

Develop A Mathematical Model to Measure Impact Force on Composite Woven Fabric

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I. INTRODUCTION

The contemporary demand for advanced materials in industries such as aerospace, automotive, and sports necessitates a comprehensive understanding of how composite woven fabrics respond to impact forces. This report introduces a pioneering initiative to develop a mathematical model tailored for measuring impact forces on composite woven fabrics. This model considers factors such as projectile mass, impact velocity, thickness of the composite, Fabric strength, fabric weight, density of the fabric, yield strength, cross section area of the bullet and energy absorbed by the composite.

II. LITERATURE REVIEW

Cavarallo's method measured impact force on fabric by considering factors like fabric boundaries, impact location, friction, and yarn strength [1]. Dokuz Eylul University studied woven fabric composite damage under impact. They used energy diagrams, load-deflection curves, and images to analyze the damage [2]. Naik et al. studied ballistic impact on woven fabric composites [3]. Abteu et al. found that 1/1 plain weave structures outperform others at lower velocities, while 2x2 basket weave structures are better at higher velocities [4].

Traditional impact energy measurement methods struggle with composite woven fabrics. Resin between fabric layers complicates the analysis of energy absorption. This limits the use of these methods for composite materials.

III. MATERIALS AND METHODS

A. Testing Method

Inserting the pin into the designated holes within the mechanism increases the stored elastic potential energy. This energy can subsequently be converted into the kinetic energy of the projectile upon release.



Fig. 1: Experimental projectile propulsion device

The projectile is inserted in front of the instrument. The instrument is rigidly fixed. By dragging the spring to the positions various speeds can be given to the projectile. By using lathe machine, the projectile was made with standard measurements.



Fig. 2: projectile

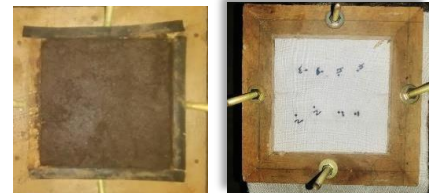


Fig. 3: Clay box

A box was built and filled with clay to collect the projectile and measure the depth the projectile goes through. This experiment utilizes a modeling clay formulation designed to mimic human tissue characteristics, enhancing the realism of the ballistic impact simulation. A meticulous calibration process ensures the clay's suitability for testing.

B. Creation of composite woven fabric

This fabric consists of Dyneema fabric layers and Kevlar fabric layers with a polyester fusing matrix sandwiched between them. For this process, use the flat press machine at 137°C for 15 seconds under 3.5 bar pressure.



Fig. 4: Kevlar

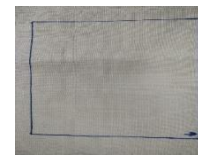


Fig.5: Dyneema



Fig. 6: Double side fusing material

C. Mathematical model development

Models focus on influence of projectile weight, velocity of projectile, contact pressure, bursting strength of the fabric, contact surface area of projectile, weight of the fabric, number

of layers and temperature on impact forces. Study emphasizes effects of velocity and temperature, for varying multiple thickness.

Assumptions for the model development:

1. Projectile moves linearly in a vector direction.
2. Projectile twist is negligible (zero twist).
3. Spring force becomes zero upon release from pin.

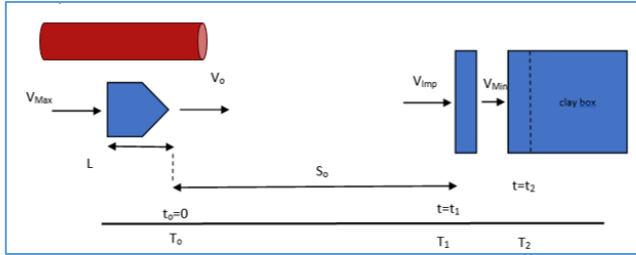


Fig. 7: Movement of projectile from the experimental propulsion device

V_{max} - Initial velocity of the projectile

V_0 - Velocity of the projectile when it leaves the pipe

V_{imp} - Impact Velocity of the projectile

S_0 - The gap between end of the gun tube and fabric sample

L - Length of the projectile

T_0 - Initial Temperature of the projectile

T_1 - the temperature of the projectile before hit the fabric

T_2 - post-impact projectile temperature

Three primary equations which were utilized in computations are given below (Department of Textile & Apparel Engineering, University of Moratuwa, Sri Lanka et al., 2024).

$$V_{max} = \sqrt{\frac{2k}{m} \times x} \quad \text{---(1)} \quad V_0 = \sqrt{v_{max}^2 - 2F \frac{l_0}{m}} \quad \text{---(2)}$$

$$V_{imp} = V_0 \times e^{-\frac{\rho A C d S}{2m}} \quad \text{---(3)}$$

Considering temperature changes the following equations were derived according to energy conservation law.

When the projectile leaves the propulsion tube,

$$\frac{m}{2} (v_{max}^2 - v_0^2) = (\Delta Q)_1 = ms(\Delta T)_1 \quad \text{---(4)}$$

When the projectile hit the fabric,

$$\frac{m}{2} (v_0^2 - v_{imp}^2) = \epsilon \sigma A (T_1^4 - T_{amb}^4) \quad \text{---(5)}$$

When the projectile penetrates the fabric,

$$\frac{m}{2} (v_{imp}^2 - v_{min}^2) = (\Delta Q)_3 \quad \text{---(6)}$$

The equation to measure the impact force on composite woven fabric.

Impact Force x thickness(d) = Absorbed Energy from fabric

$$I = \frac{(KE_P - H_P) - (E_{BM} + H_{BM})}{d} \quad \text{---(7)}$$

I = Impact Force

KE_P = Projectile Kinetic Energy

H_P = Projectile Heat

E_{BM} = Absorb Energy of Backing Materials

H_{BM} = Heat of Backing Material

d – penetration depth or movement distance of fabric when hit the projectile.

$$\text{Penetration Depth } (d) = \frac{\Delta \frac{1}{2} m v_{imp}^2 - k w}{\sum_{i=1}^n P_i A_{eff}} \quad \text{---(8)}$$

$$p_i = \frac{1}{2} \rho_t u_i^2 + \gamma_t \quad \text{---(9)} \quad w = A t \rho g \quad \text{---(10)}$$

t = Thickness of fabric, (m)

m = mass of the projectile, (kg)

ρ_t = density of the fabric, (kg/m³)

i = number of fabric layers

γ_t = yield strength of fabric, (Pa)

ρ = Volumetric density

u_i = penetration speed, (m/s)

A_{eff} = Contacted surface area, (m²)

g = Acceleration due to gravity, (m/s²)

v_{imp} = initial speed of the projectile, (m/s)

k = strength of fabric, (m) (find via experiment)

A = Cross sectional area of the projectile, (m²)

p_i = contact pressure between projectile & fabric, (Pa)

w = weight of fabric material acting top of the projectile, (N)

$$\begin{aligned} f &= ma \\ k A p g &= m f \\ f &= \frac{k A p g}{m} \end{aligned} \quad v_{out} = \sqrt{v_{in}^2 - \frac{2k A p g t}{m}} \quad \text{---(11)}$$

$$u_i = \frac{v_{in} + \sqrt{v_{in}^2 - \frac{2k A p g t}{m}}}{2} \quad \text{---(12)}$$

v_{out} = After penetrating velocity of i^{th} layer, (m/s)

u_{in} = Before penetrating velocity of i^{th} layer, (m/s)

$k \propto$ bursting strength dependent value

$$k = k' S_b \quad \text{---(13)}$$

k' = constant, (m²s²/kg)

S_b = bursting strength, (Pa)

$$I = \frac{(KE_P - H_P) - (E_{BM} + H_{BM})}{\sum_{i=1}^n \left(\frac{1}{2} \rho_t \frac{v_{in} + \sqrt{v_{in}^2 - \frac{2S_b k' A p g t}{m}}}{2} + \gamma_t \right) A_{eff}} \quad \text{---(14)}$$

IV. RESULTS & DISCUSSION

Summary of details of velocity at different points, kinetic energy of projection and indentation depth for different extensions of springs are shown below.

Table 1

Hole No	Extension of springs (cm)	$V_{max}(ms^{-1})$	$V_0(ms^{-1})$	$V_{imp}(ms^{-1})$	$E_{imp}(J)$	Indentation depth		
						2D1K (mm)	3D2K (mm)	4D3K (mm)
1	18.2	68.78	68.78	68.677	16.504	3.2	3	2.5
2	23.8	89.95	89.95	89.77	28.205	4.35	4	3.5
3	29.4	111.12	111.12	110.89	43.03	6	5	4
4	35	132.28	132.28	132.015	60.99	8	6	4.9
5	40.6	153.45	153.45	153.143	82.084	10	7	5.5

2D1K – 2 dyneema fabric layer with 1 kevlar fabric layer composite
 3D2K – 3 dyneema fabric layer with 2 kevlar fabric layer composite
 4D3K – 4 dyneema fabric layer with 3 kevlar fabric layer composite

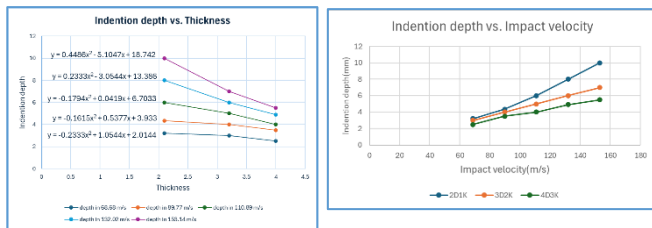


Fig. 8: Indentation depth vs thickness

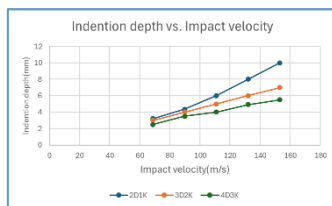


Fig. 9: Indentation depth vs impact velocity

Indentation depth vs. Impact energy which was absorbed by the backing material was plotted. A linear relationship could be noted in the graph.

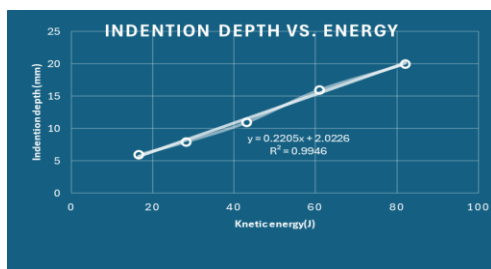


Fig. 10: Indentation depth vs Energy

Impact force for different elongations of springs of 2D1K, 3D2K and 4D3K

Table 2
2D1K

Hole Number	Average indentation depth value (with fabric sample (mm))	Energy absorbed by backing material (J)
1	3.2	5.326
2	4.35	10.5355
3	6	18.01
4	8	27.07
5	10	36.13

Table 4
4D3K

Hole Number	Average indentation depth value (with fabric sample (mm))	Energy absorbed by backing material (J)
1	2.5	2.155
2	3.5	6.685
3	4	8.95
4	4.9	13.027
5	5.5	15.745

Table 3
3D2K

Hole Number	Average indentation depth value (with fabric sample (mm))	Energy absorbed by backing material (J)
1	3	4.42
2	4	8.95
3	5	13.48
4	6	18.01
5	7	22.54

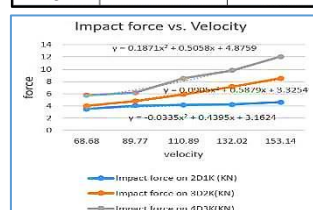


Fig. 11: Impact force vs Velocity

The graph effectively demonstrates the relationship between impact velocity and impact force for different composite woven fabrics. The clear differentiation in performance among the 2D1K, 3D2K, and 4D3K composites underscores the importance of material selection based on specific impact resistance requirements.

V. CONCLUSION

The mathematical model successfully incorporated various parameters, including fabric properties, impact conditions, and material characteristics. Model predictions for impact forces on composite woven fabrics were validated through testing. A direct relationship was observed between impact velocity and impact force on the fabric. Impact force increases with impact velocity until the fabric breaks. Beyond the breaking point, the projectile can penetrate the fabric. The energy absorbed by the fabric increases with the impact force. Maximum impact force corresponds to the fabric's breaking point. Identifying the breaking points is essential for assessing fabric strength and durability under various impact conditions. Other forms of energy losses (heat, noise, etc.) are minimal and do not significantly affect the overall energy balance. These losses are considered negligible and are not included in the calculations.

VI. LIMITATION & RECOMMENDATIONS

The limitations of the available apparatus prevented the direct measurement of bursting strength (S_b) for the composite woven fabrics studied. Consequently, the constant k' in our mathematical model could not be determined empirically. Future research should employ a more suitable instrument with a higher measurement capacity to enable the complete characterization of the model and the accurate determination of k' .

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