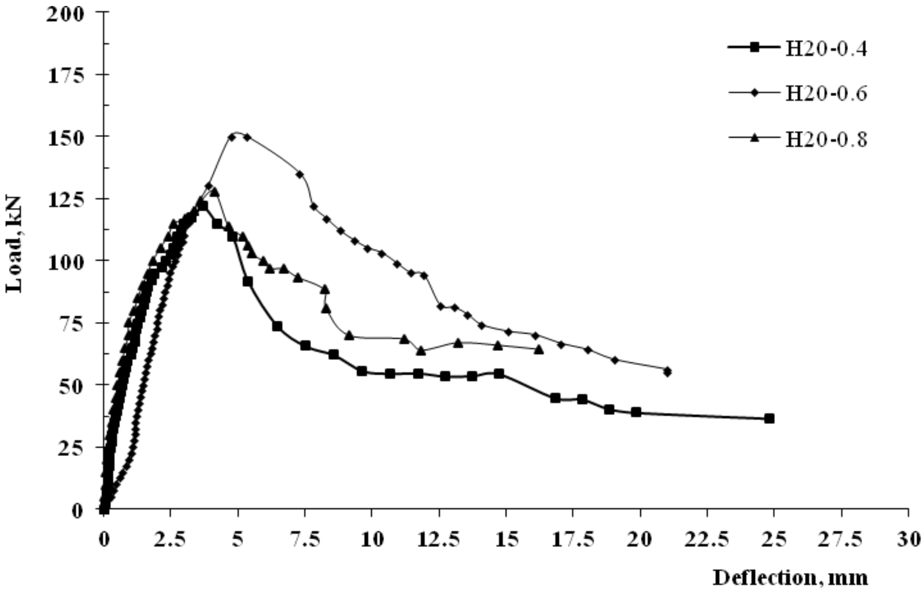

Figure 3. Load vs. Deflection in beams (D = 200mm).

Load vs. Central Deflections responses are shown in figures 3 and 4. Figures 3 and 4 demonstrate that the minimum shear reinforcement provided by the codes of practice looks sufficient for small size beams as adequate ductility has been observed. However, it seems insufficient for large size beams, where beam depth exceeds 600 mm with HSC as the shear reinforcement legs snapped after reaching the ultimate load when the shear reinforcement index $r*f_y = 0.4$ and 0.6.

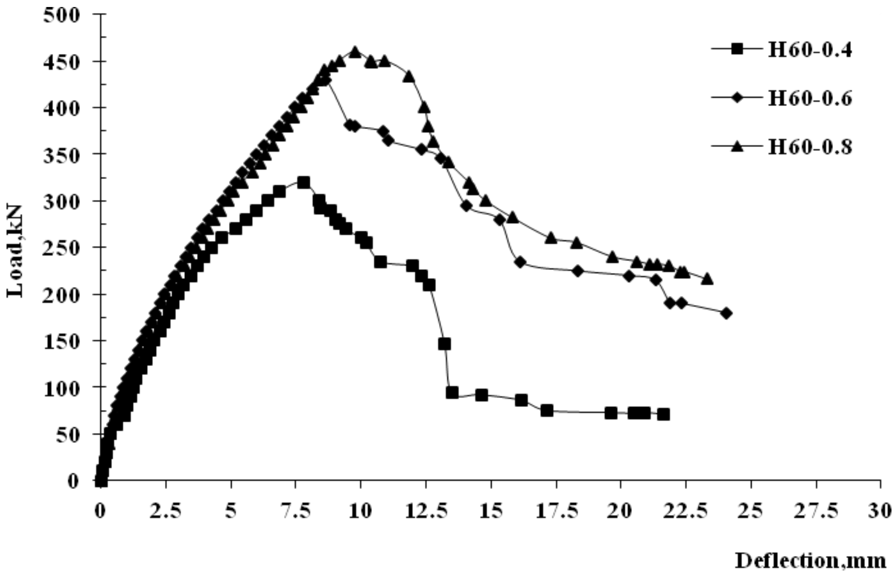

Figure 4. Load vs. Deflection in beams (D = 600mm).

6. Conclusion

The minimum amount of web reinforcement specified by IS and BS codes are independent of compressive strength of concrete and seems to be inadequate for HSC beams. However, ACI, CSA and AASHTO provisions represent as a function of compressive strength of concrete but are independent of a/d ratio and percentage of longitudinal reinforcement, ρ_l. The model proposed in this study predicts reasonably well the minimum shear reinforcement in terms of the variables compressive strength of concrete, a/d ratio and percentage of the longitudinal reinforcement, ρ_l.

References

Lin, C.H., and Lee, F.S. (2001), "*Ductility of High Performance Concrete Beams with High Strength Lateral Reinforcement*", ACI Str Jl, V. 98, N. 4, Jul-Aug, pp. 600-608.

Lin, C.H., and Lee, F.S. (2003), "*Shear Behaviour of High Workability Concrete Beams*", ACI Structural Journal, V. 100, N. 5, Sep.-Oct., pp. 599-608.

Xie. Y., Ahmad, S.H., Yu, T., and Chung, W. (1994), "*Shear Ductility of RC Beams of Normal and High Strength Concrete*" ACI Journal, V. 91 N.2, pp. 191-198.

Johnson, M.K., and Ramirez, J.A. (1989), "*Minimum Shear Reinforcement in Beams with HSC*", ACI Structural Journal, V. 86 N. 4, July-Aug, pp. 376-382.

Frosch, R. (2000), "*Behaviour of large Scale Reinforced Concrete Beams with Minimum Sear Reinforcement*", ACI Structural Journal, V. 97 N. 6, Nov.-Dec., pp. 814-820.

Yoon, Y.S., Cook, W.D., and Mitchell, D. (1996), "*Minimum Shear Reinforcement in Normal Medium and HSC Beams*", ACI St Jl, V. 93 N. 5, Sep-Oct., pp. 576 - 584.

CSA, Rexdale, (1984), "*Design of Concrete Structures for Buildings*" 1984

Roller, J.J., and Russell, H.G. (1990), "*Shear Strength of HSC Beams with Web Reinforcement*", ACI Structural Journal, V. 87 N. 2, March-April, pp. 191-198.

Ozcebe, G., and Ersoy, U., and Tankut, T. (1999), "*Evaluation of Minimum Shear Reinforcement Requirements for HSC*", ACI Journal, V. 96 N. 3, May-June, pp. 361-368.

ACI Committee 318, (1999), "*Building Code requirements for Structural Concrete (ACI 318-99),* American Concrete institute, Michigan.

Agelakos, D., Bentz, E.C., and Collins, M.P. (2001), "*Effect of concrete strength and Min. Stirrups on Shear Strength of Large Members*" ACI St Jl, V.98, N. 3, pp. 290–300.

AASHTO LRFD (2000), *Bridge Design Specifications and Commentary*, 2^{nd} Ed. American Association of State Highway Transportation Official, Washington DC.

IS 456, (2000), "*Code of Practice for Plain and Reinforced Concrete*", 4^{th} Rev, Bureau of Indian Standards, New Delhi.

BS 8110 (Part 1), (1985), "*Code of Practice for Design and Construction*," British Standards

Injaganeri, S.S., (2007), "*Studies on size effect on design strength and ductility of reinforced concrete beams in shear",* Ph.D. Thesis, Department of Civil Engineering, Indian Institute of Technology Madras.