APPLICABILITY OF GEOMETRIC BROWNIAN MOTION AND GEOMETRIC FRACTIONAL BROWNIAN MOTION TO FORECAST SHARE PRICES OF TELECOMMUNICATION SERVICES SECTOR IN SRI LANKA

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DECLARATION

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ABSTRACT

The Brownian motion is a Mathematical concept which European botanist Robert Brown introduced in 1827 to study the behaviour of molecules. The Brownian motion concept was transformed into many versions, and Geometric Brownian Motion (GBM) and Geometric Fractional Brownian Motion (GFBM) is the latest transformation of this concept. The GBM and GFBM are mathematical models used to forecast prices of stocks, commodities, etc. In this study, the GBM and GFBM were tested to estimate the share prices of telecommunication industry companies in Sri Lanka. The two sample companies were selected by representing 18% of the population of the telecommunication industry group. The five-year share prices were collected from sample companies: Sri Lanka Telecom PLC and Dialog Axiata PLC. The two models were implemented by estimating parameters such as the drift, the volatility, probability measurement and the time interval. In addition, the Hurst component was generated by a MATLAB program for GFBM. This study is concluded that GBM is the most accurate model for forecasting share prices of the telecommunication industry group with minimum mean absolute percentage error (MAPE).

Affectionately dedicated

to My Loving family members

for their enthusiastic encouragement,

which always helped me to achieve success

in my life.

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LIST OF ABBREVIATIONS

GBM	:	Geometric Brownian Motion
GFBM	:	Geometric Fractional Brownian Motion
CSE	:	Colombo Stock Exchange
GICS	:	Global Industry Classification Standard
ASPI	:	All Share Price Index
SLT	:	Sri Lanka Telecom PLC
DIALOG	:	Dialog Axiata PLC
TRC	:	Telecommunication Regulatory Commission of Sri Lanka
St Dev	:	Standard Deviation
Var	:	Variance
MAPE	:	Mean Absolute Percentage Error

Chapter 1

Introduction

1.1. Background

1.1.1. Stock Exchange

The stock market is a vital contributor to the economy of any country. The stock market or stock exchange is a market utilized for transacting shares. The transactions are issuing shares by public companies and buying and selling the shares. These financial activities are performed through stock exchanges (whether physical or electronic) or over-the-counter (OTC) marketplaces according to the set of legislation. Since the evolution of technology, all financial activities are now conducted via online platforms.

The operations in the stock exchange can be divided into two groups: primary and secondary. In the primary market, the stock market works with organizations to issue and offer their shares to the general society for the very first time through the procedure of an initial public offering (IPO). In the secondary market, these primary shares are transacted through a large number of investors according to the standard rule of “supply and demand”. As the advantage of the stock market, the public companies can raise their capital while the public grabs the opportunity for the ownership of a public company. In addition, dividend payments are made to shareholders.

There may be one or more stock exchanges in a country. The stock exchange of Sri Lanka is named “Colombo Stock Exchange (CSE)”. The CSE was not built overnight, and it dates back to 1986. Its inception goes back to 1896, when it was known as the “Colombo Share Brokers Association (CSBA)” and since then has passed through many milestones.

There are 297 public companies which are registered under the companies act No.07 of 2007. These public companies represent 20 Global Industry Classification Standard (GICS) industry groups with a market capitalization of Rs. 5815.46 billion as at the 31st of January 2022. The public companies integrated under the Companies Act No.7 of 2007 or any other state corporations, inaugurated under the laws of Sri Lanka or initiated under the laws of any other state (subject to Exchange Control approval) are eligible for the registration of CSE according to the Securities & Exchange Commission Act No.36 of 1987 and the legislation of the exchange. The CSE maintains two indexes: (All Share Price Index) ASPI and S&P SL20, and manages two main systems a central depository system and an automated trading system. The fifteen broker companies have been engaged with CSE by empowering the vision “To be the preferred choice for the creation of wealth and value”.

This study is focused on the two companies in the telecommunication services industry group: Sri Lanka Telecom PLC and Dialog Axiata PLC. According to the GICS hierarchy, telecommunication services are the leading industry group in the communication services sector. This industry group is divided into two industries: diversified telecommunication services and wireless telecommunication services. The diversified telecommunication services industry has two sub industries: alternative carriers and integrated telecommunication services. The wireless telecommunication services industry has one sub industry. The sample industry group of this study is presented in figure 1.1.

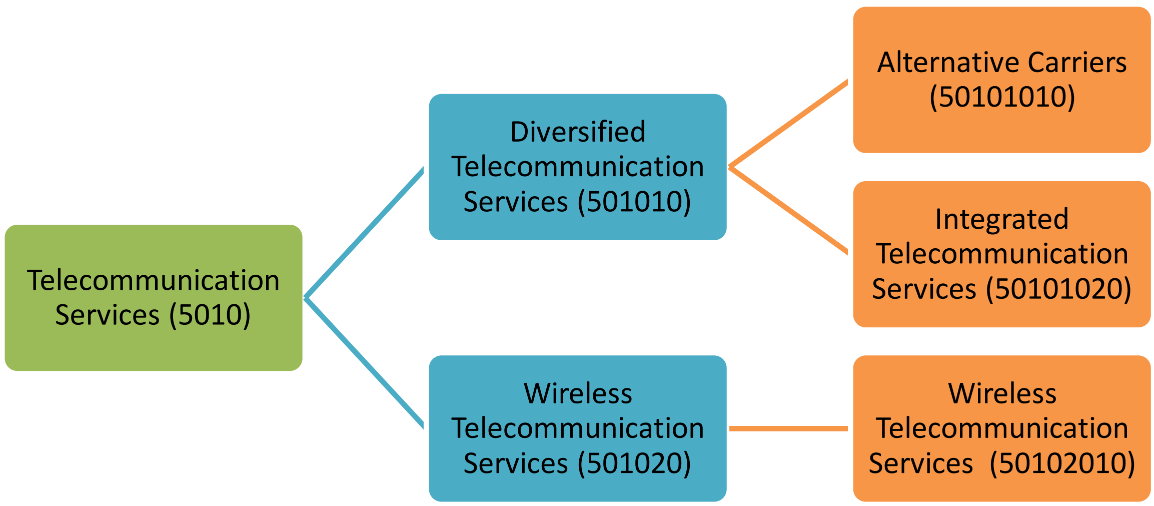

Figure 1.1 : The industry group of telecommunication services

1.1.2. Sri Lanka Telecom PLC

Sri Lanka Telecom PLC is a leading information & communication technology service provider in Sri Lanka. SLT serves a consumer base of over nine million islands wide by empowering their latest technology with ultimate-speed fibre, copper and wireless access network. SLT has vastly contributed to the Sri Lankan economy, as well, by diversifying its strategies of land and portable telephone, broadband, data services, Internet Protocol Television (IPTV), cloud computing and hosting services, and networking solutions. SLT has subsidiaries such as eChanelling PLC, Mobitel (Pvt) Ltd and the Sri Lanka Technological Campus. SLT has a market capitalization of 1.69% in CSE by issuing 1,804,860,000 shares.

1.1.3. Dialog Axiata PLC

Dialog Axiata PLC is a subsidiary of Axiata Group Berhad (Axiata) and a well-known mobile telecommunication service provider in Sri Lanka. Dialog serves through their high-level portable communication and high-speed mobile broadband services to a consumer base of over 17 million Sri Lankans. Besides, Dialog demonstrated the first 5G pilot transmission in South Asia as well. Dialog has a market capitalization of 1.99% in CSE by issuing 117,399,866 shares.

1.1.4. Brownian Motion Concept

The Brownian motion is the concept of this study. It was founded by a European botanist Robert Brown in 1827. He introduced this concept to study the behaviour of molecules based on the kinetic–molecular conception of matter. In 1905 Albert Einstein gave a scientific approach, and Norbert Winner developed a mathematical expression for it in 1918. The French Mathematician Bachelier had improved this concept for modelling share price fluctuations. The Geometric Brownian Motion (GBM) model is the latest outcome of this concept, and it can be generalized by the Geometric Fractional Brownian Motion (GFBM) model. The transformation of the Brownian motion is clearly discussed in chapter 2.

This study refers to the applicability of the Brownian motion concept to the telecommunication services sector in Sri Lanka. It broadly tested the GBM and GFBM in order to select the most accurate model for forecasting share prices of the telecommunication services sector.

1.2. Objectives

1.2.1. Primary objective

- To test the most accurate model between GBM and GFBM for forecasting share prices in the telecommunication service sector companies in Sri Lanka.

1.2.2. Specific objectives

- To study the concept of Brownian motion in the field of Financial Mathematics.
- To study the GBM and GFBM models.
- To estimate the parameters of sample companies for GBM and GFBM models.
- To apply both models for sample companies.
- To analyze the most accurate model by comparing Mean Absolute Percentage Error (MAPE).

1.2.3. Research question

What is the most accurate model between GBM and GFBM for forecasting share prices in the telecommunication service sector companies in Sri Lanka?

1.3. Significance of the study

The significance of the study is mainly focused on the stakeholders of the telecommunication services industry group. The beneficiaries are current investors, people who wish to invest in the future, two sample companies and other companies in the population.

The profit-making in the stock exchange is based on the fluctuations of the share prices. After investors buy some shares, they look for a higher price to sell them in order to make a profit. In this situation, forecasting future share prices is valuable to the first two beneficiary groups for rational decision making. The forecasted future share prices are practicable for sample companies to determine their capacity in the future. Besides, it will be adequate for the market competition of these two sample companies to gain their capital. As the final beneficiary group, the other nine companies in the population can use the findings of this study to determine the potential for entering the stock market.

1.4. Research outline

The thesis is summarized chapter wise under this topic.

Chapter 2 : Literature Review

The chapter consists of three major approaches: Conceptual, Theoretical and empirical. The information related to research methodology is included in conceptual literature. The theoretical literature refers to the transformation of the Brownian concept. The empirical literature is referred to in such a manner that other researchers realize precisely what was done and what was found in a particular research study.

Chapter 3 : Methodology

Initially, the methodology chapter adopts this research to the six layers of Saunders's research onion model: research philosophy, research approach, methodological choice, research strategy, time horizon and data collection & analysis. The sample, population, sampling method, data collection method and tools are broadly described under the topic of data collection and analysis. As the analytical framework, the formulas which were used in descriptive analysis and parameter estimations are presented.

Chapter 4 : Analysis

The results which were obtained in descriptive analysis, GBM and GFBM models and MAPE calculations are clearly presented in this chapter. In addition, the graphical representations of actual time series and forecasted time series are demonstrated.

Chapter 5 : Discussion and Recommendations

This chapter brings the conclusions and recommendations which can be helpful for further research. The final discussion is concluded in this chapter.

Chapter 2

Literature Review

2.1. Introduction

This chapter contains three major parts; Conceptual, Theoretical and Empirical. The information related to develop the research methodology is included in conceptual literature. The theoretical literature refers to the transformation of the Brownian concept. The empirical literature refers in such a manner that other researchers realize precisely what was done and what was found in a particular research study.

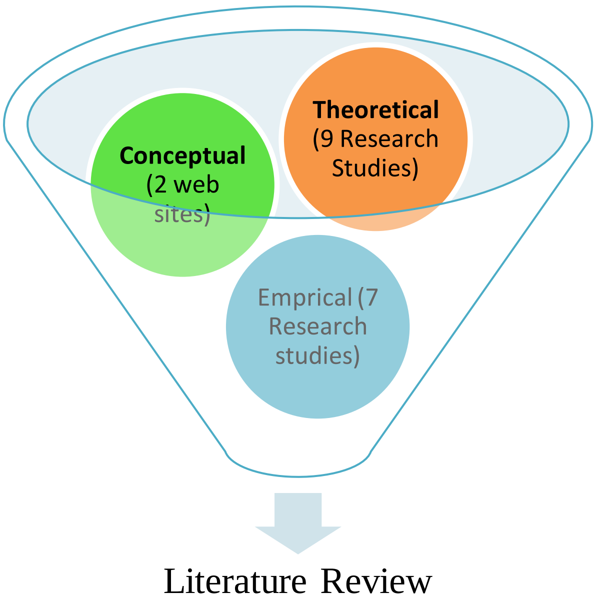

Figure 2.1 : The summary of literature review

2.2. Conceptual Framework

In this conceptual framework, the selection of population and sample is mainly described. Because it is the foundation of the entire research and affects the design of the research methodology.

The research question is to find the most accurate model between GBM and GFBM for forecasting share prices. In order to develop the research, the range of the study should be limited. Therefore an industry group is selected. The 297 registered companies in CSE are classified according to GICS. The companies listed and traded under particular industry groups are presented in table 2.1. (Colombo Stock Exchange, 2020)

GICS code	Symbol	Industry Group Name	Traded Companies	Listed Companies
1010	EGY	Energy	3	3
1510	MAT	Materials	20	24
2010	CG	Capital Goods	27	30
2020	CPS	Commercial & Professional Services	3	6
2030	TRP	Transportation	3	3
2510	A&C	Automobiles & Components	1	1
2520	CDA	Consumer Durables & Apparel	11	14
2530	CS	Consumer Services	29	37
2550	RET	Retailing	11	14
3010	FSR	Food & Staples Retailing	3	4
3020	FBT	Food, Beverage & Tobacco	38	50
3030	HPP	Household & Personal Products	1	2
3510	HES	Health Care Equipment & Services	9	9
4010	BNK	Banks	15	16
4020	DF	Diversified Financials	36	53
4030	INS	Insurance	12	13
4510	S&S	Software & Services	1	1
5010	TEL	Telecommunication Services	2	2
5510	UTI	Utilities	7	10

6010	RE	Real Estate	14	20

Table 2.1 : CSE industry groups

Since the Covid 19 pandemic, the CSE was closed for some durations, and many companies demonstrated a reduction in profit line. During this pandemic, a high revolution in the telecommunication industry group could be noticed since all state, and private platforms were transformed into online mode. Therefore the telecommunication services industry group is selected as the population.

There are 11 telecommunication services companies according to Sri Lanka Telecommunication Regulatory Commission act. They are;

- Sri Lanka Telecom PLC
- Lanka Bell Ltd
- Dialog Broadband Networks (Pvt) Ltd
- Hutchinson Telecommunication Lanka (Pvt) Ltd
- Bharti Airtel Lanka (Pvt) Ltd
- Mobitel (Pvt) Ltd
- Dialog Axiata PLC
- TATA Communications Lanka Ltd
- Lanka Communication Services (Pvt) Ltd
- Societe International Telecommunications Aeronautiques (SITA)
- Lanka Education & Research Network (LEARN)

(Telecommunication Regolatory Commission of Sri Lanka, 2014)

Among them, only two companies issue shares to the general public. Therefore Sri Lanka Telecom PLC and Dialog Axiata PLC are selected as the study sample.

2.3. Theoretical Literature

2.3.1. Random Walk (Drunkard's Walk)

For an integer, the Random Walk process at the time t, $\{W_n(t), t > 0\}$ can be interpreted as follows:

i. The initial value of the process is zero. $W_n(0) = 0$

ii. The difference between two successive jumps is equal to $\frac{1}{n}$

iii. The high and low jumps are equal and of size $\frac{1}{\sqrt{n}}$, with similar probability.

$$W_n\left(\frac{i}{n}\right) = W_n\left(\frac{i-1}{n}\right) + \frac{X_i}{\sqrt{n}} = \frac{1}{\sqrt{n}}\sum_{i=1}^{nt} X_i \qquad (2.1)$$

(Dmouj, 2006)

2.3.2. Brownian Motion (Wiener Process)

The Brownian motion (B_t) performs the following properties:

I. B_t is continuous and $B_t(0) = 0$

II. Normality condition: The increment of the Brownian process in the interval of time of length t between the two movements s and $s + t$ is $B(s + t) - B(s)$. This increment is normally distributed with zero mean and variance of time difference t. $B(s + t) - B(s) \sim N(0, t)$

III. Markov property: The information are available in this continuous distribution $B(t)$ up to time $s < t$ which depends only on the $B(s)$.

(Agustini, Affianti, & Putri, 2018)

2.3.3. Generalized Random Walk

A continuous stochastic process $\{B(t), t \geq 0\}$ is called Brownian motion with drift μ and volatility σ if,

I. $B(0) = 0$

II. $\{B(t)\}$ has independent stationary increments. Thus $B(t) - B(s)$ is normally distributed with mean of $\mu(t - s)$ and variance of $\sigma^2(t - s)$

III. $B(t)$ has a normal distribution with mean of μt and variance of $\sigma^2 t$

$$B(t) = \mu t + \sigma W_t$$

Where $W_t = z\sqrt{t}$ is random walk, $E\big(B(t)\big) = \mu t$ and $Var\big(B(t)\big) = t$

(Dmouj, 2006)

2.3.4. Stock Return

There is uncertainty in Brownian motion as a worldwide model for the stock market due to 2nd property of the Brownian motion, which expresses that the process follows normality condition with mean of zero while the price of stock typically gains at some particular rate. One method for overcoming this issue is to display the stock price as an amount of a positive, deterministic function of time and a Brownian motion.

$$dS(t) = \mu S(t)\, dt + \sigma S(t)\, dB_t \qquad \text{or} \qquad \frac{dS(t)}{S(t)} = \mu\, dt + \sigma\, dB_t \tag{2.2}$$

Calculation

$$\frac{d\big(\ln\big(S(t)\big)\big)}{dS(t)} = \frac{1}{S(t)} \Longrightarrow \frac{dS(t)}{S(t)} = d\big(\ln\big(S(t)\big)\big)$$

$$d\big(\ln\big(S(t)\big)\big) = \ln\big(S(t)\big) - \ln\big(S(t-1)\big) = \ln\left(\frac{S(t)}{S(t-1)}\right)$$

$$\ln\left(\frac{S(t)}{S(t-1)}\right) = \mu\, dt + \sigma\, dB_t \tag{2.3}$$

(Si & Bishi, 2020)

2.3.5. Ito's Lemma

Let $G(X,t)$ be function of multiple variables and $dX = a\,dt + b\,dB_t$. According to the Taylor expansion of second-order:

$$dG \approx \frac{\partial G}{\partial X}\,dX + \frac{\partial G}{\partial t}\,dt + \frac{1}{2}\left(\frac{\partial^2 G}{\partial X^2}\,(dX)^2 + \frac{\partial^2 G}{\partial t^2}\,(dt)^2\right)$$

$$dG = \frac{\partial G}{\partial X}(a\,dt + b\,dB_t) + \frac{\partial G}{\partial t}\,dt + \frac{1}{2}\left(\frac{\partial^2 G}{\partial X^2}\,b^2 dt\right)$$

$$dG = \left(\frac{\partial G}{\partial X}\,a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial X^2}\,b^2\right)dt + \frac{\partial G}{\partial X}\,bdB_t \qquad (2.4)$$

The formula 2.4 demonstrate the Ito's Lemma which is applicable to any stochastic variable.

(Dmouj, 2006)

2.3.6. Root of Stochastic Differential Equation (SDE)

The root of the above SDE can be obtained by substituting Ito's formula. Here, $G = \ln\big(S(t)\big)$ where function of $S(t)$ and t. And $dS(t) = \mu S(t)\,dt + \sigma S(t)\,dB_t$

In order to apply Ito's formula,

$$dG = d\big(\ln\big(S(t)\big)\big)$$

$$\frac{\partial G}{\partial X} = \frac{\partial\big(\ln\big(S(t)\big)\big)}{\partial S(t)} = \frac{1}{S(t)}$$

$$\frac{\partial^2 G}{\partial X^2} = \frac{\partial\left({}^1/_{S(t)}\right)}{\partial S(t)} = -\frac{1}{[S(t)]^2}$$

$$\frac{\partial G}{\partial t} = \frac{\partial\big(\ln\big(S(t)\big)\big)}{\partial t} = \ln\big(S(t)\big)\cdot\frac{\partial(1)}{\partial t} = 0$$

$$a = \mu S(t)$$

$$b = S(t)$$

$$B_t = z\sqrt{t} \implies dB_t = z\sqrt{dt}$$

Apply above values to Ito's formula,

$$dG = \left(\frac{\partial G}{\partial X}\ a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial X^2}\ b^2\right) dt + \frac{\partial G}{\partial X}\ b\ dB_t$$

$$d\big(\ln(S(t))\big) = \left(\frac{1}{S(t)} \cdot \mu S(t) + 0 + \frac{1}{2} \cdot \left(-\frac{1}{[S(t)]^2}\right) \cdot \sigma^2 [S(t)]^2\right)\ dt + \frac{1}{S(t)} \cdot \sigma \cdot S(t) \cdot\ z\sqrt{dt}$$

$$d\big(\ln(S(t))\big) = \left(\mu - \frac{1}{2}\sigma^2\right)\ dt + \sigma z\sqrt{dt}$$

We have, $\quad d(\ln(\,S(t))) = \ln(S(t)) - \ln(S(t-1)) = \ln\left(\frac{S(t)}{S(t-1)}\right)$

$\ln(S(t)) - \ln(S(t-1)) = \left(\mu - \frac{1}{2}\sigma^2\right)\ dt + \sigma z\sqrt{dt}$ This is the geometric Brownian motion model

$$\ln\left(\frac{S(t)}{S(t-1)}\right) = \left(\mu - \frac{1}{2}\sigma^2\right)\ dt + \sigma z\sqrt{dt}$$

$$\frac{S(t)}{S(t-1)} = e^{\left(\mu - \frac{1}{2}\sigma^2\right)\ dt + \sigma z\sqrt{dt}}$$

$$S(t) = S(t-1) \cdot e^{\left(\mu - \frac{1}{2}\sigma^2\right)\ dt + \sigma z\sqrt{dt}} \tag{2.5}$$

(Dmouj, 2006), (Agustini, Affianti, & Putri, 2018)

2.3.7. Drift & Volatility

Let S_i be the stock price at the end of i^{th} merchandising period and dt is the length of the time difference between two consecutive merchandising periods in years, $dt = t_i - t_{i-1}$ for i non negative integer. Let u_i is the logarithm value of the daily stock return over the short time difference dt, $\bar{u}$ and v be the mean and standard deviation of u_i.

$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right) \qquad$ for $i = 1 \ldots n$

Volatility : $\sigma = \frac{v}{\sqrt{dt}}$ (2.6)

Drift : $\bar{u} = \left(\mu - \frac{1}{2}\sigma^2\right) dt$

$\Longrightarrow \mu = \frac{\bar{u}}{dt} + \frac{\sigma^2}{2}$ (2.7)

(Ibrahim, Misiran, & Laham, 2021)

2.3.8. Log Normal Distribution

A continuous random variable S_t act in accordance with a lognormal distribution, if its natural logarithm, $\ln\big(S(t)\big)$ undergoes a normal distribution. Here $S(t)$ follows lognormal distribution, since $\mathrm{Y} = \ln\big(S(t)\big)$ follows the normal distribution. Let the mean and the standard deviation of $\mathrm{Y} = \ln\big(S(t)\big)$ data set is m & s

$$\ln\big(S(t)\big) \sim N(m, s^2) \quad \text{then} \quad S(t) \sim lognormal$$

The probability density function:

$$f(S(t), m, s) = \frac{1}{\sqrt{2\pi} \cdot s \cdot S(t)} e^{-\frac{[\ln(S(t)) - m]^2}{2s^2}} \tag{2.8}$$

Where mean $\quad \mu' = e^{\left(m + \frac{s^2}{2}\right)}$

variance $\quad \sigma'^2 = \left[e^{s^2} - 1\right] e^{(2m + s^2)}$

$$z = Inverse\ of\ standard\ normal\ distribution\ of\ f$$

(Pacati, 2010) and (Stojkoski, Sandev, Basnarkov, Kocarev, & Metzler, 2020)

2.3.9. The Hurst Component

According to Ampofi (2020), the Hurst component is introduced to generalize Geometric Brownian Motion. The algorithm of the calculation procedure is explained in the methodology chapter. After the calculation of the Hurst component, the GFBM can be presented as 2.3.10.

According to Largo (2017) the Hurst Exponent (denoted by H) is a measurement of self-similarity property and long-term nonlinearity. According to Brătian (2021), the persistent and anti-persistent behaviour of the time series of share prices is demonstrated by the Hurst component. This long memory phenomenon is described in mathematically by "Fractal Dimension (D)". The relationship between fractal dimension and Hurst component can be given as,

$$D = 2 - H \tag{2.9}$$

If there is an Efficient Market, the Hurst component should be between 0 and 1. But the real market is varied from the efficient market hypothesis (EMH); the value of the Hurst component can deviate as its normal range $0 \leq H \leq 1$. (Bhatt, Shah, & Dedania, 2005)

2.3.10. Geometric Fractional Brownian Motion Model

The GBM can be generalized by the following formula. It's called as GFBM.

$$S_H(t + \Delta t) = S(t) \cdot exp\left[\left(\hat{\mu}\Delta t - \frac{1}{2}\hat{\sigma}^2(\Delta t)^{2H} + \hat{\sigma}z\sqrt{(\Delta t)^{2H}}\right)\right] \quad (2.10)$$

Where

H - Hurst component

$\hat{\mu}$ - Annual Drift with Hurst component

$\hat{\sigma}$ - Annual Volatility with Hurst Component

Δt - Time difference between two consecutive trading periods (years)

$S(t)$ - Share price in the previous time period

(Brătian, Acu, Stan , Dinga, & Ionescu, 2021)

2.3.11. Drift & Volatility with Hurst Component

The volatility and drift for GFBM can be given as follows. Here v and $\bar{u}$ is the same as 2.3.7.

Volatility with Hurst component : $\hat{\sigma} = \frac{v}{\sqrt{(\Delta t)^{2H}}}$ (2.11)

Drift with Hurst component : $\hat{\mu} = \frac{\bar{u}}{\Delta t} + \frac{\hat{\sigma}^2}{2}$ (2.12)

(Ibrahim, Misiran, & Laham, 2021)

2.4. Empirical Literature

Ibrahim et al (2020) investigated the accuracy of GBM and GFBM in modelling Malaysia’s crude palm oil price simulation. In their research, persistent and anti-persistent behaviour across different periods was displayed. The Hurst component was calculated in three different methods such as absolute moment, aggregate variance and Higuchi method.

According to Areerak (2014) the Brownian Motion model with adaptive parameters (BMAP) and the Fractional Brownian Motion model with adaptive parameters (FBMAP) were tested to forecast future share prices in two companies of the Thai stock market. The ADVANC Info Service Public Company Limited (ADVANC) and Land and Houses Public Company Limited (LH) closed prices are concerned in the paper. He concluded that FBMAP was the most suitable model for forecasting the share prices of his sample companies. As a new point, the parameters were adapted due to avoid the effect of closed days of the share market.

The review looks at the way of behaving of the South African monetary business sectors with respect to the GBM process. It utilizes the daily, weekly, and monthly stock returns time series of a few significant protections exchanging the South African monetary market, all the more explicitly the US dollar/Euro, JSE ALSI Total Returns Index, South African All Bond Index, Anglo American Corporation, Standard Bank, Sasol, US dollar Gold Price, Brent spot oil cost, and South African white maize not so distant future. It likewise uncovers the presence of both long memory, and random walk or GBM in the South African monetary business sectors returns when the Hurst component estimation is utilized and observes that the Currency market is the most proficient of the South African monetary business sectors. That's what the review infers albeit a few presumptions hidden the cycle is abused, the Brownian motion as a model in South African monetary business sectors can't be dismissed. It very well may be acknowledged in certain occasions if a few boundaries, for example, the Hurst component, are added. (Karangwa, 2008)

According to Agustini (2018), the GBM model is the highest accurate prediction technique for forecasting share prices with the confidence level of 95%. It was proved with a MAPE value of less than 20%. In this study, the share prices are obtained from Indonesian stock exchange under the Jakarta Composite Index, which are: Charoen

Pokphand Indonesia Tbk, Harum Energy Tbk, Media Nusantara Citra Tbk, PP London Sumatra Indonesia Tbk, Vale Indonesia Tbk, Indo Tambangraya Megah Tbk, Indocement Tunggal Prakasa Tbk.

It is realized that the market records of Saudi Arabia, which is named the Tadawul All Share Index (TASI) mirror the exhibition of economic development and monetary solidness of Saudi Arabia. Consequently, determining the performance is very significant. In this experimental review, we determined the daily index prices of TASI for the year 2018. To act this, we relied upon two models, including GBM and GFBM. Further, the estimation of each model was acquired dependent on three unique approaches of calculating volatility such as simple volatility, log volatility and stochastic volatility. In the interim, the assessment of the presentation of each model was determined by utilizing mean absolute percentage error (MAPE). The outcomes uncovered that all models have high precision with paltry distinction. This demonstrates that everything models can be accustomed to anticipating the exhibition of TASI. (Alhagyan & Alduais, 2020)

Rahul (2020) used GBM to forecast share prices in the Bombay Stock Exchange (BSE). He proved it with a MAPE value of less than 10% i.e. 5.41%. Before forecasting the stock price using the GBM model, Kolmogorov-Smirnov test and Q – Q plot technique were run on the sample data for verifying that the data are normally distributed and practicable to forecast. (Si & Bishi, 2020)

Until the 1970's the most of the Muslim people are not associated with the share market venture due to the Islamic preclusions of specific exercises. During the 1990s, a significant advancement happened in a portion of the arrangements of regulation connecting with religious equity ventures, and from that point forward, the Islamic equity funds began working. The securities exchange is where partakes in organizations are traded under strict standards, guidelines and rules. The Malaysian stock exchange is called Bursa Malaysia. Syria-based equity is essentially portions of the organization meeting the standards of Islamic law. Roughly 88% of the ongoing effects of posting on BM are Syria agreeable and addresses two-third of the market capitalization of Malaysia. The securities exchange accepted to follow Brownian motion, where the assets are ceaselessly assigned in tiny timespans and position, a value in the assets of the country are being varied by irregular number. The paper conjectures the Syria counter in four area which is development, money, agriculture and trading services. The technique involved was Geometric Brownian movement as far as instability models. This strategy can be applied by the analyst to gauge the stock prices. (Omar & Jaffar, 2011)

This empirical literature was helpful in this research study to develop the theoretical background. Moreover the structure of this study is sketched by using the researches which was conducted in many different countries not only for stock market but for commodity market.

Chapter 3

Methodology

3.1. Introduction

According to Saunders, Lewis & Thornhil (2007) a research is designed in the form of the "research onion". The research onion consists of six layers: research philosophy, research approach, methodological choice, research strategy, time horizon and data collection and analysis. This study used Saunders's research onion model to explain the research methodology. The layer of the following research onion is peeled from the outside to the inner side.

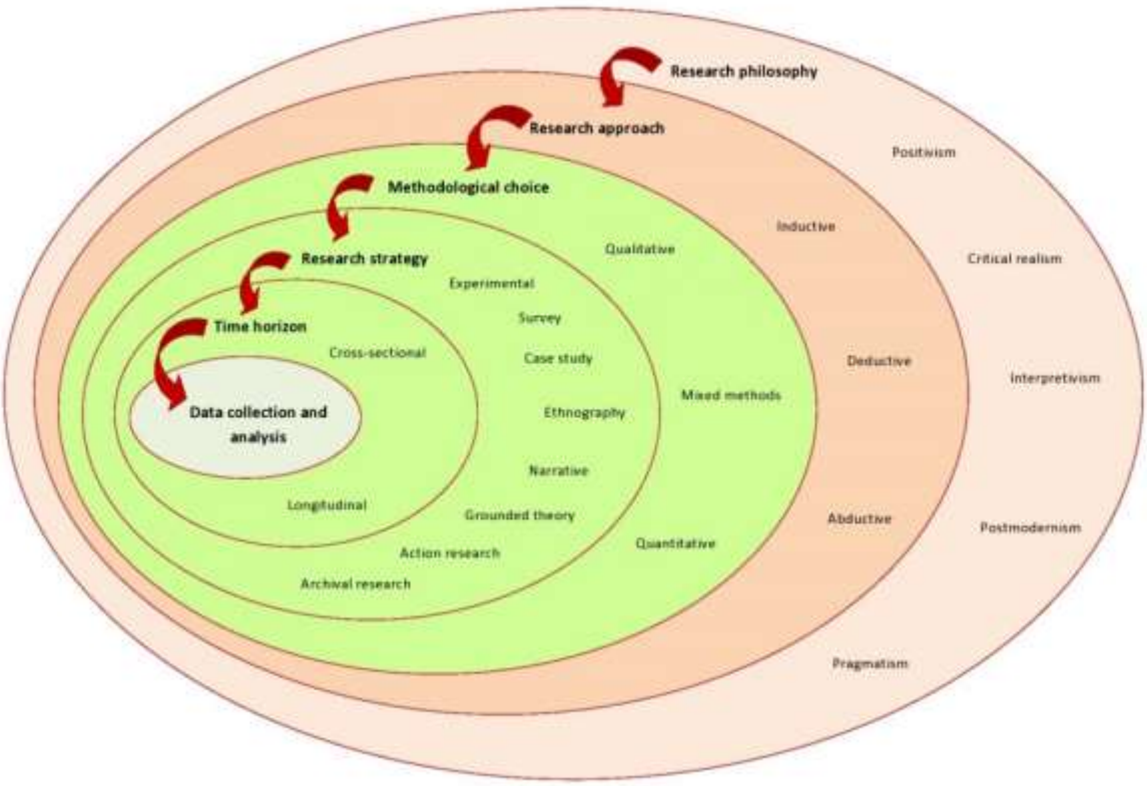

Figure 3.1 : Saunders's research onion model

3.2. Research Philosophy

According to Saunders research onion model, the research philosophy is the exterior ring of the research onion. It is mainly divided into three categories such as positivism, critical realism and interpretivism. The critical realism is concerned with the previous researches and the theories and concepts can be revised according to the view point of the researcher. Interpretivism is used in researches which the personal behavior is tested due to various changes in the society. In positivism, previous research findings, theories and concepts are used to develop new research objectives and research questions.

In this study, GBM and GFBM models are tested to find the most suitable model to forecast share prices of the telecommunication sector in Sri Lanka. Therefore, the philosophy of this research is positivism.

3.3. Research Approach

The research approach is the next outward layer. It includes 3 approaches: inductive, deductive and abduction. In the inductive approach, the researcher builds a new theory/concept/finding on the relevant subject without getting any information from previous researches. If the researcher refers previous information and adopts it to some new finding then it is known as deductive approach. The abduction approach includes some existing incomplete theory/ or concept and the researcher can generate / modify the existing one.

In this study, GBM and GFBM models are analyzed and the most suitable model with minimum error is tested. Since the previously published model is used here, this study has the deductive approach.

3.4. Methodological Choice

There are three different types of methodological choices such as quantitative, qualitative and mixed methods. The data collection process and analysis is in the numerical format in quantitative choice. In qualitative choice, the research is based on people's thoughts, concepts or experiences. This study has become a quantitative choice since the data collection & analysis is based on the numerical method. The details of data collection are given under 3.7.

3.5. Research Strategy

The six strategies are introduced using the research onion model. They are survey, experimental, case study, action research, grounded theory and ethnography. In survey strategy, data is collected using sample of individuals. It can be done by interviewing, giving questionnaires or document survey. The set of data for this research is gathered by a document survey. Therefore, the survey strategy is followed in this study.

3.6. Time Horizon

The two varieties of time horizon are cross sectional and longitudinal. If the investigation process is concerned with a particular phenomenon at a specific time frame then it is known as cross sectional. The cross sectional time frame is named as “snap shot” time collection. If the researcher maintains a diary frame work instead of snapshots, it is referred to as longitudinal. In the longitudinal framework, the data collection is processed in different time frames. In this study, since the data collection is done in a single time frame it is a cross sectional study.

3.7. Data Collection & Analysis

3.7.1. Population

The population of the research is based on the Global Industrial Classification Standard (GICS). GICS was developed by an independent international financial data and investment services company called S&P Dow Jones indices. The GICS hierarchy consits 11 sectors, 24 industry groups, 69 industries and 158 sub industries. Therefore, GICS has become the universal classification for financial investment researches, industry analysis, portfolio management and asset allocation. According to the methodology of GICS, a single sector can be categorized as follows.

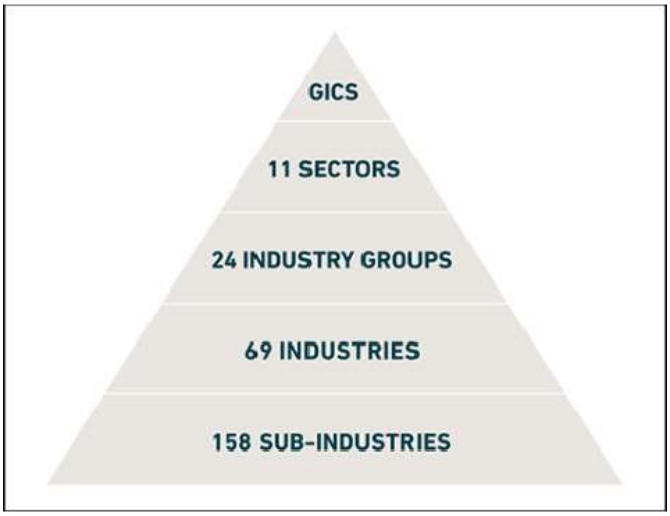

Figure 3.2 : The structure of GICS

There are two industry groups in the communication services sector and one of these industry groups is based on the population. The hierarchy of communication services sector can be figured as follows.

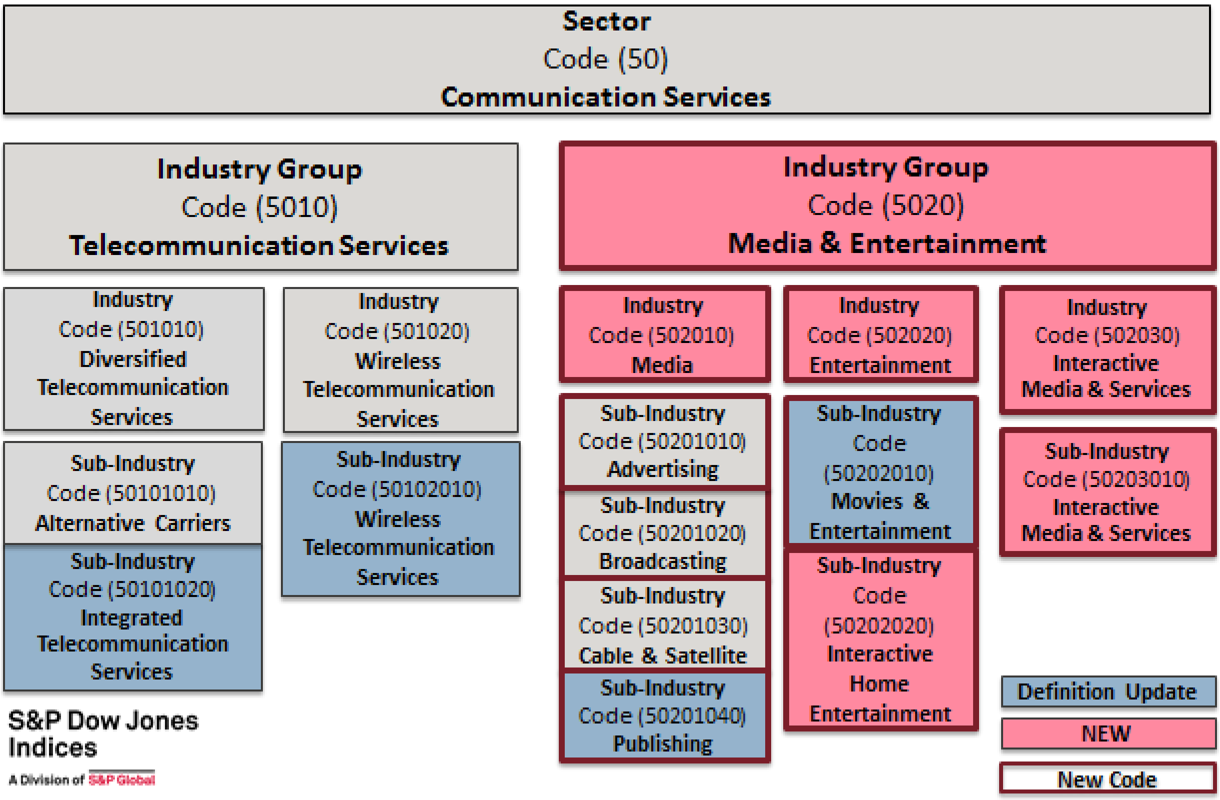

Figure 3.3 : The hierarchy of communication services

The industry group “Telecommunication Services” consists of two industries such as diversified telecommunication services and wireless telecommunication services. The industry of diversified telecommunication services has two sub industries called alternative careers and integrated telecommunication services. The 11 companies which have obtained the license of the Telecommunication Regulatory Commission of Sri Lanka (TRC) were selected as the population of the research. These 11 companies are as follows.

- Sri Lanka Telecom PLC
- Lanka Bell Ltd
- Dialog Broadband Networks (Pvt) Ltd
- Hutchinson Telecommunication Lanka (Pvt) Ltd
- Bharti Airtel Lanka (Pvt) Ltd
- Mobitel (Pvt) Ltd
- Dialog Axiata PLC
- TATA Communications Lanka Ltd
- Lanka Communication Services (Pvt) Ltd
- Societe International Telecommunications Aeronautiques (SITA)
- Lanka Education & Research Network (LEARN)

3.7.2. Sample

The sample of the research is selected by assuring 18.2% of the population. The following two companies which are registered in the Colombo Stock Exchange (CSE) are selected as the sample.

- Sri Lanka Telecom PLC
- Dialog Axiata PLC

Sri Lanka Telecom PLC is a leading information & communication technology solutions provider in Sri Lanka. SLT serves a consumer base of over nine million island wide. SLT has a market capitalization of 1.69% in CSE by issuing 1,804,860,000 shares.

Dialog Axiata PLC is a subsidiary of Axiata Group Berhad (Axiata) and a reputed mobile telecommunication service provider in Sri Lanka. Dialog serves through their high quality portable telecommunication and advanced mobile broadband services to a consumer base of over 17 Million Sri Lankan. Dialog has a market capitalization of 1.99% in CSE by issuing 117,399,866 shares.

3.7.3. Sampling Method

In statistics, there are four non- probability sampling methods such as convenience sampling, judgmental /purposive sampling, snowball sampling and quota sampling. The purposive sampling is a sampling method which is purely built according to the purpose of the research. The accuracy of a mathematical model for forecasting share prices is tested in this research. Therefore, public limited companies should be selected for the sample. The two companies which are registered in CSE is purposively selected among the population of 11 telecommunication services companies.

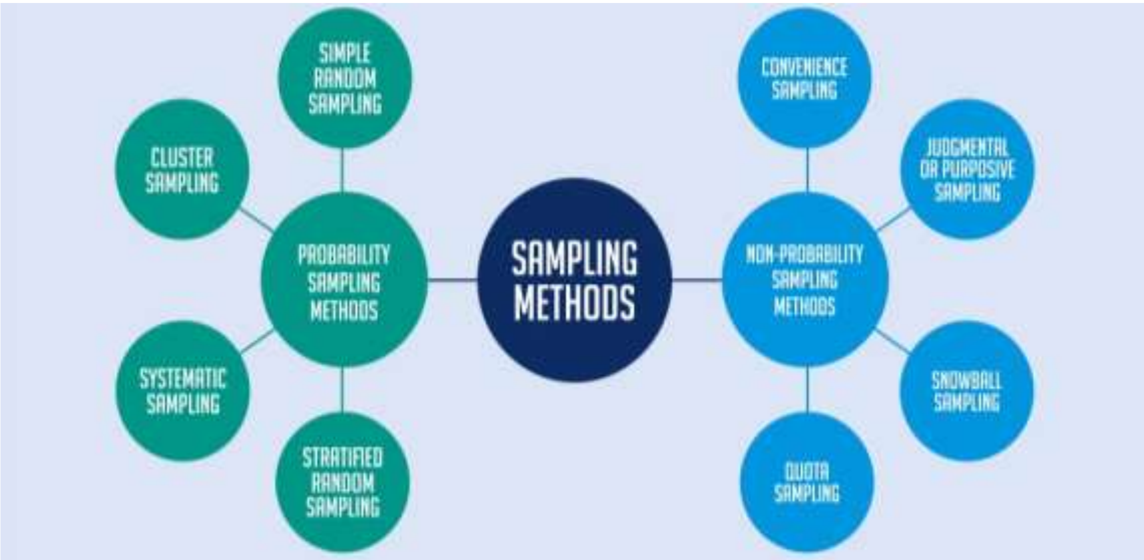

Figure 3.4 : Sampling methods

3.7.4. Data Collection Method

The two kinds of data are introduced in the research methodology. They are primary data and secondary data. The primary data is obtained directly by the data source and secondary data is the kind of data which has been gathered in the past by someone else and shared publicly. The share prices of sample companies are taken from www.investing.com website. Therefore, the data of the research is secondary.

The daily share prices of each company for the five years period (from 01st Jan 2016 to 31st Dec 2020) are collected. The actual data for SLT is from 05/01/2016 to 31/12/2020 and the actual data for Dialog is from 04/01/2016 to 31/12/2020.

	2016	2017	2018	2019	2020	Average
SLT	235	233	237	237	208	230
Dialog	239	241	240	241	209	234

Table 3.1 : No of share prices per year

3.7.5. Tools

The MATLAB software package is used to calculate Hurst component as explained in 3.8.3.1 and MINITAB is used for the descriptive analysis. The entire model is analyzed with Microsoft Excel.

3.8. Analytical Framework

The theoretical formulas which were used for the analysis are clearly presented under the following sub topics.

3.8.1. GBM Model

The final formula for GBM can be presented as follows. The $S(t + \Delta t)$ be the forecasted share price.

$$S(t + \Delta t) = S(t) \cdot exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)\ \Delta t + \sigma z\sqrt{\Delta t}\right] \qquad (3.1)$$

Where

μ - Annual Drift

σ - Annual Volatility

z - Inverse of the Standard Normal Probability

Δt - Time difference between two consecutive trading periods (years)

$S(t)$ - Share price in previous time period

In order to forecast share prices using GBM the parameters of annual volatility, annual drift and standard normal probability should be estimated.

3.8.1.1. Calculation of Annual Volatility

The volatility is a fixed parameter of a stock and expressed annually. The opinion of the stability of share prices is given by the volatility. The high volatility refers to the stock price which changes continuously within a relatively high interval. The most common and accepted method of measuring the stock volatility is the standard deviation of the price returns. The following algorithm is used to calculate the annual volatility.

Let S_i be the stock price at the end i^{th} merchandising period and u_i be the logarithm of the daily return as follows.

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right) \qquad \text{for } i = 1 \dots n \qquad (3.2)$$

The $\bar{u}$ be the mean and v be the standard deviation of u_i. The annual volatility (σ) can be obtained by using estimated $\bar{u}$ and v.

$$annual\ volatility\ (\sigma) = \frac{v}{\sqrt{\Delta t}} \qquad (3.3)$$

3.8.1.2.Calculation of Annual Drift

$$Annual\ Drift(\mu) = \frac{\bar{u}}{\Delta t} + \frac{\sigma^2}{2} \quad (3.4)$$

3.8.1.3.Calculation of Probability Measurement

A continuous random variable S_t act in accordance with a lognormal distribution, if its natural logarithm, $\ln\big(S(t)\big)$ undergoes a normal distribution. Here $S(t)$ follows lognormal distribution, since $\mathrm{Y} = \ln\big(S(t)\big)$ follows the normal distribution. Let the mean and the standard deviation of $\mathrm{Y} = \ln\big(S(t)\big)$ data set is m & s

$$\ln(S(t)) \sim N(m, s^2) \Longrightarrow S(t) \sim lognormal$$

Probability density function of log normal distribution,

$$f(S(t), m, s) = \frac{1}{\sqrt{2\pi} \cdot s \cdot S(t)}\ exp\left[-\frac{[\ln(S(t)) - m]^2}{2s^2}\right] \quad (3.5)$$

Where Mean $\mu' = exp\left[\left(m + \frac{s^2}{2}\right)\right]$

Variance $\sigma'^2 = \left[e^{s^2} - 1\right] exp[(2m + s^2)]$

The standard normal probability (z) for each share price at t can be obtained by the inverse of probability density function which is calculated in formula 3.5.

3.8.2. GFBM Model

The final formula for GFBM can be presented as follows. The $S_H(t + \Delta t)$ be the forecasted share price.

$$S_H(t + \Delta t) = S(t) \cdot exp\left[\left(\hat{\mu}\Delta t - \frac{1}{2}\hat{\sigma}^2(\Delta t)^{2H} + \hat{\sigma} z\sqrt{(\Delta t)^{2H}}\right)\right] \quad (3.6)$$

Where

H - Hurst component

$\hat{\mu}$ - Annual Drift with Hurst component

$\hat{\sigma}$ - Annual Volatility with Hurst Component

Δt - Time difference between two consecutive trading periods (years)

$S(t)$ - Share price during the previous time period

In order to forecast share prices using GFBM the parameters of $H, \hat{\mu}, \hat{\sigma}$ should be estimated. The calculation of z is similar as 3.8.1.3

3.8.3.1. Calculation of Hurst Component

There are nine methods to estimate the Hurst component. The rescaled range analysis is the most used and highest accurate method to estimate the Hurst component. In this study, the rescaled range analysis is applied to find the Hurst component and the algorithm is given here.

1. •Bisect the time series $X = \{x_1, x_2, \cdots, x_n\}$ of length n into d subseries of length l where l is an integral divisor of n. For each subseries $p = 1,2, \dots, d$
2. •The logarithmic values of each share prices $\ln(x_i)$ are obtained.
3. •The mean value (E_p) and standard deviation (S_p) are calculated for each subseries.
4. •The set of data is normalized by subtracting the mean of each subseries as follows.
   •$A_{i,p} = x_{i,p} - E_p \; for \; i = 1,2, \dots, l$
5. •The cumulative series is calculated for each subseries.
   •$B_{i,p} = \sum_{j=1}^{i} A_{j,p} \;\; for \; i = 1,2, \dots, l$
6. •Tha range of each subseries is calculated as follows.
   •$R_p = max\{B_{1,p}, \dots, B_{l,p}\} - min\{B_{1,p}, \dots, B_{l,p}\}$
7. •The ranges of each subseries (R_p) are rescaled by dividing it by the standard deviation of each subseries (S_p).
8. •The mean value of the rescaled ranges of all subseries $\{(R/S)_l\}$ is calculated.

The above algorithm is processed by the MATLAB program which is attached in Annex 1. The $(R/S)_l$ components for relevant divisions of time series can be obtained by the program.

The $(R/S)_l$ statistic follows the relation, $(R/S)_l = c * l^H$. Thus, the value of H can be generated by using a simple linear regression. The variable $\log(l)$ as the independent and the variable $\log((R/S)_l)$ is the dependent. The slope of the resulting equation in 3.7 is the value of the Hurst component.

$$(R/S)_l = c * l^H$$

$$\ln[(R/S)_l] = \ln[c * l^H]$$

$$\ln[(R/S)_l] = \ln[c] + H * \ln[l]$$

$$\ln[(R/S)_l] = H * \ln[l] + \ln[c] \tag{3.7}$$

3.8.3.2.Calculation of Volatility with the Hurst Component

The volatility formula is slightly varied due to the Hurst component as follows. Here v and t are same values as obtained in 3.8.1.1.

$$\hat{\sigma} = \frac{v}{\sqrt{(\Delta t)^{2H}}} \tag{3.8}$$

3.8.3.3.Calculation of Drift with the Hurst Component

The drift with Hurst component can be obtained by substituting values for the formula 3.9

$$\hat{\mu} = \frac{\bar{u}}{\Delta t} + \frac{\hat{\sigma}^2}{2} \tag{3.9}$$

3.8.3. Calculation of MAPE

This is the most important calculation for the final result. The minimum MAPE values are obtained to choose the most accurate model. The MAPE is calculated by using the following formula.

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{(A-F)}{A} \quad (3.10)$$

Where A - Actual share price

F - Forecasted share price

n - No of data points

The interpretation of MAPE is shown in table 3.2.

MAPE Value	**Interpretation**
$< 10\%$	A highly accurate forecasting
$10 - 20\%$	A good forecasting
$20 - 50\%$	A reasonable forecasting
$> 50\%$	An inaccurate forecasting

Table 3.2 : Interpretation of MAPE

(Brătian, Acu, Stan , Dinga, & Ionescu, 2021)

CHAPTER 4

Analysis

4.1. Descriptive Statistical Analysis

The abstract of descriptive analysis is presented in table 4.1. The number of data points of SLT and Dialog is 1150 and 1170 respectively. The mean share price and standard deviation of SLT are 30.232 and 5.65 respectively. The mean share price and standard deviation of Dialog are 11.514 and 1.375 respectively.

Statistic	**SLT**	**Dialog**
No of variables	1150	1170
Mean	30.232	11.514
Standard Deviation	5.650	1.375
Variance	31.927	1.892
Minimum Value	19.000	8.000
First Quartile	26.100	10.600
Median	30.000	11.400
Third Quartile	34.125	12.400
Maximum Value	46.400	14.800
Mode	30	11.4
Frequency for mode	39	59
Skewness	0.14	0.29
Kurtosis	-0.64	-0.40

Table 4.1 : The summary of descriptive analysis

4.1.1. Graphical representation of actual data

The time series plot of SLT and Dialog are presented in figure 4.1 and figure 4.2 respectively.

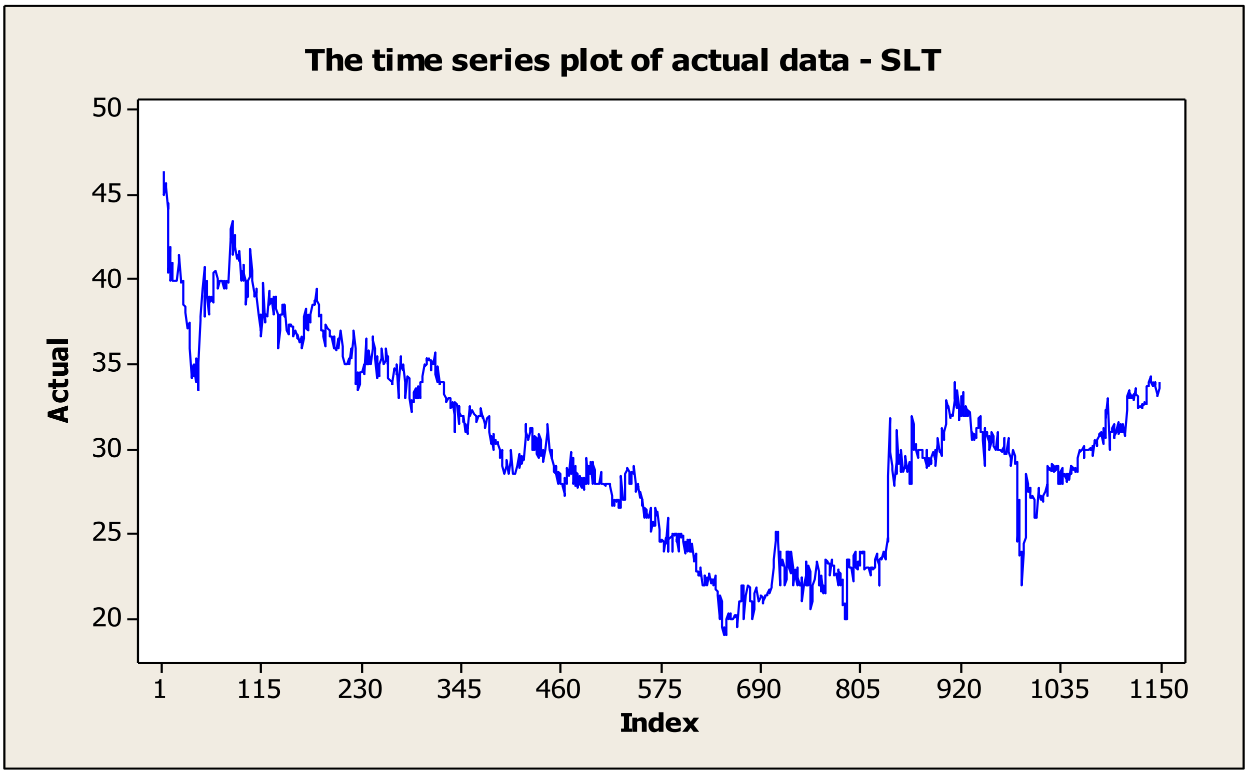

Figure 4.1 : The time series plot of actual data – SLT

A negative trend can be continuously observed from 115th to 630th. A high variance can be seen in whole 5 years.

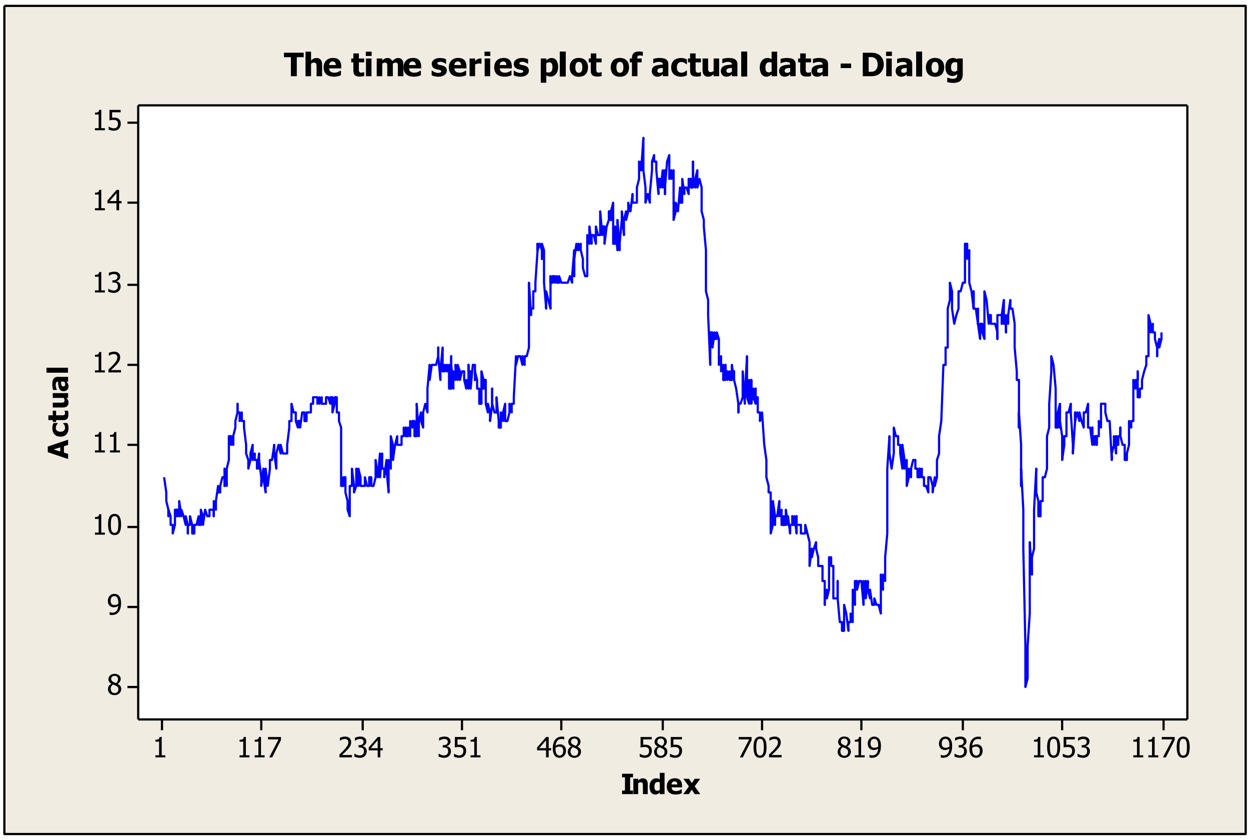

Figure 4.2 : The time series plot of actual data – Dialog

A positive trend can be observed in the initial duration up to around 600^{th} and after that there is a negative trend up to 820^{th} data point.

4.2. GBM & GFBM Models

The concept of GBM and GFBM is explained in 3.8.2 and 3.8.3 respectively. Only the formulas are presented herein.

$$S(t+\Delta t) = S(t) \cdot exp\left[\left(\mu - \tfrac{1}{2}\sigma^2\right)\Delta t + \sigma z\sqrt{\Delta t}\right]$$

$$S_H(t+\Delta t) = S(t) \cdot exp\left[\left(\hat{\mu}\Delta t - \tfrac{1}{2}\hat{\sigma}^2(\Delta t)^{2H} + \hat{\sigma} z\sqrt{(\Delta t)^{2H}}\right)\right]$$

The estimated parameters for the above two models are given in table 4.2. The theoretical explanations for $\bar{u}, v, \sigma, \mu, H, \hat{\sigma}$ and $\hat{\mu}$ are given under 3.8 and the formulas are presented herein.

Parameter	Formula
u_i	$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$ for $i = 1 \ldots n$
$\bar{u}$	$\bar{u} = \frac{1}{n}\sum_{i=1}^{n} u_i$
v	$v = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2}$
σ	$\sigma = \frac{v}{\sqrt{\Delta t}}$
μ	$\mu = \frac{\bar{u}}{\Delta t} + \frac{\sigma^2}{2}$
$\hat{\sigma}$	$\hat{\sigma} = \frac{v}{\sqrt{(\Delta t)^{2H}}}$
$\hat{\mu}$	$\hat{\mu} = \frac{\bar{u}}{\Delta t} + \frac{\hat{\sigma}^2}{2}$

Table 4.2 : Table of formulas

The estimation of Δt is taken by assuming the average number of days per year for SLT and Dialog as 230 and 234 respectively. Since the estimation of z is varied due to the share prices it has a series of results which are attached in annex 2.

Parameter	SLT	Dialog
$\bar{u}$	-0.000273	0.000134
v	0.024671	0.014378
σ	0.374	0.2199
μ	0.007148	0.0555
Δt	$\frac{1}{230}$	$\frac{1}{234}$
H	1.034817	1.003783
$\hat{\sigma}$	6.857155	3.4250
$\hat{\mu}$	23.4475	5.8967

Table 4.3 : Parameter estimation for GBM & GFBM

4.2.1. Graphical representation of forecasted data

The time series plots of actual and forecasted share prices using GBM and GFBM of SLT are demonstrated in 4.3 and 4.4 respectively. Furthermore the time series plots of actual and forecasted share prices using GBM and GFBM of Dialog are demonstrated in 4.5 and 4.6 respectively.

For further clarification, the last five forecasted data by using GBM and GFBM with actual data of SLT and Dialog is presented in table 4.4 and table 4.5 respectively.

Actual	33.3	33.1	33.6	33.6	33.9
GBM	32.4	32.0	31.8	32.3	32.3
GFBM	35.8	35.4	35.2	35.7	35.7

Table 4.4 : Comparison of last data points of SLT

Actual	12.3	12.2	12.3	12.4	12.4
GBM	12.1	12.2	12.1	12.2	12.3
GFBM	12.4	12.5	12.4	12.5	12.6

Table 4.5 : Comparison of last data points of Dialog

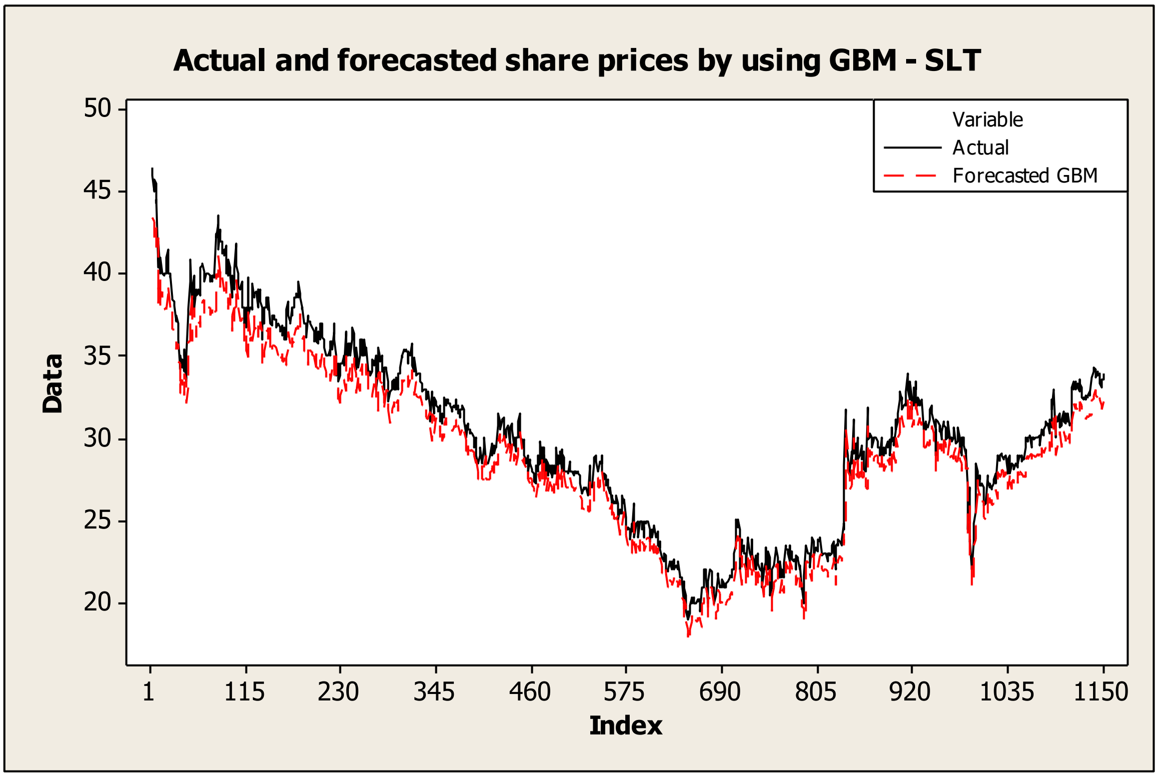

Figure 4.3 : Actual and forecasted share prices using GBM - SLT

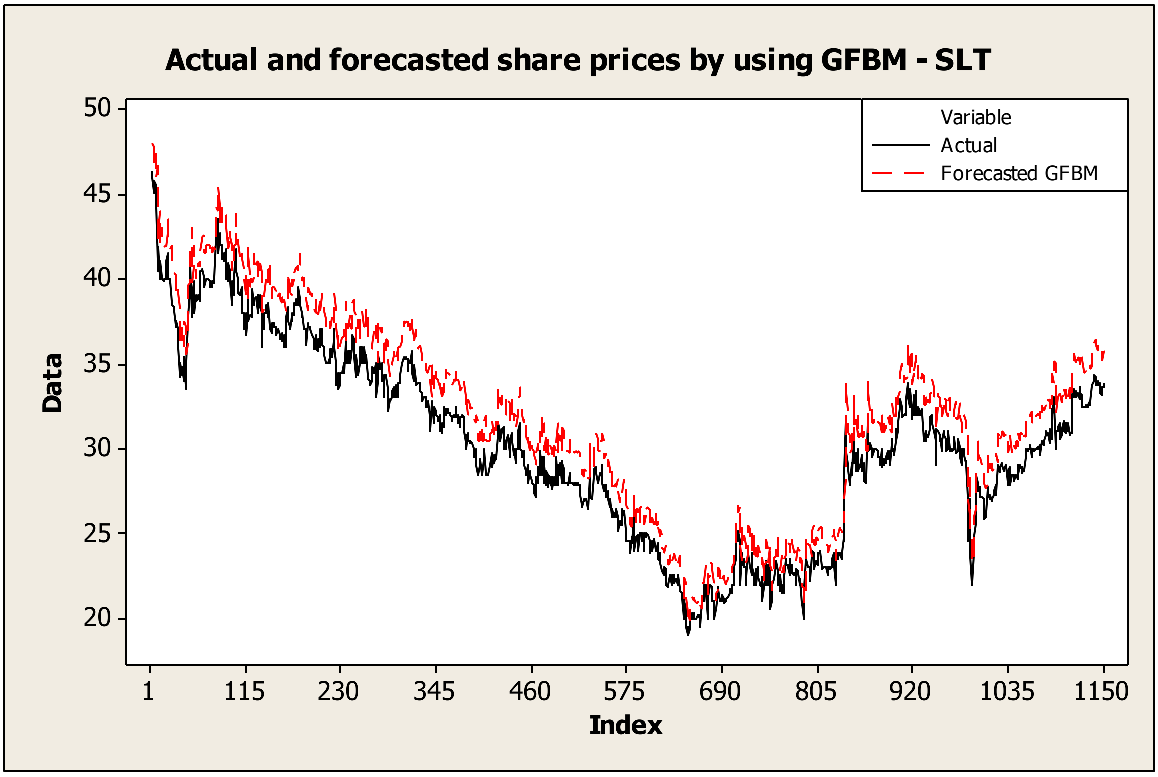

Figure 4.4 : Actual and forecasted share prices using GFBM - SLT

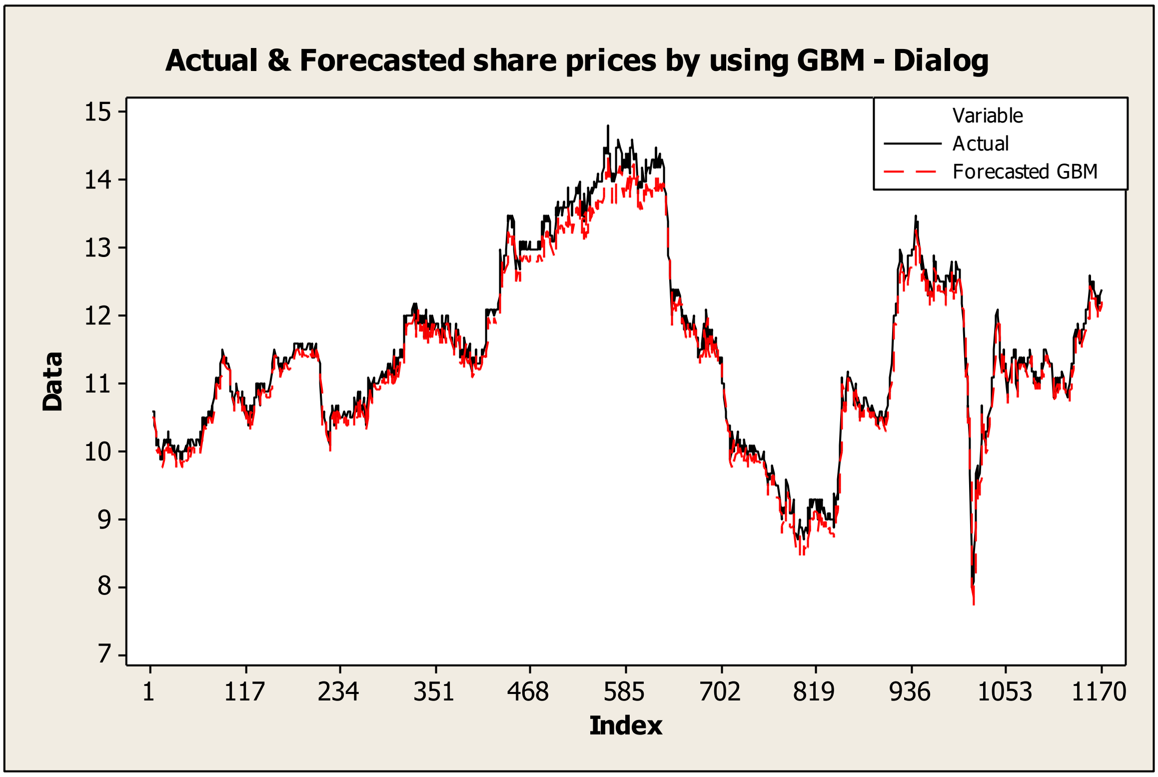

Figure 4.5 : Actual and forecasted share prices using GBM – Dialog

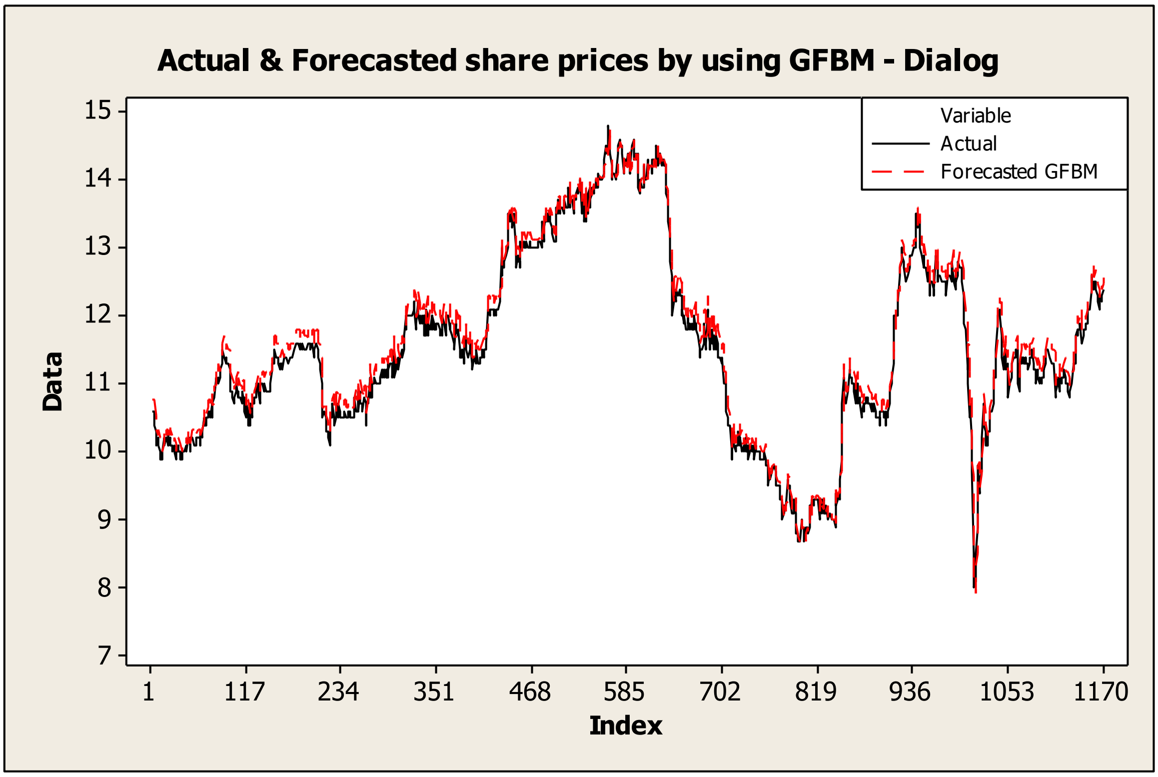

Figure 4.6 : Actual and forecasted share prices using GFBM – Dialog

Chapter 5

Discussion

5.1. Discussion

In this study, two mathematical models are tested to forecast share prices. The daily share prices for 5 years of two companies in Telecommunication sector were analyzed. In this chapter, the summary of the research is presented.

As the first sample company, 1150 daily share prices for 5 years are obtained from Sri Lanka Telecom PLC. SLT has 230 trading days per year. The mean share price is Rs.30.232 and the variance is 31.927. The share prices are varied from Rs.19 to Rs.46.4 by representing minimum and maximum value respectively within 5 year time period.

As the second sample company, 1170 daily share prices for 5 years are obtained from Dialog Axiata PLC. Dialog has 234 trading days per year. The mean share price and variance become Rs.11.514 and 1.892 respectively. The minimum and maximum share prices are Rs.8 and Rs.14.8.

By comparing these descriptive statistics of both companies, the mean value of single share of SLT is greater than the Dialog. As well, high liquidity of SLT can be noticed more than Dialog. The both companies were affected by Covid 19 pandemic and it leads for reduction of average trading days per year. Since the CSE was closed at the beginning of 2020, there were only 208 and 209 trading days in 2020 for SLT and Dialog respectively.

As the ultimate result of this study, the share prices of each company are estimated using GBM and GFBM models. In order to estimate share prices by using GBM, parameters of $S(t), \mu, \sigma, \Delta t \; and \; z$ should be calculated for each company. The parameters of $\mu, \sigma \; and \; \Delta t$ are independent of time and have single constant value for whole time series. The parameters of $S(t) \; and \; z$ are dependent variables of time and have series of values. After applying time independent variables to GBM, the final formula for SLT and Dialog are presented in 5.1 and 5.2 respectively.

$$S(t + \Delta t) = S(t) \cdot exp[-0.000273 + 0.0247z] \quad (5.1)$$

$$S(t + \Delta t) = S(t) \cdot exp[0.000134 + 0.0144z] \quad (5.2)$$

In order to estimate share prices by using GFBM, parameters of $S(t), \hat{\mu}, \hat{\sigma}, \Delta t \ and \ z$ should be calculated. The parameters of $\hat{\mu}, \hat{\sigma} \ and \ \Delta t$ are independent of time and have single constant value for whole time series. The parameters of $S(t) \ and \ z$ are dependent variables of time and have series of values. After applying time independent variables to GFBM, the final formula for SLT and Dialog are presented in 5.3 and 5.4 respectively.

$$S_H(t + \Delta t) = S(t) \cdot exp[0.1016 + 0.0247z\,] \quad (5.3)$$

$$S_H(t + \Delta t) = S(t) \cdot exp[0.0251 + 0.0144z\,] \quad (5.4)$$

The estimated share prices were obtained for both companies by using GBM and GFBM. The mean absolute percentage error can be presented in table 5.1.

Company	GBM	GFBM
SLT	4.2%	6.3%
Dialog	1.5%	1.6%

Table 5.1 : MAPE values

Since the minimum MAPE values GBM is the most accurate model for forecasting share prices of both companies SLT and Dialog.

5.2. Conclusion

The GBM is the most accurate model between GBM and GFBM for forecasting share prices of the telecommunication sector in Sri Lanka.

5.3. Recommendations

The four stake holders can be introduced through this study. They are:

- Two sample companies
- The other telecommunication companies in the population
- The current shareholders of sample companies
- The people who wish to invest in telecommunication sector

The recommendation which can be given to all above stake holders is to forecast share prices by using GBM model and make a rational decision making.

5.4. Recommendations for Further Researches

The GBM and GFBM models are tested to forecast share prices of telecommunication sector companies in Sri Lanka. The same models can be used to forecast share prices of other sectors of GICS classification in CSE.

In this research, the rescaled range analysis was used to estimate Hurst component. There are more than 8 methods to calculate Hurst component such as aggregate variance method, Higuchi method, detrended fluctuation analysis etc. This research can be followed by estimating Hurst component by using other methods.

The concept of GBM and GFBM can be applied for forecasting prices of commodity market. Although it is not practicable for Sri Lanka but it can be implemented for commodity market in other countries.

The research can be enhanced to test these GBM and GFBM models when the market is affected by external matters such as Covid 19 pandemic, economic crisis and terrorist attacks.

The time series forecasting techniques such as Auto Regressive Integrated Moving Average (ARIMA) models can be used for forecasting share prices as well as this Geometrical Brownian motion concept.

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Appendix A

```
% MATLAB code by Kalpana Athukorala
% M.Sc. in Financial Mathematics (UOM)
%Rescaled Range Analysis to find Hurst component

arr = [2,5,10,23,25,46,50,115,230,575,1150];     % Length of a
sub series

for i =1:numel(arr)
    N = arr(i);
    l = 0;
    f_answer=0;
Size_DataSet = numel(A);
    while N <= Size_DataSet
          A(1+l:N);
          Mu=mean(A(1+l:N));
          S=std(A(1+l:N));
          K=A(1+l:N)-Mu;
          sum = zeros( 1, numel(K));

for j = 2 :numel(K)
    sum(1) = K(1);
    sum(j) = sum(j-1)+ K(j);

end
    R=max(sum)-min(sum);
    RS=R/S;

    f_answer=f_answer+RS;

    l = l+ arr(i);
    N = N+ arr(i);

    end
    b=Size_DataSet/arr(i);
    RS_avg=f_answer/b


end
```