EMPIRICAL FORMULAS TO ESTIMATE THE ORDER OF 2-D FIR FAN FILTERS

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DECLARATION

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ABSTRACT

Two-dimensional (2-D) finite impulse response (FIR) fan filters belong to a special class of 2-D filters which has the capability of directional filtering. They are used in many applications such as geological and seismological data processing, and array signal processing. In this dissertation, three accurate formulas are proposed to estimate the order of 2-D FIR fan filters designed using the windowing technique in conjunction with the 2-D separable Kaiser window. Maximum passband ripple A_p, minimum stopband attenuation A_a, half of the fan angle θ, passband width *B*, transition width *T* and Kaiser window parameter β are used as the main parameters in the derivation of formulas.

Here, three estimation formulas are proposed for three different values of transition width, that is *T*=0.01π rad/sample, 0.05π rad/sample and 0.1π rad/sample by employing two steps. In the first step, a set of filters with different specifications are designed in order to experientially determine the Kaiser Window parameter β and the minimum order of the filter required to satisfy the given specifications. In the second step, the formulas for the Kaiser window parameter β and the minimum order of the filter are empirically derived through multiple linear regression using the data obtained in the first step.

In numerical evaluation, statistical means of the absolute error between the estimated and true values of the Kaiser window parameter β and the minimum filter order are calculated. It is found that mean of absolute error of estimated β and true β is less than one. Also, for minimum filter order it is slightly greater. Mean of absolute error of estimated order is varied from 2-14 of true filter order which are 13.85 for 0.01π rad/sample, 3.11 for 0.05π rad/sample and 2.33 for 0.1π rad/sample. The proposed formulas provide very good accuracy for widely employed 2-D fan filter specifications. By using these formulas, a 2-D fan filter can be designed without trial and error to determine the Kaiser window parameter β and the minimum filter order saving significant time in the design process.

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LIST OF ABBREVIATIONS

1-D	One-Dimensional
2-D	Two-Dimensional
FIR	Finite Impulse Response
IIR	Infinite Impulse Response

CHAPTER ONE

1 INTRODUCTION

In signal processing applications, it is often very important to perform tasks such as eliminating unwanted signals or noise, getting rid of certain frequencies, and transmitting others, shaping the signal spectrum, and so on. The technique used in these applications is usually called as filtering. Digital devices designed through different mathematical calculations combining with digital signal processing theories which are applicable in filtering applications is called digital filters. There are various types of digital filters, most commonly low pass, high pass, band pass, and band stop, which are employed for applications involving one-dimensional (1-D), two-dimensional (2-D) and higher-dimensional signals based on the requirements of a particular application such as 3-D velocity filtering [1] [2] [3], beamforming [4] [5], 4-D depth filtering [6] [7], 4-D light field refocusing [8] [9] [10] and 5-D depth-velocity filtering [11] [12].

The field of 2-D digital signal processing has been growing rapidly in the last three decades. Images such as satellite photographs, radar and sonar maps, medical X-ray pictures, radiographs, electron micrographs, and data from seismic, gravitational, and magnetic records are typical examples of 2-D signals that might need to be processed. [1]. 2-D digital filters are classified to recursive or non-recursive if the output of the filter depends on previous values of the output. Alternatively, these filters can be divided into infinite-impulse response (IIR) or finite-impulse response (FIR) filters. Impulse response of FIR or non- recursive filter possesses only a finite number of nonzero samples. Therefore, impulse response of these filters is absolutely summable and they are always stable [13, Ch. 3].

FIR filters are widely used in various applications of signal processing and communications compared to IIR filters [13]. Some of the important applications of the 2-D filters in image and seismic data processing are discussed below. 2-D filters for image enhancement and image restoration are most widely used. The process which an image is manipulated or transformed to extract useful information from it is called as image enhancement. Thus, it improves the visual effect of the image. Changing the form of the original image into some other form can be used for further analysis of the images. 2-D filters for image enhancement can be used in the applications where often required visions systems to determine the exact geometric shape of the object. For instance, most of robotics applications to handle the successful robot motion, drone applications and most automated industrial applications can be considered. In this case, two image enhancement techniques which are removal of noise from a digitized image by using low pass and enhancement of edges by using high pass filters based on the use of 2-D filters are used to strengthen the contour of the object. In most of the image processing applications, images are often corrupted during the formation of the images due to the nonlinearity of the recording medium (e.g., photographic film) and noise introduced during the transmission. The purpose of image restoration is to improve a recorded image according to some norm or criterion [1]. There are most used 2-D digital filters named as directional, fan-shaped also known as wedge-shaped filters, diamond-shaped and circular filters. Diamond filters are commonly used as anti-aliasing filters which is used in the signal sample conversion on the rectangular sampling grid and the quincunx sampling grid [1]. These filters are used to detect the perpendicular lines from an image.

1.1 Background

A 2-D spatio-temporal signal can be passed or rejected according to its direction. Direction is a special characteristic of a 2-D spatio-temporal signal and it cannot be defined for 1-D signal. It can pass seismic events whose apparent velocities on the earth's surface fall within a wedge-shaped region in the frequency wavenumber plane [14].

1.1.1 Fan filter

Ordinary fan-type filter, quadrant fan-type filter and general fan-type filters are the three major kinds of conventional fan type filters. Ordinary fan-type filter and quadrant fan-type filter also called as 90° fan filters. They are shown below in Figure 1.1 (a), (b) and (c) respectively.

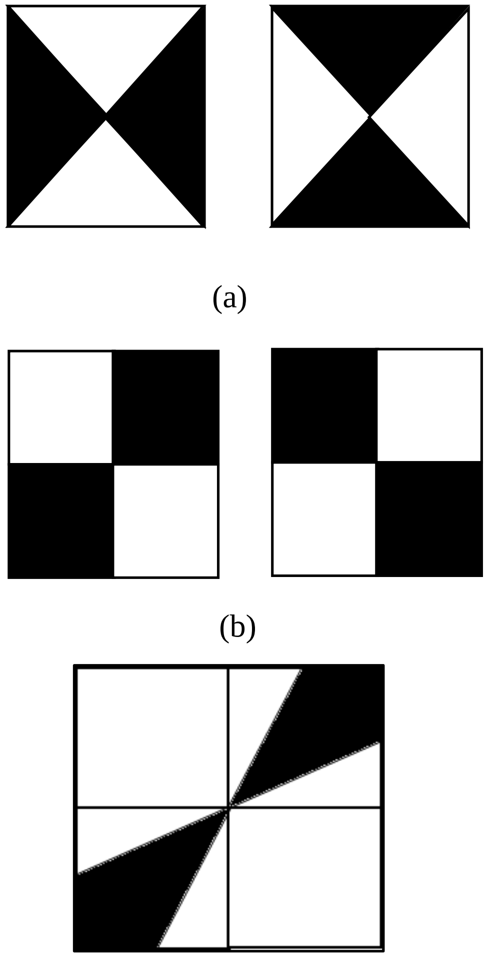


Figure 1.1: (a) Ordinary fan type filter (b) Quadrant fan type filter (c) General fan type filter

Fan-type filters have been used in many applications such as geological and seismological data processing. As an example, these filters are used to enhance the quality of seismic signals by eradicating signal components that are not associated with the subsurface ground formations [1]. Seismic signals are essentials for oil prospecting and some other geological applications such as lithological mapping, mineralogical identification, mineral and ground water exploration and engineering geological studies like volcano monitoring. In addition, these filters can detect the edge information in all the given directions very successfully. An application of linear 2-D fan filters to seismic signal processing is simply explained below [1, Ch. 13.4].

1.2 Motivation

Design and implementation methodologies of FIR fan filter are imperative when their applications in different areas are considered. Order estimation is very important as filters with shorter filter length have more advantages over longer filter length which are including fewer circuit components in hardware implementation and lower run rime in software implementation. Therefore, methods which estimate the minimum order of a fan filter that satisfies prescribed specifications leads to significant saving of design time compared to the usual trial-and-error method.

A filter design problem can be an optimization problem to find a filter which satisfies the given specifications with a minimum filter order. Many recent works and research proposed distinct methods to design different 2-D filters and only 2-D circular filter order estimation is discussed [1] [15]. Therefore, the order estimation methodology for FIR fan filter is required to discuss and come up with more accurate formula to get the filter order. Further, the windowing method for designing multidimensional FIR filter is philosophically identical to its 1-D counterpart. Most recent work has used rectangular and Hamming windowing techniques than Kaiser and other types of windows [16]. Therefore, this research study motivates to derive empirical formulas to estimate the order of FIR fan filter along with the Kaiser window technique to minimize the design time [17].

1.3 Contribution of the dissertation

The purpose of this study is to propose empirical formulas to estimate the order of the 2-D FIR fan filter based on the experimental results. Estimation of filter order is presented for three different transition width in three cases as case 01: transition width = 0.01π rad/sample, case 02: transition width = 0.05π rad/sample and case 03: transition width = 0.1π rad/sample. First, two empirical formulas are suggested for each case to estimate the filter order in terms of low complexity and good accuracy. Then evaluating each suggested formulas for random data set best formula is proposed to estimate the order of 2-D FIR fan filter.

In accordance with rotated fan filter, basic filter parameters are recognized as pass band ripple, stop band attenuation, half-width angle, and transition width and passband width. Then using the Kaiser window as a one-dimensional prototype, 2-D separable windows are formed. Using the ideal impulse response and the 2-D window functions 2-D FIR fan filters are designed. In total, 567 2-D fan filters are designed with various half-width angles and pass band ripple to obtain minimum filter order that satisfies the given filter specifications.

The minimum orders obtained with the design of 567 2-D fan filters are exploited to identify relations of the Kaiser window parameter β and the minimum filter order with respect to the specifications of 2-D fan filters. Then variations of the filter order versus filter parameters are graphed to analyze how the filter order is affected by the filter parameters. Then the relationships of filter order against the filter parameters are determined. Next, two empirical formulas are suggested to estimate the filter order by curve fitting using multiple linear regression. Finally, one formula is chosen with very good accuracy out of the two by numerically evaluating the estimated filter order for random data set. In conclusion, limitation and errors of the research study and the good accuracy of the selected polynomials is discussed. Further, future works to be done is summarized.

1.4 Related work

Early work in filter order estimation is not much interested topic as filter designing and optimization. It is evidenced by several 1-D filter order estimation examples and only one 2-D filter order estimation [17] [15] [18]. Also, it is observed that more 1-dimensional filter designs, optimization cases and less related works of filter order estimation on 2-D filters. Specifically, for 2-D filter order estimation only one study is found which is order estimation of 2-D circular filter [18]. Main objective of the most filter order estimation cases is to obtain a filter which satisfies the filters specification with the minimum order as minimized filter order have advantages over longer filter order in that they have fewer circuit elements in a hardware implementation or less computational cost in a software implementation and another main influence of the minimum order is shorter runtime in the implementation [19].

1.4.1 1-D Filter order estimation

In general, any filter design, order estimation or any filter optimization problem specify the filter specifications beforehand and in many practical filter design cases, passband edge frequency, stopband edge frequency, passband ripple and stopband ripple (usually) are set as the main parameters of a target filter. Then many filters are designed to see how long the minimum filter order, N must be [20].

To conjecture an appropriate filter order from given specifications in advance, two estimation formulas given by the equations (1) and (2) [21] [20]. Formulas have been proposed by Herrmann and Kaiser to design FIR low-pass filters in 1970 [13]. Those formulas estimate the filter order moderately well. However, they cannot achieve enough accuracy because lack of some considerations especially for longer filters. In 1970s, implementation of longer filter has been quite hard as it was the beginning of digital filters. Therefore, it has taken so much time to filter implementation and trouble to establish a more accurate formula and no longer has other formula been proposed. The conventional formulas, equations given by (1) and (2) are shown below [17]. These formulas have been used in practice to estimate the order of filters until a decade ago.

$$N_1\left(\Delta F, \delta_{p,}\delta_s\right) = \langle \frac{D_\infty\widehat{(\delta_{p,}\delta_s)}}{\Delta F} - f\left(\delta_{p,}\delta_s\right).\Delta F + 1\rangle, \qquad (1)$$

where,

$$D_\infty\left(\delta_{p,}\delta_s\right) = \left\{a_1\left(\log_{10}\delta_p\right)^2 + a_2\left(\log_{10}\delta_p\right) + a_3\right\}(\log_{10}\delta_s) + \{a_4\left(\log_{10}\delta_p\right)^2 + a_5\log_{10}\delta_p + a_6\}$$

$$f\left(\delta_{p,}\delta_s\right) = b_1 + b_2(\log_{10}\delta_p - \log_{10}\delta_s)$$

$a_1 = 5.309 \times 10^{-3}$ $\quad a_2 = 7.114 \times 10^{-2}$

$a_3 = -4.761 \times 10^{-1}$ $\quad a_4 = -2.660 \times 10^{-3}$

$a_5 = -5.941 \times 10^{-1}$ $\quad a_6 = -4.278 \times 10^{-1}$

$b_1 = 11.01217$ $\quad b_2 = 0.51244.$

In (1), ΔF denotes the transition width, $f_s - f_p$.

$$N_2\left(\Delta F, \delta_{p,}\delta_s\right) = \left\lceil \frac{-20\log_{10}\sqrt{\delta_{p,}\delta_s} - 13}{14.6\,\Delta F} + 1 \right\rceil. \quad (2)$$

The order is rounded to the nearest odd number to have integer-valued group delay. For even orders, group delay is not an integer. Although, estimation formulas have established for both odd and even filter order, FIR filters longer filter order (approximately more than 150) are not feasible. The estimation accuracies of equations given by (1) and (2) become worse as the filter order becomes long. Hence the formulas are only applicable for the FIR filters of short filter order. Therefore, accuracy improvement of longer filter is required to deliberate. Furthermore, one drawback of the above formulas is that the passband edge frequency f_p has not been considered [17].

In accordance with [17], new estimation formula has proposed to design of high-pass, band-pass, and band-stop filters. These estimated formulas cannot be used to design of other types of filters since the conventional estimation formulas can only be applied to the estimation of low-pass filters design. In comparison with the conventional estimation formulas given in equations given by (1) and (2) , the proposed formula by [17] realized much better accuracy. But the estimation formulas for band-pass and band-stop filters still leads to some wrong estimation. For instance, when minimum filter order estimation of a particular band-pass filter with the narrow passband gives longer filter order than required. Therefore, behavior of the minimum filter order of such filters should be studied further and their accuracy must be improved.

1.4.2 2-D filter order estimation

It can be found that primary motivation of 2-D filter designing, and order estimation is led by the fact that which is the better window formulation to design 2-D FIR filters and what are the differences between 1-D and 2-D window designs. An important related works of 2-D filter order estimation can be found on, [15] and same approach has been used in design example in [15, Ch. 3.3.3]. It has considered a one-dimensional window as a prototype to design 2-D filters. Generally, 2-D windows can be formulated having either a square region of support or a circular one. Furthermore, effects of window formulations for 2-D FIR filter design and derivation of formula to estimate filter order in terms of design specifications, using a Kaiser window as a prototype is explained in [15].

In this research study, to design the 2-D FIR filter windowing design procedure was considered. The ideal impulse response $i(m, n)$ is multiplied by $w(m, n)$ which is a finite area of window array to approximate an ideal frequency response by an FIR filter. So, it produces the filter impulse response $h(m, n)$ [15]. Two related formulations of window functions are generated. The first is a 2-D circularly symmetric window function and the second is a 2-D separable window obtained though the outer product of two 1-D windows.

Then designing total 4032 filters whose half of the filters having circular window and half having the square window, with various cutoff frequencies, filter orders which are always odd, and window function parameters β. Using the ideal impulse response and the window functions, the maximum passband ripples, stopband loss, and transition bandwidth have measured. Accounting previous 1-D research studies, transition bandwidth and amount of passband/stopband ripple in the filter frequency response have served as the, parameters of the design relation of order estimation in the 2-D case [22].

After completing the measurement process, they have observed that the filter order as a function of the transition bandwidth and attenuation of the filter, but not of the cutoff frequency. Finally, they have found two relationships to estimate filter order for circular window function and square window function which predicted the correct filter order, an error is less than 2 for 85% of the filters in the data base. By fitting data, reasonably well relation for estimating the Kaiser window parameter β also has determined.

By considering research works in 2-D filter design for circular window and square window, almost identical approach is used in this study to find an empirical formula to estimate order of the fan filter. Based on the results of related work, parameters which directly depend on the filter order are pre-determined along with the fan filter specifications. Then designing 567 FIR fan filters, filter order estimation formulas are developed. The derived empirical formulas provide good accuracy.

1.5 Outline of the dissertation

In this dissertation, four chapters are included. First chapter basically provides an introduction of the research and it generally discuss about the 2-D filters including main target filter in this research study which is 2-D FIR fan filter and their applications. It further reviews the requirement of the order estimation and main motivation to do this research. Then after it describes contribution of the thesis which is the summary of the entire research works. After the contribution of the thesis related work of order estimation has discussed. Mainly in 1-D order estimation, techniques used, and their successes and failures are briefly discussed and in the case of 2-D filter order estimation, symmetrical 2-D circular filter is considered.

In the chapter two, 2-D FIR fan filter design using windowing method is explained. At the beginning of the second chapter, fan filter specifications are clearly described, and then ideal impulse response of a general fan-type filter is calculated referring the research paper written by Pei and Jaw in 1994 [18]. Hereafter the definition of the Kaiser windowing technique and main design method is explained.

In the chapter three, suggested empirical formula to estimate the order of the 2-D fan filter in three different cases are explained. It includes all the assumptions made during the calculations and basic steps to estimate the filter order. It shows results of MATLAB simulations and it clearly discuss the variations of the filter order against with the other filter specifications. Then it suggests two formulas to estimate the order of the fan filter in terms of the low complexity and good accuracy. Next, each suggested formulas are numerically evaluated examining the error statistics and empirical formulas for each case is proposed. Finally, method of error minimization and the limitations of the results are discussed.

In last chapter, three models proposed for order estimation in each case and their accuracy has discussed with the help of comparison between theoretical and experimental results. Furthermore, it suggests future works and extension of the order estimation for 3-D case.

CHAPTER TWO

2 2-D FAN FILTER DESIGN USING KAISER WINDOW

In chapter two, design of the fan filter using Kaiser window method is discussed. The design of the fan filter is based on rotated fan filter presented in [18] and one filter designing example is demonstrated with the given filter specifications. As the window function design procedure for 2-D FIR filters is a straightforward extension of the one-dimensional case, the design of 2-D FIR fan filter is carried out using both 2-D window formulations based upon a 1-D Kaiser window prototype. Therefore, design procedure of the 2-D fan filter and definition of the Kaiser window is discussed as below.

2.1 Specification of general fan-type filter

Specifications for the general fan-type filter design illustrates in the Figure 2.1. The shaded region of the figure represents the passband, and the remained unshaded region represents the stopband. Two straight boundary lines bounds the passband, and it passes through the origin. These two boundary lines can point to arbitrary directions.

The parameter B is the passband width of the fan filter which is greater than 0 rad/sample and less than or equal to π rad/sample. The half-width angle θ is the spread angle of the passband. For ordinary and quadrant fan-type filters, the half-width angle or spread angle is fixed to be 90°. But for general fan-type filters it can be arbitrarily assigned from 0° to 180°. To control the aliasing effect which is the leakage beyond the folding frequency and reflects into the other end the passband width can be used [18].

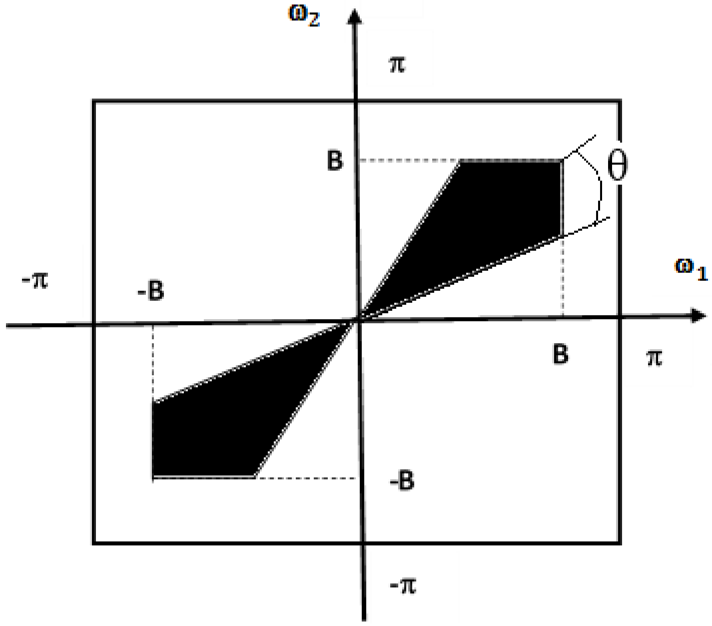


Figure 2.1: Specifications of general fan type filter

2.1.1 Modified general fan-type filter

Origin point in general fan filter is on the boundary between passband and stopband. Therefore, it is attenuated when multiplied with the window function. To overcome this difficulty of the attenuation, origin can be located the in the guard passband region. The zero-frequency response is thus improved.

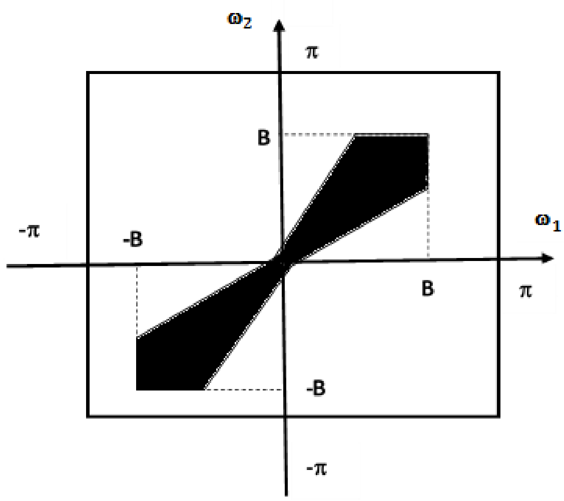


Figure 2.2: Modified general fan type filter with guard passband

When ideal impulse response is calculated for the newly define passband region, it gives very complicated expression and leads to some numerical difficulty. Therefore, a new horizontal fan-type filter is defined as in Figure 2.3. For newly defined filter, ideal impulse response can be easily calculated. Then using the coordinate transformation, the calculated ideal impulse response can be rotated to any direction. This is called as the rotated fan-type filter [18].

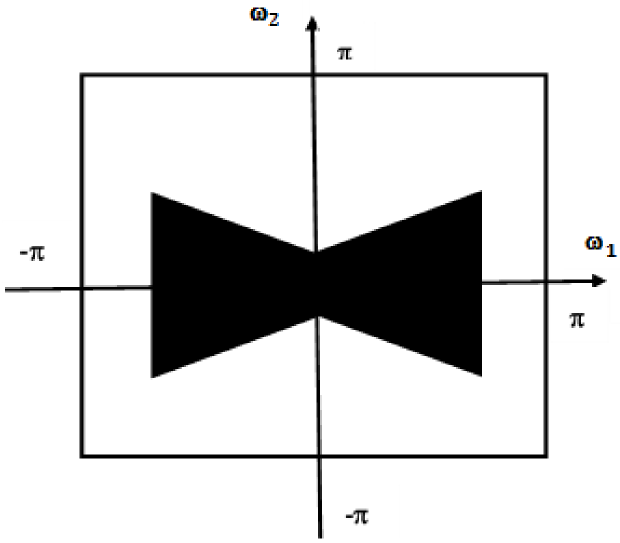
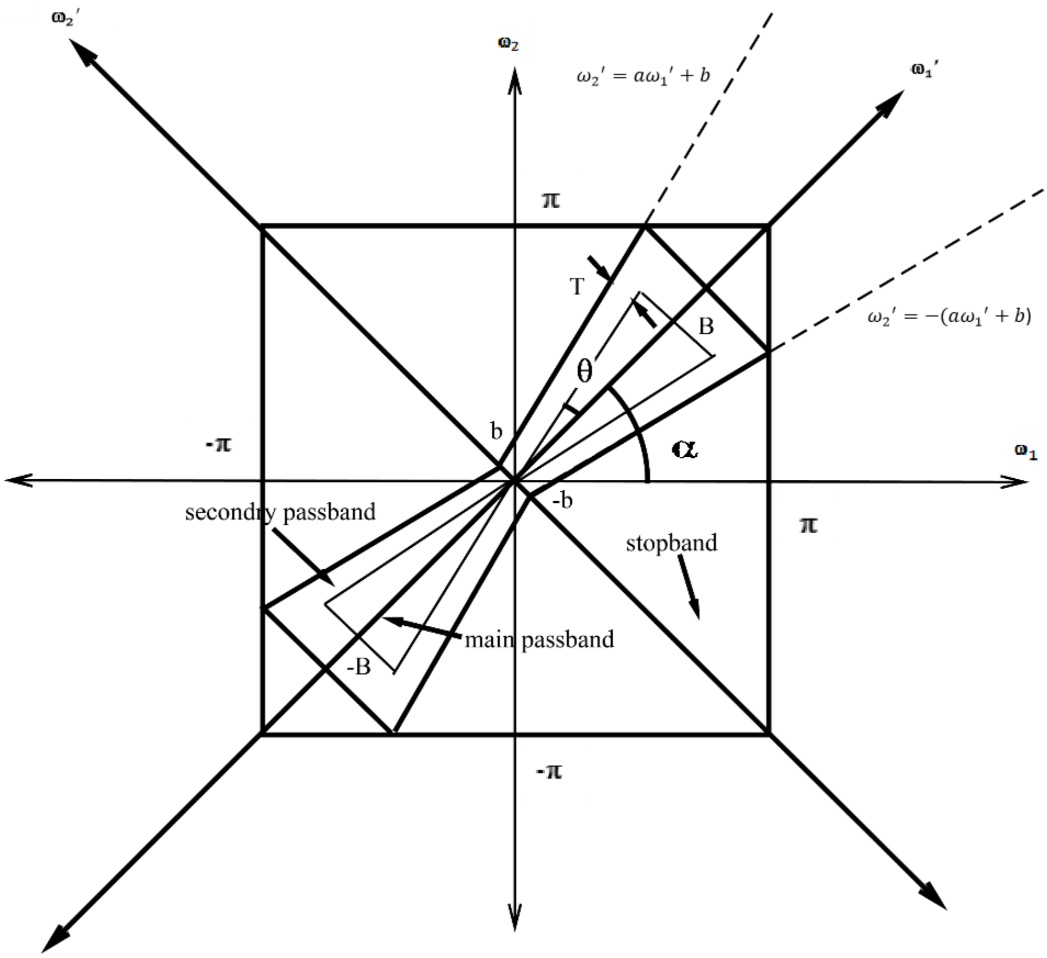


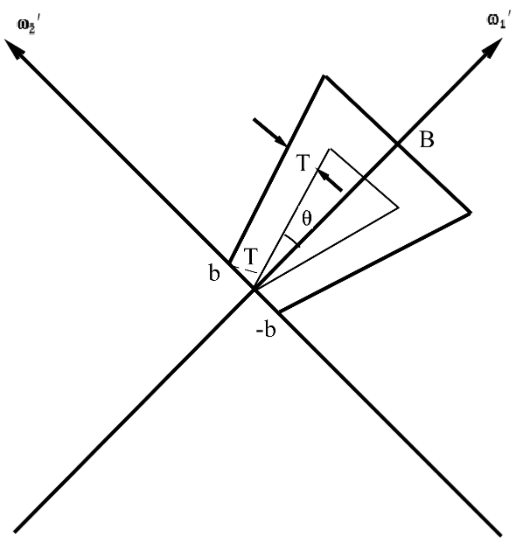
Figure 2.3: Horizontal fan-type filter

2.1.2 Definition of rotated fan-type filter

More information of a rotated fan-type filter is shown in Figure 2.4(a). To get rid of the attenuation at the zero-frequency response, secondary-passband is defined with the value one [18].



(a)



(b)

Figure 2.4: (a) Detailed definition of a rotated fan-type filter. (b) Enlarge portion of fig 9(a)

2.2 Ideal impulse response of rotated fan-type filter

Two parameters *a*, *b* are defined from Figure 2.4 (b) as,

$$a = \tan\theta, \tag{1}$$

$$b = \frac{T}{\cos\theta}, \tag{2}$$

where *T* is width of the secondary passband and *θ* is half of the fan angle, then the ideal impulse response $h(n_1, n_2)$ can be written as

$$h(n_1, n_2) = \frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left[\mathrm{H}\left(e^{j\omega_1}, e^{j\omega_2}\right)e^{j(n_1\omega_1 + n_2\omega_2)}\right] d\omega_1 d\omega_2. \tag{3}$$

In the original (ω_1, ω_2) coordinate system, equation given by (3) is difficult to integrate. But in the rotated co-ordinate system (ω_1', ω_2'), the shape of the passband changes to a horizontal fan filter which is easier to integrate in the (ω_1', ω_2') space. The relation between these two-coordinate systems, (ω_1, ω_2) and (ω_1', ω_2') is just a rotation operation and it can be written as,

$$(\omega_1', \omega_2') = (\omega_1, \omega_2)\begin{bmatrix}\cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha\end{bmatrix}, \tag{4}$$

where α is the angle between the two axes ω_1 and ω_1'. Taking (ω_1', ω_2') into (ω_1, ω_2), and using the Jacobian

$$d\omega_1 d\omega_2 = \begin{vmatrix}\dfrac{\partial\omega_1}{\partial\omega_1'} & \dfrac{\partial\omega_2}{\partial\omega_1'} \\ \dfrac{\partial\omega_1}{\partial\omega_2'} & \dfrac{\partial\omega_2}{\partial\omega_2'}\end{vmatrix}$$

$$= \begin{bmatrix}\cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha\end{bmatrix} d\omega_1' d\omega_2'$$

$$= d\omega_1' d\omega_2', \tag{5}$$

then equation given by (3) can be rewritten as

$$h(n_1, n_2) = \frac{2}{4\pi^2} \int_0^B \int_{-(a\omega_1'+b)}^{a\omega_1'+b} \cos(n_1\omega_1'\cos\alpha - n_1\omega_2'\sin\alpha + n_2\omega_1'\sin\alpha + n_2\omega_2'\cos\alpha)\, d\omega_2' d\omega_1'$$

$$= \frac{2}{4\pi^2} \int_0^B \int_{-(a\omega_1'+b)}^{a\omega_1'+b} \cos(p\omega_1' + q\omega_2')\, d\omega_2' d\omega_1' \tag{6}$$

$$= g(p, q),$$

where

$$(p, q) = (n_1, n_2) \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}. \tag{7}$$

Under various special conditions equation given by (6) can be integrated and simplified into several closed form solutions.

$$g(p, q) = \frac{1}{4\pi^2 q} \left[\frac{\cos(qb) - \cos(pB + qaB + qb)}{p + qa} - \frac{\cos(qb) - \cos(pB - qaB - qb)}{p - qa} \right],$$

for $q \neq 0, p + qa \neq 0, p - aq \neq 0,$ (8)

$$g(0,0) = \frac{B}{4\pi^2} (aB + 2b) \quad \text{for } p = q = 0, \tag{9}$$

$$g(p, 0) = \frac{1}{2\pi^2 p} [(aB + b)\sin(pB) + \frac{a(\cos(pB) - 1)}{p} \quad \text{for } q = 0, \tag{10}$$

$$g(p, q) = \frac{1}{4\pi^2 q} [(B\sin(qb) - \frac{\cos(qb) - \cos(pB - qaB - qb)}{p - qa}$$

for $p + qa = 0,$ (11)

$$g(p, q) = \frac{1}{4\pi^2 q} \left[B\sin(qb) + \frac{\cos(qb) - \cos(pB + qaB + qb)}{p + qa} \right]$$ (12) [18].

for $p - qa = 0$,

2.3 2-D fan filter design method

The window techniques used to design of multidimensional FIR filters is philosophically identical to its 1-D counterpart [10]. It is a spatial domain method. Here $i\ (n_1, n_2)$ and $I(\omega_1, \omega_2)$ represents the ideal impulse response and ideal frequency response of the ideal filter. Also $h(n_1, n_2)$and $H(\omega_1, \omega_2)$ represents the impulse response and frequency response of the filter designed by the algorithm. Then with the window method the coefficients $h(n_1, n_2)$ are given by,

$$h(n_1, n_2) = i\ (n_1, n_2) * \ w\ (n_1, n_2). \tag{13}$$

The sequence $w\ (n_1, n_2)$ is called a window function or window array [13]. The ideal impulse response, $i\ (n_1, n_2)$ can be found by computing the inverse 2-D discrete-space fourier transform of $I(\omega_1, \omega_2)$. Ideal impulse response for the fan filter is explained in section 2.2. Kaiser window is selected as the window function and two-dimensional window is formed as the outer product of two one-dimensional Kaiser windows as,

$$w\ (n_1, n_2) = w\ (n_1) * \ w\ (n_2). \tag{14}$$

This window is separable in the two independent variables. Then *finite-extent impulse response* of the FIR fan filter is achieved by multiplying 2-D Kaiser windows with ideal impulse response of the fan filter as equation given in (13).

2.3.1 Definition of Kaiser window

The Kaiser window is a generally flexible and easy to compute window and it is, widely used in filter designing.

Kaiser has shown that the one-dimensional I_o –sinh functions defined by,

$$w(n) = \begin{cases} \dfrac{I_0\left[\beta\sqrt{1-\left({n}/{N}^2\right)}\right]}{I_0(\beta)} & , |n| < N \\ 0 & , |n| > N. \end{cases}$$

Here, $I_0(\beta)$ is the modified Bessel function of the first kind of order zero and β is an independent parameter. It defines a frequency domain trade-off between main lobe width and sidelobe ripple. It can be computed to any degree of precision by using the rapidly converging series,

$$I_0(x) = 1 + \sum_{k=1}^{\infty} \left[\frac{1}{k!}\left(\frac{x}{2}\right)^k\right]^2 \text{ [23].}$$

2.3.2 Design of 2-D FIR fan filter –An example

Here, design example for 2-D FIR fan filter is considered in accordance with the design method discussed in section 2.3 along with the ideal impulse response of the rotated fan type filter in section 2.2. The design specifications of the 2-D FIR fan filter are given in Table 2.1. The filter orders, which are always odd and equal, varied from 5×5 to 1023×1023. For the design example $N_1 \times N_2$ which is the order of filter for n_1 dimension and n_2 dimension is considered as 337×337. For given passband ripple $A_p(dB)$, stopband attenuation $A_a, (dB)$ is calculated using the formula given in

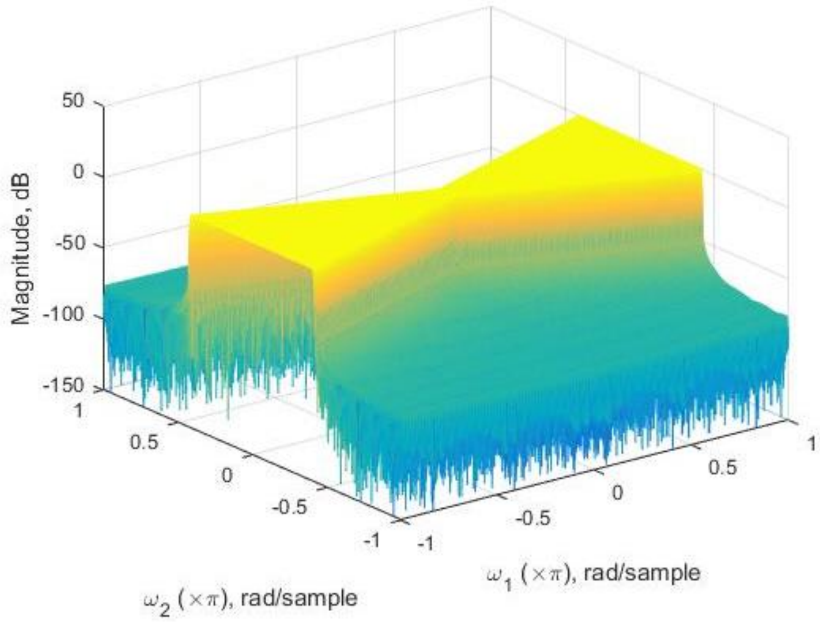
$$A_a = -20\log_{10}\left({10^{(0.05*\mathrm{Ap}-1)}}/{10^{(0.05*\mathrm{Ap}+1)}}\right) \text{ [17].}$$

Table 2.1: Example design specifications of 2-D FIR fan-type filter

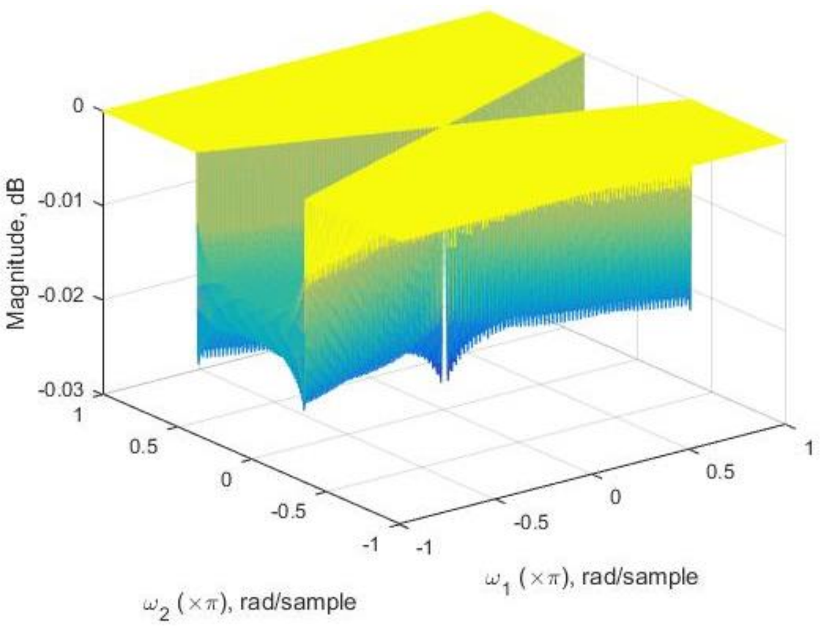
Design specification	Value
Maximum passband ripple, A_p	0.027 dB
Minimum stopband attenuation, A_a	65.935dB
Half of the fan angle, θ	$20°$
Passband width, B	π rad/sample
Transition width, T	0.05π rad/sample

Kaiser window parameter, β	4
Rotation of the fan, α	0°

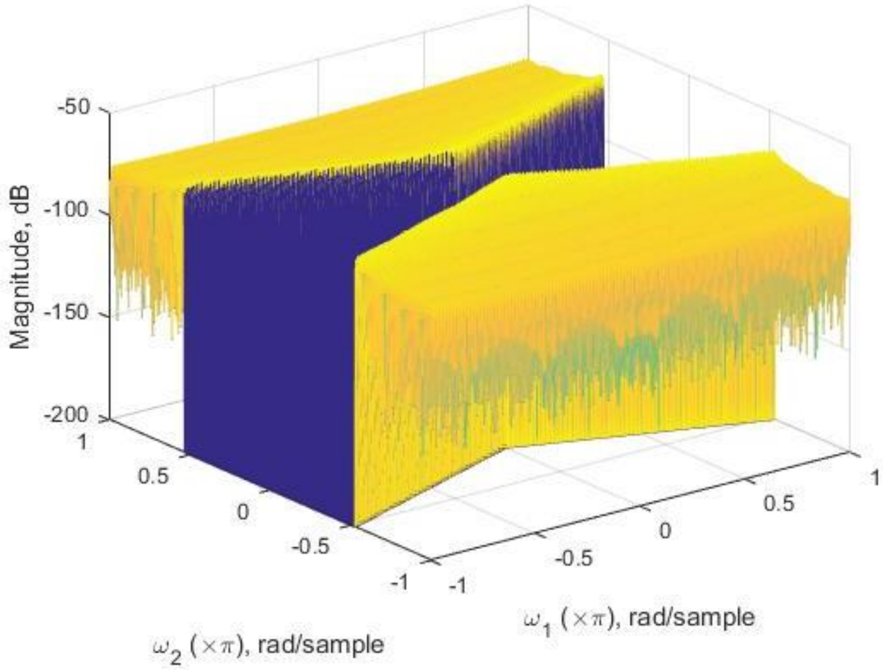
Frequency response using rotated method and its passband frequency and stop band frequency responses are demonstrated in Figure 2.5. Passband and stop band frequency responses for the given specification in Table 2.1 are illustrated in Figure 2.5.



(a)

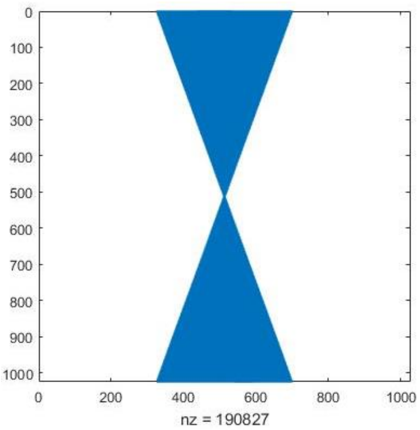


(b)

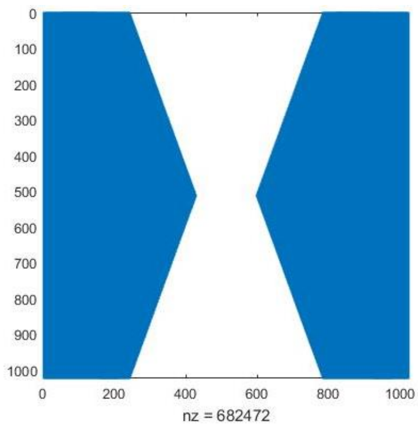


(c)

Figure 2.5: (a) Frequency response of 2-D FIR fan filter for given specification in Table 2.1 (b) Passband frequency of Fig. (a) 2-D FIR fan filter. (c) Stopband frequency of Fig. (a) 2-D FIR fan filter



(a)



(b)

Figure 2.6: (a) Passband frequency of FIR fan Filter (b) Stopband frequency of FIR fan filter for given specification in Table 2.1

CHAPTER THREE

3 EMPIRICAL FORMULA TO ESTIMATE THE MINIMUM ORDER OF THE 2-D FIR FAN FILTER

3.1 Introduction

Fan-type filters have been used in many applications as described in previous chapters. Therefore, it is worthwhile to have a specific predefined method of order estimation of 2-D FIR fan filters and it directly leads to cost estimation and run time of the filter. This chapter proposed empirical formulas to estimate the order of 2-D FIR fan filter for three cases under given filter specifications. This chapter organized as follows, all assumptions and limitations considered to derive empirical formulas for order estimation of 2-D FIR fan filter is mentioned. Then proposed approach of order estimation is presented for each case with the relevant statistical analysis. Finally, 864 fan filters are generated using the suggested formulas to evaluate the accuracy. Then best fitted, good accuracy formula is proposed to estimate the order of the filter for each case. At last, predicted values from proposed estimation formula and observed data through the experiment is compared and error is reviewed.

3.2 Proposed approach of order estimation of 2-D FIR fan filter

Order estimation of 2-D FIR fan filter is proposed in three subsections whereas it propose estimation formulas under three different transition width as case 01: T=0.01 π rad/sample, case 02: T = 0.05 π rad/sample and case 03: T = 0.1 π rad/sample. For each case, assumptions and limitations made are similar and only transition width is changed. For each case following procedure is followed. Proposed approach for estimating an empirical formula to order estimation of 2-D FIR fan filter is implemented with experimental data along with the linear regression technique. To implement the order estimation formula, experimentally 567 fan filters are generated for each mentioned case which are achieved minimum filter order satisfying given filter specifications. During the experiment filter specifications are varied as follows.

i. β varied from 0 to 8 as values giving 0,1,2,3,4,5,6,7,8.

ii. θ varied from 5° to 35° as values giving $5^{\circ}, 10^{\circ}, 15^{\circ}, 20^{\circ}, 25^{\circ}, 30^{\circ}, 35^{\circ}$.

iii. A_p varied from 0.001 dB to 0.7 dB as values giving 0.001 dB, 0.004 dB, 0.007 dB, 0.01 dB, 0.04 dB, 0.07 dB, 0.1 dB, 0.4 dB, 0.7 dB.

iv. $N_1 \times N_2$ Is varied from 5×5 to 1023×1023.

v. T varied for case 01: T= 0.01π rad/sample, case 02: T = 0.05π rad/sample and case 03: T = 0.1π rad/sample.

As there are many parameters related to 2-D FIR fan filter, design of an empirical formula is quite complicated. To reduce the complexity few assumptions made in the implementation process as mentioned below.

i. B is considered as π rad/sample.

ii. α is considered as 0 degrees.

iii. N_1 and N_2 Are always odd and equal filter order.

Experimental data obtained for each above-mentioned case are tabulated below. Considering the variations of minimum filter order $N_1 \times N_2$, β, θ and A_p, order estimation process is divided into two steps as minimum filter order given β estimation and order estimation. In β estimation, minimum filter order given β is considered for given A_p, and θ. Then relationship among variables is converted to estimation formula using multiple linear regression. Then similar approach carried out for the order estimation process. For given A_p, and θ minimum filter order is observed from experimental data and their relationship is taken as estimation formula using multiple linear regression. In both estimation procedure, relations among the filter parameters considered as follows:

Minimum filter order given β is determined by two parameters, A_p and θ. Hence, β can be expressed as,

$$\beta = f(A_p, \theta).$$

And minimum filter order $N_1 \times N_2$ is determined by two parameters, A_p and θ. Hence, $N_1 \times N_2$ can be expressed as,

$$N_1 \times N_2 = f(A_p, \theta).$$

Design procedures for each case is explained as given below.

3.2.1 Case 01: Design of an empirical formula to estimate order of 2-D FIR fan filter when T = 0.01 π rad/sample

Accordingly, section 3.2 given filter specification satisfied, $N_1 \times N_2$ is obtained when T = 0.01 π rad/sample. 567 filters are generated by varying filter order $N_1 \times N_2$ from 5 × 5 to1023×1023. Experimental minimum filter orders which satisfied givenA_p, θ and β when T=0.01 π rad/sample is given in Table 3.1. As an example, 2nd row, 10th column, Table 3.1 data indicates that the minimum filter order is 703 when A_p = 0.001 dB, θ=5° , β = 8 and T = 0.01 π rad/sample.

Experimental results:

Table 3.1: Experimental $N_1 \times N_2$ values for given θ, β and A_p when T = 0.01 π rad/sample

A_p	θ = 5°								
(dB)	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001	-	-	-	-	-	-	-	-	703
0.004	-	-	-	-	-	-	-	447	493
0.007	-	-	-	-	-	-	623	439	483
0.01	-	-	-	-	-	-	475	433	475
0.04	-	-	-	-	733	323	363	401	437
0.07	-	-	-	913	387	311	349	383	417
0.1	-	-	-	539	267	303	337	371	403
0.4	-	511	325	201	223	247	383	309	331
0.7	491	313	161	183	207	231	255	275	295
	θ = 10°								

A_p (dB)	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001	-	-	-	-	-	-	-	-	561
0.004	-	-	-	-	-	-	-	501	499
0.007	-	-	-	-	-	-	601	447	467
0.01	-	-	-	-	-	-	455	417	461
0.04	-	-	-	-	547	309	349	387	421
0.07	-	-	-	893	371	299	335	369	401
0.1	-	-	-	-	255	291	325	357	387
0.4	-	499	313	191	217	245	271	295	317
0.7	481	303	153	173	197	219	281	261	281
A_p (dB)	$\theta = 15^{\circ}$								
	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001	-	-	-	-	-	-	-	-	539
0.004	-	-	-	-	-	-	915	471	511
0.007	-	-	-	-	-	-	443	457	495
0.01	-	-	-	-	-	897	431	447	483
0.04	-	-	-	-	525	367	367	391	415
0.07	-	-	-	867	353	315	335	353	385
0.1	-	-	-	-	333	293	309	341	371
0.4	-	487	301	181	205	231	255	279	301
0.7	467	293	145	163	183	207	227	247	265
A_p (dB)	$\theta = 20^{\circ}$								
	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001	-	-	-	-	-	-	-	-	541
0.004	-	-	-	-	-	-	561	471	515
0.007	-	-	-	-	-	-	415	459	501
0.01	-	-	-	-	-	691	409	449	487
0.04	-	-	-		349	339	365	395	423
0.07	-	-	-	659	329	315	341	367	391
0.1	-	-	-		309	299	323	345	365
0.4	-	469	285	177	191	217	241	263	283
0.7	453	281	133	151	171	193	213	231	249

A_p (dB)	$\theta = 25^{\circ}$								
	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001	-	-	-	-	-	-	-	701	551
0.004	-	-	-	-	-	-	521	471	519
0.007	-	-	-	-	-	825	411	459	503
0.01	-	-	-	-	-	499	405	451	493
0.04	-	-	-	979	321	335	369	401	431
0.07	-	-	-	621	301	315	345	373	397
0.1	-	-	-	-	277	301	325	349	371
0.4		445	265	181	189	201	225	245	263
0.7	443	265	121	137	157	177	197	213	229
A_p (dB)	$\theta = 30^{\circ}$								
	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001	-	-	-	-	-	-	-	655	581
0.004	-	-	-	-	-	939	483	477	527
0.007	-	-	-	-	-	467	411	463	509
0.01	-	-	-	-	-	459	405	453	495
0.04	-	-	-	919	293	331	369	407	441
0.07	-	-	-	429	281	315	351	383	413
0.1	-	-	-	-	271	305	337	367	395
0.4	-	417	243	199	183	199	261	281	299
0.7	407	245	207	167	219	241	213	229	243
A_p (dB)	$\theta = 35^{\circ}$								
	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001	-	-	-	-	-	-	-	605	593
0.004	-	-	-	-	-	727	439	491	537
0.007	-	-	-	-	-	433	417	471	515
0.01	-	-	-		879	419	409	459	499
0.04	-	-	-	715	285	329	369	407	439
0.07	-	-	-	253	273	313	349	381	413
0.1	-	-	-	245	265	301	335	367	395
0.4	-	393	219	197	219	241	271	293	313
0.7	377	225	177	175	195	215	233	251	269

3.2.1.1 Estimation of minimum order given β

From Table 3.1, minimum filter order given β value is selected out of 0 to 8 for each of given A_p and θ. Then selected β is tabulated and given in Table 3.2. As an example, when given θ =5° and $A_p = 0.7$dB minimum filter order, 161 given β is 2. To determine the relationship among given A_p and θ with minimum filter order given β, multiple linear regression is used. Hence minimum filter order given β is drawn using curve fitting tool considering passband ripple and theta as predictors and β as the response.

Table 3.2: Minimum filter order given β for given A_p and θ selected from experimental data given in Table 3.1 when T = 0.01π rad/sample

A_p(dB)	θ =5°	θ =10°	θ =15°	θ =20°	θ =25°	θ =30°	θ =35°
0.001	8	8	8	8	8	8	8
0.004	7	8	7	7	7	7	6
0.007	7	7	6	6	6	6	6
0.01	7	7	6	6	6	6	6
0.04	5	5	5	5	4	4	4
0.07	5	5	5	5	4	4	4
0.1	4	4	5	5	4	4	3
0.4	3	3	3	3	3	3	3
0.7	2	2	2	2	2	3	3

Minimum order achieved β variation with respect to given θ and A_p is examined. Then considerable relationship of β with respect to given θ and A_p cannot be identified. In consequence, log value of passband ripple, log (A_p) is considered instead of A_p. Most early research studies of filter order estimations and variations analysis have been used log scale of passband ripple as an alternative of passband ripple [17]. So then, measurable, worthwhile relationship among log (A_p), θ with β could be achieved. Therefore, in every consideration of A_p of determining relationship is replaced with logA_p.

Applying curve fitting tool in MATLAB for the data set given in Table 3.2, suitable fitted curves are observed to obtain the strongest relationship among the variables, log *(A_p)*, θ and β. Then four polynomials as *Poly13, Poly14, Poly23, and Poly24* are selected with good fitness. Goodness-of-fit statistics of selected polynomials are tabulated in Table 3.3. Comparing goodness-of-fit statistics of chosen four polynomials, well fitted polynomial is proposed to determine the β which achieved minimum filter order.

Table 3.3: Goodness-of-fit statistics of selected polynomials of β for given log *(A_p)* and θ when T = 0.01π rad/sample

Statistics	*Poly11*	*Poly13*	*Poly14*	*Poly23*
Sum of squares due to error (SSE)	11.495	7.607	7.076	7.555
R-square	0.946	0.964	0.967	0.965
Adjusted R-square	0.944	0.961	0.962	0.959
Root mean squared error (RMSE)	0.362	0.369	0.362	0.374

When compare the goodness-of-fit statistics of chosen four polynomials, *Poly14* gives the least SSE and RMSE while producing high R-square and adjusted R-square. Therefore, based on the goodness of fit statistics, *Poly14* is proposed as the best fitted curve to gain minimum filter order achieved β for the given set of experimental data with the lower complexity and good accuracy. Behavior of minimum filter order given β for given θ and log *(A_p)* with linear model *Poly14* is given in Figure 3.1. Therefore, proposed estimation formula to estimate β could be shown as follows.

- **Estimation formula of minimum order given β when T = 0.01π rad/sample**

$$f\ (\theta, \log_{10} A_p) = a_1 + a_2\theta + a_3(\log_{10} A_p) + a_4\theta(\log_{10} A_p) + a_5(\log_{10} A_p)^2 + a_6\theta(\log_{10} A_p)^2 + a_7(\log_{10} A_p)^3 + a_8\theta(\log_{10} A_p)^3 + a_9(\log_{10} A_p)^4,$$

Coefficients (with 95% confidence bounds):

$a_1 = 6.018 \times 10^{-1}$ $(-0.3041, 1.508)$

$a_2 = 5.128 \times 10^{-2}$ $(0.01646, 0.08609)$

$a_3 = -7.395$ $(-11, -3.794)$

$a_4 = 1.339 \times 10^{-1}$ $(0.0348, 0.233)$

$a_5 = -4.774$ $(-9.02, -0.5273)$

$a_6 = 5.448 \times 10^{-2}$ $(-0.01927, 0.1282)$

$a_7 = -1.875$ $(-3.79, 0.04008)$

$a_8 = 5.24 \times 10^{-3}$ $(-0.01015, 0.02063)$

$a_9 = -2.768 \times 10^{-1}$ $(-0.57, 0.01629)$.

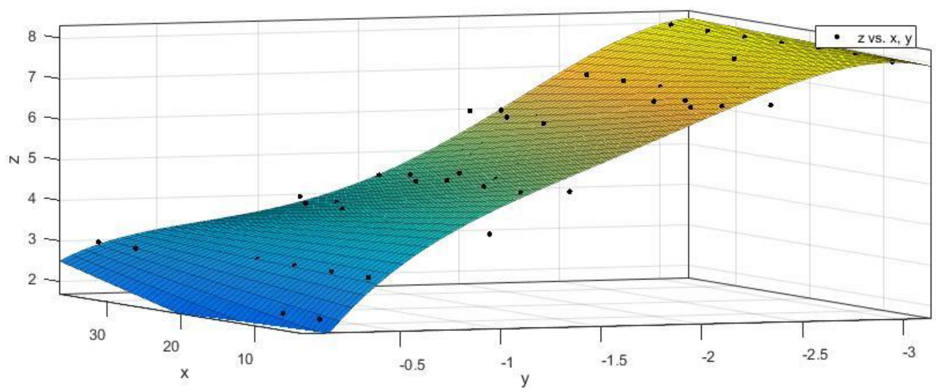


Figure 3.1: Behavior of N given β for given θ and log (A_p) with linear model *Poly14*

3.2.1.2 Design of empirical formula for order estimation

Like the method of minimum filter order achieved β estimation, relationship among θ, log (A_p) and minimum filter order can be obtained. Minimum filter orders are selected from Table 3.1 for given θ and A_p. It is given in Table 3.4. Fitting Table 3.4 data, minimum filter order as the response and log value of passband ripple and theta as predictors, best fitted curves are observed to obtain the strongest relationship among the variables, log (A_p), θ and N.

Table 3.4: Minimum filter order selected from Table 3.1 for given θ and A_p when T = 0.01π rad/sample

A_p(dB)	θ =5°	θ =10°	θ =15°	θ =20°	θ =25°	θ =30°	θ =35°
0.001	703	561	539	541	551	581	593
0.004	447	499	471	471	471	477	439
0.007	439	447	443	415	411	411	417
0.01	433	417	431	409	405	405	409
0.04	323	309	367	339	321	293	285
0.07	311	299	315	315	301	281	253
0.1	267	255	293	299	277	271	245
0.4	201	191	181	177	181	199	197
0.7	161	153	145	133	121	167	175

Observing multiple fitted curves, four good, fitted curves are selected and their goodness-of-fit statistics are tabulated in Table 3.5. Comparing goodness-of-fit statistics, two best fitted polynomials to determine the minimum order of the fan filter is suggested.

Table 3.5: Goodness-of-fit statistics of selected polynomials of N for given θ and log A_p when T = 0.01π rad/sample

Statistics	*Poly13*	*Poly14*	*Poly21*	*Poly23*
The sum of squares due to error (SSE)	3.41e +04	3.34e +04	3.36e +04	3.14e +04
R-square	0.969	0.969	0.969	0.971
Adjusted R-square	0.966	0.965	0.967	0.967
Root mean squared error (RMSE)	24.68	24.89	24.08	24.13

According to the statistics of selected polynomials two formulas are suggested in terms of the lower complexity and the good accuracy. In the context of low complexity *Po1y13* provides fairly very good R-square, adjusted R square and comparatively less error. Meanwhile *Poly21* shows same R-square, greater adjusted R square and least error with better goodness compared to other polynomials. Therefore, *Poly13* is suggested as low complexity empirical formula to estimate the order of the fan filter and *Poly21* is suggested to estimate the order of the fan filter with good accuracy. Behavior of N for given θ and log (A_p) with linear model *Poly13* and *Poly21* are illustrated in Figure 3.2 and Figure 3.3, respectively. Suggested empirical formulas to estimate the order of the filter are presented as follows.

- **Low complexity empirical formula to estimate order of FIR fan filter**

Linear model *Poly13*:

$$f((\log_{10} A_p), \theta) = a_1 + a_2 (\log_{10} A_p) + a_3(\theta) + a_4(\theta)(\log_{10} A_p) + a_5(\theta)^2 + a_6(\log_{10} A_p)(\theta)^2 + a_7(\theta)^3,$$

Coefficients (with 95% confidence bounds):

$a_1 = 1.094 \times 10^2 \quad (44.59, 174.3)$

$a_2 = -1.776 \times 10^2 \quad (-206.4, -148.7)$

$a_3 = 1.194 \quad (-9.687, 12.08)$

$a_4 = 3.261 \quad (-0.04534, 6.567)$

$a_5 = 3.901 \times 10^{-2} \quad (-0.5177, 0.5957)$

$a_6 = -6.71 \times 10^{-2} \quad (-0.1479, 0.01368)$

$a_7 = -1.975 \times 10^{-3} \quad (-0.01095, 0.006996).$

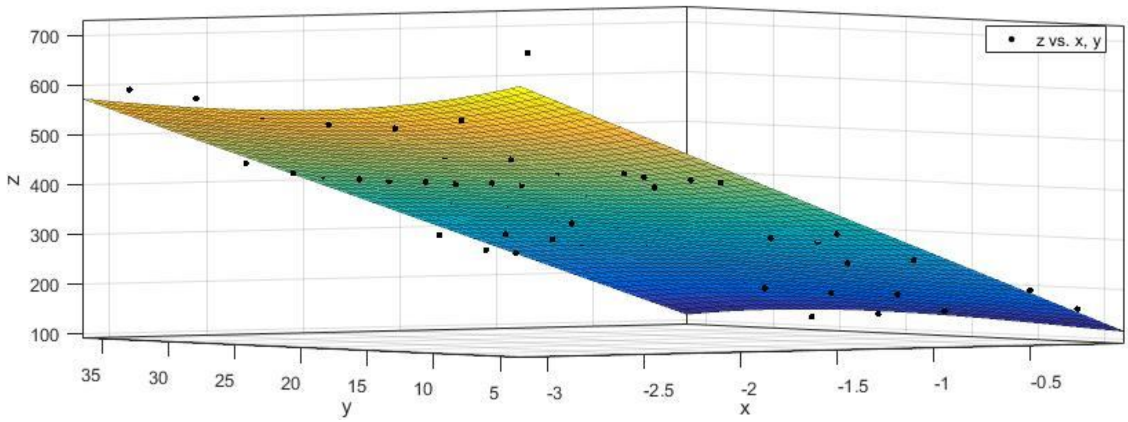


Figure 3.2 : Behavior of N for given θ and $\log_{10}(A_p)$ when T = 0.01π rad/sample with linear model *Poly13*

- **Accurate empirical formula to estimate order of FIR fan filter**

Linear model *Poly21*:

$$f\big((\log_{10} \mathrm{A_p}), \theta\big) = a_1 + a_2(\log_{10} \mathrm{A_p}) + a_3(\theta) + a_4(\log_{10} \mathrm{A_p})^2 + a_5(\log_{10} \mathrm{A_p})(\theta),$$

Coefficients (with 95% confidence bounds):

$a_1 = 137.5$ $(107.9, 167.1)$

$a_2 = -131.7$ $(-160.7, -102.6)$

$a_3 = 0.03883$ $(-1.161, 1.239)$

$a_4 = 8.425$ $(0.3549, 16.5)$

$a_5 = 0.5768$ $(-0.1052, 1.259)$.

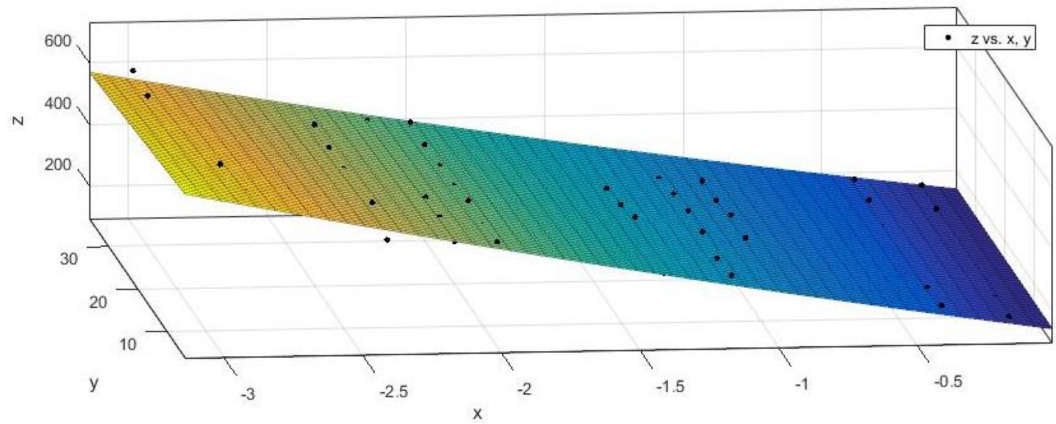


Figure 3.3: Behavior of *N* for given θ and $\log_{10}(A_p)$ when *T* = 0.01π rad/sample with linear model *Poly21*

3.2.1.3 Evaluation of proposed formulas of minimum order given β

Next, the accuracy of proposed β estimation formula of 2-D FIR fan filter is evaluated by designing 108 new 2-D FIR fan filters using proposed formula. To design fan filters different specifications selected as given below.

I. Nine random A_p values are selected within 0.001 dB and 0.7 dB values taken as 0.003 dB, 0.005 dB, 0.009 dB, 0.02 dB, 0.06 dB, 0.03 dB, 0.2 dB, 0.3 dB, 0.5 dB.

II. Twelve θ values are selected within 0° and 45° as $2°, 3°, 7°, 12°, 17°, 21°, 26°, 28°, 31°, 36°, 41°, 43°$

Other specifications are same as mentioned in section 3.2. Then error between observed β by experimental results and predicted β by proposed formula is calculated. To numerically evaluate the accuracy of proposed β estimation formula of 2-D FIR fan filter, the standard deviation of the error and the average of the error are calculated. Using proposed formula of β estimation in *Poly14*, minimum filter order given β is predicted for 108 filter for above given specifications. It is given in Table 3.6. Experimental data obtained for same fan filter specification is given in Table 3.7. Observed experimental data is attached in Appendix A-I. The error between predicted β and experimental β is shown in Table 3.8.

Table 3.6: Predicted β from proposed estimation polynomial *Poly13*

A_p (dB)	θ											
	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°	43°
0.003	8	8	8	7	7	7	7	7	7	7	7	7
0.005	7	7	7	7	7	7	7	7	6	6	6	6
0.009	7	7	7	6	6	6	6	6	6	6	5	5
0.02	6	6	6	6	6	5	5	5	5	5	4	4
0.06	5	5	5	5	5	4	4	4	4	4	4	4
0.03	6	6	6	5	5	5	5	5	5	4	4	4
0.2	4	4	4	4	4	4	4	4	3	3	3	3
0.3	3	3	3	3	3	3	3	3	3	3	3	3
0.5	2	2	3	3	3	3	3	3	3	3	3	3

Table 3.7: Experimental minimum filter order given β

A_p (dB)	θ											
	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°	43°
0.003	8	8	7	7	7	7	7	7	6	6	6	6
0.005	7	7	7	7	7	7	7	7	6	6	6	6
0.009	7	7	7	7	6	6	6	6	6	6	5	5
0.02	6	6	6	6	5	5	5	5	5	5	4	4
0.06	5	5	5	5	5	5	4	4	4	4	4	4
0.03	5	5	5	6	5	5	5	4	4	4	5	5
0.2	4	5	4	4	4	4	4	3	3	3	3	3
0.3	3	3	3	3	3	4	3	3	3	3	2	3
0.5	2	2	2	2	2	3	3	3	3	3	2	2

Table 3.8: Error between predicted β and experimental β

A_p (dB)	θ											
	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°	43°
0.003	0	0	1	0	0	0	0	0	1	1	1	1
0.005	0	0	0	0	0	0	0	0	0	0	0	0
0.009	0	0	0	-1	0	0	0	0	0	0	0	0

0.02	0	0	0	0	1	0	0	0	0	0	0	0
0.06	0	0	0	0	0	-1	0	0	0	0	0	0
0.03	1	1	1	-1	0	0	0	1	1	0	-1	-1
0.2	0	-1	0	0	0	0	0	1	0	0	0	0
0.3	0	0	0	0	0	-1	0	0	0	0	1	0
0.5	0	0	1	1	1	0	0	0	0	0	1	1

Numerically calculated mean of the absolute error, $\bar{X}_{error}$ and standard deviation of the absolute error σ_{error} found to be 0.2314 and 0.4237. According to the statistical figures obtained it seems that there is less error in predicted β and observed β.

3.2.1.4 Evaluation of suggested formulas for 2-D FIR fan filter order estimation when *T* = 0.01*π* rad/sample

Next, the accuracy of proposed order estimation formulas of 2-D FIR Fan filter is evaluated by designing 108 new 2-D FIR fan filters using proposed formulas. To design fan filters different specifications selected as given below.

I. Nine random A_p values are selected within 0.001 dB and 0.7 dB values taken as 0.003 dB, 0.005 dB, 0.009 dB, 0.02 dB, 0.06 dB, 0.03 dB, 0.2 dB, 0.3 dB, 0.5 dB.
II. Twelve θ values are selected within 0° and 45° as $2°, 3°, 7°, 12°, 17°, 21°, 26°, 28°, 31°, 36°, 41°, 43°$

Other specifications are same as mentioned in section 3.2. Then error between observed filter order from experimental results and predicted filter order from suggested formulas is calculated. To numerically evaluate the accuracy of suggested order estimation formula of the 2-D FIR fan filter, the standard deviation of the error and the average of the error are calculated. Two estimation formulas suggested for minimum order estimation is evaluated in subsection as given below.

A. Evaluation of 2-D FIR fan filter order estimation polynomial *Poly13*

Using suggested low complexity empirical formula of order estimation *Poly13*, minimum filter order is predicted for 108 filters considering the above given specifications. It is given in Table 3.9. Experimental filter order obtained for same fan filter specification is given in Table 3.10. The error between predicted filter order and experimental filter order is shown in Table 3.11.

Table 3.9: Predicted minimum filter order from suggested polynomial *Poly13*

A_p	θ											
(dB)	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°	43°
0.003	544	538	518	500	488	483	481	480	481	482	483	483
0.005	506	501	483	467	457	453	450	450	449	449	448	447
0.009	462	458	442	429	421	417	415	414	414	412	408	406
0.02	403	399	388	378	372	369	367	366	365	361	354	350
0.06	321	319	312	307	304	303	301	300	297	291	279	273
0.03	373	370	360	352	347	345	343	342	340	335	326	322
0.2	232	231	229	229	230	230	229	227	224	214	197	189
0.3	202	201	202	203	205	206	204	203	199	188	170	160
0.5	164	164	167	171	174	175	174	172	168	155	135	124

Table 3.10: Experimental minimum filter order for given filter specification

A_p	θ											
(dB)	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°	43°
0.003	507	503	523	499	475	473	477	479	483	445	515	489
0.005	453	449	437	479	465	465	467	469	467	427	491	461
0.009	443	441	429	443	423	409	407	406	409	413	379	373
0.02	385	383	371	381	391	367	347	347	345	341	363	345
0.06	323	319	311	295	317	321	303	291	283	275	281	267
0.03	335	333	323	349	373	349	341	313	295	285	349	335
0.2	257	287	247	233	219	237	241	257	241	215	217	207
0.3	215	213	205	195	183	205	205	219	213	201	201	197
0.5	175	173	167	159	149	159	171	197	187	179	175	163

Table 3.11: Error between predicted filter order and experimental filter order

A_p	θ											
(dB)	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°	43°

0.003	37	35	-5	1	13	10	4	1	-2	37	-32	-6
0.005	53	52	46	-12	-8	-12	-17	-19	-18	22	-43	-14
0.009	19	17	13	-14	-2	8	8	8	5	-1	29	33
0.02	18	16	17	-3	-19	2	20	19	20	20	-9	5
0.06	-2	0	1	12	-13	-18	-2	9	14	16	-2	6
0.03	38	37	37	3	-26	-4	2	29	45	50	-23	-13
0.2	-25	-56	-18	-4	11	-7	-12	-30	-17	-1	-20	-18
0.3	-13	-12	-3	8	22	1	-1	-16	-14	-13	-31	-37
0.5	-11	-9	0	12	25	16	3	-25	-19	-24	-40	-39

Numerically calculated mean of the absolute error, $\bar{X}_{error}$ and standard deviation of the absolute error σ_{error} found to be 17.0277 and 13.6124.

A. Evaluation of 2-D FIR fan filter order estimation polynomial *Poly21*

Using suggested empirical formula of order estimation *Poly21*, minimum filter order is predicted for 108 filters considering above given specifications. It is given in Table 3.12. Experimental filter order obtained for same fan filter specification is given in Table 3.13. The error between predicted filter order and experimental filter order is shown in Table 3.14.

Table 3.12: Predicted filter order from suggested polynomial of *Poly21*

A_p (dB)	θ											
	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°	43°
0.003	521	519	513	506	499	494	487	484	479	472	465	462
0.005	483	481	476	470	463	458	452	449	445	439	432	430
0.009	440	439	434	428	423	418	413	410	407	401	395	393
0.02	384	383	379	374	370	366	361	359	356	352	347	345
0.06	310	309	306	303	300	297	294	292	290	287	284	282
0.03	356	355	352	348	343	340	336	334	332	327	323	322
0.2	233	233	231	229	227	226	224	223	222	221	219	218
0.3	208	208	207	206	204	203	202	201	201	199	198	197
0.5	178	178	177	176	176	175	174	174	174	173	172	172

Table 3.13 : Error between predicted filter order and experimental filter order

A_p (dB)	θ											
	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°	43°
0.003	14	16	-10	7	24	21	10	5	-4	27	-50	-27
0.005	30	32	39	-9	-2	-7	-15	-20	-22	12	-59	-31
0.009	-3	-2	5	-15	0	9	6	4	-2	-12	16	20
0.02	-1	0	8	-7	-21	-1	14	12	11	11	-16	0
0.06	-13	-10	-5	8	-17	-24	-9	1	7	12	3	15
0.03	21	22	29	-1	-30	-9	-5	21	37	42	-26	-13
0.2	-24	-54	-16	-4	8	-11	-17	-34	-19	6	2	11
0.3	-7	-5	2	11	21	-2	-3	-18	-12	-2	-3	0
0.5	3	5	10	17	27	16	3	-23	-13	-6	-3	9

Numerically calculated mean of the absolute error, $\bar{X}_{error}$ and standard deviation of the absolute error σ_{error} found to be 13.8518 and 11.8706. Summary statistics of *Poly13* and *Poly21* is given in Table 3.14 to determine best estimation formula for order estimation of 2-D FIR fan filter.

Table 3.14: $\bar{X}_{error}$ and σ_{error} for suggested formulas of 2-D FIR filter order

Statistics	Estimation *Poly13*	Estimation *Poly21*
$\bar{X}_{error}$	17.0277	13.8518
σ_{error}	13.6124	11.8706

According to the error statistics, it is found that estimation polynomial *Poly21* gives considerably lower absolute mean and standard deviation of the error distribution. Therefore, *Poly21* is proposed as the best fitted formula to determine filter order of 2-D FIR filter when T=0.01π rad/sample.

3.2.2 Case 02: Design of an empirical formula to estimate order of 2-D FIR fan filter when T = 0.05π rad/sample

The same specifications of 2-D fan filter and assumptions specified in the section 3.2 is considered when the T is 0.05π rad/sample and same analysis is used in the following analysis too, and, for brevity, same content is not mentioned here again.

3.2.2.1 Estimation of minimum order given β

In case 02, computation of minimum filter order achieved β for given θ, $\log_{10} A_p$ is presented when the T is 0.05π rad/sample. To determine the relationship among given θ, $\log_{10} A_p$ with β, minimum filter order achieved β values are selected from the experimental data attached in Appendix A-II and it is tabulated in Table 3.15.

Table 3.15: Experimental minimum filter order achieved β for given θ and A_p when T = 0.05π rad/sample

A_p (dB)	θ =5°	θ =10°	θ =15°	θ =20°	θ =25°	θ =30°	θ =35°
0.001	8	8	8	8	8	8	7
0.004	8	7	7	7	7	6	6
0.007	7	7	6	6	6	6	5
0.01	7	7	6	6	6	6	5
0.04	5	6	5	5	4	4	4
0.07	5	5	5	4	4	4	4
0.1	4	4	5	4	4	3	3
0.4	3	3	3	3	3	3	3
0.7	2	2	2	2	3	3	2

Applying MATLAB curve fitting tool for data given in Table 3.15, suitable fitted curves are observed to obtain the strongest relationship among the variables, log (A_p), θ and β. Then four polynomials as *Poly11, Poly13, Poly14, and Poly23* are selected with good fitness. Goodness-of-fit statistics of selected polynomials are tabulated in Table 3.16.

Comparing goodness-of-fit statistics of chosen four polynomials, well fitted polynomial is proposed to determine the minimum filter order achieving β.

Table 3.16: Goodness-of-fit statistics of selected polynomials of β from curve fitting tool when T =0.05π rad/sample

Statistics	*Poly11*	*Poly13*	*Poly14*	*Poly23*
Sum of squares due to error (SSE)	12.14	7.48	7.35	7.14
R-square	0.94	0.96	0.97	0.97
Adjusted R-square	0.94	0.96	0.96	0.96
Root mean squared error (RMSE)	0.45	0.37	0.37	0.36

Analyzing statistics in Table 3.16, *Poly23* shows least error and highest R square and adjusted R square value with good accuracy. However, with the same R square and adjusted R square *Poly14* gives good accuracy with lower complexity. Therefore, based on the goodness of fit statistics, *Poly 14* is suggested to determine the minimum filter order achieved β for given theta and passband ripple with good accuracy and lower complexity. Behavior of minimum filter order given β for given θ and log A_p with linear model *Poly14* is given in Figure 3.4. Therefore, proposed estimation formula to estimate β could be shown as follows.

- **Estimation formula of minimum filter order given β when T = 0.05π rad/sample**

$$f\ (\theta, \log_{10} A_p) \ = \ a_1 \ + a_2\theta + a_3(\log_{10} A_p) + a_4\theta(\log_{10} A_p) + a_5(\log_{10} A_p)^2 + a_6\theta(\log_{10} A_p)^2 + a_7(\log_{10} A_p)^3 + a_8\theta(\log_{10} A_p)^3 + a_9(\log_{10} A_p)^4,$$

Coefficients (with 95% confidence bounds):

$a_1 \ = \ 1.144$ (0.2208, 2.067)

$a_2 \ = \ 3.65 \times 10^{-2}$ (0.001028, 0.07197)

$a_3 = \ -5.231$ (-8.9, -1.562)

$a_4 = \ 1.025 \times 10^{-1}$ (0.001518, 0.2035)

$a_5 =$ -2.289 (-6.615, 2.038)

$a_6 =$ 2.567×10^{-2} (-0.04947, 0.1008)

$a_7 =$ -8.21×10^{-1} (-2.833, 1.069)

$a_8 =$ -6.567×10^{-4} (-0.01634, 0.01503)

$a_9 =$ -0.1451 (-0.4438, 0.1535).

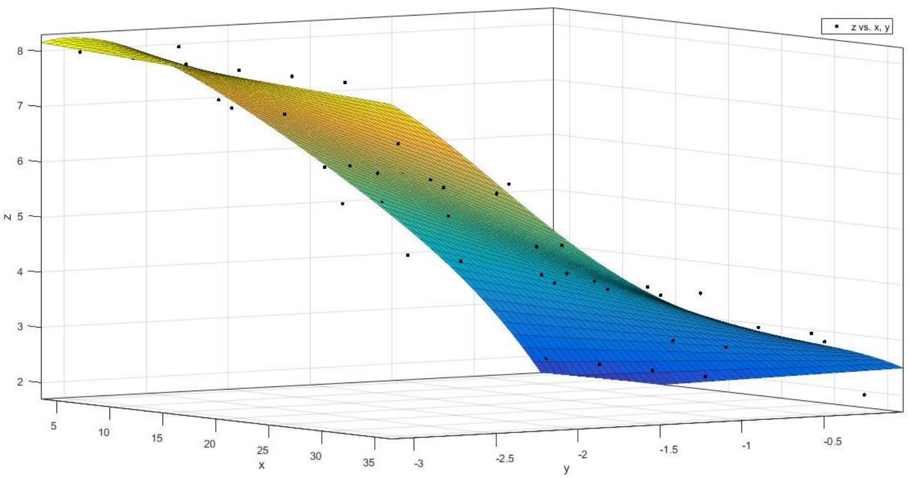


Figure 3.4: Behavior of minimum filter order given β for given θ and log (A_p) with linear model *Poly14*

3.2.2.2 Design of empirical formula for order estimation

Minimum filter orders selected from attached in Appendix A-III for given θ and A_p is given in Table 3.17. Fitting Table 3.17 data, minimum filter order as the response and log value of passband ripple and theta as predictors best fitted curves are observed to obtain the strong relationship among the variables, log A_p , θ and *N*.

Table 3.17: Minimum filter order selected for given θ and A_p when *T* = 0.05π rad/sample

A_p (dB)	θ =5°	θ =10°	θ =15°	θ =20°	θ =25°	θ =30°	θ =35°
0.001	143	137	111	123	113	123	123

0.004	101	103	97	97	97	99	91
0.007	89	93	91	85	85	85	87
0.01	89	91	87	85	83	85	85
0.04	67	71	75	71	65	61	69
0.07	65	61	67	67	61	59	59
0.1	55	53	63	61	59	55	49
0.4	41	39	37	43	43	43	43
0.7	33	31	31	29	35	39	35

Considering observations of multiple fitted polynomials, four polynomials are selected with good fitness and their goodness-of-fit statistics are tabulated in Table 3.18. Comparing goodness-of-fit statistics of chosen polynomials, best fitted polynomial to determine the minimum order of the fan filter is proposed.

Table 3.18: Goodness-of-fit statistics of selected polynomials of N

Statistics	*Poly11*	*Poly13*	*Poly23*	*Poly25*
The sum of squares due to error (SSE)	1854	1463	1138	764
R-square	0.96	0.97	0.98	0.98
Adjusted R-square	0.96	0.97	0.97	0.98
Root mean squared error (RMSE)	5.56	5.11	4.59	3.99

According to the statistics of selected polynomials two formulas are suggested in terms of the lower complexity and the good accuracy. In the context of low complexity *Po1y13* provides fairly good R-square, adjusted R square and comparatively less error. Meanwhile, *Poly25* shows greater R-square, adjusted R square and least error with better goodness compared to other polynomials. However, *Poly23* provides same R-square and nearly equal adjusted R-square. Therefore, *Poly13* is suggested as low complexity empirical formula to estimate the order of the fan filter and *Poly23* is suggested to estimate the order of the fan filter with good accuracy. Behavior of N for given θ and log (A_p) with linear model *Poly13* and *Poly23* are illustrated in Figure 3.5 and Figure 3.6 respectively. Suggested empirical formulas to estimate the order of the filter are presented as follows.

- **Low complexity empirical formula to estimate order of 2-D FIR fan filter**

Linear model *Poly13*:

$$f((\log_{10} A_p),\theta) = a_1 + a_2(\log_{10} A_p) + a_3(\theta) + a_4(\theta)(\log_{10} A_p) + a_5(\theta)^2 + a_6(\log_{10} A_p)(\theta)^2 + a_7(\theta)^3,$$

Coefficients (with 95% confidence bounds):

$a_1 =$	15.69	(2.263, 29.12)
$a_2 =$	-40.4	(-46.37, -34.42)
$a_3 =$	1.274	(-0.9795, 3.527)
$a_4 =$	0.952	(0.2673, 1.637)
$a_5 =$	-3.234×10^{-2}	(-0.1476, 0.08296)
$a_6 =$	-1.81×10^{-2}	(-0.03483, -0.001371)
$a_7 =$	1.481×10^{-4}	(-0.00171, 0.002006).

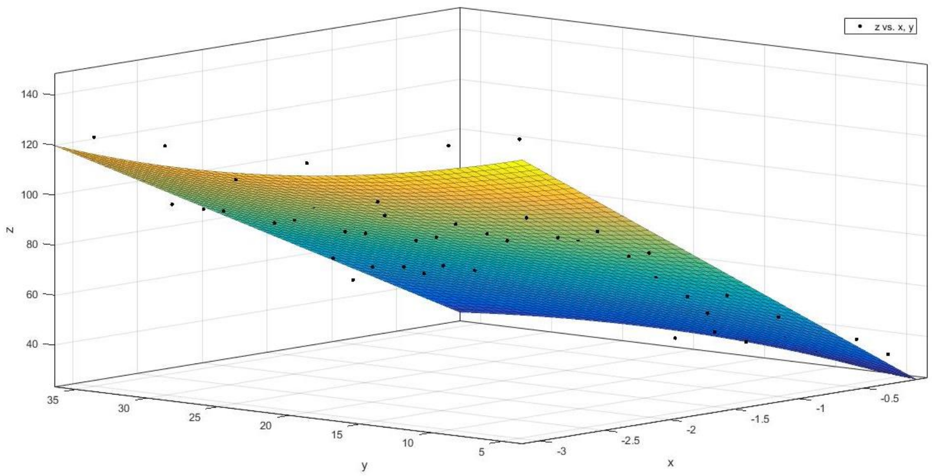


Figure 3.5 : Behavior of *N* for given θ and log A_p when *T* = 0.05π rad/sample with linear model *Poly13*

- **Accurate empirical formula to estimate order of 2-D FIR fan filter**

Linear model *Poly23*:

$$f\left((\log_{10} \mathrm{A_p}), \theta\right) = a_1 + a_2(\log_{10} \mathrm{A_p}) + a_3(\theta) + a_4(\log_{10} \mathrm{A_p})^2 + a_5(\log_{10} \mathrm{A_p})(\theta) + a_6(\theta)^2 + a_7(\log_{10} \mathrm{A_p})^2(\theta) + a_8(\log_{10} \mathrm{A_p})(\theta)^2 + a_9(\theta)^3,$$

Coefficients (with 95% confidence bounds):

$a_1 = \quad 20.55 \quad (7.348, 33.75)$
$a_2 = \quad -30.81 \quad (-42.65, -18.97)$
$a_3 = \quad 1.265 \quad (-0.7745, 3.305)$
$a_4 = \quad 3.132 \quad (-0.3138, 6.578)$
$a_5 = \quad 0.9346 \quad (0.1591, 1.71)$
$a_6 = \quad -0.03234 \quad (-0.136, 0.0713)$
$a_7 = \quad -0.005685 \quad (-0.1598, 0.1484)$
$a_8 = \quad -0.0181 \quad (-0.03314, -0.003063)$
$a_9 = \quad 0.0001481 \quad (-0.001522, 0.001818).$

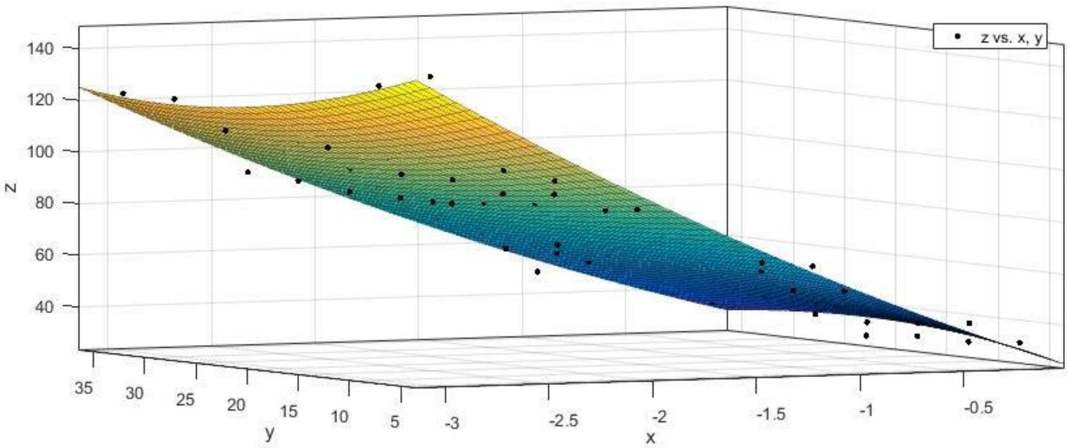


Figure 3.6: Behavior of *N* for given θ and log A_p and *T* = 0.05π rad/sample with linear model *Poly23*

3.2.2.3 Evaluation of proposed formulas for minimum order given β when T=0.05π rad/sample

Similar evaluation as mentioned in case 01 is performed with same specifications and for brevity, same content is not included in this section again and simply evaluation with data and calculations are included. Predicted β using proposed estimation formula *Poly14*, is given in Table 3.19 and experimental data obtained for same fan filter specification is given in Table 3.20. Then error between predicted β and experimental β is shown in Table 3.21.

Table 3.19: Predicted β from proposed estimation polynomial *Poly14*

A_p (dB)	θ											
	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°	43°
0.003	8	8	8	7	7	7	7	7	7	6	6	6
0.005	8	8	7	7	7	7	6	6	6	6	5	5
0.009	7	7	7	7	6	6	6	6	5	5	5	5
0.02	6	6	6	6	6	5	5	5	5	4	4	4
0.06	5	5	5	5	5	4	4	4	4	4	3	3
0.03	6	6	6	5	5	5	5	5	4	4	4	4
0.2	4	4	4	4	4	3	3	3	3	3	3	3
0.3	3	3	3	3	3	3	3	3	3	3	3	3
0.5	3	3	3	3	3	3	3	3	3	3	3	3

Table 3.20: Experimental minimum filter order given β

A_p (dB)	θ											
	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°	43°
0.003	8	8	8	7	7	7	7	7	7	7	6	6
0.005	8	7	8	7	7	6	7	7	6	6	6	6
0.009	7	7	7	6	6	6	6	6	6	5	5	6
0.02	6	6	6	6	5	5	5	5	5	5	4	5
0.06	5	5	5	5	5	4	4	4	4	4	4	4
0.03	4	4	4	4	5	5	5	5	4	4	4	4
0.2	3	3	4	4	4	4	4	3	3	3	3	3

0.3	3	3	3	3	4	4	3	3	3	2	3	3
0.5	2	2	2	2	3	3	3	3	3	2	2	2

Table 3.21: Error between predicted β and experimental β

A_p (dB)	θ											
	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°	43°
0.003	0	0	0	0	0	0	0	0	0	-1	0	0
0.005	0	1	-1	0	0	1	-1	-1	0	0	-1	-1
0.009	0	0	0	1	0	0	0	0	-1	0	0	-1
0.02	0	0	0	0	1	0	0	0	0	-1	0	-1
0.06	0	0	0	0	0	0	0	0	0	0	-1	-1
0.03	2	2	2	1	0	0	0	0	0	0	0	0
0.2	1	1	0	0	0	-1	-1	0	0	0	0	0
0.3	0	0	0	0	-1	-1	0	0	0	1	0	0
0.5	1	1	1	1	0	0	0	0	0	1	1	1

Numerically calculated mean of the absolute error, $\bar{X}_{error}$ and standard deviation of the absolute error σ_{error} found to be 0.3425 and 0.5323. According to the statistical figures obtained it seems that there is less error in predicted β and observed β.

3.2.2.4 Evaluation of suggested formulas for 2-D FIR fan filter order estimation When *T* = 0.05π rad/sample

Evaluation of two estimation formulas is illustrated below as mentioned in previous section.

A. Evaluation of low complexity empirical formula to estimate order of FIR fan filter

Using suggested low complexity empirical formula of order estimation polynomial *Poly13*, minimum filter order is predicted for 108 filters considering the above given specifications. It is given in Table 3.22. Experimental filter order obtained for same fan filter specification is given in Table 3.23. The error between predicted filter order and experimental filter order is shown in Table 3.24.

Table 3.22: Predicted N from proposed estimation *Poly13*

A_p (dB)	θ											
	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°	43°
0.003	115	114	110	106	103	101	100	100	100	101	104	106
0.005	107	106	103	99	96	95	94	94	94	95	97	98
0.009	97	96	94	91	89	88	87	86	86	87	89	90
0.02	84	83	82	80	79	78	77	77	77	77	78	78
0.06	65	65	65	65	65	64	64	63	63	63	63	63
0.03	77	77	76	75	74	73	72	72	72	72	72	73
0.2	45	46	47	49	49	49	49	49	48	47	46	45
0.3	38	39	41	43	44	44	44	44	43	42	40	40
0.5	30	31	33	36	38	38	38	38	37	35	33	32

Table 3.23: Experimental N for given filter specification

A_p (dB)	θ											
	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°	43°
0.003	103	103	101	101	97	97	97	99	101	105	103	103
0.005	101	91	99	97	95	85	97	97	95	89	99	99
0.009	91	89	87	91	87	85	83	85	85	85	77	93
0.02	79	79	77	79	79	75	73	73	71	71	73	73
0.06	67	65	63	63	69	67	61	61	59	57	59	57
0.03	77	77	75	77	77	71	71	71	61	59	71	71
0.2	47	45	51	49	51	53	53	51	49	45	45	45
0.3	45	43	43	41	43	49	45	47	45	45	41	43
0.5	37	35	35	33	35	37	41	41	41	41	37	35

Table 3.24: Error between predicted N and experimental N

A_p (dB)	θ											
	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°	43°

0.003	12	11	9	5	6	4	3	1	-1	-4	1	3
0.005	6	15	4	2	1	10	-3	-3	-1	6	-2	-1
0.009	6	7	7	0	2	3	4	1	1	2	12	-3
0.02	5	4	5	1	0	3	4	4	6	6	5	5
0.06	-2	0	2	2	-4	-3	3	2	4	6	4	6
0.03	0	0	1	-2	-3	2	1	1	11	13	1	2
0.2	-2	1	-4	0	-2	-4	-4	-2	-1	2	1	0
0.3	-7	-4	-2	2	1	-5	-1	-3	-2	-3	-1	-3
0.5	-7	-4	-2	3	3	1	-3	-3	-4	-6	-4	-3

Numerically calculated mean of the absolute error, $\bar{X}_{error}$ and standard deviation of the absolute error σ_{error} found to be 3.5648 and 2.9521.

B. Evaluation of empirical formula to estimate order of 2-D FIR fan filter

Using suggested empirical formula of order estimation polynomial *Poly23*, minimum filter order is predicted for 108 filters considering the above given specifications. It is given in Table 3.25. Experimental filter order obtained for same fan filter specification is given in Table 3.26. The error between predicted filter order and experimental filter order is shown in Table 3.27.

Table 3.25: Predicted filter order from proposed polynomial *Poly23*

A_p	θ											
(dB)	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°	43°
0.003	116	115	111	107	104	102	100	100	100	102	105	106
0.005	106	105	102	99	96	94	93	93	93	94	96	98
0.009	95	95	92	90	87	86	85	85	85	86	87	88
0.02	81	81	80	78	76	75	75	74	74	75	76	76
0.06	63	63	63	63	63	62	62	61	61	61	61	61
0.03	74	74	73	72	71	70	70	69	69	69	70	70
0.2	45	45	47	48	49	49	49	48	48	47	46	45
0.3	39	40	42	44	45	45	45	45	44	43	41	40
0.5	32	33	36	38	40	40	40	40	39	38	35	34

Table 3.26 : Error between predicted N and experimental N

A_p (dB)	θ											
	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°	43°
0.003	13	12	10	6	7	5	3	1	-1	-3	2	3
0.005	5	14	3	2	1	9	-4	-4	-2	5	-3	-1
0.009	4	6	5	-1	0	1	2	0	0	1	10	-5
0.02	2	2	3	-1	-3	0	2	1	3	4	3	3
0.06	-4	-2	0	0	-6	-5	1	0	2	4	2	4
0.03	-3	-3	-2	-5	-6	-1	-1	-2	8	10	-1	-1
0.2	-2	0	-4	-1	-2	-4	-4	-3	-1	2	1	0
0.3	-6	-3	-1	3	2	-4	0	-2	-1	-2	0	-3
0.5	-5	-2	1	5	5	3	-1	-1	-2	-3	-2	-1

Numerically calculated mean of the absolute error, $\bar{X}_{error}$ and standard deviation of the absolute error σ_{error} found to be 3.1111 and 2.7896.

Table 3.27: $\bar{X}_{error}$ and σ_{error} for suggested formulas of 2-D FIR fan filter order

Statistics	*Poly13*	*Poly23*
$\bar{X}_{error}$	3.5648	3.1111
σ_{error}	2.9521	2.7896

According to the error statistics, *Poly23* gives significant less mean and less standard deviation of the absolute error distribution than *Poly13*. Therefore, *Poly23* is proposed as the best fitted formula to determine filter order of 2-D FIR filter when T = 0.05π rad/sample.

3.2.3 Case 03: Design of an empirical formula to estimate order of 2-D FIR fan filter when T = 0.1π rad/sample

The same specifications of 2-D fan filter and assumptions specified in the section 3.2 is considered when the T is 0.1π rad/sample and same analysis is used in the following analysis too, and, for brevity, same content is not mentioned here again.

3.2.3.1 Estimation of minimum order given β

In case 03, computation of minimum filter order achieved β for given θ, log (A_p) is presented when T is 0.1π rad/sample. In order to determine the relationship among given θ, log (A_p) with β, minimum filter order achieved β values are selected from the experimental data given in Appendix A-IV and it is tabulated in Table 3.28.

Table 3.28: Minimum filter order achieved β for given θ and A_p from experimental data when T = 0.1π rad/sample

A_p(dB)	θ =5°	θ =10°	θ =15°	θ =20°	θ =25°	θ =30°	θ =35°
0.001	8	8	8	8	8	8	7
0.004	8	7	7	7	7	6	6
0.007	7	7	7	6	6	6	6
0.01	7	6	6	6	6	6	5
0.04	5	6	5	5	4	4	4
0.07	5	5	5	4	4	4	4
0.1	4	5	4	4	3	3	3
0.4	3	3	3	3	3	3	2
0.7	2	2	2	2	2	2	2

Applying curve fitting tool in MATLAB for data given in Table 3.28, two polynomials, *Poly13* and *Poly23* are selected from others with fairly good fitness. Goodness-of-fit statistics of selected polynomials are tabulated in Table 3.29.

Table 3.29: Goodness-of-fit statistics for selected polynomials of β from curve fitting tool when $T = 0.1\pi$ rad/sample

Statistics	*Poly13*	*Poly23*
Sum of squares due to error (SSE)	6.956	6.823
R-square	0.970	0.970
Adjusted R-square	0.966	0.966
Root mean squared error (RMSE)	0.352	0.355

When compare the goodness-of-fit statistics of chosen two polynomials, *Poly13* gives the better R-square and adjusted R-square with less error. Therefore, based on the fit statistics, *Poly13* is proposed as the best fitted curve to gain minimum filter order achieved β for the given set of experimental data with the lower complexity and good accuracy. Behavior of minimum filter order given β for given θ and log (A_p) with linear model *Poly13* is given in Figure 3.7 and proposed estimation formula is given as follows.

- **Estimation formula of minimum order given β when $T = 0.1\pi$ rad/sample**

Linear Model *Poly13*

$$f\left(\theta, \log_{10} A_p\right) = a_1 + a_2\theta + a_3\left(\log_{10} A_p\right) + a_4\theta\left(\log_{10} A_p\right) + a_5\left(\log_{10} A_p\right)^2 + a_6\theta\left(\log_{10} A_p\right)^2 + a_7\left(\log_{10} A_p\right)^3,$$

Coefficients (with 95% confidence bounds):

$a_1 = 1.7$ $(1.09, 2.311)$

$a_2 = 5.629 \times 10^{-3}$ $(-0.01976, 0.03102)$

$a_3 = -3.435$ $(-4.657, -2.213)$

$a_4 = 7.12 \times 10^{-2}$ $(0.03366, 0.1088)$

$a_5 = -2.614 \times 10^{-1}$ $(-1.017, 0.4939)$

$a_6 = 2.078 \times 10^{-2}$ $(0.008959, 0.0326)$

$a_7 = 5.024 \times 10^{-2}$ $(-0.09948, 0.2)$.

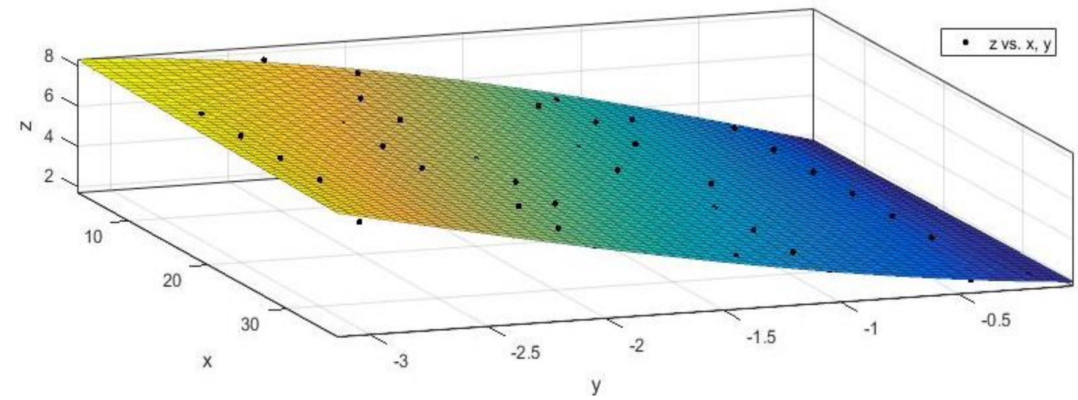


Figure 3.7: Behavior of minimum filter order given β for given θ and log *(A_p)* with linear model *Poly13*

3.2.3.2 Design of empirical formula for order estimation

Minimum filter orders selected from attached in Appendix A-V for given θ and A_p and it is given in Table 3.30. Fitting Table 3.30 data, minimum filter order as the response and log value of passband ripple and theta as predictors best fitted curves are observed to obtain the strongest relationship among the variables, log *(A_p)*, θ and *N*.

Table 3.30: Minimum filter order selected for given θ and A_p when *T* = 0.1π rad/sample

A_p(dB)	θ =5°	θ =10°	θ =15°	θ =20°	θ =25°	θ =30°	θ =35°
0.001	87	71	67	63	57	63	63
0.004	51	53	49	49	49	51	47
0.007	45	49	49	43	43	43	45
0.01	45	47	45	43	43	43	43
0.04	35	37	37	37	33	31	31
0.07	33	31	35	33	31	31	31
0.1	29	31	33	31	31	29	25
0.4	21	21	19	21	25	23	23
0.7	17	17	15	17	17	19	17

Observing multiple fitted polynomials four polynomials are selected and their goodness-of-fit statistics are tabulated in Table 3.31. Comparing goodness-of-fit statistics of chosen polynomials, best fitted polynomial to determine the minimum order of the fan filter is proposed.

Table 3.31: Goodness-of-fit statistics for selected polynomials of order

Statistics	*Poly13*	*Poly14*	*Poly23*	*Poly24*
The sum of squares due to error (SSE)	762.2	756.9	508.6	380.5
R-square	0.946	0.946	0.964	0.973
Adjusted R-square	0.940	0.938	0.959	0.967
Root mean squared error (RMSE)	3.689	3.744	3.069	2.732

According to the statistics of selected polynomials two formulas are suggested in terms of the lower complexity and the good accuracy. In the context of low complexity *Po1y13* provides fairly good R-square, adjusted R square and comparatively less error. Meanwhile *Poly24* shows greater R-square, adjusted R square and least error with better goodness compared to other polynomials. Therefore, *Poly13* is suggested as low complexity empirical formula to estimate the order of the fan filter and *Poly24* is suggested to estimate the order of the fan filter with good accuracy. Behavior of N for given θ and log (A_p), with linear model *Poly13* and *Poly24* are illustrated in Figure 3.8 and Figure 3.9 respectively. Suggested empirical formulas to estimate the order of the filter are presented as follows.

- **Low complexity empirical formula to estimate order of 2-D FIR fan filter**

Linear model *Poly13*:

$$f((\log_{10} A_p),\theta) = a_1 + a_2(\log_{10} A_p) + a_3(\theta) + a_4(\theta)(\log_{10} A_p) + a_5(\theta)^2 + a_6(\log_{10} A_p)(\theta)^2 + a_7(\theta)^3,$$

Coefficients (with 95% confidence bounds):

$a_1 =$ 4.641 $(-5.052, 14.34)$

$a_2 =$ -24.04 $(-28.35, -19.73)$

$a_3 =$ 1.014 $(-0.6126, 2.64)$

$a_4 =$ 0.7441 $(0.2499, 1.238)$

$a_5 = \quad -2.69 \times 10^{-2} \qquad (-0.1101, 0.05632)$

$a_6 = \quad -1.389 \times 10^{-2} \qquad (-0.02596, -0.001814)$

$A_7 = \quad 1.481 \times 10^{-4} \qquad (-0.001193, 0.001489).$

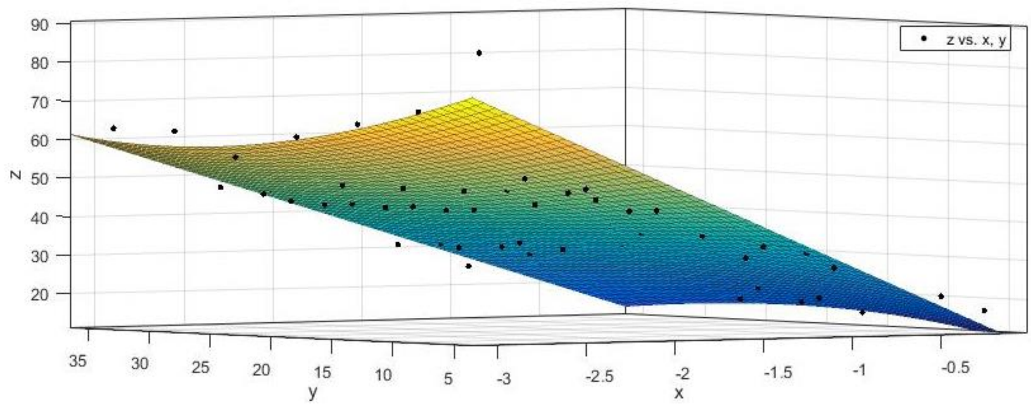


Figure 3.8: Behavior of *N* for given θ and log (A_p) when *T* = 0.1π rad/sample with linear model *Poly13*

- **Accurate empirical formula to estimate order of 2-D FIR fan filter**

Linear model *Poly24*:

$$f\big((\log_{10} \mathrm{A_p}), \theta\big) = a_1 + a_2(\log_{10} \mathrm{A_p}) + a_3(\theta) + a_4(\log_{10} \mathrm{A_p})^2 + a_5(\log_{10} \mathrm{A_p})(\theta) + a_6(\theta)^2 + a_7(\log_{10} \mathrm{A_p})^2(\theta) + a_8(\log_{10} \mathrm{A_p})(\theta)^2 + a_9(\theta)^3 + a_{10}(\log_{10} \mathrm{A_p})^2(\theta)^2 + a_{11}(\log_{10} \mathrm{A_p})(\theta)^3 + a_{12}(\theta)^4,$$

Coefficients (with 95% confidence bounds):

$a_1 =$	20.22	$(4.764, 35.68)$
$a_2 =$	10.23	$(-2.802, 23.26)$
$a_3 =$	7.368×10^{-2}	$(-3.726, 3.873)$
$a_4 =$	10.75	$(6.967, 14.54)$
$a_5 =$	-2.466	$(-4.258, -0.6741)$
$a_6 =$	-6.235×10^{-2}	$(-0.3828, 0.2581)$
$a_7 =$	-9.479×10^{-1}	$(-1.382, -0.514)$

a_8 = 6.974×10^{-2} $(-0.005291, 0.1448)$

a_9 = 3.248×10^{-3} (-0.007925, 0.01442)

a_{10} = 2.143×10^{-2} $(0.01082, 0.03203)$

a_{11} = -3.004×10^{-4} $(-0.001418, 0.000817)$

a_{12} = -4.444×10^{-5} $(-0.0001819, 9.303\text{e}-05)$.

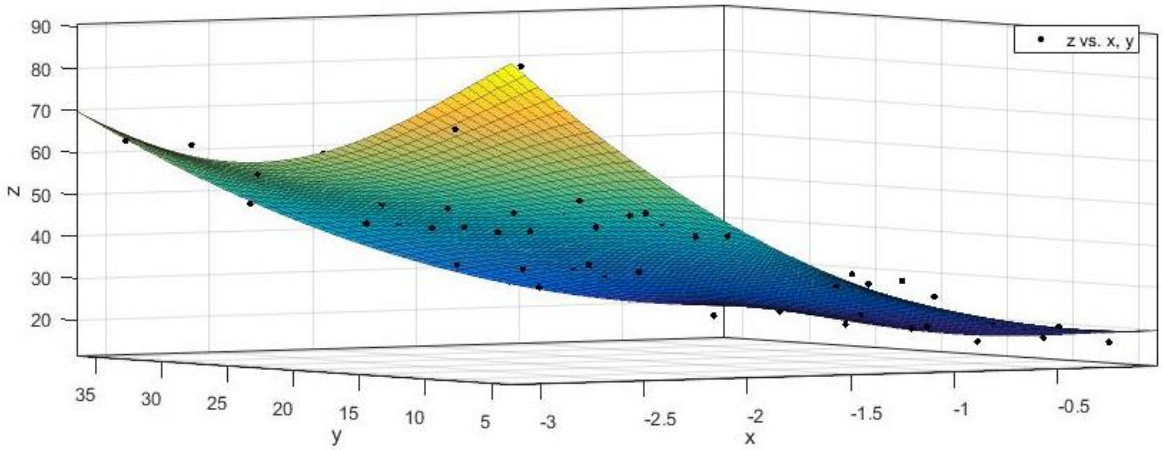


Figure 3.9: Behavior of N for given θ and log (A_p) and T = 0.1π rad/sample with linear model *Poly24*

3.2.3.3 Evaluation of proposed formulas for minimum order given β

Similar evaluation as mentioned in case 01 and case 02 is performed with same specifications and for brevity, same content is not included in this section again and simply evaluation with data and calculations are indicated. Note that for given FIR fan filter specification, experimental values of β and N for given $\theta = 43°$ could not be achieved. Therefore, same analysis performed ignoring data for given $\theta = 43°$. Predicted β using proposed estimation formula *Poly13*, is given in Table 3.32 and experimental data obtained for same fan filter specification is given in Table 3.33. Then error between predicted β and experimental β is shown in Table 3.34.

Table 3.32: Predicted β from proposed estimation formula in *Poly13*

A_p (dB)	θ										
	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°
0.003	8	8	8	7	7	7	7	7	7	6	6
0.005	8	7	7	7	7	7	6	6	6	6	6
0.009	7	7	7	7	6	6	6	6	6	5	5
0.02	6	6	6	6	6	5	5	5	5	5	4
0.06	5	5	5	5	5	4	4	4	4	4	3
0.03	6	6	6	5	5	5	5	5	4	4	4
0.2	4	4	4	4	3	3	3	3	3	3	3
0.3	3	3	3	3	3	3	3	3	3	2	2
0.5	3	3	3	3	2	2	2	2	2	2	2

Table 3.33:Experimental minimum filter order given β

A_p (dB)	θ										
	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°
0.003	7	8	8	8	7	7	7	7	6	6	6
0.005	8	8	8	7	7	7	6	6	6	6	6
0.009	7	7	7	6	6	6	6	6	6	5	6
0.02	6	6	6	6	5	5	5	5	5	5	5
0.06	5	5	5	5	5	4	4	4	4	4	4
0.03	5	6	6	5	5	5	4	5	4	4	4
0.2	3	3	3	4	3	3	3	3	3	3	3
0.3	3	3	3	3	3	3	3	3	3	2	3
0.5	1	2	3	2	3	3	3	3	2	2	2

Table 3.34 Error between predicted β and experimental β

A_p (dB)	θ										
	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°
0.003	1	0	0	-1	0	0	0	0	1	0	0
0.005	0	-1	-1	0	0	0	0	0	0	0	0
0.009	0	0	0	1	0	0	0	0	0	0	-1

0.02	0	0	0	0	1	0	0	0	0	0	-1
0.06	0	0	0	0	0	0	0	0	0	0	-1
0.03	1	0	0	0	0	0	1	0	0	0	0
0.2	1	1	1	0	0	0	0	0	0	0	0
0.3	0	0	0	0	0	0	0	0	0	0	-1
0.5	2	1	-2	1	-1	-1	-1	-1	0	0	0

Numerically calculated mean of the absolute error, $\bar{X}_{error}$ and standard deviation of the absolute error σ_{error} found to be 0.2626 and 0.4862. According to the statistical figures obtained it seems that there is less error in predicted β and observed β.

3.2.3.4 Evaluation of proposed formulas for 2-D FIR fan filter order when T = 0.1π rad/sample

Evaluation of two estimation formulas is illustrated below as mentioned in previous section.

A. Evaluation of low complexity empirical formula to estimate order of FIR fan filter

Using suggested low complexity empirical formula of order estimation *Poly13*, minimum filter order is predicted for 99 filter for above given specifications. It is given in Table 3.35. Experimental filter order obtained for same fan filter specification is given in Table 3.36. The error between predicted filter order and experimental filter order is shown in Table 3.37.

Table 3.35: Predicted minimum filter order from proposed formula *Poly13*

A_p (dB)	θ										
	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°
0.003	64	63	60	56	54	52	51	51	51	52	54
0.005	59	58	55	53	50	49	48	48	48	48	50
0.009	53	52	50	48	46	45	44	44	44	44	46
0.02	45	45	44	42	41	40	39	39	39	39	40

0.06	34	34	34	34	34	33	33	32	32	32	32
0.03	41	41	40	39	38	38	37	37	36	36	37
0.2	22	23	24	25	26	26	25	25	25	24	23
0.3	18	19	21	22	23	23	23	23	22	21	20
0.5	13	14	16	18	19	20	20	19	19	18	16

Table 3.36: Experimental minimum filter order for given filter specification

A_p	θ										
(dB)	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°
0.003	69	53	53	53	49	49	51	51	51	47	55
0.005	51	51	51	51	49	49	51	51	49	45	53
0.009	47	45	45	47	45	43	43	43	43	43	47
0.02	41	41	39	41	41	39	37	37	37	37	37
0.06	37	33	33	35	35	31	31	31	31	31	31
0.03	35	39	39	41	39	37	33	37	37	41	37
0.2	23	23	23	25	31	29	27	25	25	25	23
0.3	23	23	23	21	23	25	25	23	23	23	23
0.5	17	19	21	17	19	21	21	23	23	21	19

Table 3.37: Error between predicted filter order and experimental filter order

A_p	θ										
(dB)	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°
0.003	-5	10	7	3	5	3	0	0	0	5	-1
0.005	8	7	4	2	1	0	-3	-3	-1	3	-3
0.009	6	7	5	1	1	2	1	1	1	1	-1
0.02	4	4	5	1	0	1	2	2	2	2	3
0.06	-3	1	1	-1	-1	2	2	1	1	1	1
0.03	6	2	1	-2	-1	1	4	0	-1	-5	0
0.2	-1	0	1	0	-5	-3	-2	0	0	-1	0
0.3	-5	-4	-2	1	0	-2	-2	0	-1	-2	-3
0.5	-4	-5	-5	1	0	-1	-1	-4	-4	-3	-3

Numerically calculated mean of the error, $\bar{X}_{error}$ and standard deviation of the error σ_{error} found to be 2.3333 and 2.0751.

B. Evaluation of empirical formula to estimate filter order of 2-D FIR fan filter

Using suggested empirical formula, *Poly24*, minimum filter order is predicted for 99 filters considering the above given specifications. It is given in Table 3.37. The error between predicted filter order and experimental filter order is shown in Table 3.38.

Table 3.38: Predicted filter order from proposed formula *Poly24*

A_p (dB)	θ										
	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°
0.003	63	63	61	57	54	52	51	51	51	52	53
0.005	55	55	55	52	50	49	48	47	47	47	46
0.009	46	47	48	47	46	45	44	44	43	42	38
0.02	36	37	40	41	40	40	39	39	38	35	30
0.06	27	28	31	33	33	33	32	32	31	28	22
0.03	32	33	36	38	37	37	36	36	35	32	26
0.2	21	22	24	25	25	25	25	25	25	23	17
0.3	20	20	22	23	23	23	23	23	23	21	16
0.5	19	20	20	20	20	20	20	21	21	20	16

Table 3.39 : Error between predicted filter order and experimental filter order

A_p (dB)	θ										
	2°	3°	7°	12°	17°	21°	26°	28°	31°	36°	41°
0.003	-6	10	8	4	5	3	0	0	0	5	-2
0.005	4	4	4	1	1	0	-3	-4	-2	2	-7
0.009	-1	2	3	0	1	2	1	1	0	-1	-9
0.02	-5	-4	1	0	-1	1	2	2	1	-2	-7
0.06	-10	-5	-2	-2	-2	2	1	1	0	-3	-9
0.03	-3	-6	-3	-3	-2	0	3	-1	-2	-9	-11
0.2	-2	-1	1	0	-6	-4	-2	0	0	-2	-6

0.3	-3	-3	-1	2	0	-2	-2	0	0	-2	-7
0.5	2	1	-1	3	1	-1	-1	-2	-2	-1	-3

Numerically calculated Mean of the absolute error, $\bar{X}_{error}$ and standard deviation of the absolute error σ_{error} found to be 2.6868 and 2.5620.

Table 3.40: $\bar{X}_{error}$ and σ_{error} for suggested formulas of 2-D FIR filter order

Statistics	*Poly13*	*Poly24*
$\bar{X}_{error}$	2.3333	2.6868
σ_{error}	2.0751	2.5620

According to the error statistics, estimation *Poly13* gives considerable low mean and standard deviation than the *Poly24* of the error distribution. Therefore, *Poly13* is proposed as the best fitted formula to determine filter order of 2-D FIR filter when T is 0.1π rad/sample.

3.2.4 Discussion

In this chapter, I proposed empirical formulas to estimate order of 2-D FIR fan filter for three different values of transition width of the fan filter. In section 3.2, order estimation approach of the 2-D FIR fan filter is explained in two steps for given θ and $\log A_p$. In the first step, minimum order given β is calculated and in the second steps two estimation formulas are suggested in the context of lower complexity and good accuracy by studying their goodness of fit statistics. Then each suggested polynomials are evaluated by predicting β and N from suggested polynomials. From evaluating the error of suggested formula, statistically best polynomial is proposed to estimate the order of 2-D FIR fan filter. This analysis is performed for transition width of 0.01π rad/sample, 0.05π rad/sample and 0.1π rad/sample. All the assumptions considered during the estimation procedure is mentioned before stating the analysis. Summary of the proposed estimation formulas is given in Table 3.40 and Table 3.41.

Table 3.41: Summary of the proposed β estimation formula

T	β estimation formula	Statistical results
0.01π rad/sample	$f\ (\theta, \log_{10} A_p)\ =\ 0.602\ + 0.051(\theta) - 7.395(\log_{10} A_p) +\ 0.134(\theta)(\log_{10} A_p) - 4.774(\log_{10} A_p)^2 + 0.054(\theta)(\log_{10} A_p)^2 - 1.875\ (\log_{10} A_p)^3 + 0.005(\theta)(\log_{10} A_p)^3 - 0.277\ (\log_{10} A_p)^4$	$\bar{X}_{error}$ = 0.2314 σ_{error} = 0.4237
0.05π rad/sample	$f\ (\theta, \log_{10} A_p) =\ 1.144\ +\ 0.037\ (\theta) - 5.231(\log_{10} A_p) +\ 0.103(\theta)(\log_{10} A_p) - 2.289\ (\log_{10} A_p)^2 +\ 0.025\ (\theta)(\log_{10} A_p)^2 - 0.821(\log_{10} A_p)^3 - 0.145(\log_{10} A_p)^4$	$\bar{X}_{error}$ = 0.3425 σ_{error} = 0.5323
0.1π rad/sample	$f\ (\theta, \log_{10} A_p) =\ 1.7\ +\ 0.006\ (\theta) - 3.435(\log_{10} A_p) + 0.071(\theta)(\log_{10} A_p) - 0.261(\log_{10} A_p)^2 +\ 0.002(\theta)(\log_{10} A_p)^2\ + 0.05(\log_{10} A_p)^3$	$\bar{X}_{error}$ = 0.2626 σ_{error} = 0.4862

Table 3.42: Summary of the proposed N estimation formula

T	N estimation Formula	Statistical results
0.01π rad/sample	$f((\log_{10} \mathrm{A_p}), \theta) =\ 137.5 - 131.7\ (\log_{10} \mathrm{A_p})\ + 0.039\ (\theta) +\ 8.425\ (\log_{10} \mathrm{A_p})^2 + 0.577\ (\log_{10} \mathrm{A_p})\ (\theta)$	$\bar{X}_{error}$ = 13.8518 σ_{error} = 11.8706
0.05π rad/sample	$f((\log_{10} A_p), \theta) = 20.55 - 30.81\ (\log_{10} \mathrm{A_p}) + 1.265\ (\theta)\ +\ 3.132\ (\log_{10} \mathrm{A_p})^2\ + 0.935\ (\log_{10} \mathrm{A_p})\ (\theta) - 0.032\ (\theta)^2 - 0.006\ (\log_{10} \mathrm{A_p})^2\ (\theta) - 0.018\ (\log_{10} \mathrm{A_p})\ (\theta)^2$	$\bar{X}_{error}$ = 3.1111 σ_{error} = 2.7896
0.1π rad/sample	$f((\log_{10} A_p), \theta) =\ 4.641 - 24.04\ (\log_{10} A_p) + 1.014(\theta) + 0.744(\theta)(\log_{10} A_p) - 0.027(\theta)^2 - 0.014(\log_{10} A_p)(\theta)^2$	$\bar{X}_{error}$ = 2.3333 σ_{error} = 2.0751

When designing 2-D FIR fan filter, few assumptions are considered. They are B is equal to π rad/sample and α is considered as 0 degrees. Also N_1 and N_2 are always odd filter order and equal in order. So proposed formula is established only considering odd filter order of the fan filter and even filter order are not considered. When consider the error distribution of the proposed β estimation formulas, it could be found that maximum difference of two, between estimated and the observed. It is evidenced by maximum σ_{error} of 0.5323 for T = 0.05π rad/sample given in Table 3.40. Further, it shows less deviation between estimated and the observed for lower T. Therefore, statistical figures of β estimation formulas indicate that there is very small, negligible error between estimated and observed.

Next, statistics of errors of order estimation formulas are summarized in Table 3.41. It is shown that significant error between predicted and the observed for given specification of 2-D FIR fan filter. However, it shows nearly $\bar{X}_{error}$ of 13.8518 and maximum σ_{error} of 11.8706. So, it is significant.

This low accuracy for T=0.01π rad/sample might be due to the relationships between the filter specifications and β and the order N is highlight nonlinear when the transition band is less than 0.05π rad/sample. So, we need higher order polynomials in order to have more accurate estimates, and these will be long and not user friendly. Furthermore, in the observed filter order designing, β employed as a direct input. Nevertheless, order estimation by proposed formula is irrespective of β. So, to minimize errors above considerations need to be addressed.

CHAPTER FOUR

4 CONCLUSION AND FUTURE WORK

4.1 Conclusions

In this thesis, accurate and low complexity estimation formulas are proposed to estimate the order of 2-D FIR fan filter for three different cases of transition width. Furthermore, another three formulas are proposed to determine Kaiser window parameter β which is given minimum order of the 2-D fan filter. As the first case, minimum order of 2-D FIR fan filter and relevant minimum order given β is estimated when 0.01π rad/sample of transition width. Same approach is carried out to the remaining two cases when transition widths are 0.05π rad/sample and 0.1π rad/sample. These proposed formulas facilitate anyone to easily estimate the accurate order of 2-D FIR fan filter avoiding trial and error. So then design time could be reduced and directly leads to less computational cost in a software implementation.

First, minimum filter order is observed which achieve filter specification for given θ, β and A_p under the predefined assumptions for three different transition width of 0.01π rad/sample, 0.05π rad/sample and 0.1π rad/sample. Then observing the data relationship among the variables converted to estimation formula in two steps whereas, estimating minimum filter order given β and then the relevant order of 2-D FIR fan filter. In the design procedure, two well fitted estimation formulas are suggested for each case. Then numerically evaluating the suggested formulas, one empirical formula is proposed to estimate the order of the fan filter for each case as given below.

When T = 0.01π rad/sample, empirical formula of 2-D FIR fan filter order estimation

$$f\big((\log_{10} \mathrm{A_p}), \theta\big) = 137.5 - 131.7(\log_{10} \mathrm{A_p}) + 0.039(\theta) + 8.425\,(\log_{10} \mathrm{A_p})^2 + 0.577\,(\log_{10} \mathrm{A_p})\,(\theta)$$

When T = 0.05π rad/sample, empirical formula of 2-D FIR fan filter order estimation

$$f((\log_{10} A_p), \theta) = 20.55 - 30.81\,(\log_{10} \mathrm{A_p}) + 1.265\,(\theta) + 3.132\,(\log_{10} \mathrm{A_p})^2 + 0.935\,(\log_{10} \mathrm{A_p})\,(\theta) - 0.032\,(\theta)^2 - 0.006\,(\log_{10} \mathrm{A_p})^2\,(\theta) - 0.018\,(\log_{10} \mathrm{A_p})\,(\theta)^2$$

When T = 0.1π rad/sample, empirical formula of 2-D FIR fan filter order estimation

$$f((\log_{10} A_p), \theta) = 4.641 - 24.04\,(\log_{10} A_p) + 1.014(\theta) + 0.744(\theta)(\log_{10} A_p) - 0.027(\theta)^2 - 0.014(\log_{10} A_p)(\theta)^2$$

Mean and the standard deviation of the error which is the result of estimated and observed considered as the numerical measures and summary is given in section 3.3. When consider the statistical mean of absolute error between estimated and the observed, β and N which is given in Table 4.1 realize much better accuracy. It is found that the mean of absolute error of estimated β and observed β is less than one and for minimum filter order it is slightly different. Mean of absolute error for transition width of 0.05π rad/sample and 0.1π rad/sample is varied by 2-3 of required order and only for transition width of 0.01π rad/sample it takes around 14. However, these statistical figures realize very good accuracy of estimated empirical formula for order estimation.

Table 4.1: Summary statistical mean of absolute error between estimated and the observed values of β and N

T	Absolute $\bar{X}_{error}$ of β	Absolute $\bar{X}_{error}$ of N
0.01π rad/sample	0.23	13.85
0.05π rad/sample	0.34	3.11
0.1π rad/sample	0.26	2.33

4.2 Future work

The design order estimation formulas for 2-D FIR fan filter, only based on odd and equal filter order. In the design procedure even filter order and different filter order of N_1 and N_2 is not considered. Since the design formulas established for 2-D FIR fan filters of odd filter order, the estimation formulas for every integer filter order can be newly considered. In this proposed approach of order estimation, we considered only *β,* θ and A_p for given T and estimation formula is proposed as a function of θ andA_p. Here we assume few of fan filter parameters whereas B is equal to π rad/sample and α is 0 degrees to reduce the design complexity. However, varying these filter parameter alike θ and A_p new estimation formula can be derived. Then single formula can be designed to obtain filter order of the 2-D FIR fan filter providing all the filter parameters at once. Furthermore, this approach can be extended to estimate the order of 3-D FIR fan filter and multidimensional fan filters as well.

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APPENDICES

A. Experimental minimum filter order obtained for given filter specifications

Appendix A-I : Experimental minimum filter order for 864 filters, for given β and A_p when T = 0.01π rad/sample

A_p	2							
(dB)	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	-	-	667	507
0.005	-	-	-	-	-	-	453	499
0.009	-	-	-	-	-	487	443	487
0.02	-	-	-	-	591	385	427	467
0.06	-	-	-	399	323	361	397	431
0.03	-	-	-	-	335	377	417	455
0.2	-	713	351	257	289	321	351	379
0.3	-	509	215	245	275	303	331	357
0.5	503	175	199	227	253	279	303	327
A_p	3							
(dB)	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	-	-	663	503
0.005	-	-	-	-	-	-	449	495
0.009	-	-	-	-	-	485	441	483
0.02	-	-	-	-	589	383	425	465
0.06	-	-	-	397	319	357	395	429
0.03	-	-	-	-	333	375	414	453
0.2	-	711	349	255	287	319	347	375
0.3	-	507	213	243	273	301	329	353
0.5	503	173	197	225	251	277	301	323
A_p	7							
(dB)	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8

0.003	-	-	-	-	-	-	523	491
0.005	-	-	-	-	-	-	437	483
0.009	-	-	-	-	-	471	429	471
0.02	-	-	-	-	575	371	413	453
0.06	-	-	-	387	311	347	383	417
0.03	-	-	-	-	323	363	403	441
0.2	-	701	339	247	277	309	337	365
0.3	-	333	205	235	263	291	317	343
0.5	495	167	191	217	243	267	291	313
A_p (dB)	12							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	-	-	499	515
0.005	-	-	-	-	-	759	479	497
0.009	-	-	-	-	-	451	443	467
0.02	-	-	-	-	413	381	397	437
0.06	-	-	-	371	295	333	369	401
0.03	-	-	-	-	399	349	387	425
0.2	-	683	325	233	265	295	323	349
0.3	-	321	195	223	251	277	303	327
0.5	483	159	181	205	229	253	277	297
A_p (dB)	17							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	-	-	475	519
0.005	-	-	-	-	-	573	465	507
0.009	-	-	-	-	-	423	451	487
0.02	-	-	-	-	391	391	421	449
0.06	-	-	-	349	317	325	351	383
0.03	-	-	-	691	373	375	399	421
0.2	-	661	309	219	249	279	307	331
0.3	669	307	183	209	235	263	287	311

0.5	469	149	169	191	215	239	261	281
A_p (dB)	21							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	-	-	473	523
0.005	-	-	-	-	-	545	465	511
0.009	-	-	-	-	687	409	453	493
0.02	-	-	-	-	367	391	427	461
0.06	-	-	-	331	321	347	371	395
0.03	-	-	-	663	349	379	411	439
0.2	-	469	291	237	251	267	293	317
0.3	649	293	205	201	225	251	273	295
0.5	453	269	159	181	205	227	247	265
A_p (dB)	26							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	-	665	477	529
0.005	-	-	-	-	-	505	467	515
0.009	-	-	-	-	493	407	453	497
0.02	-	-		637	347	391	431	467
0.06	-	-	621	303	321	353	383	409
0.03	-	-	-	467	341	379	415	447
0.2	-	443	269	241	263	283	303	323
0.3	619	273	205	219	237	255	271	287
0.5	431	251	171	181	191	211	229	247
A_p (dB)	28							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	-	507	479	531
0.005	-	-	-	-	-	491	469	517
0.009	-	-	-	-	479	407	457	501
0.02	-	-		619	347	393	437	475
0.06	-	-	605	291	327	365	399	431

0.03	-	-	-	313	341	385	425	461
0.2	-	431	257	259	287	315	341	365
0.3	605	263	219	243	267	291	315	335
0.5	419	241	197	215	235	255	273	291
A_p (dB)	31							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	-	483	487	539
0.005	-	-	-	-	767	467	473	521
0.009	-	-	-	-	455	409	457	501
0.02	-	-	-	441	345	391	431	469
0.06	-	-	425	283	319	355	389	421
0.03	-	-	-	295	337	379	417	453
0.2	-	413	241	249	277	305	329	353
0.3	583	251	213	233	253	281	303	323
0.5	403	277	187	207	227	245	263	281
A_p (dB)	36							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	-	445	507	549
0.005	-	-	-	-	569	427	485	529
0.009	-	-	-	-	417	413	463	505
0.02	-	-	-	403	341	391	435	471
0.06	-	-	529	275	315	355	389	419
0.03	-	-		285	333	379	419	453
0.2	-	375	215	241	269	295	319	343
0.3	545	229	201	223	249	273	297	317
0.5	369	203	179	201	223	243	261	279
A_p (dB)	41							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	665	515	531	569
0.005	-	-	-	-	515	491	513	553

0.009	-	-	-	-	379	453	491	531
0.02	-	-	-	363	359	417	461	499
0.06	-	-	349	281	333	377	415	449
0.03	-	-	795	351	349	403	445	481
0.2	789	341	217	257	295	329	359	387
0.3	499	201	209	245	279	309	337	363
0.5	333	175	195	227	255	281	305	327
A_p	43							
(dB)	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	637	489	513	551
0.005	-	-	-	-	493	461	495	533
0.009	-	-	-	-	373	431	471	509
0.02	-	-	-	345	347	397	439	475
0.06	-	-	335	267	317	359	395	429
0.03	-	-	763	337	335	383	423	459
0.2	-	329	207	245	281	313	341	367
0.3	479	193	197	233	265	295	321	345
0.5	325	163	185	215	243	267	291	311

Appendix A-II : Experimental minimum filter orders for given θ, β and A_p when T = 0.05π rad/sample

A_p	θ =5°								
(dB)	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001	-	-	-	-	-	-	1017	349	143
0.004	-	-	-	-	-	619	231	107	101
0.007	-	-	-	-	857	343	125	89	99
0.01	-	-	-	-	579	227	97	89	97
0.04	-	-	737	341	113	67	75	81	89
0.07	-	777	419	183	79	65	71	79	85
0.1	697	577	299	143	55	63	69	73	75

0.4	289	139	65	41	47	53	55	59	63
0.7	99	63	33	37	43	47	53	57	61
A_p (dB)	θ =10°								
	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001	-	-	-	-	-	-	925	343	137
0.004	-	-	-	-	-	571	191	103	105
0.007	-	-	-	-	845	299	121	93	101
0.01	-	-	-	-	571	223	93	91	97
0.04	-	-	727	335	143	79	71	79	85
0.07	1003	767	413	179	75	61	69	75	81
0.1	689	569	295	105	53	59	63	73	75
0.4	213	173	63	39	45	51	55	57	59
0.7	97	61	31	37	41	45	49	53	57
A_p (dB)	θ =15°								
	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001	-	-	-	-	-	-	829	261	111
0.004	-	-	-	-	-	521	185	97	107
0.007	-	-	-	-	751	291	91	95	103
0.01	-	-	-	-	519	181	87	93	101
0.04	-	-	713	289	105	75	77	83	89
0.07	945	827	403	175	71	67	73	77	83
0.1	675	557	287	101	67	63	65	69	73
0.4	207	237	121	37	43	47	49	53	57
0.7	95	59	31	33	39	43	47	51	55
A_p (dB)	θ =20°								
	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001	-	-	-	-	-	-	617	215	123
0.004	-	-	-	-	-	429	113	97	107
0.007	-	-	-	-	653	209	85	95	105
0.01	-	-	-	-	429	139	85	93	103
0.04	-	-	691	279	99	71	79	85	93
0.07	957	693	391	133	67	69	75	81	87
0.1	655	505	279	97	61	65	69	71	75

0.4	203	163	57	43	45	47	49	51	53
0.7	91	57	29	31	35	41	45	47	51
A_p	θ =25°								
(dB)	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001	-	-	-	-	-	-	449	173	113
0.004	-	-	-	-	953	305	105	97	107
0.007	-	-	-	-	557	165	85	95	105
0.01	-	-	-	991	375	101	83	93	103
0.04	-	-	557	233	65	69	79	85	93
0.07	885	667	341	125	61	67	75	81	87
0.1	595	487	231	91	59	65	67	73	75
0.4	161	189	53	43	47	51	53	55	59
0.7	87	53	45	35	37	41	43	45	49
A_p	θ =30°								
(dB)	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001	-	-	-	-	-	1013	325	133	123
0.004	-	-	-	-	-	257	99	99	109
0.007	-	-	-	-	427	95	85	97	105
0.01	-	-	-	807	289	93	85	95	103
0.04	-	-	531	187	61	69	79	87	93
0.07	775	637	289	117	59	67	75	83	89
0.1	603	463	219	55	57	65	67	69	73
0.4	279	117	49	43	49	53	59	61	63
0.7	83	49	41	39	43	45	49	53	57
A_p	θ =35°								
(dB)	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001	-	-	-	-	-	925	305	123	123
0.004	-	-	-	-	737	223	91	103	113
0.007	-	-	-	959	335	87	87	99	107
0.01	-	-	-	663	239	85	85	97	105
0.04	-	-	435	173	59	69	79	87	95
0.07	699	535	239	79	59	67	75	83	89
0.1	601	371	173	49	57	65	69	75	79

0.4	141	139	45	43	49	55	59	65	69
0.7	77	99	35	39	43	49	53	57	59

Appendix A-III : Experimental minimum filter order for 864 filters ,for given β and A_p when T = 0.05π rad/sample

A_p	2							
(dB)	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	861	347	135	103
0.005	-	-	-	-	503	195	107	101
0.009	-	-	-	701	267	99	91	99
0.02	-	-	-	-	119	79	87	95
0.06	-	-	-	81	67	73	81	87
0.03	-	-	-	-	87	77	85	93
0.2	-	-	47	53	59	65	71	77
0.3	-	69	45	51	57	63	67	73
0.5	101	37	41	47	53	57	63	67
A_p	3							
(dB)	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	861	309	135	103
0.005	-	-	-	-	503	195	91	101
0.009	-	-	-	701	267	99	89	99
0.02	-	-	-	-	119	79	87	95
0.06	-	-	-	81	65	73	81	87
0.03	-	-	-	-	87	77	85	93
0.2	-	-	45	53	59	65	71	77
0.3	-	103	43	51	57	61	67	73
0.5	101	35	41	47	51	57	61	67
A_p	7							
(dB)	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	853	305	131	101
0.005	-	-	-	-	459	191	103	99
0.009	-	-	-	655	263	97	87	95

0.02	-	-	-	-	117	77	85	93
0.06	-	-	-	79	63	71	79	85
0.03	-	-	-	-	85	75	83	89
0.2	-	-	69	51	57	63	69	75
0.3	-	101	43	49	55	59	65	71
0.5	99	35	39	45	51	55	59	65
A_p	12							
(dB)	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	761	261	101	105
0.005	-	-	-	-	451	153	97	103
0.009	-	-	-	605	221	91	91	97
0.02	-	-	-	-	85	79	85	89
0.06	-	-	-	75	63	69	75	81
0.03	-	-	-	-	81	77	79	87
0.2	-	-	65	49	55	61	67	71
0.3	-	97	41	47	51	57	63	67
0.5	97	33	37	43	47	53	57	61
A_p	17							
(dB)	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	667	217	97	107
0.005	-	-	-	971	363	117	95	105
0.009	-	-	-	-	213	87	93	101
0.02	-	-	-	-	79	81	89	97
0.06	-	-	-	71	69	75	79	85
0.03	-	-	-	-	77	79	87	93
0.2	-	-	63	51	55	57	63	67
0.3	-	95	45	43	49	55	59	63
0.5	95	57	35	39	45	49	53	57
A_p	21							
(dB)	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	537	143	97	107
0.005	-	-	-	835	317	85	95	105
0.009	-	-	-	-	171	85	93	103

0.02	-	-	-	-	75	81	89	97
0.06	-	-	-	67	69	75	81	87
0.03	-	-	-	-	71	79	87	95
0.2	-	-	59	53	59	63	67	71
0.3	-	59	51	49	53	57	61	63
0.5	91	55	37	39	43	47	51	55
A_p (dB)	26							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	409	135	97	109
0.005	-	-	-	731	233	103	97	107
0.009	-	-	-	-	99	83	93	103
0.02	-	-	-	-	73	81	91	97
0.06	-	-	-	61	69	75	83	89
0.03	-	-	-	95	71	79	87	95
0.2	-	89	55	53	59	65	71	75
0.3	-	55	45	51	55	61	65	69
0.5	87	51	41	45	49	53	55	59
A_p (dB)	28							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	999	401	131	99	111
0.005	-	-	-	681	193	99	97	109
0.009	-	-	-	-	97	85	95	105
0.02	-	-	-	-	73	83	91	99
0.06	-	-	-	61	69	77	85	91
0.03	-	-	-	91	71	81	89	97
0.2	-	87	51	55	61	67	73	77
0.3	-	53	47	51	57	63	67	71
0.5	85	49	41	47	51	55	59	63
A_p (dB)	31							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	969	321	125	101	111
0.005	-	-	-	557	185	95	99	109
0.009	-	-	-	-	91	85	95	105

0.02	-	-	-	119	71	81	91	99
0.06	-	-	117	59	69	77	83	91
0.03	-	-	-	61	71	79	89	97
0.2	-	83	49	53	61	67	73	77
0.3	-	51	45	51	57	63	67	71
0.5	81	47	41	47	51	55	59	63
A_p	36							
(dB)	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	849	269	115	105	113
0.005	-	-	-	493	143	89	101	109
0.009	-	-	-	-	85	85	97	105
0.02	-	-	-	109	71	81	91	99
0.06	-	-	107	57	67	75	83	89
0.03	-	-	-	59	69	79	89	95
0.2	-	77	45	53	59	67	71	77
0.3	109	45	45	51	57	63	67	73
0.5	75	41	41	47	51	57	61	65
A_p	41							
(dB)	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	667	105	103	109	117
0.005	-	-	-	425	103	99	105	113
0.009	-	-	-	247	77	93	101	109
0.02	-	-	-	73	75	85	95	103
0.06	-	-	71	59	69	77	85	93
0.03	-	-	-	71	73	83	91	99
0.2	-	95	45	53	61	67	75	79
0.3	-	41	43	51	57	65	69	75
0.5	95	37	41	47	53	59	63	67
A_p	43							
(dB)	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	649	269	103	107	115
0.005	-	-	999	441	155	99	105	111
0.009	-	-	-	239	99	93	101	107

0.02	-	-	-	-	73	85	93	101
0.06	-	-	-	57	67	77	85	91
0.03	-	-	-	71	71	83	91	97
0.2	-	119	45	53	61	67	73	79
0.3	-	87	43	51	57	63	69	73
0.5	-	35	41	47	53	57	63	67

Appendix A-IV : Experimental minimum filter orders for given θ, β and A_p when T = 0.1π rad/sample

A_p	θ =5°								
(dB)	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001							509	175	85
0.004					787	329	117	55	51
0.007					469	181	63	45	51
0.01				707	309	115	49	45	49
0.04		687	409	189	57	35	39	41	45
0.07		429	229	111	39	33	37	41	43
0.1		269	169	71	29	33	35	39	43
0.4		51	33	21	25	27	31	33	35
0.7		33	17	19	23	25	27	29	31
A_p	θ =10°								
(dB)	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001							443	153	71
0.004					757	305	95	53	53
0.007				1013	403	169	61	49	51
0.01				737	285	93	47	47	51
0.04		757	383	167	55	41	37	41	43
0.07		403	227	109	39	31	35	39	41
0.1		285	147	53	37	31	35	37	41
0.4		69	33	21	23	27	29	31	33

0.7		31	17	19	21	23	25	29	29
A_p (dB)	θ =15°								
	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001							395	131	67
0.004					703	241	93	49	55
0.007				935	375	127	59	49	53
0.01				665	279	73	45	47	51
0.04		685	375	163	53	37	41	43	47
0.07		395	221	69	37	35	37	41	43
0.1		279	125	51	33	33	35	37	39
0.4		67	31	19	23	25	27	29	31
0.7		31	15	17	21	23	25	27	29
A_p (dB)	θ =20°								
	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001						871	327	109	63
0.004					589	215	57	49	55
0.007				815	327	105	43	49	53
0.01				609	233	87	43	49	53
0.04		647	327	159	51	37	41	45	47
0.07		365	177	83	33	35	39	41	45
0.1		271	121	49	31	33	37	39	43
0.4		65	29	21	23	25	27	29	31
0.7		29	17	17	19	21	23	25	27
A_p (dB)	θ =25°								
	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001						767	243	87	57
0.004					459	153	53	49	55
0.007				713	297	83	43	49	53
0.01				567	205	51	43	49	53
0.04		569	297	133	33	37	41	45	47

0.07		405	189	63	31	35	39	43	45
0.1		225	133	31	31	33	37	41	43
0.4		63	27	23	25	27	29	31	33
0.7		27	17	19	21	21	23	25	27
A_p (dB)	$\theta=30^\circ$								
	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001						611	197	67	63
0.004					403	111	51	51	57
0.007				611	231	79	43	49	55
0.01				421	145	47	43	49	53
0.04		543	283	93	31	37	41	45	49
0.07		335	145	43	31	35	39	43	45
0.1		231	93	29	31	33	37	41	43
0.4		59	25	23	25	29	31	33	35
0.7		39	19	21	23	25	27	29	29
A_p (dB)	$\theta=35^\circ$								
	β=0	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.001						493	121	63	63
0.004				905	331	73	47	53	57
0.007				495	183	45	45	51	55
0.01			889	365	135	43	45	49	55
0.04		563	251	87	31	37	41	45	49
0.07		317	151	39	31	35	39	43	47
0.1		235	87	25	29	33	37	41	45
0.4		55	23	23	25	29	31	33	35
0.7		37	17	21	23	25	27	29	31

Appendix A-V : Experimental minimum filter order for 864 filters ,for given β and A_p when T = 0.1π rad/sample

A_p (dB)	2							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-		-	-	-	69	53
0.005	-	-	-	-	-	81	55	51
0.009	-	-	-	-	115	51	47	51
0.02	-	-	-	-	47	41	45	49
0.06	-	-	-	41	35	37	41	45
0.03	-	-	-	95	35	39	43	47
0.2	-	53	25	27	31	33	37	39
0.3	91	35	23	27	29	33	35	37
0.5	-	-	-	-	-	-	-	-

A_p (dB)	3							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	-	-	55	53
0.005	-	-	-	-	-	81	55	51
0.009	-	-	-	-	-	51	45	51
0.02	-	-	-	-	59	41	45	49
0.06	-	-	-	29	33	37	41	45
0.03	-	-	-	93	45	39	43	47
0.2		71	23	27	31	33	37	39
0.3	91	35	23	27	29	31	35	37
0.5	-	-	-	-	-	-	-	-

A_p (dB)	7							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	-	-	67	53
0.005	-	-	-	-	-	79	53	51
0.009	-	-	-	-	113	49	45	49
0.02	-	-	-	-	45	39	43	47

0.06	-	-	111	39	33	37	41	43
0.03	-	-	-	94	43	39	43	45
0.2		71	23	27	29	33	35	39
0.3	89	51	23	25	29	31	33	37
0.5	-	-	-	113	27	29	31	33
A_p (dB)	12							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	-	-	65	53
0.005	-	-	-	-	-	77	51	53
0.009	-	-	-	-	111	47	47	51
0.02	-	-	-	-	43	41	45	47
0.06	-	-	89	39	35	35	39	41
0.03	-	-	-	91	41	41	43	45
0.2	-	69	33	25	29	31	35	37
0.3	87	49	21	25	27	29	33	35
0.5	49	17	19	23	25	27	29	31
A_p (dB)	17							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	-	109	49	55
0.005	-	-	-	-	-	59	49	53
0.009	-	-	-	-	89	45	49	53
0.02	-	-	-	-	41	43	47	49
0.06	-	-	105	37	35	39	41	45
0.03	-	-		69	39	41	45	47
0.2	-	67	31	27	29	31	33	35
0.3	85	31	23	23	25	29	31	33
0.5	47	29	19	21	23	25	27	29
A_p (dB)	21							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	-	89	49	55

0.005	-	-	-	-	-	57	49	55
0.009	-	-	-	-	69	43	49	53
0.02	-	-	-	103	39	43	47	51
0.06	-	-	83	35	35	39	43	45
0.03	-	-	-	67	37	41	45	49
0.2	-	47	29	29	31	33	35	37
0.3	83	45	25	25	27	29	31	33
0.5	47	27	21	21	23	25	27	29
A_p (dB)	26							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	-	67	51	55
0.005	-	-	-	-	-	43	49	55
0.009	-	-	-	-	65	43	49	53
0.02	-	-		81	37	43	47	51
0.06	-	-	63	31	35	39	43	47
0.03	-	-	-	33	37	41	45	49
0.2	115	45	27	29	31	35	37	39
0.3	79	43	25	27	29	31	33	37
0.5	43	25	41	23	25	27	29	31
A_p (dB)	28							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	-	65	51	57
0.005	-	-	-	-	113	51	51	55
0.009	-	-	-	-	49	43	49	53
0.02	-	-	-	79	37	43	47	51
0.06	-	-	77	31	35	39	43	47
0.03	-	-		47	37	41	45	49
0.2	113	43	25	29	31	35	37	39
0.3	77	27	23	27	29	33	35	37
0.5	43	25	23	25	27	29	31	33

A_p (dB)	31							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	-	51	51	57
0.005	-	-	-	-	93	49	51	55
0.009	-	-	-	-	47	43	49	53
0.02	-	-	-	59	37	43	47	51
0.06	-	-	59	31	35	39	43	47
0.03	-	-	-	31	37	41	45	49
0.2	109	57	25	29	31	35	37	41
0.3	75	25	23	27	29	33	35	37
0.5	34	23	21	25	27	29	31	33
A_p (dB)	36							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	103	47	55	59
0.005	-	-	-	-	57	45	53	57
0.009	-	-	-	-	43	45	49	55
0.02	-	-	-	41	37	43	47	51
0.06	-	-	39	31	35	39	43	47
0.03	-	-	117	41	37	41	45	49
0.2	103	39	25	27	31	35	37	41
0.3	69	23	23	27	29	33	35	37
0.5	37	21	21	25	27	29	31	33
A_p (dB)	41							
	β=1	β=2	β=3	β=4	β=5	β=6	β=7	β=8
0.003	-	-	-	-	-	55	55	59
0.005	-	-	-	-	81	53	53	57
0.009	-	-	-	-	51	47	51	55
0.02	-	-	-	51	37	43	49	53
0.06	-	-	63	31	35	39	43	47
0.03	-	-	-	37	37	43	47	41

0.2	-	49	23	27	31	35	39	41
0.3	65	33	23	27	31	33	35	39
0.5	49	19	21	25	27	31	33	35