A MULTI-PRODUCT CAPACITATED VEHICLE ROUTING PROBLEM MODEL WITH SEASONALITY FOR THE STATIONERY INDUSTRY

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ABSTRACT

This dissertation presents an optimisation approach to deliver stationery commodities with highly seasonal demand. The problem is structured as a capacitated vehicle routing problem (CVRP) consisting of one distribution centre to numerous customer locations (one-to-many) with multi-products, where each customer requires various product mixes. A mixed-integer linear programming model was used to formulate the capacitated vehicle routing problem, and the Gurobi solver with customised heuristics algorithms was used to arrive at the solution. Intending to gain the advantage of separating the distributing points according to geographical areas, K means clustering was engaged. The solution generated through this model determined that annual distance has diversified by 44% between the peak and off-peak periods. The main findings show that the annual distance savings is around 28%, while the annual capacity saving is 22% when using the CVRP model compared to current practices.

Further, it is determined that it is adequate to have 15 and 53 trips per week starting and ending at the distribution centre for off-season and season, respectively. Moreover, the route sequence of every vehicle was illustrated cluster-wise and season-wise separately. Two experiments were done, changing vehicle capacity and time horizon to have better outcomes, which will be additional guidance for the organisation. The dissertation offers a guide to improving the use of optimising techniques in the distribution network aiming at seasonal demand variations while providing a sound basis for future research directions.

Keywords - Seasonality, Multi-product, Capacitated vehicle routing problem, Distance optimisation, Stationery industry

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ALNS	-	Adaptive Large-Neighbourhood Search
API	-	Application Programming Interface
CBM	-	Cubic Meters (m^3)
CPU	-	Central Processing Unit
CSO	-	Central Statistics Office Ireland
CVRP	-	Capacitated Vehicle Routing Problem
DC	-	Distribution Centre
DCVRP	-	Distance-Constrained and Capacitated Vehicle Routing Problem
GA	-	Genetic Algorithm
GIS	-	Geographic Information System
LKR	-	Sri Lankan Rupee
LNS	-	Large-Neighbourhood Search
MILP	-	Mixed-Integer Linear Programming
MIP	-	Mixed Integer Programming
NP	-	Nondeterministic Polynomial
OR	-	Operations Research
PSO	-	Particle Swarm Optimisation
PVRP	-	Periodic Vehicle Routing Problem
RAM	-	Random Access Memory
SA	-	Simulated Annealing
SAP	-	System Applications and Products In Data Processing
TSP	-	Travelling Salesman Problem
UAV	-	Unmanned Aerial Vehicle
VRAP	-	Vehicle Routing-Allocation Problems
VRP	-	Vehicle Routing Problem
VRPPD	-	Vehicle Routing Problem with Pickup and Delivery
VRPTW	-	Vehicle Routing Problem with Time Windows

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1 CHAPTER 1: INTRODUCTION

1.1 Background of Rationale of the Study

The transportation and logistics industry is continuously employing optimisation techniques to improve productivity. The industry is challenging an era of extraordinary changes as digitisation takes hold and customer expectations grow. The espousal of data science and machine learning techniques lets companies optimise logistics and determine factors that affect performance. The logistics industry is primarily driven by factors such as on-time delivery, fuel expense, security measures, offshoring, supply chain trustworthiness, and domestic distribution networks.

Operation research (OR) techniques are famous for obtaining efficient solutions for operational problems. One of more attractive titles in OR can be identified as Vehicle Routing Problem (VRP). It is of paramount importance for thousands of organisations and companies transporting commodities or people. Vehicle routing optimisation is a more significant field because keeping a better routing network helps companies to reduce cost effects while increasing efficiency. Thus, in the operational research area, VRP has numerous applications.

In practical scenarios, VRP is not only about providing the shortest possible route. Many other factors contribute to planning the optimised routes. The complexity in VRP arises due to multiple constraints and its unpredictable nature. Therefore, any suggested solution must be designed to be flexible enough to accommodate any complicated problems. The problem can be well-sophisticated and complex, depending on an exceptionally distinctive set of input variables [1].

Many commodities and appliances in real-world businesses experience seasonal fluctuations in sales. As a result, firms must expand their logistical resources during certain months when demand is higher than during the rest of the year to fulfil all customer orders. Moreover, in most practical cases, the corporation must distribute a variety of products with varying quantities and weights. A Capacitated Vehicle Routing Problem (CVRP) model is addressed in this study, considering the disparity between seasonal and off-seasonal demand and different varieties of order fulfilment. This model provides applicability to commercial needs. When comparing past studies on VRP applications, taking into account seasonality and multi-products together are less.

Moreover, when it comes to Sri Lankan manufacturing companies, using these kinds of analytical solutions for Vehicle Routing Problems is a rare occurrence [2].

1.2 Problem Description

The dataset utilised in this research came from one of Sri Lanka's leading book and stationery businesses. Currently, the firm delivers its items according to a manually established weekly schedule. The same route schedule is generally employed in peak and off-peak planning horizons. Without the use of technology, the routes are usually chosen based on the drivers' gut feelings and experiences.

Demand for these commodities aligns with the beginning of new school terms, resulting in a highly seasonal sales pattern for these items, as illustrated in "Figure 1.1".This reduces vehicle capacity utilisation during the off-season periods. Using separate route plans for two planning horizons might decrease distance-related costs by utilising truck capacity.

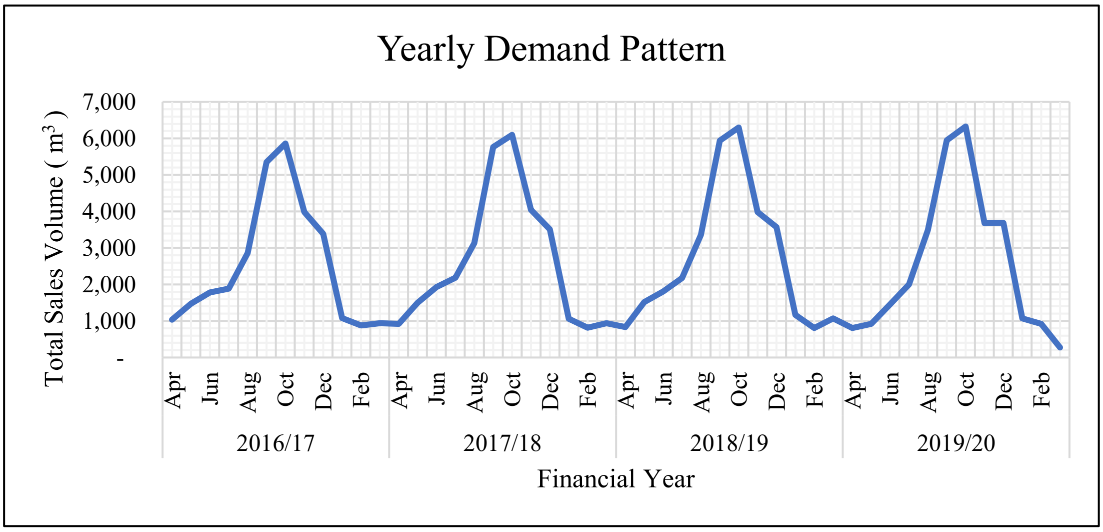

Figure 1.1:Yearly total demand pattern

When it comes to product variations, a single client mandates a variety of stationery items. As a result, the organisation must deliver many items of varying quantities to a single consumer. Both requirements are taken into account in the CVRP solution related to this study. The CVRP is formulated as a Mixed-integer linear programming (MILP), and the Gurobi commercial solver in the python platform has been used for solving the MILP problem.

1.3 Research Objectives

The main objective of the study is to determine the optimal number of vehicles utilised to complete the distribution procedure in both off-peak and peak periods, together with the route sequence of each tour, in order to minimise distance-related outlays while managing effective vehicle capacity utilisation.

In order to achieve the main objective, there are three sub-objectives formulated as follows.

I. Investigate the different types of VRP applications under various types of products portfolio and a highly seasonal demand pattern.
II. Identity nature of the distribution process, sales pattern and product nature related to the stationery company to develop appropriate VRP solution.
III. Determine the VRP variables and constraints in order to formulate the mixed integer linear programming.
IV. Determine VRP solutions for off-peak and peak planning horizons.

1.4 Research Process and structure of the Dissertation

The remainder of the thesis is structured as follows: The literature review is included in Chapter 2. The methodology is described in Chapter 3 while the problem definition and mathematical models are presented in Chapter 4. In Chapter 5, computational results and the analysis are explained. Discussion and recommendations are given in Chapter 6. Finally, Chapter 7 concludes the main findings.

2 CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

Chapter 1 offered an overview of the research. The purpose of this chapter is to investigate the available literature in the field of research. Mainly, this chapter includes vehicle routing problems (VRP) and their classifications, capacitated vehicle routing problem (CVRP), solutions approach for VRPs, VRPs related to multi-product and seasonal product distribution and mixed integer linear programming (MILP).

2.2 Routing Problem

Because of the economic importance of the transportation sector, academics have made great efforts to investigate the most efficient methods of performing it. Alternatively, OR has been an invaluable ally in our endeavour. One of the most outstanding achievements of OR has been the resolution of transportation-related issues. This study is relevant to the vehicle routing problem type of transportation problem. When focusing on routing problems there are different kinds of routing problems. These are Vehicle Routing Problem (VRP), Travelling Salesman Problem (TSP) [2] and Tour Location Problems [3] or Vehicle Routing-Allocation Problems (VRAP) [4].

The Vehicle Routing Problem is a generic name given to various kinds of problems which are under this group. Examples can be mentioned as capacitated, time windows, pickup and delivery, backhauls, and split delivery. Vehicle routes are designed to minimise particular objectives, such as total distance travelled. This will be covered in greater detail later in this chapter. The proposed VRP must tackle not only the challenge of finding the shortest visitation order for each route but also the problem of allocating the customers it will visit in the best way feasible. The TSP [2] aims for the shortest path that visits each city precisely once (Hamiltonian cycle). It is unnecessary to visit all the places listed in the VRAP [4]. Each of these is a combinatorial optimisation problem [5]. A combinatorial problem has discrete and finite domains for its decision variables. Figure 2.1 show the above-discussed routing problems.

This study focuses on the visit to customers with a suitable number of vehicles in order to cater the all the customers according to their demands. When looking at those routing problems, VRP is the most suitable combinatorial optimisation problem for this study.

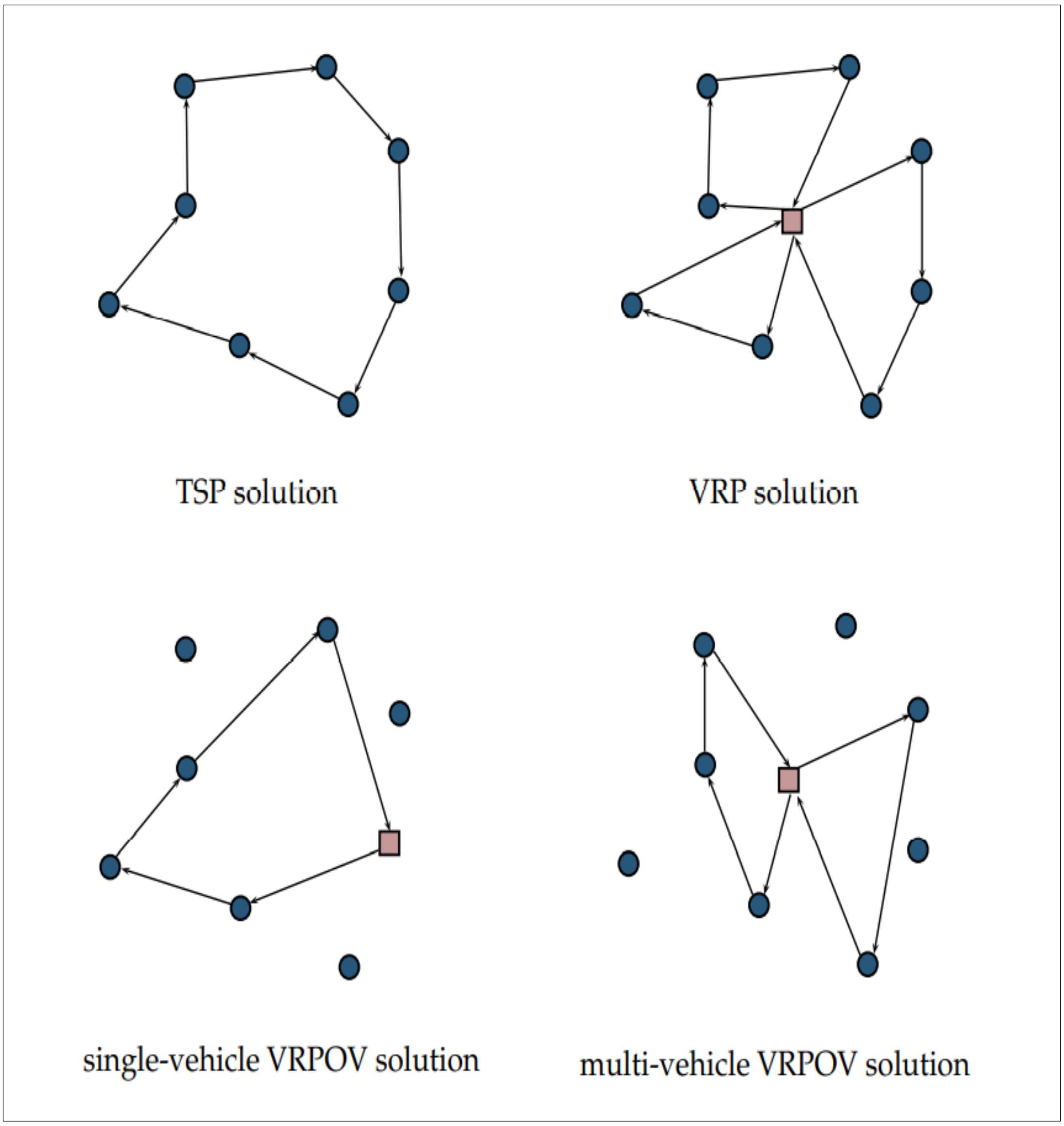

Figure 2.1: Different types of routing problems [6]

2.2.1 Vehicle Routing Problem (VRP)

The Vehicle Routing Problem is defined as the problem of establishing optimal distribution or collection routes from one or more distribution centres to several geographically dispersed cities or customers while considering side restrictions [7]. Dantzig and Ramzer [9] proposed the VRP method as the truck dispatching problem in 1959 to optimise the routing of a fleet of gasoline delivery trucks between a bulk terminal and a large number of service outlets supplied by the terminal. It was the first introduced [8] VRP. It has been more than sixty years since the topic was introduced, yet it still appears to be a tremendous obstacle for the scientific community [10] engaged in real-world circumstances.

VRP can be used to achieve a variety of goals. The primary objectives [11] are to minimise the total transportation expense, the number of vehicles (or drivers) necessary to serve the customer base, balance the routes for travel time and tonnage, and minimise the penalty associated with incomplete customer service. The whole distance travelled (or the overall journey duration), and the fixed expenses connected with the used vehicles (and the accompanying drivers) determine the global transportation cost.

The number of distribution centres, the type of vehicle used (homogeneous or heterogeneous fleet), the number of vehicles used, service time, the nature of demand, demand locations, vehicle capacity, the nature of routes (directed, undirected, or mixed), costs (variable cost/routing costs, fixed operating costs/vehicle acquisition costs), the operation performed (pick up or drop off or mixed), and the goal of the problem are the variations in the classic VRP problem [10]. From the initial paper to the present, several publications have been devoted to the precise and approximate solution of the various varieties of this issue [8] to tackle many real-world scenarios for logistics optimisation.

2.3 Variants of Vehicle Routing Problems

VRP can be categorised into different kinds of variants. The following are the Basic VRP variations and any combination of these variants. [12].

I. **Capacitated VRP (CVRP):** Because vehicle capacity is known, loading the vehicle above its capacity is not permitted.

II. **Distance-Constrained and Capacitated VRP (DCVRP):** Because vehicle capacity and maximum distance constraints are applied, each preceding journey length should not exceed.

III. **VRP with Time Windows (VRPTW):** Each customer's service must begin within a specific time range. During the servicing, the vehicle must stay at the customer's location.

IV. **VRP with Backhauls:** Customers can both deliver and return goods. One trip is required to deliver items to delivery customers and pick up goods from customers. As a result, transportation expenses are reduced.

V. **VRP with Pickup and Delivery (VRPPD):** Vehicles must transport things to clients and pick up commodities from their locations.

Some other variants have been studied in the literature recently, as follows[12].

I. **Open VRP**: Vehicles are under no obligation to return to the distribution centre. The company does not have its own vehicle fleet or does not have enough vehicles to meet client demand, which is advantageous. The organisation must contribute another party for product delivery, and those trucks will be allocated routes without returning to the firm distribution centre.
II. **Multi-depot VRP**: This is suited for businesses with more than one distribution centre.
III. **Mix Fleet VRP**: The vehicle fleet includes a variety of vehicles with varying capacities and overall costs (fixed and variable). The variable cost is proportional to the distance travelled.
IV. **Split-Delivery VRP**: If it saves money, the same client can be served by different vehicles.
V. **Periodic VRP**: A group of consumers must be visited one or more times within a specified time frame. PVRP with service choice (PVRP-SC) is a PVRP variation in which the visit frequency to nodes is a model decision variable. This can result in more efficient car excursions or improved customer service benefits.
VI. **Stochastic VRP**: This can be difficult when the components of the issues are random. Stochastic demand, stochastic consumers, stochastic travel time, and stochastic service time are examples of typical components.
VII. **Fuzzy VRP**: Some values, such as average needs, tour times, number of clients, and customer locations, are challenging to get using probability distributions. Researchers have presented successful models for this using fuzzy set theory.

Figure 2.2 shows the hierarchy of VRP variants [13]. When it comes to CVRP, it can be interconnected with other variants and built complex vehicle routing problems. Therefore, CVRP is better in the initial stage of extending a complex problem. Toth & Vigo [11] illustrated it in Figure 2.3

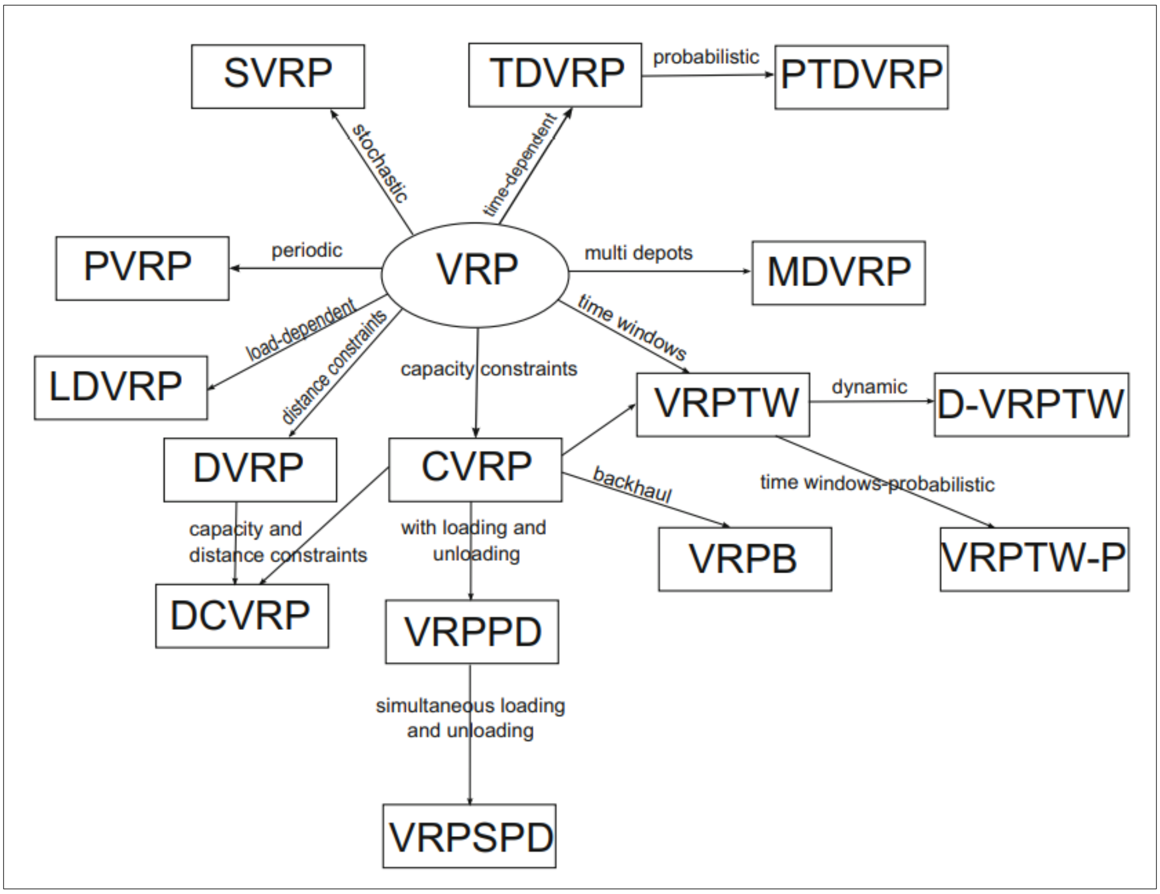

Figure 2.2:Hierarchy of VRP variants [12]

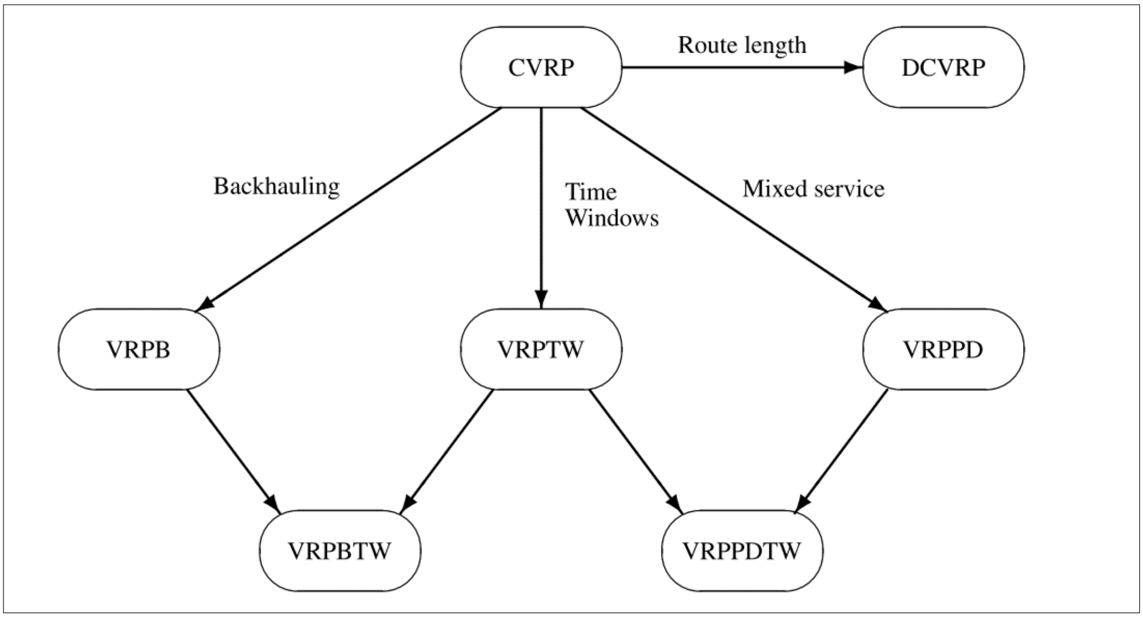

Figure 2.3:The fundamental problems of the VRP class and their interconnections [11]

2.3.1 Capacitated Vehicle Routing Problem (CVRP)

The Capacitated Vehicle Routing Problem (CVRP) is the most basic variant of the VRP. It established the concept of delivering goods to customers. As mentioned above section, it can be extended to more complex VRP.

The CVRP was presented as a generalisation of the Traveling Salesman Problem (TSP) [9]. CVRP is a VRP in which vehicles with limited capacity must obtain or transport products to different neighbourhoods [14]. The trucks are similar and situated at a single central distribution centre, and only the vehicle capacity constraints are mandatory [11]. Loading the vehicle over its limit is not authorised in CVRP because its capacity is already known [12]. Furthermore, with CVRP, all consumers correspond to deliveries. Additionally, demand is deterministic and cannot be split [11]. In this study, the CVRP is solved separately during peak and off-peak periods and assumes that the demand of considering time period is deterministic.

The CVRPs' [11] overarching goal is to reduce the overall cost of serving all clients. It might be a weighted function of the number of routes available and their distance or travel time. One research article has defined the constraints [15] of CVRP as follows. According to that study [15], constraints I, II, and III apply to any VRP definition. However, constraint (V) is unique to CVRP.

I. A single truck supplies each non-depot node.
II. Each vehicle's journey begins and terminates at the depot (node 0)
III. A vehicle cannot stop at the same non-depot node twice.
IV. No vehicle can be loaded over its maximum capacity.

Table 2.1[15] mentions specific characteristics of CVRP.

Table 2.1:Features of CVRP

Characteristics of VRP	Possibilities for the CVRP
Fleet size	Multiple vehicles with capacity constraints
Kind of fleet	Homogeneous
Origin of vehicles	Single depot
Nature of demand	Known deterministic demand
Location of demand	At each node
Nature of network	Non-oriented. Symmetrical in distances
Maximum time on route	No constraint
Activities	Only deliveries
Constraints	(I), (II), (III) and (IV)
Objective	Minimise distances (costs)

2.4 Approaches for VRP solution

Different tactics to the VRP have been discovered in the past several decades, ranging from the use of pure optimisation techniques for addressing small-to-medium-sized issues to the use of heuristics and meta-heuristics for tackling medium and large-sized problems [16].

2.4.1 Classification of Algorithms for VRP

Therefore, three types of algorithms can be identified to solve VRP problems: Exact algorithms, Heuristics algorithms and Metaheuristics algorithms. When the number of vertices rises, computation time also increases. So, heuristics and metaheuristics generate near-optimal solutions and are much quicker than exact algorithms [12].

Direct tree search approaches, dynamic programming, and integer linear programming are the three major categories of extract algorithms (Figure 2.4). Specific heuristic algorithms, such as the nearest neighbour algorithm, tour improvement procedures and insertion algorithms, can be applied directly to CVRP and DCVRP without adjustment.

When extract and heuristic algorithms are compared, extract algorithms may address relatively small cases. However, heuristic methods have proven to be highly satisfying [7]. Genetic algorithm [17], Simulated annealing (SA) [18], Tabu search [19], and evolutionary strategies are included under heuristics algorithms [20].

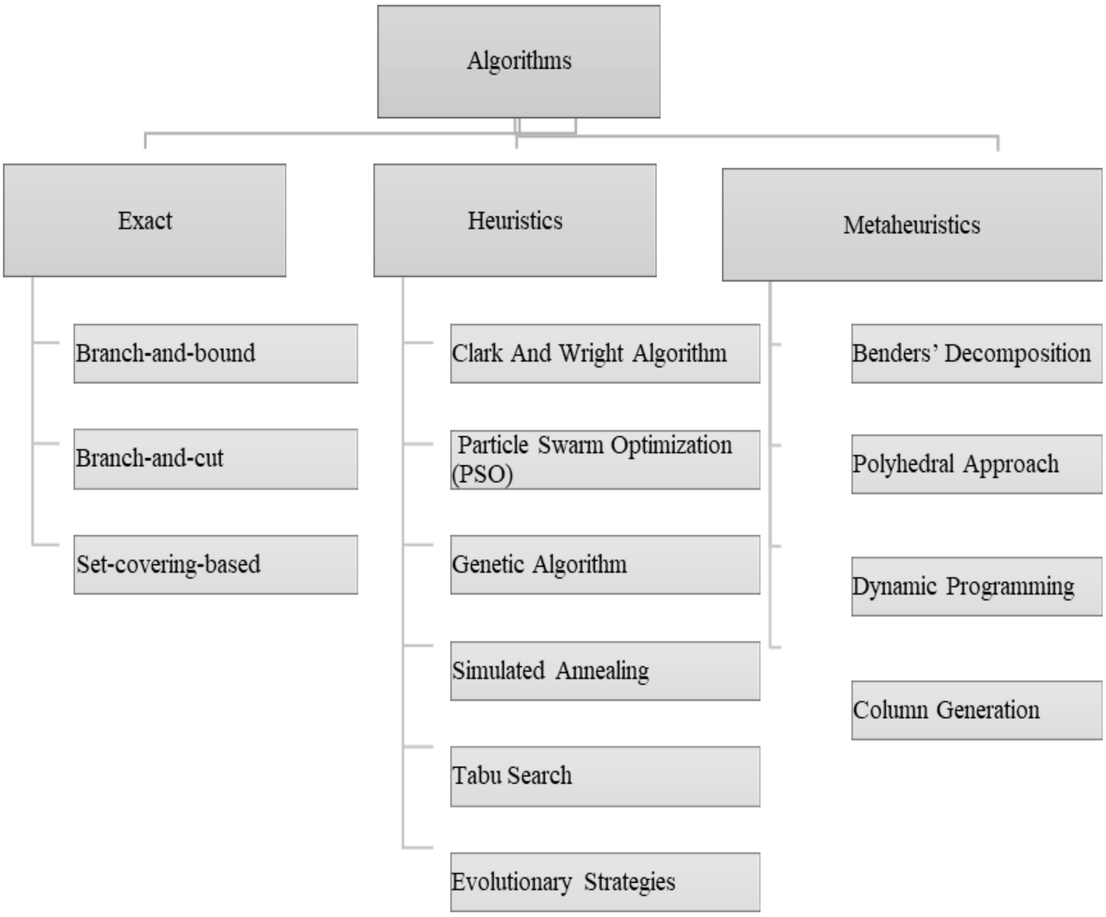

Figure 2.4: Instance for VRP algorithms

Among the various ways to address vehicle routing problems in the literature, metaheuristics are supposed to be extremely efficient and effective [21] in state-of-the-art solutions for large instances.

Metaheuristics can perform a more thorough systematic search in the solution space while producing high-quality solutions consistently, in spite of needing a longer computational time than heuristics [16]. Further, metaheuristic approaches are categorised according to the source of inspiration (natural or artificial), the initial solution (population or single solution), the objective function utilised (dynamic, static), the neighbourhood structure (single, many), and memory stability (memory, memory-free) [22]. In the past three decades, genetic algorithms [17], simulated annealing [18], tabu search [19], artificial immune system [23], ant colony [24], particle swarm [25], and artificial bee colony [26] are the repeatedly applied metaheuristic techniques for integrated optimisation problems [27].

2.4.2 Heuristics approaches

Accuracy, rapidity, adaptability, and simplicity are all essential VRP heuristics. The capability to integrate side restrictions encountered for real-world needs is referred to as flexibility. The saving method, also known as the Clark and Wright algorithm [28], is faster and simpler [29]. It is easily programmed and has no parameters; however, modified algorithms indicate that it is not the most accurate when measured as a percentage of optimum value. Additional conditions can be added, but the accuracy output is a massive failure. As a result, the algorithm's adaptability is also its weakest point [29].

Dantzig and Ramser [9], the pioneers of VRP in 1959, gave the first mathematical programming formulation and algorithmic effort at VRP in a real-world application. After that, Clark and Wright, 1964 [30] improved the Dantzig and Ramser approach by proposing a heuristic [12]. After that, there are so many attempts to improve heuristics for VRP when looking into the past literature. Developing a heuristic for the Capacitated Rural Postman problem with split delivery and time windows on arcs [31], [32],[33], [34] and successfully applying it to a problem of managing the vehicles for delivering foods on a cattle ranch, proposing a cluster and route heuristic for the crew exchange problem [35], developing a lower bound of split delivery vehicle routing problems (SDVRP) [36],[37], introducing a dynamic programming approach for VRP with split pick-ups [38] and describing a tabu search heuristic [39] are few examples. As a result of introducing of VRP by Dantzig and Ramser [9] and improving it as a heuristics by Clark and Wright 1964 [30], much of the research efforts have turned to develop powerful metaheuristics [1].

Because this problem has 87 customer locations (dealer points) and those are also divided into clusters, this can be mentioned as a medium or small-size problem. So, the heuristics method is used to solve the MILP problem relevant to CVRP. Heuristic techniques or metaheuristic-based algorithmic search strategies are widely used to address NP-hard problems[40]. The CVRP is also NP-hard [41] in the strict sense. It is justified to use a declarative modelling framework and a constraint programming environment to tackle these types of instances [40]. In those facts, the Gurobi solver [42] (section 2.5) with heuristics solutions in the python programming language platform is used for this study.

2.5 Gurobi Solver

Nowadays, several reputable companies use mathematical optimisation in their operations for better business decisions. It causes less time-consuming and potential savings in other resources. Gurobi solver [42] is one of the most popular commercial solvers in the world when it comes to mathematical optimisation. Gurobi can solve many types of complications, such as linear programming (LP), quadratic programming (QP), quadratically-constrained programming (QCP), mixed-integer linear programming (MILP), mixed-integer quadratic programming (MIQP), and mixed-integer quadratically-constrained programming (MIQCP).

More than 165 new benchmark VRP cases were carried out [43] while evaluating commercial solvers such as the CPLEX and Gurobi, with the intention of minimising the system's delayed latency by satisfying mandatory visit times. While Gurobi had beaten CPLEX in 90 cases (55%), CPLEX outperformed it in 27 (16%). The studies [44], [45] were used to solve the mathematical formulations related to VRP in the Gurobi solver's framework and for the small-size dataset, the solver found optimum solutions in a tolerable calculation time.

As indeed the Gurobi solver is a more powerful tool, limitations still exist. As the problem size increased, the computation time raised exponentially, and Gurobi could not give the optimal solution for the large-size problem comparing metaheuristics [44]. The main limitation for the research community is that Gurobi's output does not mention the heuristic method strictly used to solve the problem; it only says heuristic solution. This study compares current organisation practices with solutions from the Gurobi commercial solver. Therefore, the issue is not valid for this research.

2.6 Python Programming language

Python is a popular and good-quality programming language recommended by many operations researchers and data scientists. It is straightforward, flexible, and powerful, has excellent libraries for machine learning, optimisation, and statistical modelling, and is easy to learn for beginners [46].

Therefore, most of the research community used python programming language for VRP solutions as well. Solving VRP using the Simulated Annealing algorithm [46], solving a range of VRPs with a column generation approach [47] and solving a combinatorial bees algorithm for VRP [48] in the python platform can be given as few instances among the number of researches which were used python as a programing language.

2.7 Clustering Methods.

The clustering method was a statistical tool used in numerous applications such as bioinformatics, machine learning, pattern recognition, and data mining. Customer clustering for vehicle routing also was popular in many cases. The effort by Keeney (1972) suggested using clustering to assign customer areas to various facilities, and after that, a lot of scholars focused on this problem by using exact methods or agglomeration heuristics [49].

A clustering approach was proposed in a paper [50] for a pre-processing stage in solving a VRP with time windows, a heterogeneous fleet of vehicles, and several distribution centres. A problem with up to 100 customers could be solved using a clustering algorithm instead of 25 without using clustering. Four types of different kinds of clustering algorithms which are k-means, centroid-based heuristics, Density-based Spatial Clustering of Application with Noise (DBSCAN) and Shared Nearest Neighbour (SNN) clustering algorithm, were tested with VRPTW and K-means algorithm exhibited better performance than other algorithms taking into consideration of the travelled distance, waiting time, vehicle amount and computational period conditions [51].

2.7.1 K-means clustering algorithm

The k-means clustering algorithm is illustrated in detail by Hartigan (1975) [52]. Cluster analysis was designed primarily as a statistical data analysis technique. The K-means method divides M points in N dimensions into K groups in order to minimise the within-cluster sum of squares [52].

K-means clustering has been used as a standard method across several recent manuscripts on VRP. For a small instance set, calculating Chopra and Meindl's [53] savings matrix approach for Clark and Wright's [28] method yields a reasonable response. However, it does not deliver satisfactory outcomes for big instance sets [54]. When comparing total distance and amount of vehicles, research offered an Enhanced k-means clustering method [54], which gave a better answer than Clarke and Wright's saving strategy. K-means clustering methods are commonly employed in data mining concepts[55]. For a data mining and optimisation-based real-time mobile intelligent routing system for city logistics, the k-means method was used as an independent segment to cluster consumer locations [56]. A two-step K-means algorithm [57] was used for all the customers to be divided into several clusters, and then a border modification algorithm was applied to balance the number of customers between the groups. Customer clustering is employed according to several criteria with the intention of getting an economic advantage and reducing the complexity of optimising the logistics function To group the customers into logical divisions. The most common criteria are the total transportation costs, route duration, total distance, time or delivery constraints, and workload balancing [49]. Hence using the CVRP in this study, route duration and workload balancing are neglected. To overcome these limitations, customers clustering into geographical areas can help.

2.8 Mixed-Integer Linear Programming (MILP)

MILP theory and practice have been significantly expanded throughout the years, and it is an essential tool in enterprise and engineering [58]. Mixed integer linear programming (MILP) denotes an effective mathematical modelling tactic to solve complicated optimisation assignments and determine the potential trade-offs between conflicting objectives [59].

The main reasons for the triumph of MILP are linear programming (LP) based solvers and the modelling flexibility of MILP [60]. Today, several extremely effective state-of-the-art problem solver [42], [61], [62], [63] incorporate many advanced techniques. There can be mentioned many examples that used MILP for VRP solutions with different algorithms and solvers. Sweep algorithm and MILP for VRP with time windows [64], MILP to solve a real-life VRP with pickup and delivery [65] and MILP formulation for Green VRP [66] in IBM ILOG CPLEX optimisation studio can be given as examples. Moreover, MILP is widely used for most real-world VRP formulations, and several examples can be shown in section 2.9.

2.9 Real-world applications in VRP

Many real-world purposes of the saving method [28] have been indicated, beginning with improving the distribution of Cooperative Wholesale society vehicles in the English Midlands and saving 400 clients from Manchester. A couple of examples are mentioned herewith. Coca-Cola Enterprises has used ORTEC's savings-based software for daily route planning of about 10,000 cars, resulting in yearly cost savings of $45 million and improvements in customer service. FRILAC Company (Navarra, Spain) has applied to optimise its route network with 50 points, saving 10% on average. McDonald's has collected medical specimens using Gaskell's modified savings approach [29].

Nowadays, in real-world cases, there are so many complicating attributes in combinatorial optimisation problems, including logistics applications and transportation problems. Therefore, the science community has focused on simultaneously considering multiple attributes to introduce more representative models of real-world scenarios. In particular, VRP researchers have recently concentrated on multi-attribute VRP (MAVRP) [67]. When dealing with real-world and large-scale issues, a skilful mix of principles from many optimisation methodologies can give more efficient behaviour and more flexibility [21]. For example, hybrid metaheuristics [21], which are such methods for optimisation that combine different metaheuristics or integrate artificial intelligence or operational research methodologies into the procedure, can be mentioned.

Moreover, as technology advancements grew, VRP applications expanded in a variety of ways in response to the requirements of factual scenarios. VRP attempts with a fleet of Unmanned Aerial Vehicles (drones) to maximise customer satisfaction within weather and energy consumption restrictions [68], [69]. Additionally, a dynamical variant of the VRP with time windows (VRRTW) [70] was investigated after discovering an essential feature in which existing routings were subject to change over at any moment.

2.9.1 VRP for Seasonal Demand Pattern

In real-world scenarios, different kinds of VRPs are met, engaging with seasonal fluctuations. Because of the needs of the customers in the various periods over the year, promotions given by the sellers and the nature of the products, highly seasonal demand patterns have been created in some industries. Fashion and stationery industries can be given as instances of these kinds of industries. Because of events such as the new year and Christmas, the fashion industry get a seasonal demand pattern, while new school terms make the stationery industry get a seasonal demand pattern. This can be caused to vehicle routing with the purpose of distribution of goods and collection of raw materials. The study attempts to see how it impacts vehicle routing in order to distribute goods over the year.

Some VRP experiments carried the seasonality component into consideration. Based on the adaptive large-neighbourhood search (ALNS) context, a solution technique for a Multi-Period VRP with Seasonal Fluctuations (MPVRPSF) was obtained. The proposed solution strategy incorporates a number of original algorithmic properties, including various creative operators based on the problem characteristics [71].

The grass growing in Ireland is very seasonal, resulting in fluctuating milk production patterns (Figure 2.5) [1].

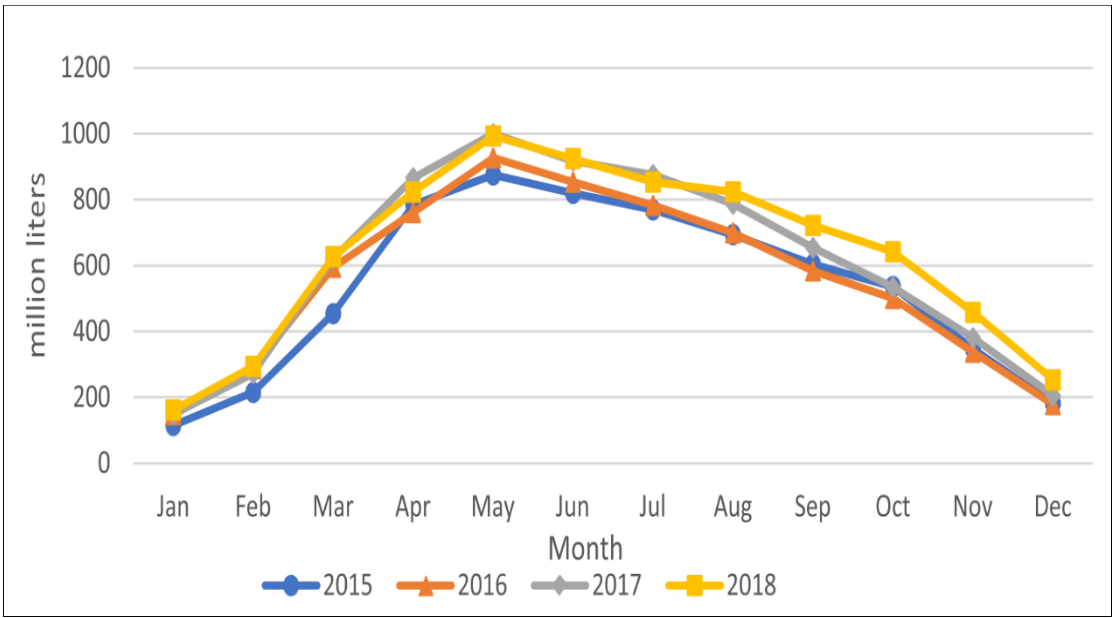

Figure 2.5:Ireland: Monthly domestic milk intake by creameries and pasteurisers for the years 2015 to 2018 (source CSO 2019) [1]

Using the large neighbourhood search (LNS) method, a publication offered a well-efficient and dynamic routing technique that may be utilised to build cost-efficient route collection plans for milk harvesting in a variable environment [72]. Another study [1] proposed and verified the seasonal transport of raw milk (SToRM) model, intending to demonstrate a method for optimising the total distance necessary to fulfil collecting routes for the significantly seasonal production patterns seen in the Irish milk business. GIS data were combined with an ALNS metaheuristic-based technique to solve the capacitated vehicle routing problem (CVRP) that occurred from collecting milk from a simulated set of fifty milk-producing farmhouse spots in that study. According to [1], the StoRM model assesses the impact of flattening the seasonal curve to align it with other European Union states.

Likewise, mathematical models linked to CVRP with fuzzy stochastic demands (CVRPFSD) and CVRP with intuitionistic fuzzy stochastic demands (CVRPIFSD) were published, demonstrating that the effort might be used to arrange the shipping of seasonal perishable products [73].

The heterogeneous VRP with temporal windows and a limited amount of resources (HVRPTW-LR) [74] was developed to successfully handle resources for exact services businesses with highly seasonal demand by utilising a hybrid variable neighbourhood descent metaheuristic based on a Tabu Search algorithm. The HVRPTW-LR might be employed when limited resources [74], such as cars, drivers, and instruments, are available.

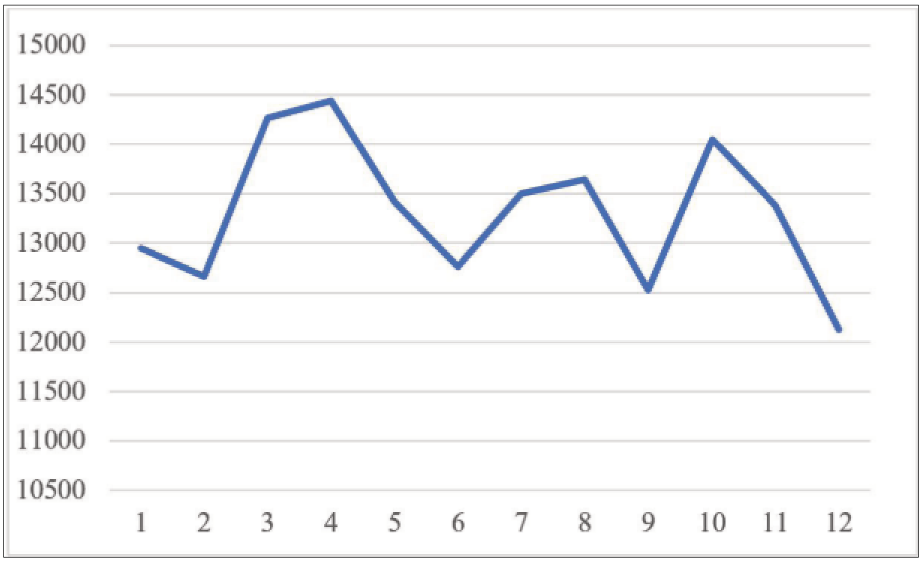

Figure 2.6:Total demand fluctuations for each period (adapted from [27] **)**

A periodic capacitated VRP was implemented with a simulated annealing algorithm under sufficient time and a fair number of vehicles providing small-scale enterprise management in the sense of exhibiting a digitalisation approach for truck arranging applications [27]. Julia Programming language was used to develop the model. A furniture roving component manufacturing company conducted a 12-period study with 30 clients and a uniform fleet size from a store. Figure 2.6 [27] shows the total demand fluctuations for each period according to the instance which was being used in that paper.

The researchers attempted to formulate the algorithms for multi-period vehicle routing problems (MPVRP) [71] as an extension of the Periodical vehicle routing problem (PVRP) [75]. There is a study [76] about the evaluation of PVRP. As in most real-world applications, the planning horizon includes multiple periods, and the time in which each route is executed is a variable problem [77]. According to [71], the classical MPVRP is closely associated with the PVRP, in which the clients specify a service rate and suitable combinations of visit dates, and also, the definition of the periods is based on seasonal production level variations.

A new rich VRP [77] that could appear in a real-life scenario was proposed and formalised as a Multi Depot Multi-Period Vehicle Routing Problem with a Heterogeneous Fleet. The goal of the problem was to reduce the total distribution charge. Each consumer is aware of the timeframe and the collection of times during which the delivery may take place. A Mixed Integer Programming (MIP) was used to formulate the problem, and an effective Adaptive Large Neighborhood Search (ALNS) based Matheuristic approach was proposed.

When it comes to taking the seasonal effect to VRP, tactical scheduling for a class of multi-period VRP (MPVRP) [78] was formulated. That problem involves optimising daily product collections from several manufacturing spots over a particular planning horizon. In the framework of the paper, a single routing strategy for the entire horizon was planned, and the seasonal fluctuations in the producers' supplies were encountered. Manufacturing variations across time are represented using a series of cycles, each associated with a production season, but intra-period volatility is ignored. The first step related to the construction of a plan, while the second stage considered the varied eras when developing the mathematical model. A branch-and-price approach was used, and the approach was able to solve instances with up to 60 producers and five periods.

A practical distribution problem [79] was faced by a third-party logistics provider who delivered pharmaceutical products to healthcare facilities in Tuscany. Multiple depots, a heterogeneous vehicle fleet, periodic demands, flexible time frames, conflicts between customers and vehicles, a maximum number of customers per route and a maximum duration for the routes were characterised when formulating the problem. The paper test an artificial case. When it comes to periodic demand character, a number of packages identified client desires and dispersed them across a 6-day time frame. Additionally, the demand was pretty strong from Monday to Friday, with a decline on Saturday.

2.9.2 Multi-Product VRP

Many organisations are involved in multi-product processing, such as food and grocery distribution, pharmaceutical goods, rubbish collection, marine products, etc [80]. The following are some examples of VRPs connected with multi-products.

A heterogeneous VRP with Multi-Trips and Multi-Products (HVRPMTMP) [81] was suggested for a pharmaceutical items distribution firm in Indonesia that delivered a range of pharmaceutical products to its 55 customers by enabling certain vehicles with the small-scale capacity to perform multi-trips. The suggested Genetic algorithm (GA) could reduce the overall cost by 48.78%. Client demand had been estimated as volume demand in that situation, based on the unit volume and quantity of the product requested by the customer.

Furthermore, when it comes to multi-product routing problems, a heterogeneous vehicle routing problem was proposed arising in soft drink distribution in a Quebec-based company. The vehicles must deliver goods and recyclable pick-up materials at customer locations. The problem was modelled as a mixed-integer program and suggested three strategies [82]. A constructive heuristic and two petal-based heuristics are included. Alternatively, an ant colony optimisation with a two-pheromone trail approach coupled with tabu search was used to build a heterogeneous VRP with multi-products with temporal windows, and the issue was represented with integer programming [21]. Both studies([82], [21]) confine their unit weight to commodities and unit volume to vehicle volume and weight capability.

A paper [83] proposed a scheduling algorithm for handling a common sub-problem faced in the practice of store-direct delivery networks, which involved the planning of a single delivery truck, multiple commodities, and a particular sequence of clients to be visited in a delivery cycle. They tested the quantities of various goods to be partially dispersed using two packing strategies, Largest Fit First (LFF) and Smallest Fit First (SFF). Because a space demand was examined in three dimensions, the publication showed that as future expansions, the three-dimension bin-packing problem could be coupled as a replacement for one-dimensional size.

Moreover, A VRP in Multi-Product Frozen Food Supply [84] included factors such as price, punishment degree, unit volume, and perishable coefficient of various frozen foods. In addition to the usual limitations of time windows and loading weight, the volume capacity of the vehicle was considered. The model was created using the Genetic Algorithm (GA).

A couple of instances were mentioned about multi-product VRP in the ice manufacturing industry products like tube ice, flake ice and cube ice are produced with the demand for ice driven by consumption, population growth and weather conditions or seasonality [85]. CVPR for multiple goods, time windows, restricted distribution time for each customer, and a heterogeneous fleet [86] to minimise overall costs, consisting of travelling fees, driver wages and a penalty expense, had presented developing to solve small-sized problems. At the same time, a differential evolution approach with a local search was established to solve large-sized applications. The VRP, considering vehicle capacities, time windows, multiple products, fleet sizes, and fleet size limits on roads (VRPTWFS) [85], was formulated as an NP-hard problem with developing the hybridisation of PSO and ALNS model to minimise transportation costs.

The pharmaceutical industry has a range of product types to distribute. A study [87] developed an approach for Capacitated Heterogeneous Vehicle Routing with Time Windows Problem for the pharmaceutical industry. This model was established in Medan City, Indonesia, to deal with the matter of a pharmaceutical trader. The resulting model was a MILP, and a neighbourhood heuristic algorithm was used to solve the model.

2.10 Chapter Summary

This chapter presents a summary of existing literature relevant to the research title. The reviewed literature has implied the importance of the vehicle routing problem (VRP) to address real-world scenarios, although it needs to account for some context-specific parameters. It was also evident that only a few research studies were conducted relevant to the stationery industry under this title. The research aims to fulfil that existing industrial gap by presenting a way to optimise the vehicle routing schedule of companies with a highly seasonal demand pattern.

The next chapter will explain the formulation of the problem to achieve the abovementioned intentions.

3 CHAPTER 3: METHODOLOGY

3.1 Introduction

Chapter 2 portrays that most relevant literature has constructed mathematical models to formulate the problem. For this sort of study, heuristic approaches are applied. This chapter highlights the procedures and methods researchers utilise to get the desired result. In addition, the chapter illustrates the limitations of the approach utilised in the analysis. Secondary data and relevant information must be retrieved from the industry to achieve the research objectives. To achieve the core objectives of the investigation, the acquired data needs to be analysed by employing computing experiments.

3.2 Research design

Figure 3.1 shows the summary of the research design discussed in this section. The study's primary goal is to control the appropriate number of vehicles utilised to complete the distribution method in both peak and off-peak planning periods, as well as the route sequence of each tour, in order to minimise distance-related outlays while enhancing vehicle capacity utilisation. This was subdivided into four sub-objectives to operationalise the study.

The first sub-objective is to investigate the different types of VRP applications under various types of products portfolio and a highly seasonal demand pattern. In order to achieve this objective, a thorough literature survey was conducted, which was discussed in detail in Chapter 2. Different kinds of advanced algorithms have been formulated for these kinds of problems. Nevertheless, in this study, as an initial solution for the company, the CVRP model will be formulated by engaging with the k-means clustering algorithm as the initial solution for the company. The CVRP can be extended by adding more real-world factors as future extensions of this study.

The second sub-objective is identifying the nature of the distribution process, sales pattern, and product nature related to the stationery company to develop an appropriate VRP solution. To get some information about truck types, dealer points, and volume measurements, the focus group interview was held with relevant departments of the company. Descriptive analysis was done to identify the sales patterns of the products, as discussed in Chapter 5.

The third sub-objective is to identify the variables and constraints of the VRP to formulate the mixed integer linear programming. Discussions held with the staff of the company and the literature review helped to get the information relevant to variables and constraints. A mixed integer linear programme was formulated and illustrated in Chapter 4 accordingly.

The last objective was to determine VRP solutions for both peak and off-peak planning horizons. The details analysis is illustrated in Chapter 5.

3.3 Data collection

This research uses secondary data for computation experiments. Furthermore, some insights have to be taken from the practitioners of the operations in the company in order to formulate the mathematical model and to have an idea about the current practices of the delivery process and sales pattern.

3.3.1 Secondary data

Mainly for this study, we need month-wise and depot-wise sales volume. The data was taken from the SAP operating system used by the selected organisation. There were 155,249 rows in the data set. The organisation has historical data from 2016 only. Since the covid pandemic happened in 2020 and 2021, the company's sales volume significantly deviated from the expected scenario impacting the seasonal effect. Therefore, the study used data from April 2016 to March 2020 only.

3.3.2 Interviews

Semi-structured focus group interviews were conducted to take information about variables and constraints required as inputs for the mathematical model and to get to know the nature of the product and sales pattern. The interviews were conducted with the resource persons mentioned in Table 3.1.

Table 3.1: Reasons for the interviews

Resource person	Reason
Logistics Team	To get knowledge about the current delivery process
Demand and supply planning Team	To identify the sales patterns and product nature and get the customer-wise sales details
Marketing Team	To get knowledge about the market strategies relevant to the current sales patterns

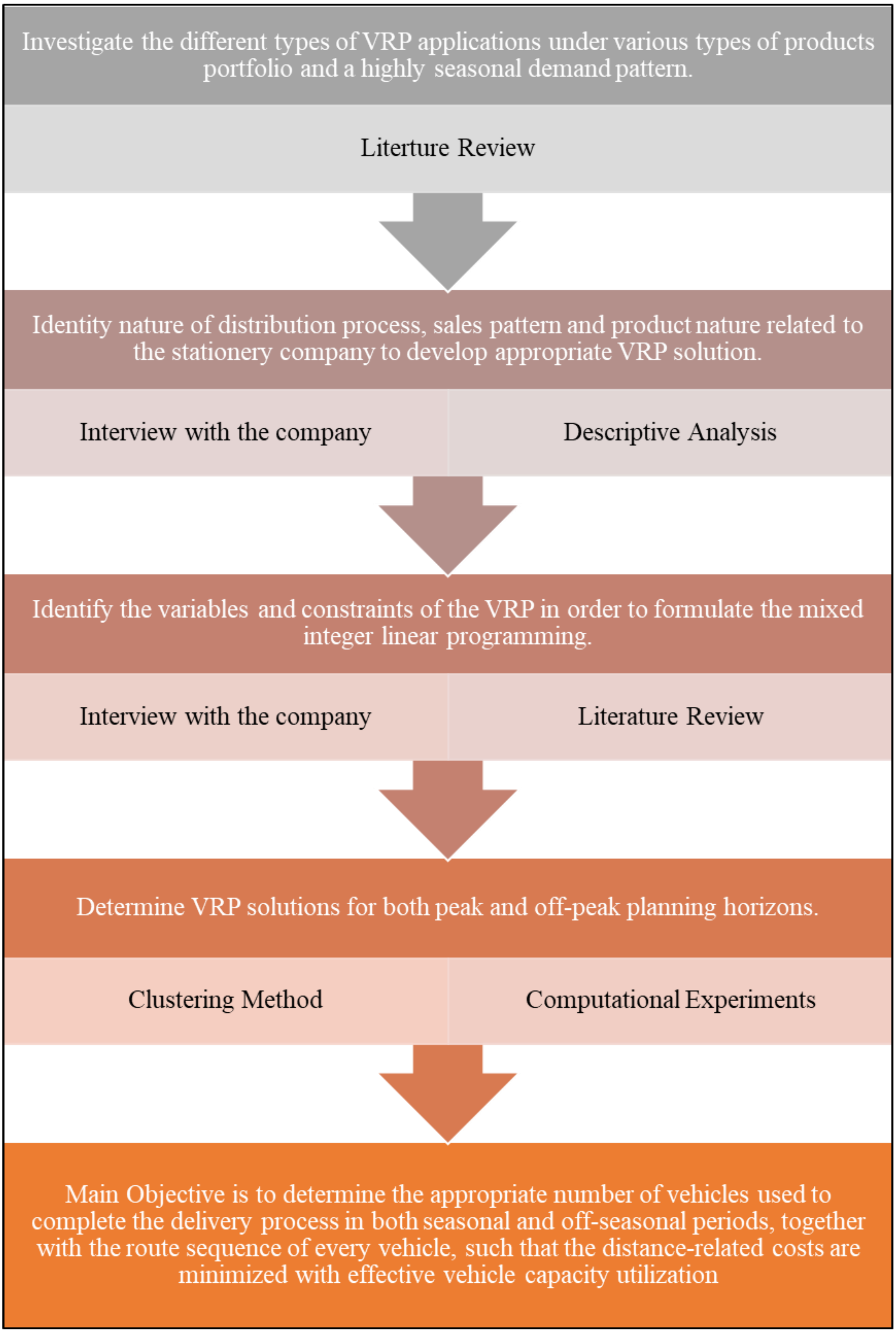

Figure 3.1: Research design

3.4 Data Processing

When taking secondary data for analysis, data cleaning is an important part. Received raw data from company systems cannot be applied to computer experiments directly. The adjustments conducted on the received data are described in the following subsections.

3.4.1 Data cleaning

The company distributes goods under three main distribution channels: general trade, modern trade and institutions. This study is only focused on general trade since other channels do not significantly impact seasonal effects and separate distribution patterns are used for them. Around 94% of sales are under the general trade, as shown in Figure 3.2.

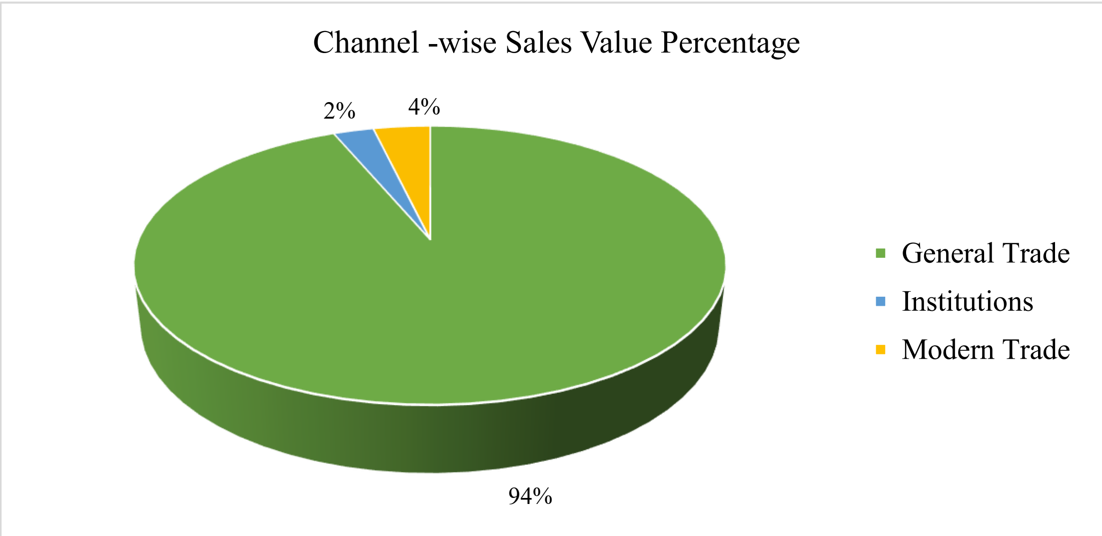

Figure 3.2:Channel -wise Sales Value Percentage

The company has divided products into six main categories, which will be explained under descriptive analysis(Section 5.2). Apart from that, another category called “Other” includes Packing items, Balls, Balloons and Tissues. It is around 3% of company sales value under general trade, as shown in Figure 3.3. These are also not impacted by seasonal effects. The volume of these products is also eliminated from this study since the main focus is on seasonal effects. The eliminated volume is only around 9.2% of the total company's yearly sales value.

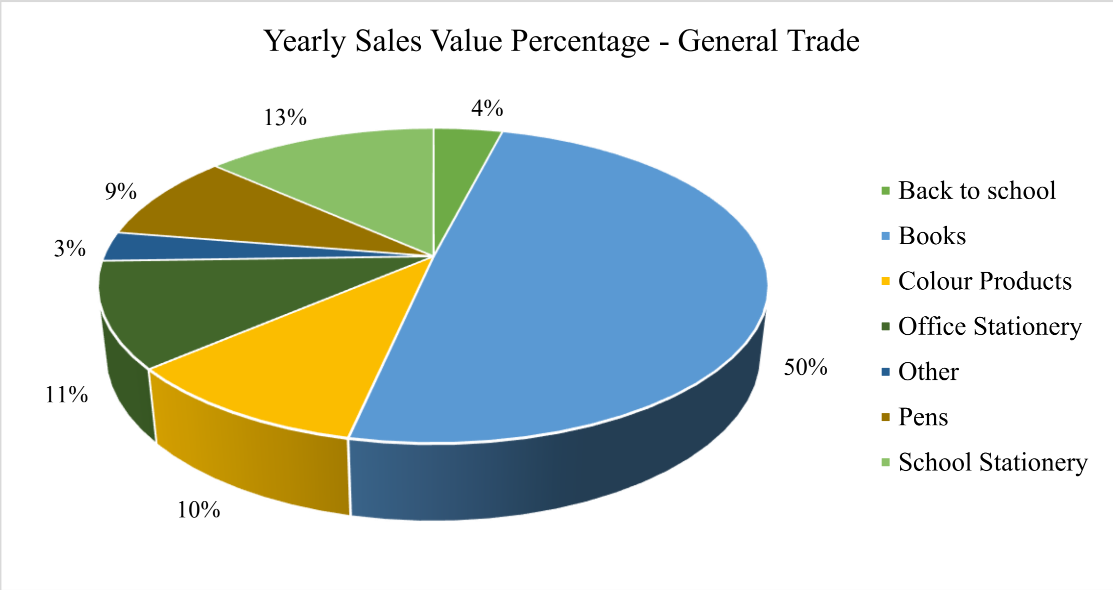

Figure 3.3:Yearly Sales Value Percentage - General Trade

Considering the above eliminations and inappropriate values in some cells, 34,096 rows had to be removed from the data set.

3.4.2 Product-wise volume calculation

The company's SAP operating system mentions sales volume in quantities and LKR values for the organisation's expectations. This study required calculating the volume of the products by CBM (m^3); hence, carton-wise length, height and width were measured for all the products in six main categories to calculate the volume by CBM.

3.5 Justification of the analytical framework

The methods of analysis and resources used are justified in this section. Most of the research carried out on similar topics has formulated mathematical models with the usage of heuristics techniques to solve the formulas. Because of the unwillingness of the organisations to provide financial information, only the number of the units of products were taken and justified findings in volumes based. Theories of computation and statistical analysis approaches employed in analysing data to achieve the aims of the study are discussed below.

3.5.1 Descriptive analysis

Descriptive analysis [88] is a statistical approach for displaying a summary of data obtained. Descriptive statistics are used to provide meaningful summaries from a set of data. Descriptive analysis can identify trends and patterns in data gathered from a sample representing the entire population. Therefore, this analysis is vital for this study because of getting the proper understanding of the sales patterns and products' nature for the purpose of providing a better outcome.

3.5.2 Mathematical formulation

Looking at the variables of the problem, as detailed in Chapter 4, there are different kinds of variables. Integer and boolean variables can be identified as integers. Variables like the distance of the routes can be identified as continuous variables. So, there are both types of variables here. Therefore, MILP is suitable for formulating the mathematical model. Further, looking at the literature, most applications used the MILP model to formulate the CVRP problem.

3.5.3 K- means cluster Analysis

Cluster analysis was conducted with the intention of gaining the benefit of the separation of zones since distance constraint was not counted in CVRP. Customer locations were divided into clusters using the k-means clustering algorithm. Locations were plotted according to longitude and latitude, and this is a kind of geographical separation. Using this method, the distance constraint can be taken into account slightly. Before running the routing problem, the consumers will be grouped using partitioning-based iterative clustering. K-means clustering is used to construct distinct clusters so that each consumer is assigned to only one group. The most significant advantage of the K-means clustering approach is that it uses less memory than other clustering strategies, such as hierarchical clustering assessment. [89].

3.5.4 Computation Experiments

The studies are based on using commercial software in order to compare the current practices of the company. Gurobi Optimiser is one of the most popular commercial solvers, and there are free versions for academic requirements. Further, since the study is limited to 87 customer locations grouped into several clusters, the problem in this study is categorised as a medium size problem. According to the literature, Gurobi gives accurate solutions for small and medium size problems [44], [45]. A student version of Gurobi optimiser [42] 9.5.1 in the python platform was employed to evaluate the various CVRP results. Python programming language is used to develop the CVRP model. Literature elaborated that Python is flexible, easy, and powerful, has excellent libraries for machine learning, optimisation, and statistical modelling, and is easy to learn for first users [46].

3.6 Chapter Summary

This chapter elaborates on the strategy for data collecting and the analytical processes utilised to achieve the primary research objectives. The mathematical model in which the problem is formulated is described in Chapter 4.

4 CHAPTER 4: FORMULATION OF PROBLEM

4.1 Introduction

Optimising the distribution network with the condition of highly seasonal demand patterns and a large product portfolio is an identified industrial gap which should be addressed. This important real-world scenario which is related to this research, is described in this chapter. Problem description, problem definition and the mathematical model will be discussed in this chapter.

4.2 Problem Description

This research problem is based on the requirement of a transport carrier to deal with the weekly distribution schedule of a firm with an extremely seasonal sales pattern. The firm uses a single distribution centre managed to store company-manufactured and imported goods. Generally, the organisation operates a fleet of vehicles with 30 m^3. However, in practice, vehicles cannot be fully loaded. The logistics manager of the organisation mentioned that according to his experience, a truck's total possible loading capacity is around 27 m^3. The organisation has to outsource additional vehicles for seasonal months, a vast expense to the firm.

Furthermore, the truck's capacity utilisation is relatively low during off-peak periods. Seasonal months are those from August through December, whereas off-seasonal months are those between August and December. The fiscal year ran from April to March of the following calendar year. We can reduce the usage of outsourced vehicles and reuse the same trucks by using a weekly timetable. This paper assumes that weekly demand remains constant throughout a planning horizon.

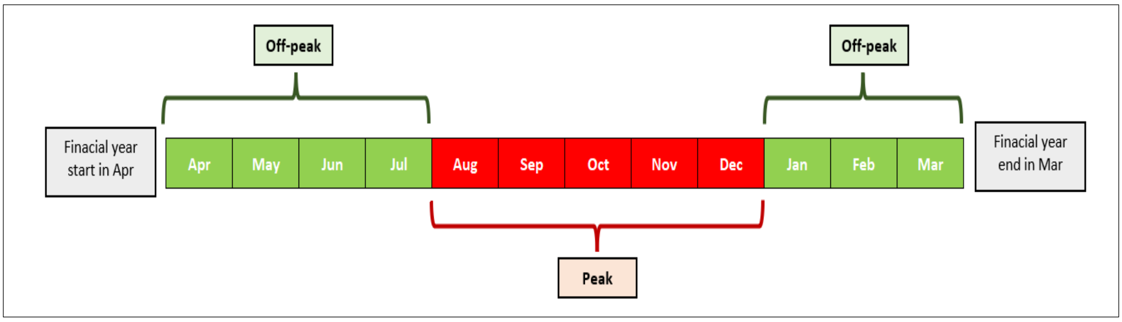

Figure 4.1: Peak and Off-peak distribution throughout the year

Because there is a considerable seasonal change in demand throughout the year, this is shown as a capacitated vehicle routing problem (CVRP) with a seasonal fluctuation (typically, several months). It is thought that a year can be divided into two periods, each reflecting a different level of demand. The study only takes into account inter-period demand changes; possible intra-period volatility is ignored. Furthermore, the distribution center (DC) is expected to have a maximum capacity to meet peak-season demand. To distribute the items, only one DC is used. This is also a Multi-Product challenge due to the many commodities that must be delivered to the same consumer. This is a CVRP with many products that capture seasonal and off-seasonal product supply. These VRP varieties are less common in the literature but common in real-world circumstances.

This type of solution is typically used by businesses that supply their goods across a vast geographic area and have a highly seasonal demand pattern. Clients come from cities and their adjacent metropolitan regions around the country. Each truck begins at the DC, drives along the delivery route to distribute items to client sites, and then returns to the DC. All distances are determined for each planning horizon and each cluster.

4.2.1 Problem definition

The CVRP considering a variety of commodities and seasonal fluctuation is formally specified in the subsection. The problem involves serving a set of clients, N, with a set of products, P, in a set of the planning horizon, H, relevant to a set of the cluster, G, at the distance-related minimum cost. The distribution centre (DC) is located at N = 0, and the number of all customer locations is $V=N\cup\{0\}$. Each customer i requires the q_{pigh}, the number of various products, p, which can be carried from the central DC where i=0. Distribution is carried out during a timeframe (week). The average velocity is made equal for each and every vehicle and known in advance. Set of routes begins and ends at the DC. Additionally, it is taken into consideration that the maximum route capacity is C. The distance d_{ij} is known for each pair of nodes.

4.2.2 Definitions and notations

Table 4.1 summarises the definitions of the variables and constants, as shown below.

Table 4.1:Description and Notations

Notation	Description
Index	
i, j, a	Customer location; i,j,a =1………n (n is an integer)
i, j, a	Location of the Depot; i , j, a = 0
H	Planning horizon; h=1…..n (n is an integer)
P	Product type; p = 1……..n (n is an integer)
G	Cuslter; g=1…..n (n is an integer)
Parameters	
P	Set of Product types
N	Set of customer locations
V	Set of nodes (V=N+{0})
H	Set of planning horizons
G	Set of Clusters
C_p	Volume of the product p (m^3)
q_{pihg}	Demand of product p at customer i in panning horizon h and cluster g (units)
d_{ij}	Travelling distance from customer i to j (km)
W_p	Weight of product type p (kg)
C	Maximum capacity of a route (vehicle capacity-m^3)
W	Maximum weight of a route-weight limit of a vehicle (kg)
T_{max}	Maximum route duration (working hours per day)
D_{ihg}	Total demand (in volume – m^3) at customer i in planning horizon h and cluster g
W_{ihg}	Total demand in weight at customer i in planning horizon h and cluster g (kg)
T	Total travel time of a route
Decision Variables	
A	Objective function
X_{ijhg}	Boolean variable equal to 1 if arc ij is included in the solution relevant to cluster g and planning horizon h

4.2.3 Assumptions

A homogenous fleet of vehicles with identical volume (m3) capacity and the exact cost per kilometer has been evaluated. The following assumptions are made in this framework:

I. Vehicle capacity is well-known.

II. The number and the customer points are known, and the distribution centre is also known.

III. Any product mixed with distinct weights can be packed into the vehicle.

IV. The average velocity is set equal for each vehicle and is known ahead of time.

4.3 The mixed-integer linear Programming model

4.3.1 Objective function

Minimising the total distance-related cost is stated as the objective function in equation (1). The total driving distance of the route network is to be minimised. The distribution path of vehicles should minimise the total costs as far as possible while meeting the customer requirements for each and every planning horizon and cluster.

$$Min(A) \sum_{i \in V} \sum_{j \in V} \sum_{h \in H} \sum_{g \in G} X_{ijhg}\, d_{ij} \qquad (1)$$

4.3.2 Constraints

The constraints illustrated in this CVRP are as follows.

Subject to :

$$\sum_{j \in N} X_{0jhg} = \sum_{i \in N} X_{i0hg};\ \forall h \in H, \forall g \in G\ \ for\ each\ route \qquad (2)$$

The first constraint (2) makes sure that each and every route begins and ends its travelling path at DC only. That means each vehicle takes the DC as the origin point, moves along the delivery route to supply the goods to the customer sites, and then comes back to the DC. This constraint should be valid for all the planning horizons and all the clusters.

$$\sum_{j \in N} X_{jihg} = 1 \,; i \neq j, \forall i \in N, \forall h \in H, \forall g \in G \tag{3}$$

$$\sum_{i \in N} X_{ijhg} = 1 \,; i \neq j, \forall i \in N, \forall h \in H, \forall g \in G \tag{4}$$

Both constraints (3) and (4) guarantee that a customer is only visited once. Every customer is serviced by one vehicle, but one can serve multiple customers. There are no partial shipments, and retailers' demand must be satisfied. Any location cannot be catered to more than one time. These constraints should be valid for all the planning horizons and all the clusters.

$$\sum_{i \in N} D_{jhg} X_{iahg} \leq C \,;\; \forall h \in H, \forall g \in G \; for\ each\ route \tag{5}$$

The total demand volume by customer i uploaded by each route must not exceed the vehicle's maximum capacity is the above constraint in equation (5). The sum of customer demand in each route in the solution should not exceed the vehicle's maximum capacity for all clusters and planning horizons.

$$X_{iahg} + X_{aihg} \leq 1; i \neq a, \forall i, a \in N, \forall h \in H, \forall g \in G \tag{6}$$

Equation (6) confirms that if a vehicle delivers to a customer on any route, that vehicle will not come back on that route again. This should not be a valid route that has only one customer, and it is ensured by equation (2).

$$X_{iihg} = 0 \,; \forall i \in N, \forall h \in H, \forall g \in G \tag{7}$$

Diagonal variables are not considered as guaranteed by constraint in equation (7). That means there is always a path to deliver, and the vehicle has to move. This constraint must be valid for all the clusters and the planning horizons.

$$X_{ijhg} \in \{0,1\};\; \forall i, j \in N, \forall h \in H, \forall g \in G \tag{8}$$

This constraint (8) specifies the variable domain. This is a boolean variable which says whether the vertical (route) is in the solution or not in the relevant planning horizon and cluster.

Total demand in volume at any customer i in the planning horizon h and cluster g is estimated as follows (9).

$$D_{ihg} = \sum_{p=1}^{P} q_{pihg} C_p \text{ ; } \forall i \in N, \forall h \in H \tag{9}$$

Two planning horizons H={1,2} are counted in this study. Planning horizon 1 is called off-peak months, which are from January to July. Months from August to December are called peak months, and those are counted under planning horizon 2. In this research, the average volume within each planning horizon related to each customer is considered as the demand volume of the relevant customer in the relevant planning horizon.

4.4 Chapter Summary

In this chapter, MILP was formulated to solve the CVRP, which is capturing multi-products and multi-periods. When it comes to real-world scenarios to operate practically, distance or time constraints and weight constraints can also be included. For this study, as an initial solution for the organisation, a CVRP is introduced, which can be extended in future considering more constraints. This will be further discussed in Chapter 5.

5 CHAPTER 5: ANALYSIS AND FINDINGS

5.1 Introduction

The main objective of the research is to achieve this through a thorough analysis of secondary data collected from several sources from the selected company. The described approach in Chapter 3 and the formulated problem in Chapter 4 are helpful to the analysis. This chapter explains the analyses that were performed as well as the results of those studies. To evaluate the data and draw conclusions, descriptive analysis, cluster analysis, and computing experiments were used.

5.2 Descriptive Analysis

The objective of the descriptive analysis is to identify the nature of the product categories and their volume to assist the data cleaning and processing and to identify the sales pattern basically to get the proper understanding of the data set. All the volumes are related to general trade as mentioned in Chapter 3.

5.2.1 Main categories

In the selected company, there are around 400 different stationery items and those have been categorised into six main groups as follows.

I. Books
II. Back to School
III. Colour Products
IV. Office Stationery
V. Pens
VI. School Stationery

Figure 5.1 shows the category-wise volume (m^3) contribution to the operation.

According to Figure 5.1, the largest category is the "books" with around 60% of the total volume."Books" category is the biggest category value-wise (LKR), and it contributes around 50% of the yearly sale. It includes B5 Books, Children's Books, CR Books, Drawing Books, Exercise Books, Note Pad, Paper Products, Practical Books, Science Books and Spiral Books. Mainly, CR and Exercise Books have a higher demand compared to other items under this category. The percentage volume distribution of the “Books” category is shown in Figure 5.2 considering four financial years.

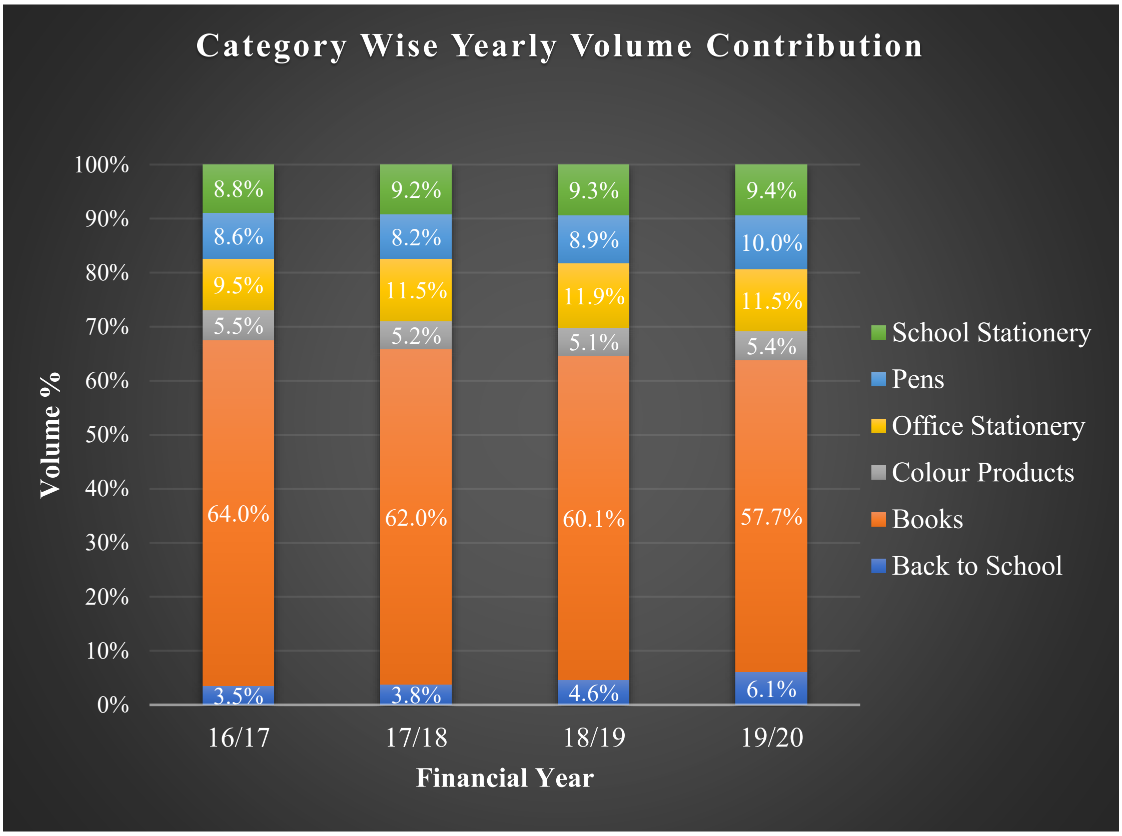

Figure 5.1: Category Wise Yearly Volume Contribution

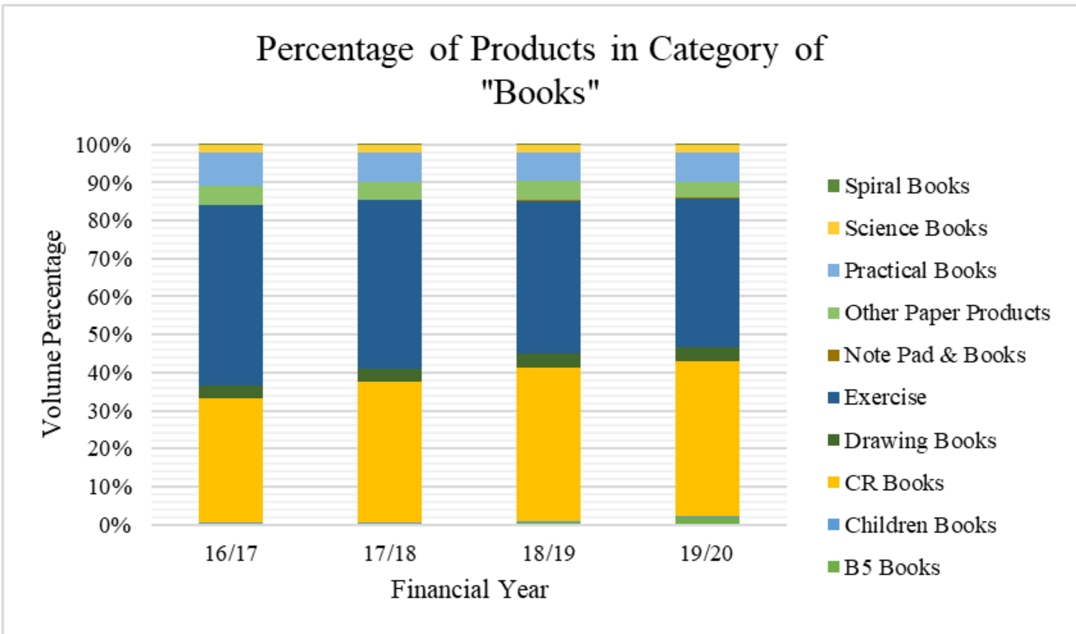

Figure 5.2:Volume distribution percentage of products in the category of "books"

Water bottles and Lunchboxes are included in the “Back to School” category. As per the experience of Demand Planners of the company, an approximate ratio of 80:20 could be observed between water bottles and lunch boxes respectively with respect to the sales pattern. This was evident when the total year volume percentage of the two categories was investigated (Figure 5.3).

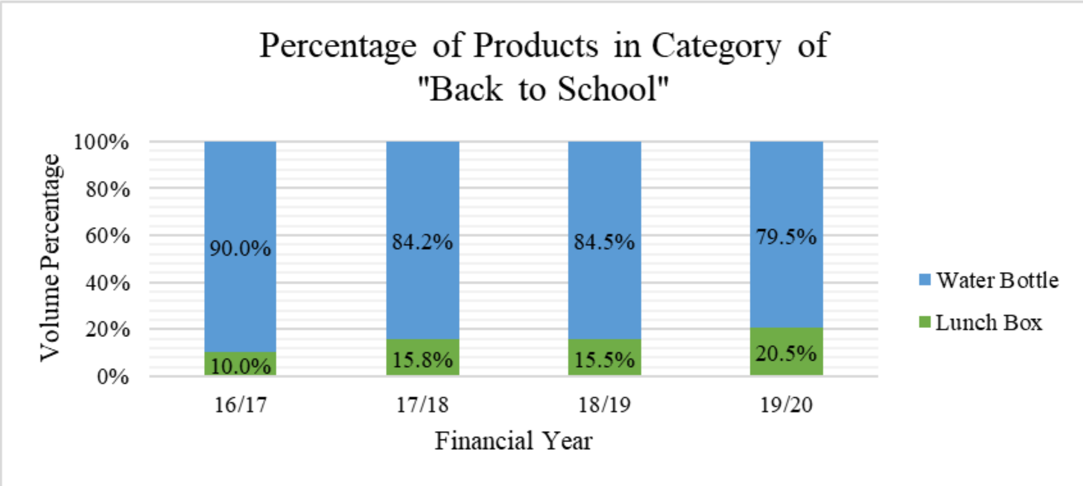

Figure 5.3:Volume distribution percentage of products in the category of "back to school"

There are various kinds of colour products included under the “Colour Products” category. Those are Poster Colour, Colour Pencils, Colour Pens, Highlighters, Pastel and Water Colour. As per Figure 5.4, Pastel is the core selling product in this category.

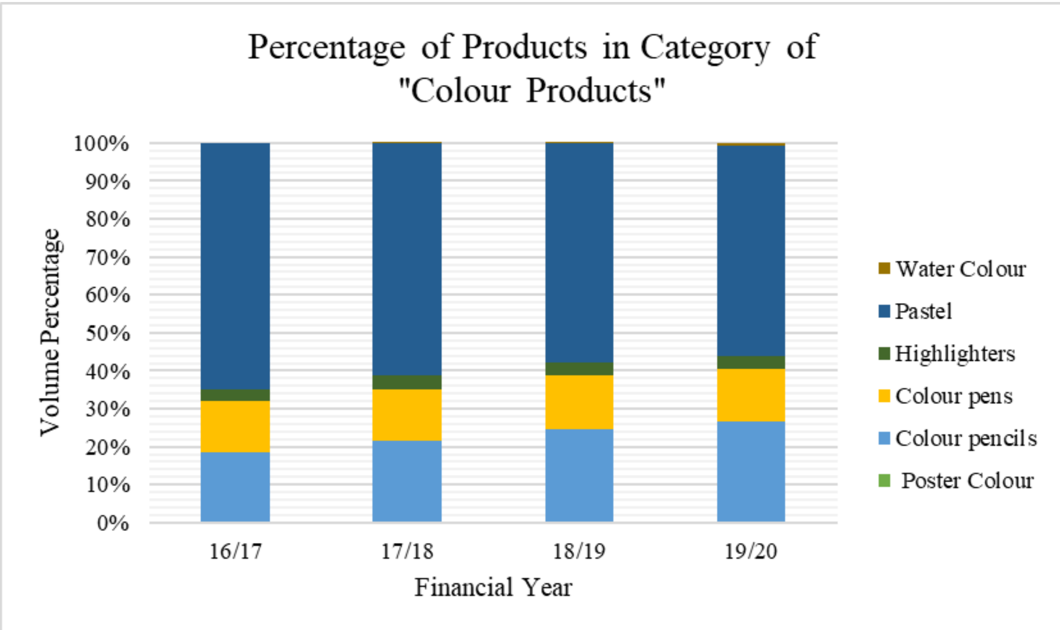

Figure 5.4:Volume distribution percentage of products in the category of "colour products"

Copy Papers, Files, Markers, Puncher, Stapler, Stapler Pin and White Ink are categorised as “office stationary”. As shown in

Figure 5.5, the category of “copy papers” has taken the highest volume in this category. The seasonal effect is created because of the school terms, but products in this category vary slightly from the school stationery products. However, the seasonal effect can be seen sightly in this category as well, because the shop owners use to order these products together with other products in order to take advantage of the seasonal discounts.

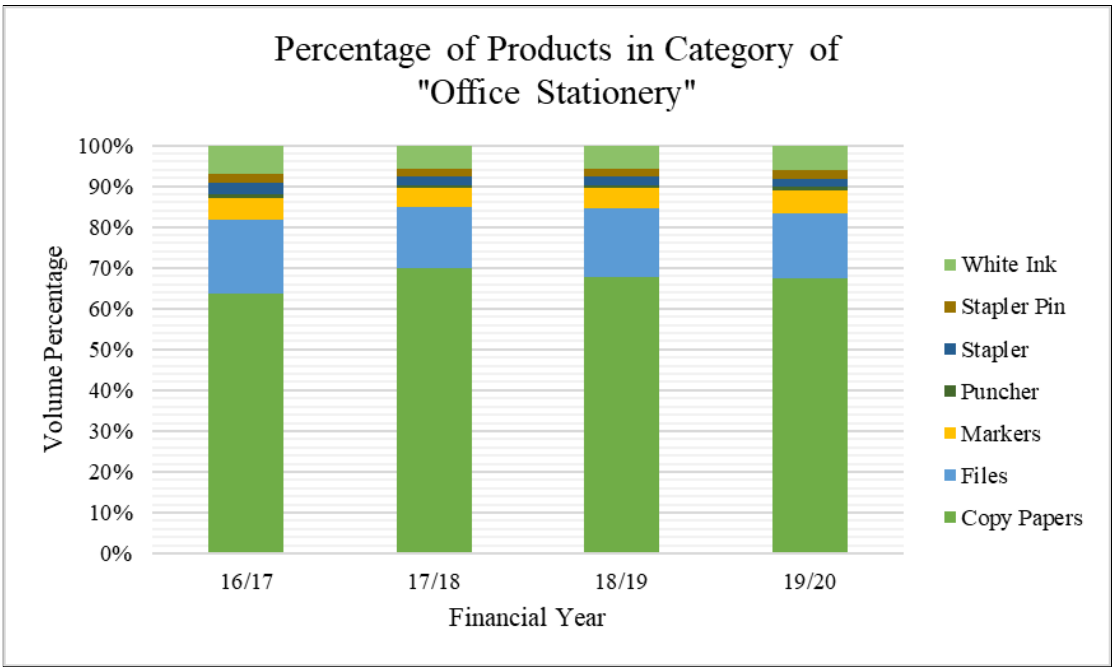

Figure 5.5:Volume distribution percentage of products in the category of “office stationery”

Pens are also categorised as a separate category because there is a considerable sales volume from pens alone. The advantage of this categorisation is mainly taken by the planning team and it has been very useful in understanding the trends. Since the volume is high, it is better to take “Pens” as a separate category for a better understanding of trends. This has been categorised into sub-levels under the main category based on price namely, Rs.10, Rs.15, Rs.20, Rs.25, and Rs.30. Figure 5.6 shows that the Rs.10 pen has the highest sales volume and Rs.20 pen has also a considerable sales volume compared to other pens.

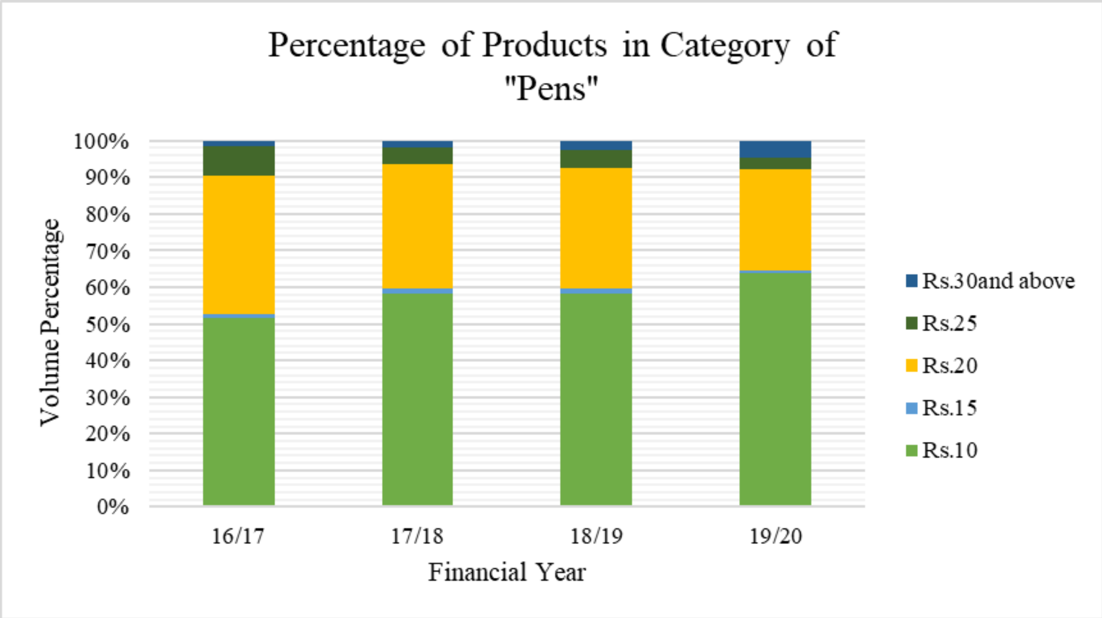

Figure 5.6:Volume distribution percentage of products in the category of "pens"

Under the "School stationery" group, Clay, Erasers, Glue, Mathematical Box, Pencils, Ruler, Scissors and Sharpeners are included. The biggest sales volume is shown under "Gule" according to Figure 5.7. When it comes to sales quantity, the highest value is shown under the "Pencils", but volume-wise it is smaller than "Glue" because of the way of packing and the nature of the product.

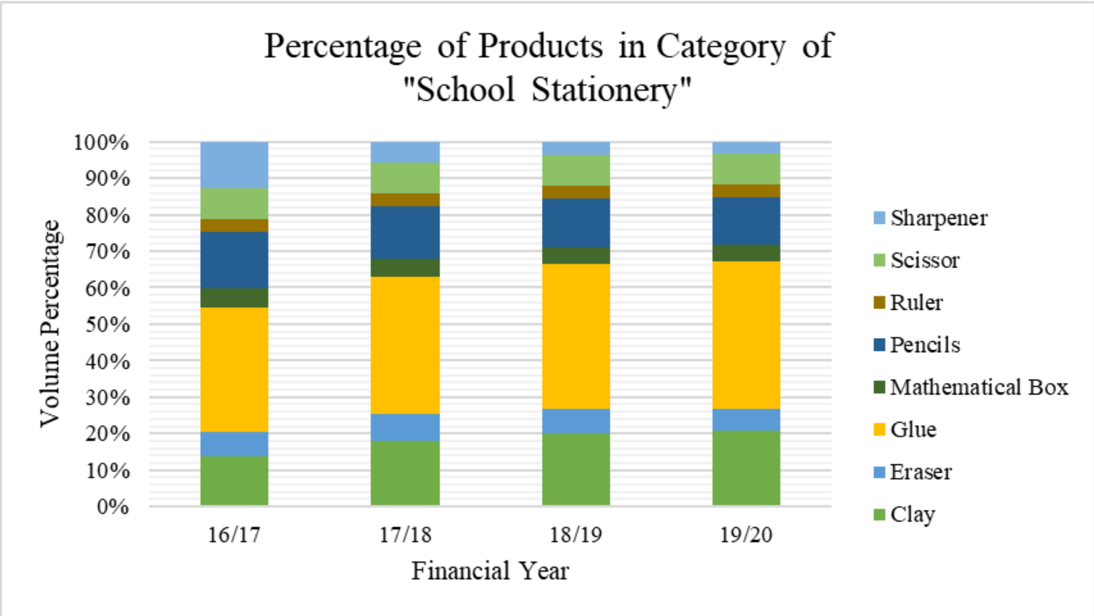

Figure 5.7:Volume distribution percentage of products in the category of "school stationery"

5.2.2 Demand pattern

Although most schools are started in January, the line chart shows the peak months are from August to December. The reason for that is the discounts given by the company for these months as their marketing strategy. These stationery products are not perishable and their expiry period is long. Considering both reasons, clients used to purchase the products for the new school term in order to get an advantage from the seasonal offers. That's how this seasonal demand pattern has been created as shown in Figure 5.8.

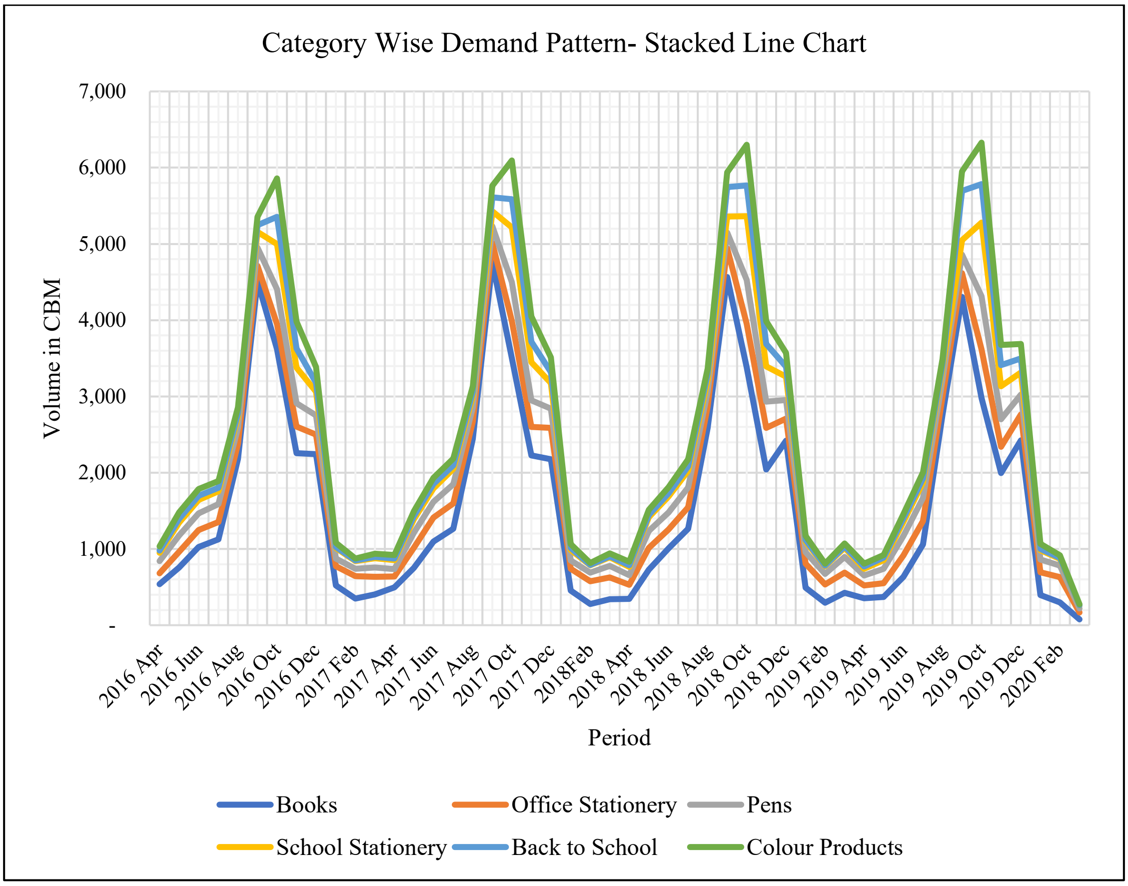

Figure 5.8:Category Wise Demand Pattern- Stacked Line Chart

When it comes to product category-wise demand patterns, the "Books" peak months are started in August and continue till December. The peak months of "Back to School" start from September while the peak months of other categories start from October. All categories' peak period ends in December. Category-wise demand patterns can be seen in Figure 5.9.

Considering these all facts, we considered the peak months as August to December while January to July is off-peak in this study (Figure 4.1).

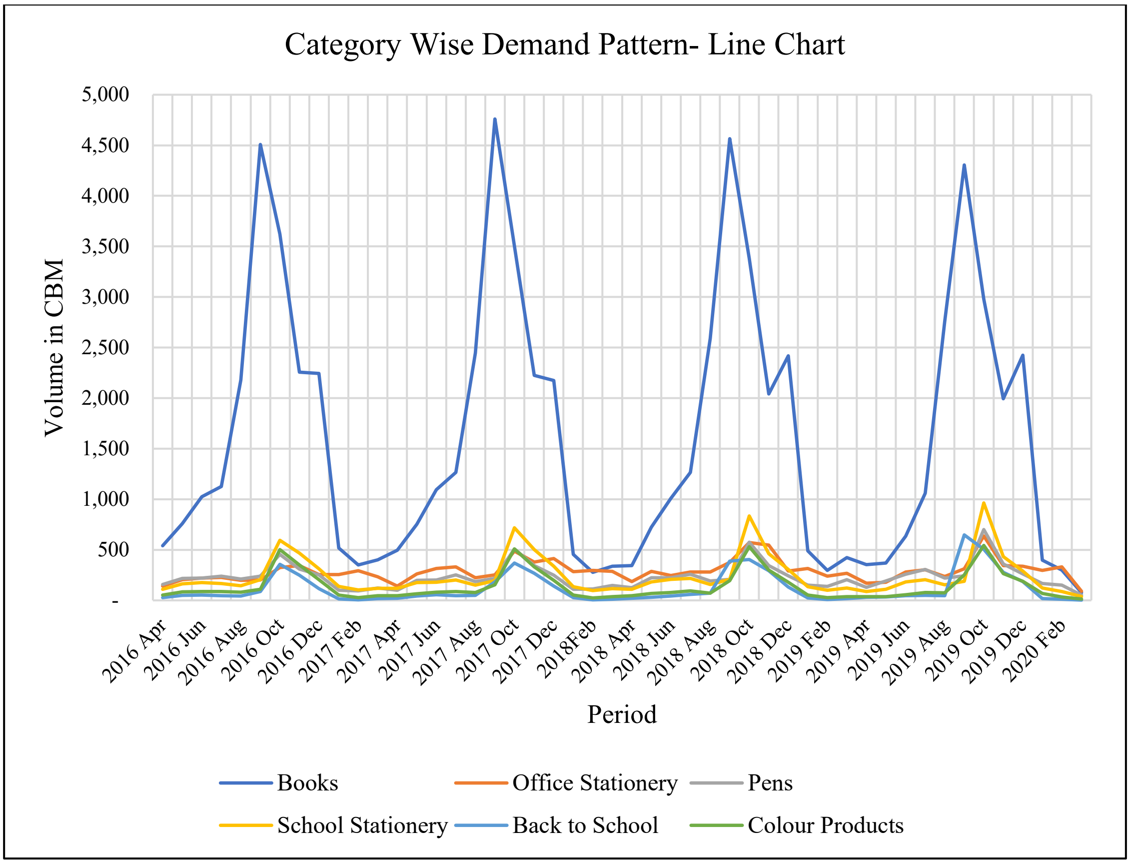

Figure 5.9:Catergory Wise Demand Pattern- Line Chart

5.2.3 Volume comparison of the seasons

Figure 5.10 shows the category-wise volume relevant to two planning horizons. Here the volume percentage has been taken as the average monthly volume percentage within the planning horizons. Products like “Back to School”, “Books”, and “Colour products” have higher sales volume in peak months compared to Off-peak. Usually, customers used to buy these kinds of products at the start of the new school year and use them during the whole year. Students rarely purchase these products mid of the year. “School stationery” has the next higher volume in peak compared to off-peak. From this category, only pencils are generally bought mid of year as well. Other products under this category are mostly one-time buying, like the above-mentioned categories. Moreover, pens are needed for young people and hence are not restricted to students. Therefore, in off-peak months, a little higher volume from the category of “pens” is sold than in the above-mentioned product categories. When it comes to “Office Stationery”, a higher volume percentage is sold in off-peak when compared to other categories. As mentioned above (section 5.2.1), this category varies slightly from the needs of the school students. They are mostly used by office workers, so there is some demand throughout the year for these products. That is why the off-peak percentage is higher than in other categories.

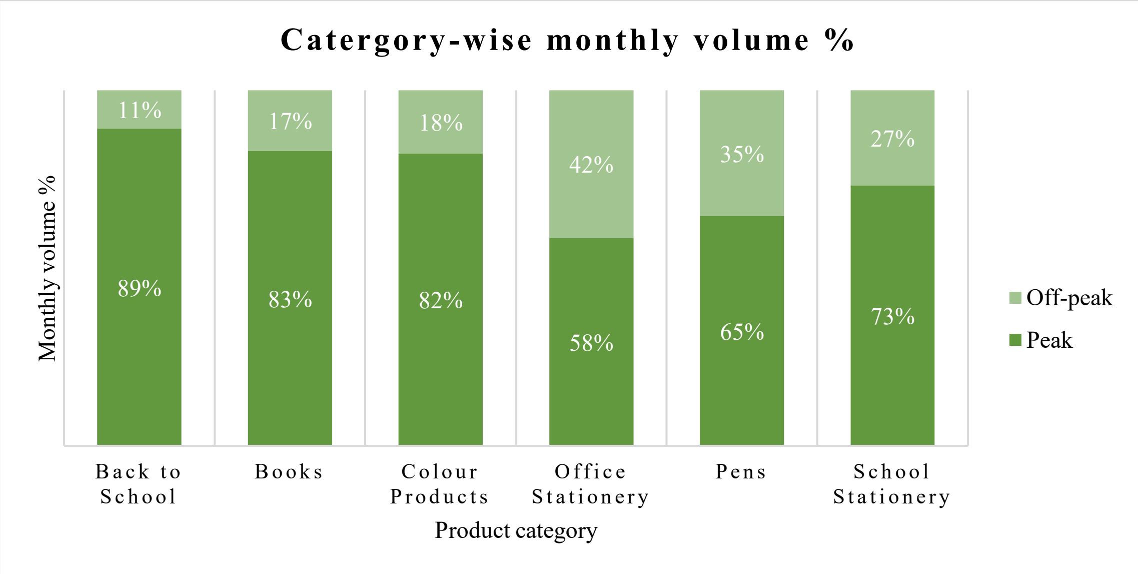

Figure 5.10:Catergory-wise monthly volume percentage

When comparing the total yearly volume in two planning horizons, "Peak" and "Off-peak", around 72% (Figure 5.12, Figure 5.11) of the total volume is sold in the five peak months. Therefore, the average monthly volume ratio is around 4:1 at peak and off-peak, respectively. According to the changing in promotion types and new marketing strategies, the company could improve their volume during peak season yearly. It can be seen in Figure 5.12. The marketing team mentioned this information during the interviews conducted to understand the nature of the products' sales patterns.

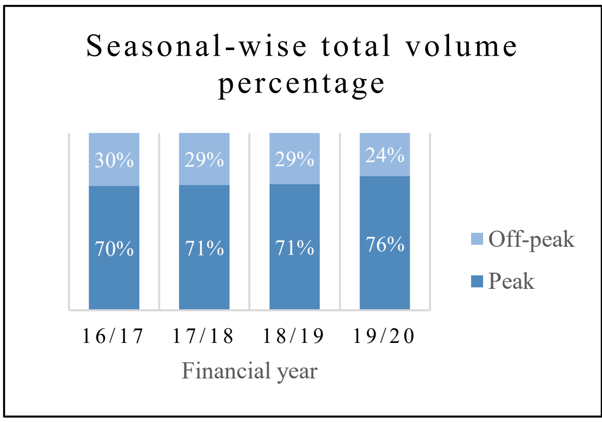

Figure 5.12:Seasonal-wise total volume percentage

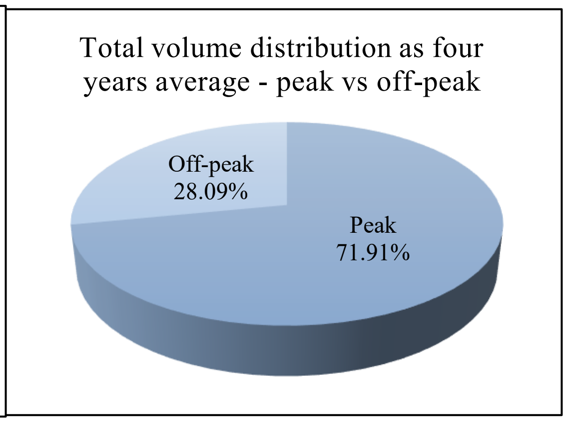

Figure 5.11:Total volume distribution as four years average - Peak vs Off-peak

5.2.3.1 Scientific justification for season-wise volume difference

There should be a scientific clarification to justify the fact that the sales volume in peak is higher than the sales volume in off-peak. Table 5.1 shows that the p-values were less than 0.05, indicating that the $H_{0:}$ the data are normally distributed, would have to be rejected. It means that data are not normally distributed. Therefore, it could be inferred that non-parametric tests would be necessary for testing this particular hypothesis. Therefore, the Mann-Whitney test is used to find whether there is a significant difference in volume between the peak and off-peak.

Table 5.1:Tests of Normality

Tests of Normality								
Year		Season	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
			Statistic	df	Sig.	Statistic	Df	Sig.
16/17	Volume	Off-peak	.232	484	.000	.647	484	.000
		Peak	.373	484	.000	.469	484	.000
17/18	Volume	Off-peak	.235	484	.000	.640	484	.000
		Peak	.351	484	.000	.465	484	.000
18/19	Volume	Off-peak	.238	484	.000	.614	484	.000
		Peak	.346	484	.000	.456	484	.000
19/20	Volume	Off-peak	.221	484	.000	.668	484	.000
		Peak	.348	484	.000	.485	484	.000
a. Lilliefors Significance Correction								

Table 5.2 shows the Mann-Whitney test ranks while Table 5.3 shows the test statistics. The Asymptotic Significance was less than 0.05, which means that the H_0: there is no significant difference in volumes between two seasons, could be rejected according to the Mann-Whitney test statistics illustrated in Table 5.3. Therefore, it is concluded that there is scientific evidence to infer that there is a significant difference in sales volume between peak and off-peak.

Table 5.2:Mann-Whitney test - Ranks

Ranks					
Year		season	N	Mean Rank	Sum of Ranks
16/17	Volume	Peak	484	550.49	266438.00
		Off-peak	484	418.51	202558.00
		Total	968		
17/18	Volume	Peak	484	558.99	270553.00
		Off-peak	484	410.01	198443.00
		Total	968		
18/19	Volume	Peak	484	560.66	271360.00
		Off-peak	484	408.34	197636.00
		Total	968		
19/20	Volume	Peak	484	584.60	282944.50
		Off-peak	484	384.40	186051.50
		Total	968		

Table 5.3:Mann-Whitney test statistics

Test Statistics[a]		
Year		Volume
16/17	Mann-Whitney U	85188.000
	Wilcoxon W	202558.000
	Z	-7.345
	Asymp. Sig. (2-tailed)	.000
17/18	Mann-Whitney U	81073.000
	Wilcoxon W	198443.000
	Z	-8.291
	Asymp. Sig. (2-tailed)	.000
18/19	Mann-Whitney U	80266.000
	Wilcoxon W	197636.000
	Z	-8.477
	Asymp. Sig. (2-tailed)	.000
19/20	Mann-Whitney U	68681.500
	Wilcoxon W	186051.500
	Z	-11.141
	Asymp. Sig. (2-tailed)	.000
a. Grouping Variable: Peak/off		

5.3 Cluster Analysis

5.3.1 Mapping of Location

According to the addresses of dealer points, longitude and latitude were found using "A. CRE Geocoding Excel Add-in". This problem contains 87 dealer points (customer locations) all over the country as shown in Figure 5.13. Further, Figure 5.14 illustrates the scatter plot of the DC and the client spots corresponding to longitude and latitude.

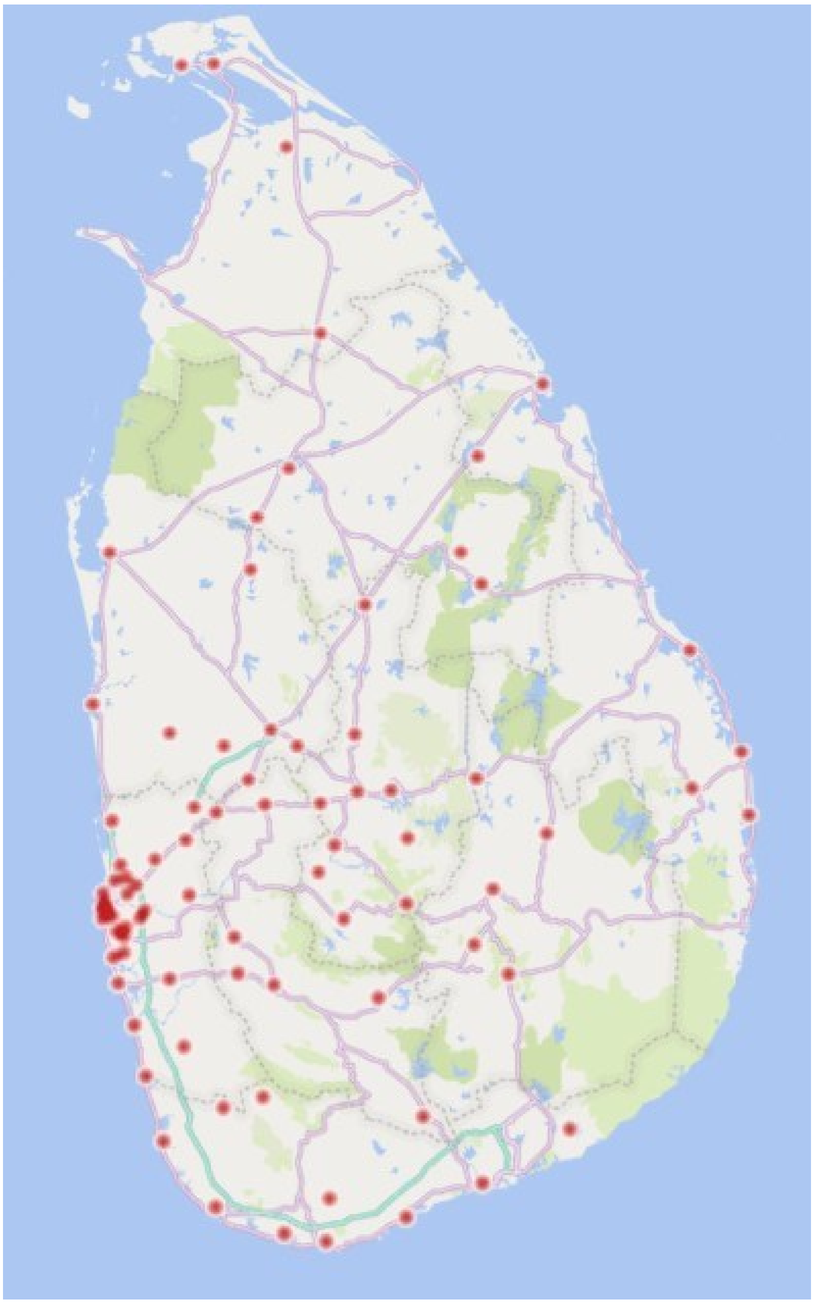

Figure 5.13: Map of customer location

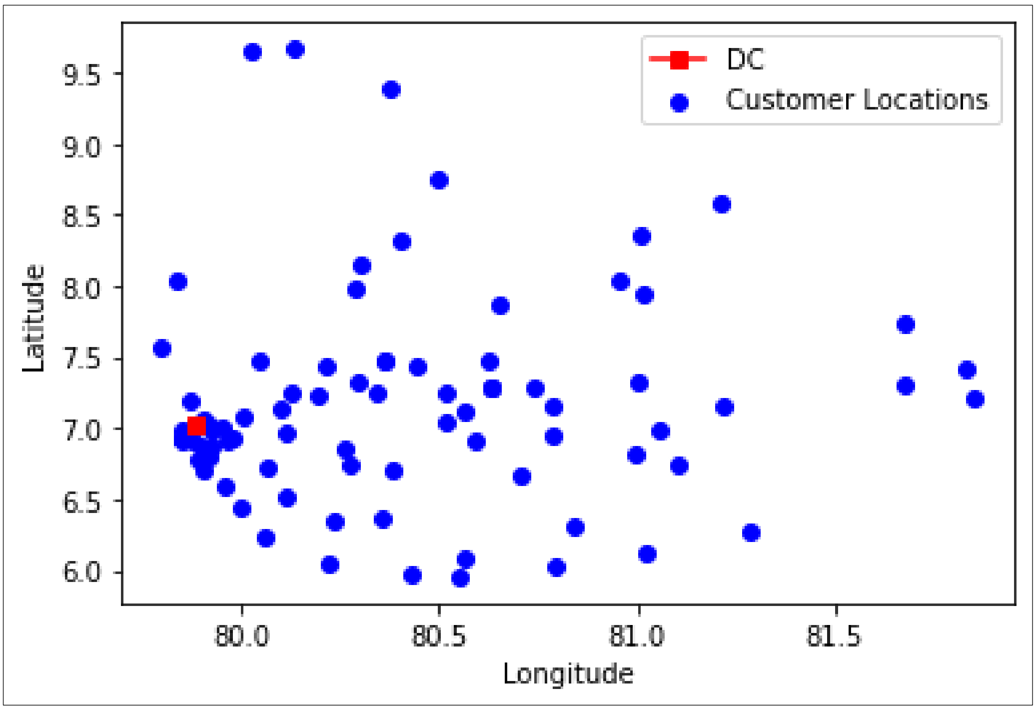

Figure 5.14: Scatter plot of customer location

5.4 K- means cluster analysis

The k-means clustering algorithm [52] was applied to allocate customers into groups according to the distance. According to the elbow curve denoted in Figure 5.15, the optimal number can't be found since the point of inflexion on the curve or elbow point is not clearly shown. However, according to the silhouette method [90], the optimal number for k =6 is between 2 and 10 since the highest silhouette score appeared (Figure 5.17) when k = 6. At the point of k = 6, the silhouette score is 0.463.

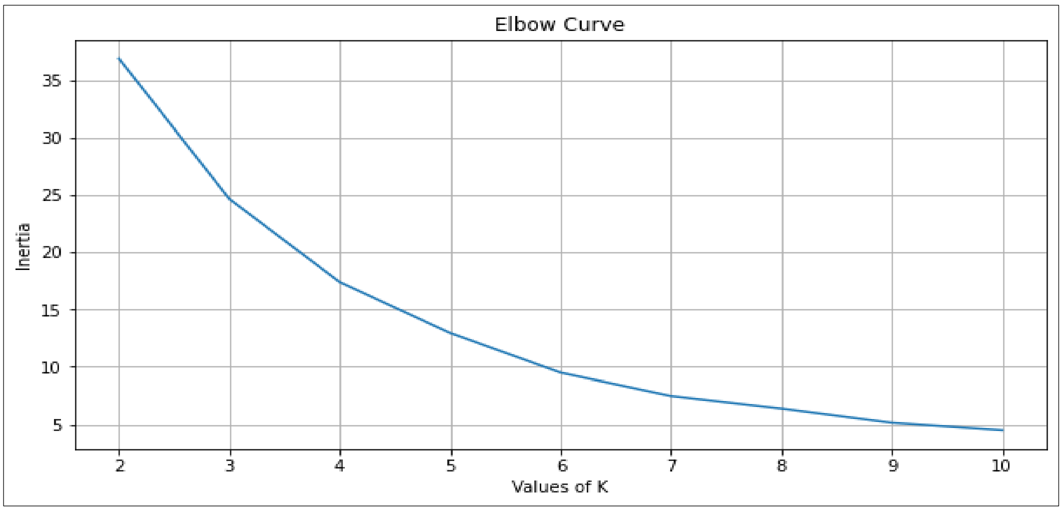

Figure 5.15:Elbow curve of the number of clusters relevant to customer locations

"Figure 5.16" displays the scatter plot of customers according to the k-means clusters in all six clusters together while Figure 5.18 shows it separately. Table 5.4 shows the number of locations included under each cluster according to k-mean clustering. These clusters will be used for VRP solutions.

```
Silhouette score for k(clusters) = 3 is 0.4215205300683153
Silhouette score for k(clusters) = 4 is 0.41545395056703033
Silhouette score for k(clusters) = 5 is 0.44819950190337887
Silhouette score for k(clusters) = 6 is 0.46339946966894663
Silhouette score for k(clusters) = 7 is 0.4478795933831698
Silhouette score for k(clusters) = 8 is 0.44503051022680506
Silhouette score for k(clusters) = 9 is 0.4508685169343837
Silhouette score for k(clusters) = 10 is 0.42001424133561105
```

Figure 5.17:Silhouette Scores

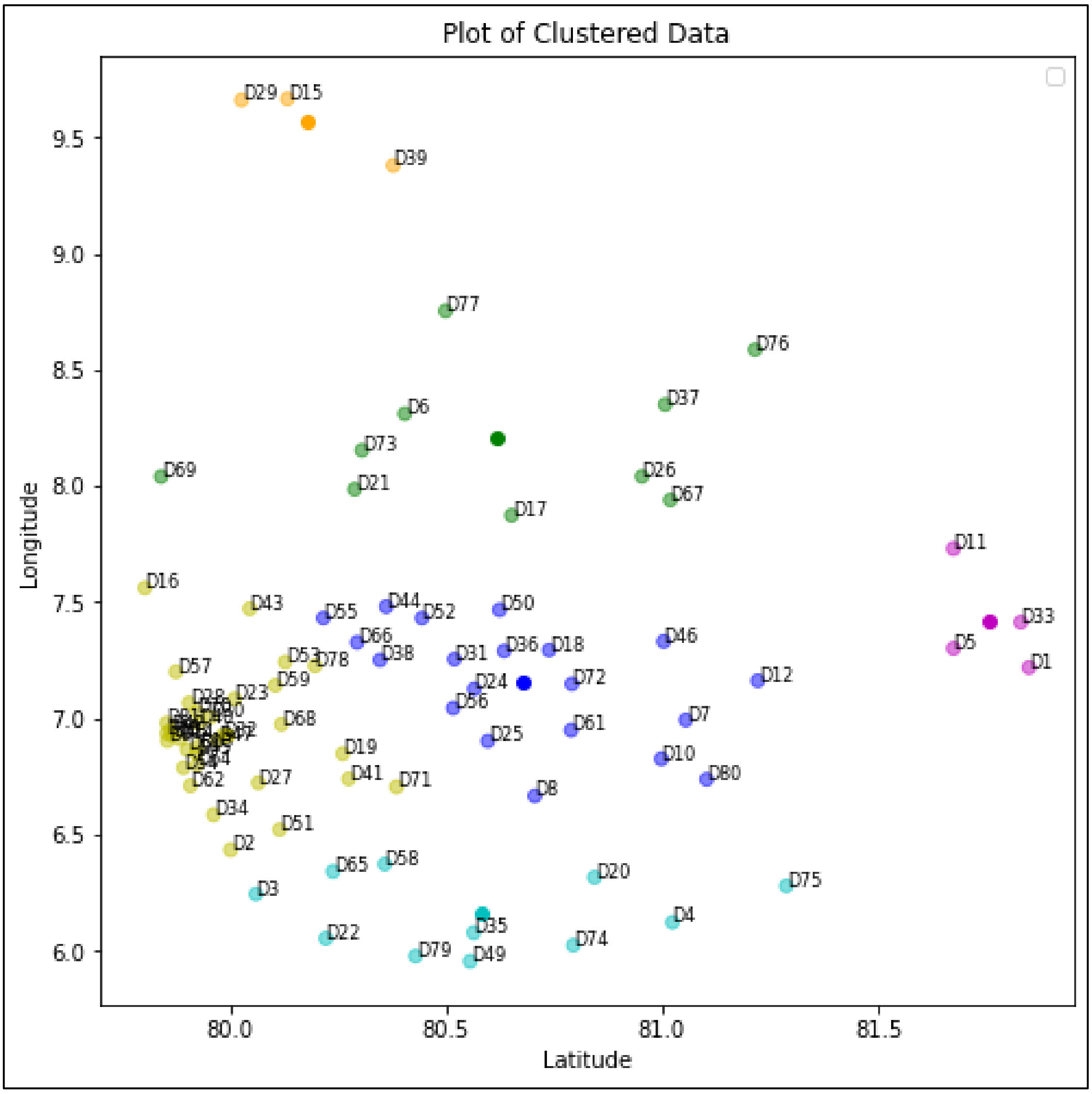

Figure 5.16:Plot of Clustered Data

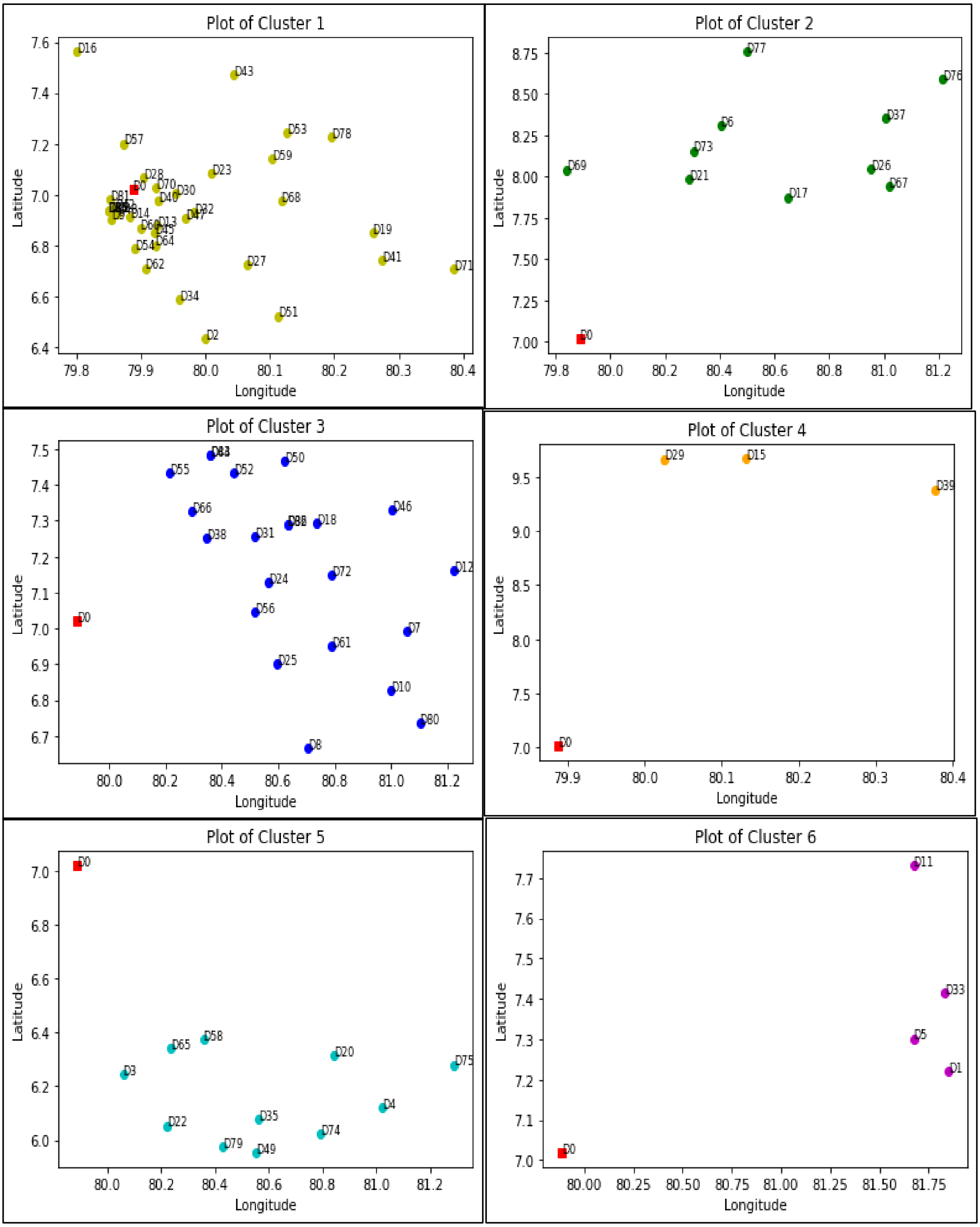

Figure 5.18:Plot of customer location according to the clusters separately

Table 5.4:Number of locations under clusters

Cluster Name	Colour of the Plots	Number of Locations
Cluster 1	Yellow	36
Cluster 2	Green	10
Cluster 3	Blue	23
Cluster 4	Orange	3
Cluster 5	Cyan	11
Cluster 6	Magenta	4

5.5 Capacitated Vehicle Routing Problem Solutions

According to the mathematical model built in Chapter 4, an optimised vehicle routing sequence will be identified. The main objective is to minimise the overall distance required to complete the delivery routes engaging with high seasonality.

5.5.1 Location-wise volume calculation

According to equation 9 (Chapter 4), the total demand in volume of each and every customer for each planning horizon is calculated. Separate bins were created for each dealer point (customer locations) according to the demand for the different products. All the volumes are calculated in cubic meters (m^3).

5.5.2 Computational Experiments

The solutions for the formulated problem were generated separately for each cluster and each planning horizon. To get solutions for the MILP model illustrated in Chapter 4, the Gurobi solver was used.

5.5.2.1 Calculation of route distance

Possible route distances were calculated using the Google API [91] in the python platform. Using Google API, actual road distances can be found as suitable for real-world operations. It has captured an unprecedented area with its mapping services and collected vast volumes of details on the local level accurately.

5.5.2.2 Time horizon

The company used to deliver orders to the dealer points weekly basis. Therefore, dealers used to place orders weekly. When looking into the data, the time horizon for this study is also considered one week. The weekly total volume of dealers over the planning horizons is taken as average. Here, it was assumed that within a planning horizon, weekly demand is equally distributed.

5.5.2.3 Type of vehicles

The company has 14 trucks with a capacity of $30m^3$. To fulfil the customer demand, the company is outsourcing more trucks based on the requirement. One of the objectives of this study is to reduce the number of trucks currently hired. When interviewing the logistics manager of the company, he mentioned that they can not be loaded with a full truck practically. According to their experience, around 27 m^3 can be loaded in a truck. Therefore, the possible loading capacity was taken as 27 m^3 per truck in this study. This study is based on a homogeneous vehicle fleet.

5.5.2.4 Route sequence

The formulated CVRP model under Chapter 4 was solved according to each cluster and two planning horizons separately using Gurobi optimiser in the python platform. Figure 5.19, Figure 5.21, Figure 5.20, Figure 5.22, Figure 5.24 and Figure 5.23 show the vehicle routing solutions during the “Peak” and the “Off-peak” under clusters 1,2,3,4,5 and 6 respectively. All the solutions were taken during one week of time horizon. The route sequence of every trip has been illustrated clearly in “Appendix A” because the figures can be unclear when the customer locations are very close to each other. Even it can be misleading as one customer is visited multiple times. However, “Appendix A” will help the reader to understand the route sequences without a doubt.

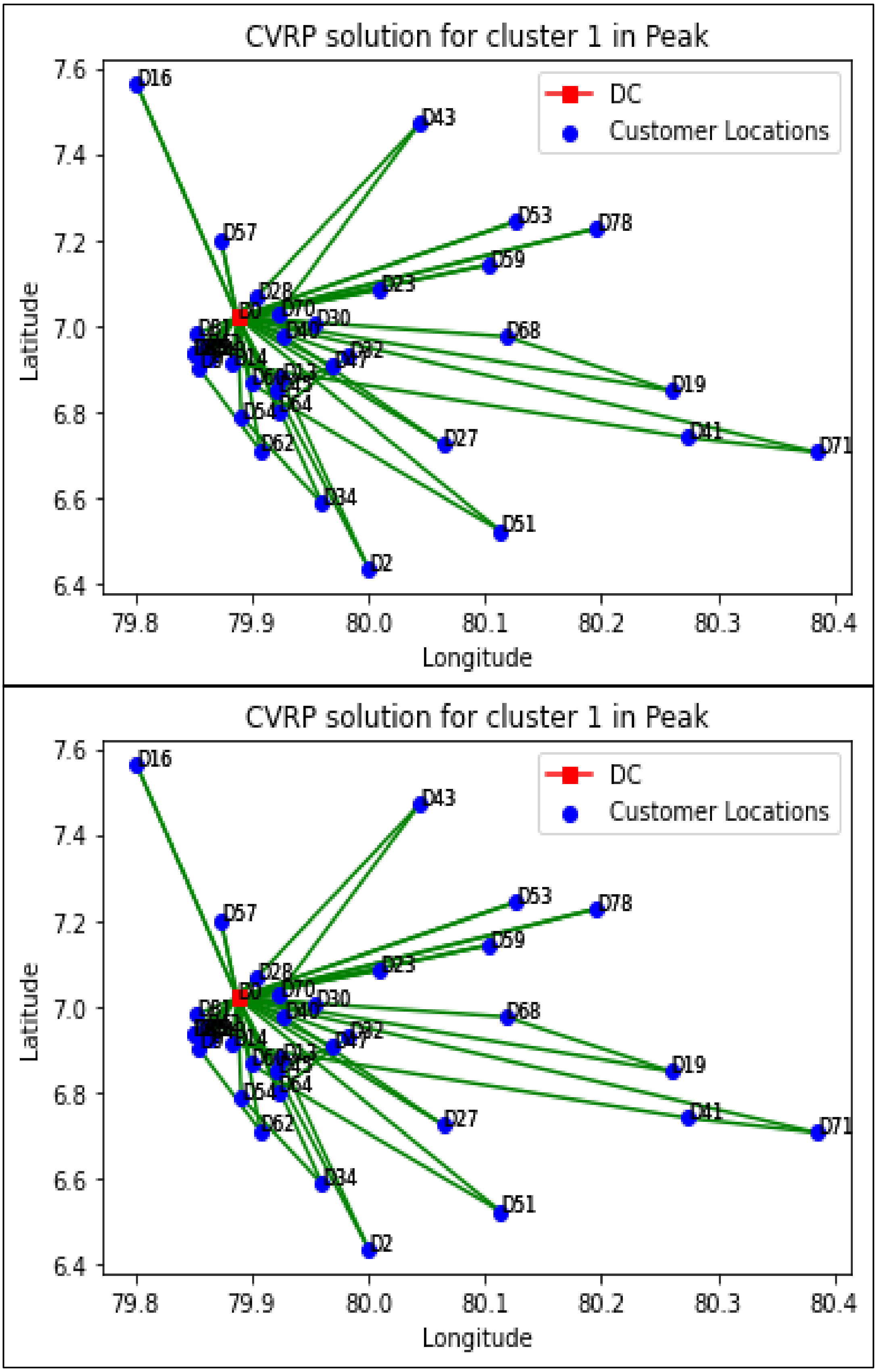

Figure 5.19:CVRP solutions for cluster 1

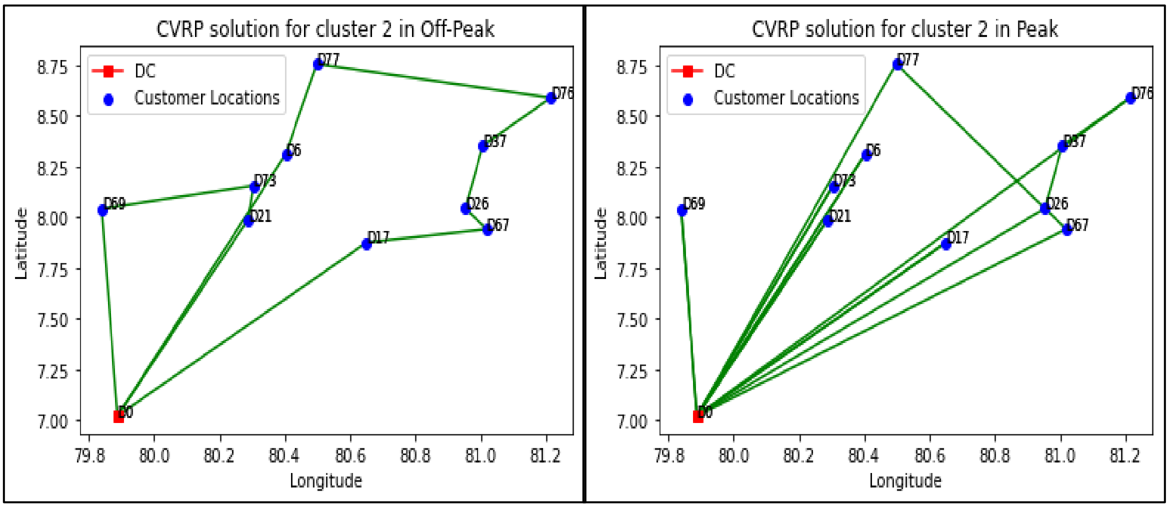

Figure 5.20:CVRP solutions for cluster 2

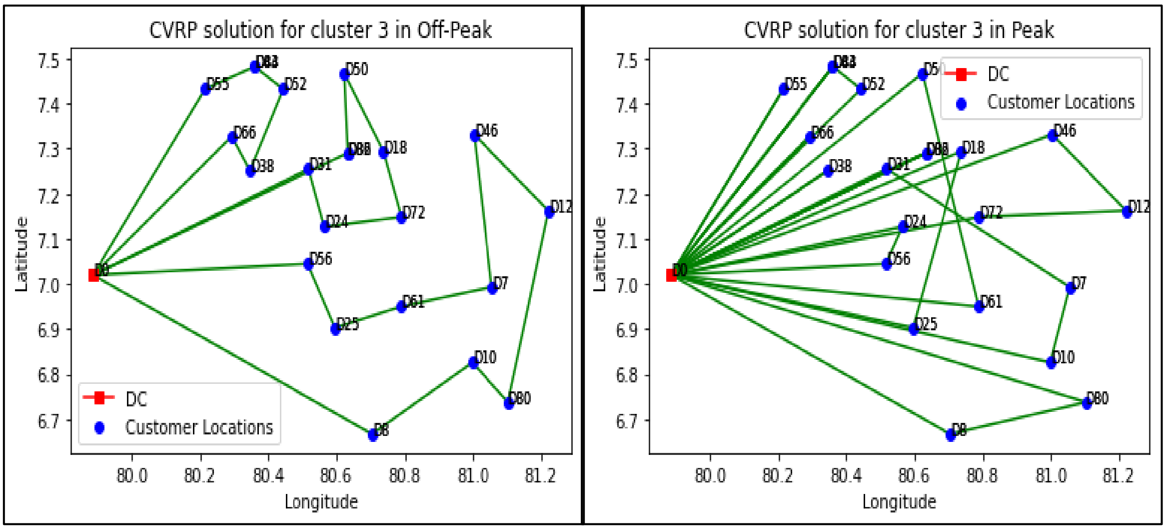

Figure 5.21:CVRP solutions for cluster 3

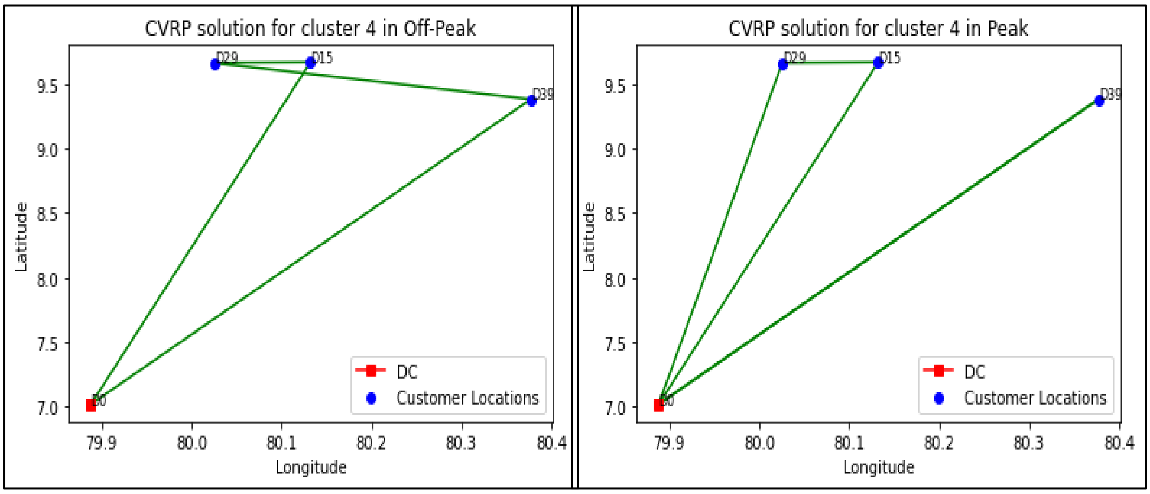

Figure 5.22: CVRP solutions for cluster 4

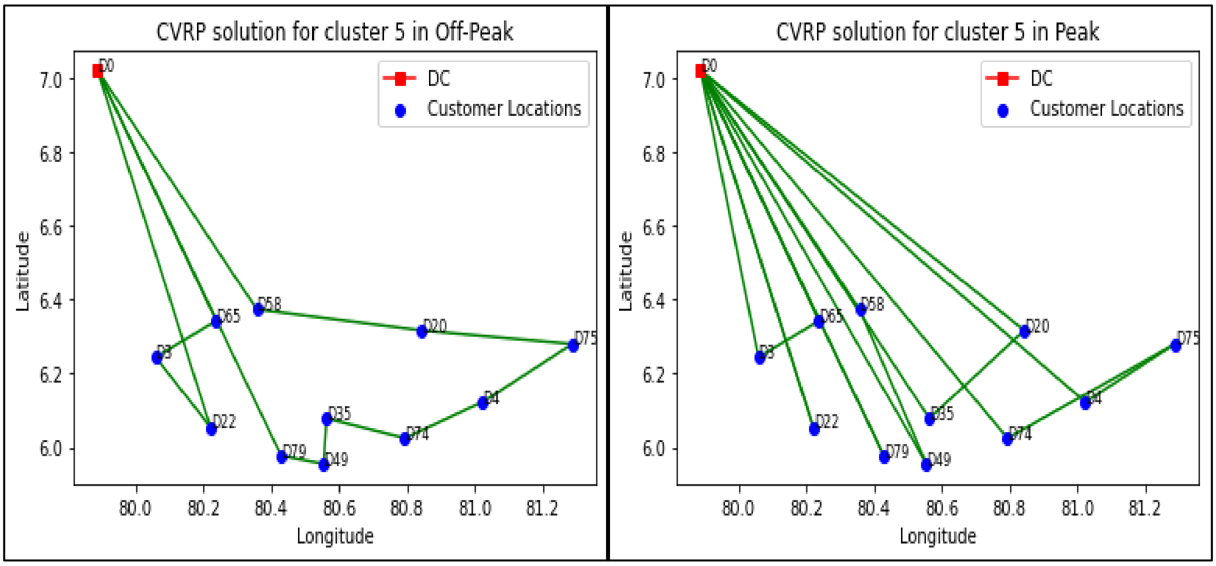

Figure 5.23: CVRP solutions for cluster 5

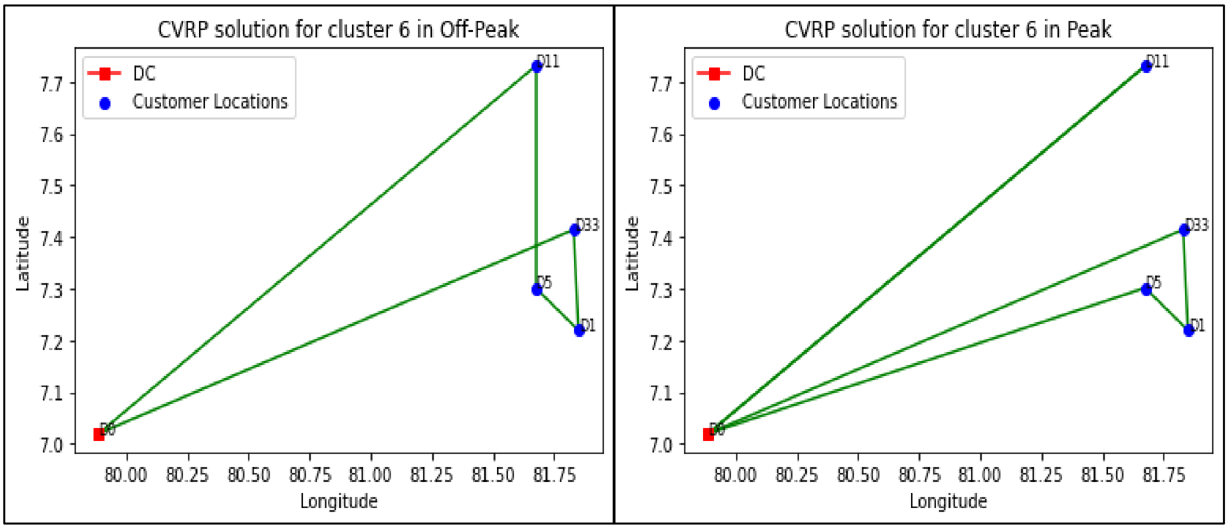

Figure 5.24:CVRP solutions for cluster 6

5.5.2.5 Planning Horizon-wise Route Distance Comparison

According to the CVRP solutions, distances were calculated at peak and off-peak planning horizons for each cluster. The total distance throughout the week is 5472 km and 13563 km off-peak and peak respectively according to the CVRP model. Table 5.5 shows the cluster-wise percentage differences of route distance of "Off-peak" respective to the route distance of peak. Differences of over 60% in total route distance required between off-peak and peak were observed within one week of the period. When it comes to yearly percentage, it is overall 44%. When comparing peak and off-peak route distances, obviously the distance of the peak is higher than off-peak hence the volume difference, but these values are considerably high. Almost all the clusters show the same behaviour comparing the peak and off-peak horizons hence the Peak season has a larger volume in every cluster compared to off-peak (Figures Figure 5.19 Figure 5.21, Figure 5.20, Figure 5.22, Figure 5.24 and Figure 5.23). Using the same timetable and routes for both planning horizons (the current practice of the company) is inefficient compared to the values in Table 5.5.

Table 5.5:Percentage difference of route distance between peak and off-peak

Cluster Name	**Distance (km)**		**Difference (%) of route distance between peak and off-peak**	
	Off-Peak	**Peak**	**Within one week**	**Throughout the year**
Cluster 1	786.82	2,002.38	61%	45%
Cluster 2	1,025.37	2,696.07	62%	47%
Cluster 3	1,236.79	3,689.22	66%	53%
Cluster 4	784.51	1,439.66	46%	24%
Cluster 5	867.79	2,145.23	60%	43%
Cluster 6	770.90	1,590.46	52%	32%
Overall	**5,472.17**	**13,563.015**	**60%**	**44%**

5.5.2.6 Planning horizon-wise vehicle capacity and fleet requirements

The number of trips needed for each cluster during a week is mentioned in Table 5.6 according to the CVRP solution. 15 trips are adequate to cater a week in the off-peak while 53 trips are needed to cater a week in the peak months. Currently, the company has to schedule around 25 trips for off-peak and around 60 trips for peak.

When it comes to total capacity utilisation, there is no huge difference between the two planning horizons according to our CVRP solution. But clusters 4 and 6 have less capacity utilisation because the total capacity of the cluster is less compared to the truck capacity as shown in Figure 5.25.

Table 5.6:Number of trips needed under CVRP and vehicle capacity utilisation

Cluster Name	Number of Trips		Vehicle Capacity Utilisation	
	Peak	Off-peak	Peak	Off-peak
Cluster 1	23	6	77%	87%
Cluster 2	7	2	77%	71%
Cluster 3	13	3	85%	94%
Cluster 4	2	1	72%	45%
Cluster 5	6	2	81%	71%
Cluster 6	2	1	67%	35%
Overall	**53**	**15**	**79%**	**78%**

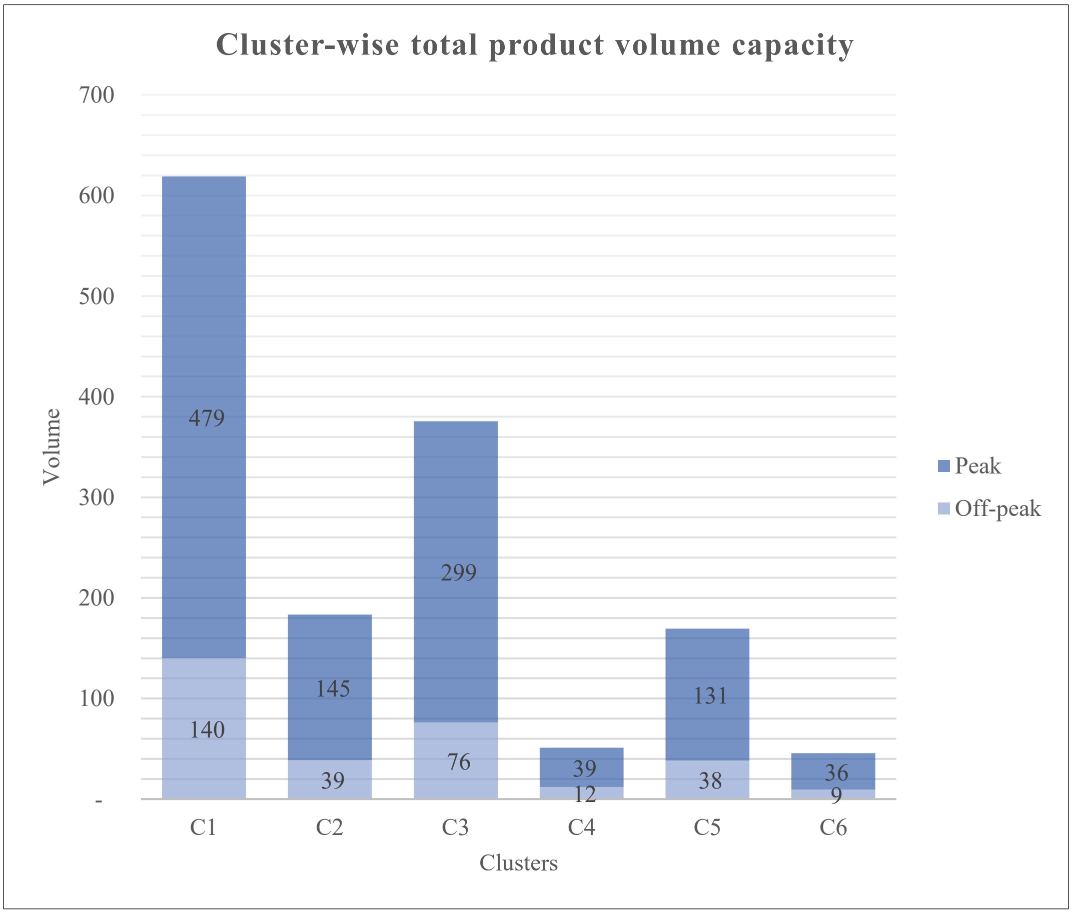

Figure 5.25:Cluster-wise total product volume capacity

5.5.2.7 Changing timeframe

Because the volume of the off-peak is less, "delivering thrice per month (10-day plan)", "delivering twice per month (15-day plan)", and "delivering once per month (monthly basis)" were considered to solve the vehicle routing problem instead of "weekly basis" (delivering four times per month) for each customer location. The 10-day plan showed the optimum solution and other time horizons can't be considered for some clusters because the capacity limit of the truck was exceeded. Figure 5.26, Figure 5.29, Figure 5.28, Figure 5.27, Figure 5.31 and Figure 5.30 show the vehicle routing solution for the "10-day plan" for off-peak and peak periods according to clusters 1,2,3,4,5, and 6 respectively. As mentioned above with figures of weekly time horizons, the route sequence of every trip has been illustrated clearly in "Appendix B" to avoid readers' unclearness and misleading.

Table 5.7 shows the number of trips and vehicle capacity utilisation according to 10 days of timeframe (10-day plan). It helps to increase vehicle capacity utilisation. Even clusters 4 and 6 show a considerable increase in capacity utilisation.

Table 5.7:Number of trips needed under CVRP and vehicle capacity utilisation (10 Days of Timeframe)

10 Days of Timeframe (delivering thrice per month)				
Cluster Name	**Number of Trips**		**Vehicle Capacity Utilisation**	
	Peak	**Off-peak**	**Peak**	**Off-peak**
Cluster 1	29	8	82%	87%
Cluster 2	9	2	79%	95%
Cluster 3	18	4	82%	94%
Cluster 4	2	1	96%	60%
Cluster 5	8	2	81%	95%
Cluster 6	2	1	89%	46%
Overall	**68**	**18**	**84%**	**86%**

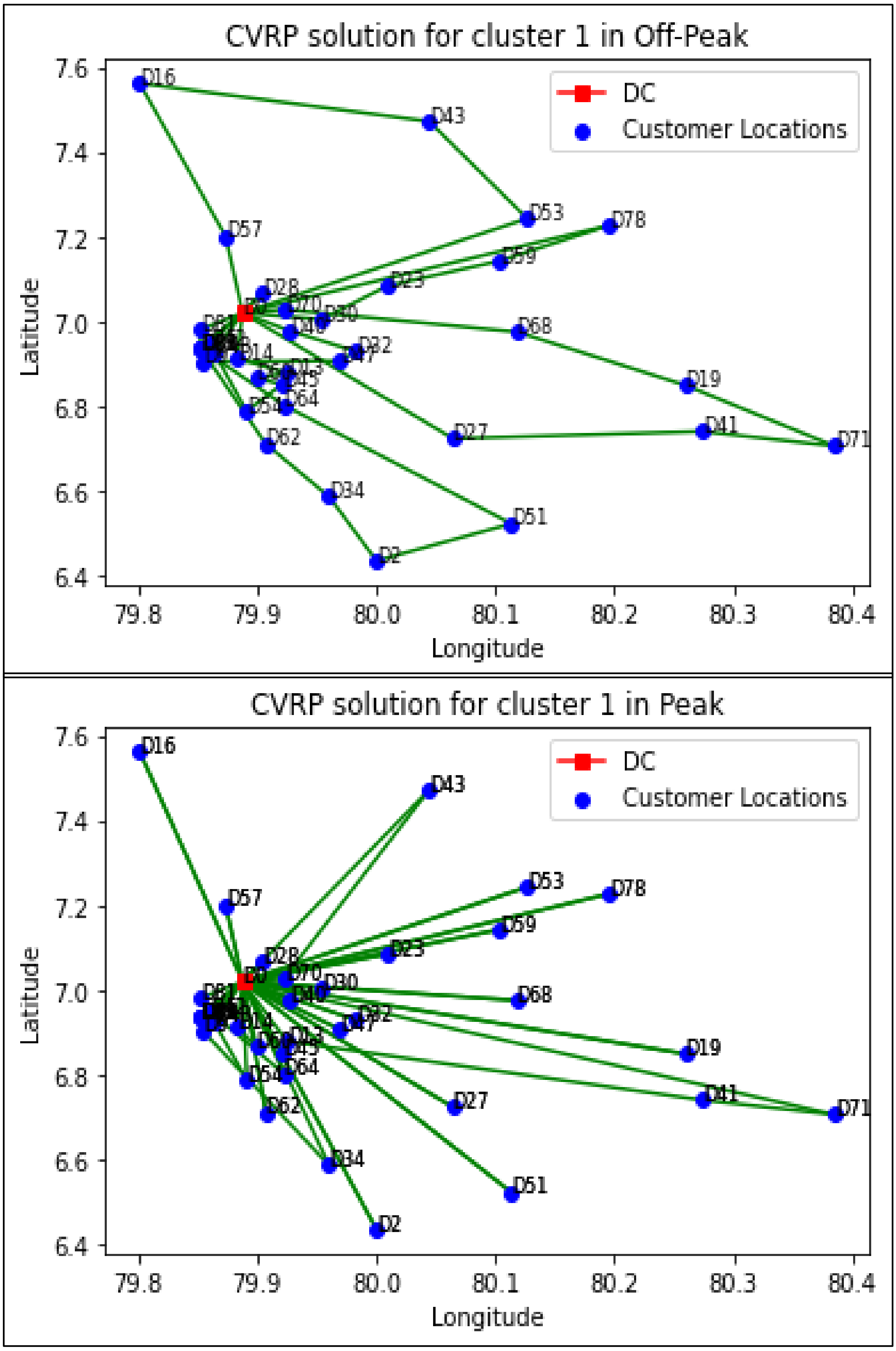

Figure 5.26:CVRP solution for cluster 1 in 10-day plan

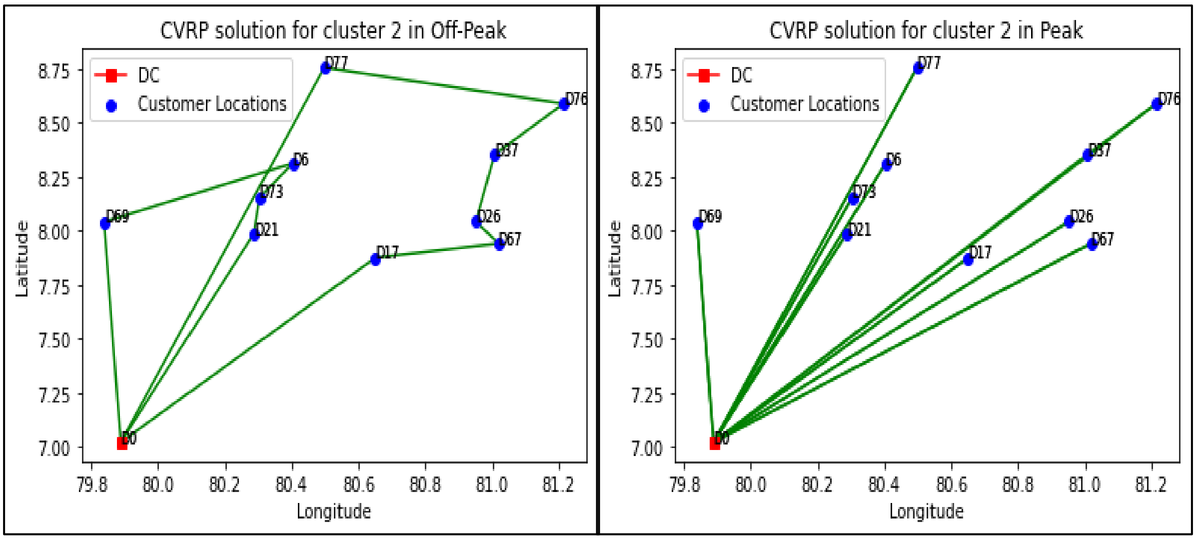

Figure 5.29:CVRP solution for cluster 2 in 10-day plan

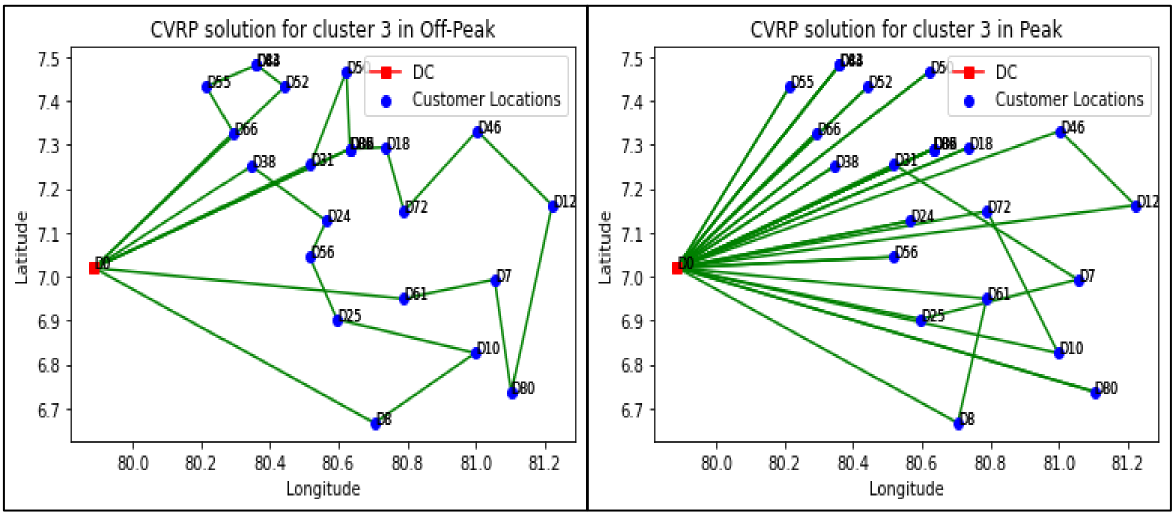

Figure 5.28:CVRP solution for cluster 3 in 10-day plan

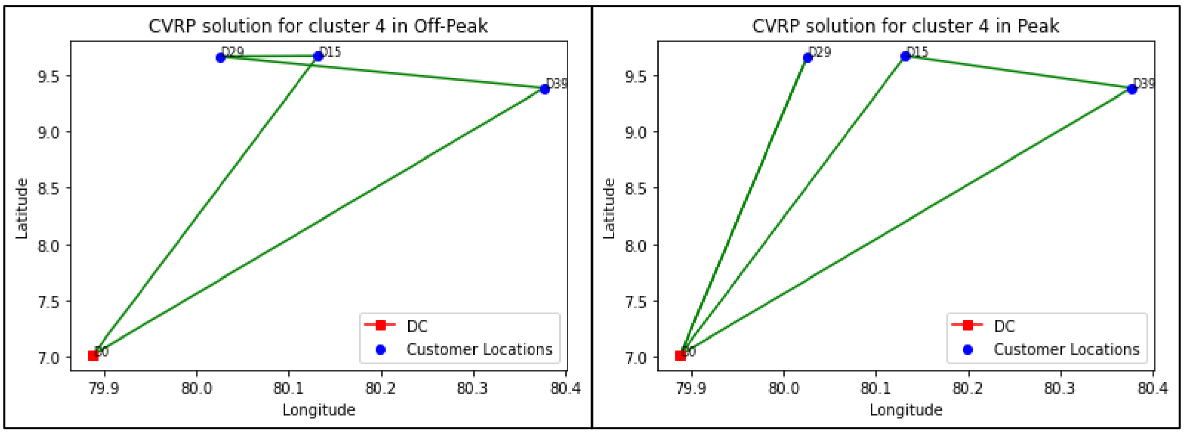

Figure 5.27:CVRP solution for cluster 4 in 10-day plan

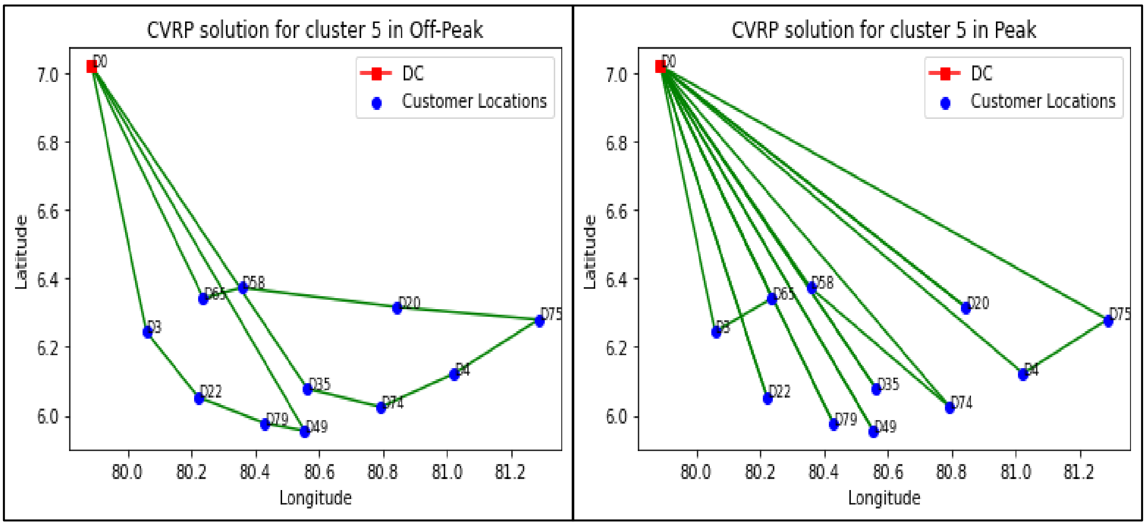

Figure 5.31:CVRP solution for cluster 5 in 10-day plan

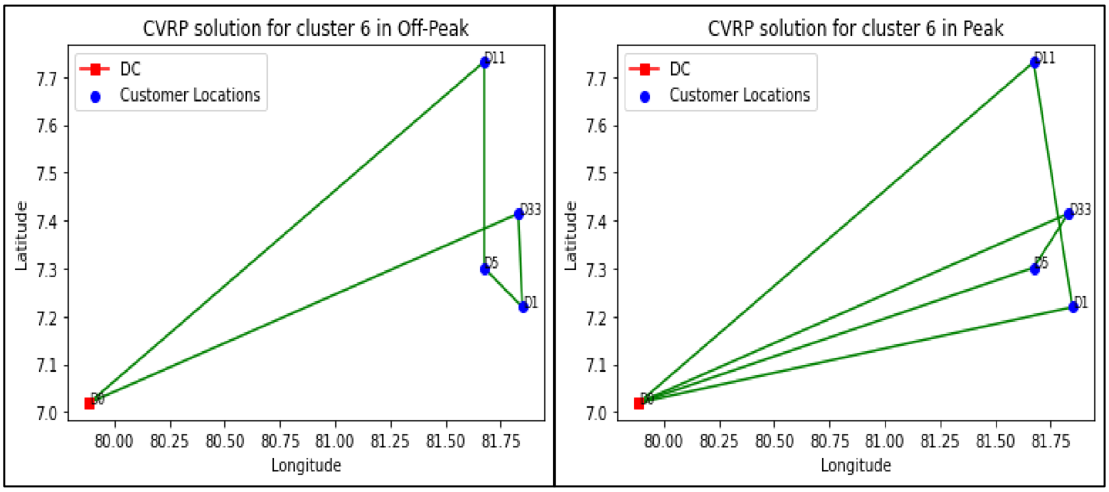

Figure 5.30:CVRP solution for cluster 6 in 10-day plan

There is a 12% distance saving when the time horizon is 10-day compared to the scenario of the weekly time horizon. And also, capacity utilisation over the year is 83% when using a 10-day of time horizon. It was 79% in the weekly time horizon. 6% of capacity saving is shown in a 10-day of time horizon than the weekly time horizon.

5.5.2.8 Comparison with current practices

Compared to the current scenario which is being practised in the company, the total yearly distance saving can be achieved at around 28 % using this CVRP model with a weekly time horizon. The figure is 36% when using the 10-day of time horizon for solutions. Furthermore, the company has a huge issue with capacity utilisation in the off-peak months and they mentioned in the interview that the value is around 40-55% for off-peak and 65-90% for peak periods. But this CVRP model gives more than 72%

when using a weekly time horizon except for clusters 4 and 6 since the total volume of the clusters is low. In the 10-day of time frame, that limitation can be overcome even in clusters 4 and 6. More than 79% of capacity utilisation can be achieved except only off-peak in clusters 4 and 6. The total capacity savings throughout the year is 22%. Table 5.8 shows the comparison with current practices and CVRP solutions.

Table 5.8: Yearly figures comparison

	Weekly Time Horizon			10-Day Time Horizon			Current Practice		
	Peak	Off-peak	Total	Peak	Off-peak	Total	Peak	Off-peak	Total
Total distance over the year (km)	271,260	153,221	424,481	250,902	124,413	375,315	342,523	243,360	585,883
Total trips needed	1,060	420	1,480	1,020	378	1,398	1,200	700	1,900
Used capacity	22,572	8,816	31,388	22,572	8,816	31,388	22,572	8,816	31,388
Total allocated capacity	28,620	11,340	39,960	27,540	10,206	37,746	32,400	18,900	51,300
Average Capacity Utilisation over the year	79%	78%	79%	82%	86%	83%	70%	47%	61%

5.5.2.9 Comparison with different capacities

Instead of 27m^3 trucks, the experiment was done by replacing the trucks with different capacities as shown in Table 5.9. because there is a constraint which is "one customer should be visited once", values can't be taken to some clusters under 15m^3 trucks. Looking at the values, 15 m^3 trucks can be used to keep high capacity utilisation, especially in clusters 4 and 6 in off-peak. Using these small trucks for other slots is not suitable because the number of trips will be increasing and that means distance is also increasing. Figure 5.33 and Figure 5.32 shows the vehicle routing solutions of cluster 4 and 6 using 15 m^3 trucks under weekly and 10-day time horizons. Cluster 6 performs well under a 10-day time horizon with high capacity utilisation compared to the weekly time horizon. When it comes to cluster 4, the distance is increasing than using 27 m^3

and occupied two trips. When using 27 m^3 trucks, only one trip is occupied. Capacity utilisation is 54% and it is also less than the weekly scenario which is 81%.

When looking at larger trucks than currently used by the company, the number of trips will be decreasing. This will not be valid for clusters with less capacity for example clusters 4 and 6 in off-peak. It will cause less capacity utilisation. Since using CVRP to solve this problem, using a larger vehicle means travel time per trip will be higher. In this study, it can be only shown that using larger vehicles can reduce the total distance over the week, especially at peak.

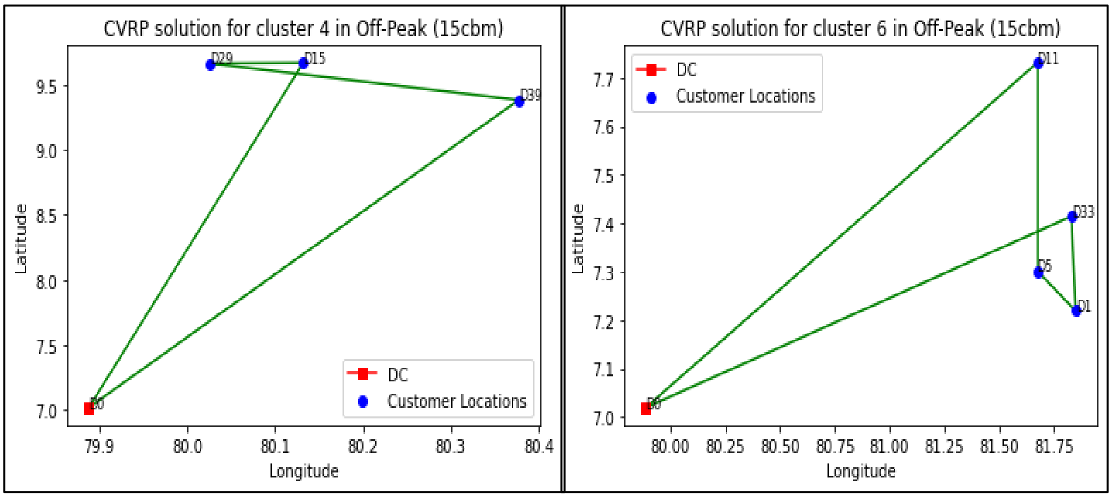

Figure 5.32:CVRP solutions for clusters 4&6 in off-peak by using 15 m3 trucks under 10 days of time horizon

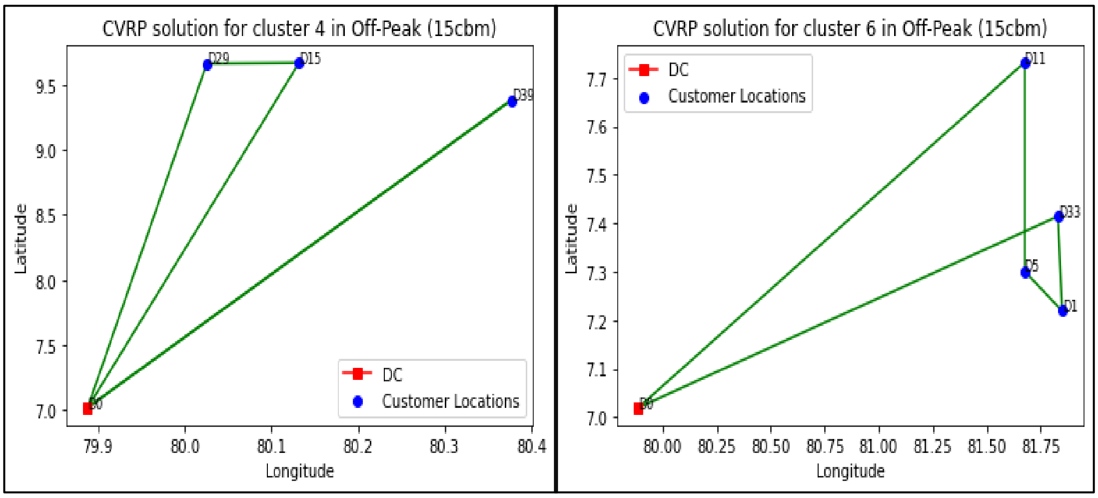

Figure 5.33:CVRP solutions for clusters 4&6 in off-peak by using 15 m3 trucks under weekly time horizon

Table 5.9: Computraisation experiment with replacing truck capacity

Truck Capacity	27 m^3		15 m^3		34 m^3		40 m^3	
Planning Horizon	Off-Peak	Peak	Off-Peak	Peak	Off-Peak	Peak	Off-Peak	Peak
Distance (km)								
Cluster 1	787	2,002	-	-	658	1,403	646	1,321
Cluster 2	1,025	2,696	1,381	-	986	2,173	708	1,773
Cluster 3	1,237	3,689	1,912	-	1,207	2,986	1,051	2,617
Cluster 4	785	1,440	785	-	785	1,440	785	785
Cluster 5	868	2,145	1,199	-	828	1,590	601	1,488
Cluster 6	771	1,590	771	2,173	771	1,422	771	771
Overall	**5,472**	**13,563**	**-**	**-**	**5,235**	**11,013**	**4,560**	**8,754**
Number of Trips								
Cluster 1	6	23	-	-	5	16	4	14
Cluster 2	2	7	3	-	2	5	2	4
Cluster 3	3	13	6	-	3	10	2	9
Cluster 4	1	2	1	-	1	2	1	1
Cluster 5	2	6	3	-	2	4	1	4
Cluster 6	1	2	1	3	1	2	1	1
Overall	**15**	**53**	**-**	**-**	**14**	**39**	**11**	**33**
Vehicle Capacity Utilisation								
Cluster 1	87%	77%	-	-	82%	88%	88%	85%
Cluster 2	71%	77%	86%	-	57%	85%	48%	91%
Cluster 3	94%	85%	85%	-	75%	88%	95%	83%
Cluster 4	45%	72%	81%	-	36%	57%	30%	97%
Cluster 5	71%	81%	85%	-	57%	96%	96%	82%
Cluster 6	35%	67%	62%	80%	28%	53%	23%	90%
Overall	**78%**	**79%**	**-**	**-**	**66%**	**85%**	**72%**	**85%**

5.5.2.10 Computation Times

This problem was implemented and solved in the python programming environment using gurobi commercial solver. The processor of the device is Intel(R) Core(TM) i5-1035G1 CPU @ 1.00GHz - 1.19 GHz with 8 GB RAM. The solution was obtained in less than 30 seconds.

5.6 Chapter Summary

This chapter portrayed how the main objective of this study was achieved by solving a formulated MILP model for CVRP.

Descriptive analysis was used to identify the nature of product volumes and product category-wise demand patterns in peak and off-peak planning horizons. The k-means clustering algorithm was used to get the advantage of the separation of areas because distance constraint was not considered in CVRP. Formulated MILP model in Chapter 4 was solved using the Gurobi solver in order to achieve the main objective of the study. 28% distance savings were obtained yearly.

6 CHAPTER 6: DISCUSSION AND RECOMMENDATION

This chapter elaborates on and discusses the conclusions of the analysis depicted in Chapter 5. This provides a coherent and logical representation of the analysis performed and conclusions reached.

6.1 Literature survey

It can be concluded from the literature review that various VRP applications are attempting to optimise various accepts of product distribution. Most of the researchers have tried to expand the VRP application by adding different kinds of features according to real-world needs. VRP applications are, therefore, not only about the routing of a vehicle fleet but also an application to solve real-world distribution problems effectively and efficiently.

Furthermore, with the advent of technology, VRP applications have grown widely in the corporate sector because of the benefits obtained through resource optimisation. Nowadays, companies have focused on implementing VRP models to optimise their resources with the aim of increasing profitability.

There are a plethora of VRP applications related to industries like milk and other beverages, pharmaceuticals and perishable products. When it comes to the stationery industry, VRP applications which are directly related to the stationery industry could not be found. It is safe to say that the stationery products distribution is easier than the above-mentioned products because of the long expiry period, less damage and fewer needs of special distribution conditions, but there are some areas, researchers need to focus on such as seasonality and large product portfolio.

In conclusion, there are considerable VRP applications engaged with multi-products, but fewer applications engaging with high seasonality. Applications of both seasonality and multi-product together with stationery product distribution were unable to be found.

6.2 Background analysis

The selected organisation is the market leader in the stationery industry in Sri Lanka. The company has more than 50% of the market share across all the product categories it is active in and hence the volume is considerable. Product-wise sales data is recorded in the company’s SAP system accurately. It was evident that the accuracy and reliability of the secondary data are assured.

Moreover, some information relevant to the constraints of the mathematical model was taken from managers of the company as mentioned in Chapters 4 and 5. The staff of the company has hands-on experience in distributing larger product volumes, and they have knowledge and experience in the current distribution process. As a result, it is fair to assume that the information's accuracy and reliability are guaranteed. Finally, the analysis is founded on credible data. As a result, the analysis's findings can also be considered credible.

6.3 Discussion of the findings

The study's main goal is to determine the suitable number of vehicles used to carry out the delivery process in both peak and off-peak phases, such that the distance-related outlays are minimised with effective vehicle capacity utilisation. Four sub-objectives were established in order to attain the primary goal. The first sub-objective is the first sub-objective to investigate the different types of VRP applications under various types of product portfolios and a highly seasonal demand pattern. This was discussed under section 6.1.

The second sub-objective is to identify the nature of the distribution process, sales pattern and product nature to develop an appropriate VRP solution. This is achieved through descriptive analysis. The company has a large product portfolio and categorises its products into six main categories. the “Books” category has the largest sales volume. The common feature of the product is the highly seasonal demand pattern. Category-wise peak to off-peak sales volume ratio is quite different from category to category but it takes in commonly 72% of sales volume is shown throughout the year while off-peak shows 28% of sales volume.

The third sub-objective is to identify the variables and constraints of the VRP in order to formulate the mixed integer linear programming. The discussion happened with the logistics, supply and demand planning and marketing teams and the inputs were used to formulate the constraints and identify the variable. According to the literature, the CVRP problem was formulated as illustrated in Chapter 4. the objective function was formulated in order to minimise the distance-related costs. A CVRP model is the best initial solution for the company because future adjustments can be easily done by encouraging more real-world features as defended in literature.

The final step was to determine CVRP solutions for both peak and off-peak planning horizons. In order to achieve this, the first step was the separation of geographical areas because CVRP does not consider distance constraints. Using k-mean clustering, the area separation can be taken into account at least sightly. According to the elbow curve and silhouette score, 87 customers were divided into 6 clusters. The study was not going to balance the customer locations within the clusters because trucks that travelled long distances can directly go to that geographical area and started delivering from there. The second step of the last sub-objective is the computerisation experiments to get the CVRP solutions.

6.3.1 Discussion and recommendations of CVRP

In this kind of case with a highly seasonal demand pattern, where the peak to off-peak average monthly sales volume ratio is around 4:1. Therefore, there can be a considerable distance difference in the distribution network within the planning horizons. In this case, the off-peak to peak weekly percentage distance difference is 60% according to the CVRP model. It is around 44% considering the complete year. According to the current practice, it is 29% considering the yearly demand. The current delivery trips during the week are 25 and 60 for off-peak and peak respectively. According to the CVRP model, it reduces to 15 and 53 in off-peak and peak respectively. Distance savings is 28% compared with the current scenario and hence the main objective, minimising the distance-related cost is achieved.

Increasing truck capacity can lead to minimising distance further in clusters with larger volumes. Instead of delivering weekly basis (four times per month), changing the 10-day basic (three times per month) plan is better especially in off-peak due to less volume. In the peak period also, the yearly distance is less in the 10-day plan than in the weekly plan. But in peak months, customer satisfaction is more important in order to compete with competitors. If the company uses the 10-day plan in order to optimise distance, they should communicate with customers about the procedure. The model tries to achieve the maximum possible capacity in order to reduce the number of trips reducing the total distance. Therefore, travelling time per trip is increased with the 10-

day plan. This happens either using the 10-day plan or increasing truck capacity. To overcome these practical issues, this CVRP model should be extended by adding constraints related to time windows. In future studies, there is a plan to extend the model so that it can take into account the variable time as well.

When it comes to vehicle capacity utilisation, the current figures are around 40-55% in off-peak and 65-90% in peak. According to the weekly scenario of this CVRP model, the vehicle capacity utilisation is more than 72% and the truck with less capacity can be used in clusters 4 and 6 at off-peak to increase vehicle capacity utilisation. Comparing current scenarios of the company, the capacity savings throughout the year is 22%. Therefore, an effective vehicle capacity utilisation routing pattern is suggested by the CVRP model further to achieve the main objective. The 10-day plan can also be used to increase vehicle capacity utilisation subjected to the above-mentioned limitation. Considering the increase of vehicle capacity utilisation under the weekly time horizon, the CVRP should be extended with a heterogeneous vehicle fleet instead of homogeneous vehicles.

6.3.2 Proposed delivery schedule

According to the weekly scenario, the company can only use their own fleet of 14 vehicles to cater for the general trade customers without hiring vehicles. In off-peak, they can arrange 3 trips per day only on weekdays to complete the 15 trips. At peak, they can cater to 10 or 11 routes in order to full fill the demand by completing 53 trips. If any route takes more than 12 hours, the driver has to take a rest or another driver can be used to continue the trip.

7 CHAPTER 7: CONCLUSION AND FUTURE DIRECTIONS

Throughout the year, the school stationery market experiences extremely seasonal demand. Despite utilising a delivery network with the same routes all year, this model contributes to analysing the weekly distribution network in demand fluctuation circumstances. The distance-related price can be reduced by using a different delivery plan. The primary goal of this strategy is to reduce the total distance required to complete the delivery process for the particularly seasonal demand patterns seen in the stationery industry. Data for this study were collected from a renowned stationery manufacturer in Sri Lanka, which has a significant seasonal demand trend. When compared to the firm's present procedures, the CVRP model saves 28% on distance and 22% on capacity annually.

As routes alter from season to season, a varied number of vehicles are required. According to this study, the needed travels range from 53 to 15 per week for peak and off-peak periods. To overcome the issue of less capacity utilisation due to less sales volume in some slots, the truck capacity and delivery time horizon can be changed. Analysis examples were illustrated in Chapter 5.

7.1 Limitations of the Study

The major limitation was the data could be taken only for four years due to Covid 19 pandemic, sales pattern is considerably different from the organisation’s normal sales pattern. Therefore, the data from 2020 April didn’t take into account. The company has data only from 2016 April, so only four-year data is taken into account.

Another limitation is that most commercial solvers offer limited features in these kinds of academic studies without financial support. This is common for Gurobi which was used in this study.

7.2 Future Research Directions

The model provided in this dissertation provides a solid foundation for future research. It may be developed further to represent schedulers' ever-changing environment while establishing and managing the distribution network.

Greater focus may be required on the function and expectations of the drivers. A time limit on routes, for example, may be addressed, as could the fact that individual drivers would often transport goods on more than one route in a specific time window, taking into account loading and unloading time as well. The limitation is depicted in Equation 10.

$$T \leq T_{max}\ for\ each\ route \tag{10}$$

The weight restriction of the vehicle can be considered as shown in equations 11 and 12.

$$\sum_{i=1}^{N} W_{ih} \leq C\ for\ each\ route\ in\ planning\ horizon\ h \tag{11}$$

$$W_{ih} = \sum_{p=1}^{P} q_{pih} W_p\ ;\ \forall i \in N, \forall h \in H \tag{12}$$

The notations and definitions relevant to equations (10),(11) and (12) are mentioned in Table 4.1 in Chapter 4.

Additionally, forecasting quantity at a variety of periods all over the year would be carefully studied when the transport implications would be an advantage to a cost-effective distribution strategy. Likewise, a heterogeneous fleet of vehicles would be considered when transportation of different volumes as elaborated in Chapter 6. It can be used together with bin-packing heuristics with the VRP models to distribute high-seasonality products for future research.

Furthermore, according to most of the literature, this can be modelled as a metaheuristics algorithm to take more real-world features into account.

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APPENDIX A

Route sequence of trips relevant to the weekly plan

A week as the Timeframe		
Cluster	Peak	Off-Peak
Cluster 1	[(D0, D14), (D14, D41), (D41, D71), (D71, D0)], [(D0, D16), (D16, D0)], [(D0, D19), (D19, D68), (D68, D0)], [(D0, D23), (D23, D0)], [(D0, D28), (D28, D0)], [(D0, D30), (D30, D0)], [(D0, D40), (D40, D27), (D27, D0)], [(D0, D43), (D43, D70), (D70, D0)], [(D0, D47), (D47, D45), (D45, D0)], [(D0, D48), (D48, D42), (D42, D0)], [(D0, D53), (D53, D0)], [(D0, D54), (D54, D34), (D34, D0)], [(D0, D57), (D57, D0)], [(D0, D59), (D59, D0)], [(D0, D60), (D60, D51), (D51, D0)], [(D0, D62), (D62, D9), (D9, D0)], [(D0, D63), (D63, D0)], [(D0, D64), (D64, D32), (D32, D0)], [(D0, D78), (D78, D0)], [(D0, D81), (D81, D13), (D13, D2), (D2, D0)], [(D0, D84), (D84, D0)], [(D0, D85), (D85, D0)]	[(D0, D40), (D40, D30), (D30, D59), (D59, D23), (D23, D28), (D28, D0)], [(D0, D47), (D47, D13), (D13, D45), (D45, D60), (D60, D14), (D14, D0)], [(D0, D48), (D48, D63), (D63, D81), (D81, D0)], [(D0, D57), (D57, D16), (D16, D43), (D43, D53), (D53, D78), (D78, D42), (D42, D0)], [(D0, D84), (D84, D64), (D64, D27), (D27, D41), (D41, D71), (D71, D19), (D19, D68), (D68, D32), (D32, D0)], [(D0, D85), (D85, D9), (D9, D54), (D54, D62), (D62, D34), (D34, D2), (D2, D51), (D51, D70), (D70, D0)]
Cluster 2	[(D0, D17), (D17,D0)], [(D0, D21), (D21,D0)], [(D0, D6), (D6,D0)], [(D0, D69), (D69,D0)], [(D0, D73), (D73,D0)], [(D0, D76), (D76,D37), (D37, D26), (D26, D0)], [(D0, D77), (D77,D67), (D67, D0)]	[(D0, D6), (D6, D77), (D77, D76), (D76, D37), (D37, D26), (D26, D67), (D67, D17), (D17, D0)], [(D0, D69), (D69, D73), (D73, D21), (D21, D0)]
Cluster 3	[(D0, D10), (D10, D7), (D7, D31), (D31, D0)], [(D0, D18), (D18, D25), (D25, D0)], [(D0, D24), (D24, D56), (D56, D0)], [(D0, D36), (D36, D0)], [(D0, D38), (D38, D0)], [(D0, D44), (D44, D0)], [(D0, D50), (D50, D61), (D61, D0)], [(D0, D55), (D55, D0)], [(D0, D66), (D66, D0)], [(D0, D72), (D72, D12), (D12, D46), (D46, D0)], [(D0, D8), (D8, D80), (D80, D0)], [(D0, D82), (D82, D0)], [(D0, D83), (D83, D52), (D52, D0)]	[(D0, D31), (D31, D24), (D24, D72), (D72, D18), (D18, D50), (D50, D82), (D82, D36), (D36, D0)], [(D0, D66), (D66, D38), (D38, D52), (D52, D44), (D44, D83), (D83, D55), (D55, D0)], [(D0, D8), (D8, D10), (D10, D80), (D80, D12), (D12, D46), (D46, D7), (D7, D61), (D61, D25), (D25, D56), (D56, D0)]
Cluster 4	[(D0, D15), (D15, D29), (D29, D0)], [(D0, D39), (D39, D0)]	[(D0, D15), (D15, D29), (D29, D39), (D39, D0)]
Cluster 5	[(D0, D22), (D22, D0)], [(D0, D35), (D35, D20), (D20, D0)], [(D0, D4), (D4, D75), (D75, D74), (D74, D0)], [(D0, D58), (D58, D49), (D49, D0)], [(D0, D65), (D65, D3), (D3, D0)], [(D0, D79), (D79, D0)]	[(D0, D65), (D65, D3), (D3, D22), (D22, D0)], [(D0, D79), (D79, D49), (D49, D35), (D35, D74), (D74, D4), (D4, D75), (D75, D20), (D20, D58), (D58, D0)]
Cluster 6	[(D0, D11), (D11, D0)], [(D0, D33), (D33, D1), (D1,D5), (D5, D0)]	[(D0, D33), (D33, D1), (D1, D5), (D5, D11), (D11, D0)]

APPENDIX B

Route sequence of trips relevant to the 10-day plan

10 Days as the Timeframe		
Cluster	Peak	Off-peak
Cluster 1	[(D0, D16), (D16, D0)], [(D0, D19), (D19, D0)], [(D0, D2), (D2, D0)], [(D0, D23), (D23, D0)], [(D0, D27), (D27, D0)], [(D0, D28), (D28, D0)], [(D0, D30), (D30, D0)], [(D0, D32), (D32, D0)], [(D0, D40), (D40, D0)], [(D0, D42), (D42, D64), (D64, D0)], [(D0, D43), (D43, D70), (D70, D0)], [(D0, D45), (D45, D0)], [(D0, D47), (D47, D0)], [(D0, D48), (D48, D0)], [(D0, D51), (D51, D0)], [(D0, D53), (D53, D0)], [(D0, D54), (D54, D81), (D81, D0)], [(D0, D57), (D57, D0)], [(D0, D59), (D59, D0)], [(D0, D60), (D60, D14), (D14, D0)], [(D0, D62), (D62, D0)], [(D0, D63), (D63, D0)], [(D0, D68), (D68, D0)], [(D0, D71), (D71, D41), (D41, D13), (D13, D0)], [(D0, D78), (D78, D0)], [(D0, D9), (D9, D34), (D34, D0)], [(D0, D84), (D84, D0)], [(D0, D85), (D85, D0)], [(D0, D87), (D87, D0)]	[(D0, D27), (D27, D41), (D41, D71), (D71, D19), (D19, D68), (D68, D70), (D70, D0)], [(D0, D28), (D28, D0)], [(D0, D30), (D30, D23), (D23, D59), (D59, D78), (D78, D0)], [(D0, D40), (D40, D32), (D32, D47), (D47, D9), (D9, D42), (D42, D0)], [(D0, D48), (D48, D54), (D54, D45), (D45, D60), (D60, D13), (D13, D14), (D14, D0)], [(D0, D53), (D53, D43), (D43, D16), (D16, D57), (D57, D0)], [(D0, D63), (D63, D81), (D81, D0)], [(D0, D84), (D84, D85), (D85, D64), (D64, D51), (D51, D2), (D2, D34), (D34, D62), (D62, D87), (D87, D0)]
Cluster 2	[(D0, D17), (D17, D0)], [(D0, D21), (D21, D0)], [(D0, D26), (D26, D0)], [(D0, D6), (D6, D0)], [(D0, D67), (D67, D0)], [(D0, D69), (D69, D0)], [(D0, D73), (D73, D0)], [(D0, D76), (D76, D37), (D37, D0)], [(D0, D77), (D77, D0)]	[(D0, D69), (D69, D6), (D6, D73), (D73, D21), (D21, D0)], [(D0, D77), (D77, D76), (D76, D37), (D37, D26), (D26, D67), (D67, D17), (D17, D0)]
Cluster 3	[(D0, D10), (D10, D72), (D72, D0)], [(D0, D18), (D18, D0)], [(D0, D24), (D24, D0)], [(D0, D25), (D25, D7), (D7, D31), (D31, D0)], [(D0, D36), (D36, D0)], [(D0, D38), (D38, D0)], [(D0, D44), (D44, D0)], [(D0, D46), (D46, D12), (D12, D0)], [(D0, D50), (D50, D0)], [(D0, D52), (D52, D0)], [(D0, D55), (D55, D0)], [(D0, D56), (D56, D0)], [(D0, D66), (D66, D0)], [(D0, D8), (D8, D61), (D61, D0)], [(D0, D80), (D80, D0)], [(D0, D82), (D82, D0)], [(D0, D83), (D83, D0)], [(D0, D86), (D86, D0)]	[(D0, D31), (D31, D50), (D50, D36), (D36, D0)], [(D0, D38), (D38, D24), (D24, D56), (D56, D25), (D25, D10), (D10, D8), (D8, D0)], [(D0, D61), (D61, D7), (D7, D80), (D80, D12), (D12, D46), (D46, D72), (D72, D18), (D18, D82), (D82, D86), (D86, D0)], [(D0, D66), (D66, D55), (D55, D44), (D44, D83), (D83, D52), (D52, D0)]
Cluster 4	[(D0, D29), (D29,D0)], [(D0,D39), (D39, D15), (D15, D0)]	[(D0, D15), (D15, D29), (D29, D39), (D39, D0)]
Cluster 5	[(D0, D20), (D20, D0)], [(D0, D22), (D22, D0)], [(D0, D3), (D3, D65), (D65, D0)], [(D0, D35), (D35, D0)], [(D0, D4), (D4, D75), (D75, D0)], [(D0, D49), (D49, D0)], [(D0, D58), (D58, D74), (D74, D0)], [(D0, D79), (D79, D0)]	[(D0, D3), (D3, D22), (D22, D79), (D79, D49), (D49, D0)], [(D0, D35), (D35, D74), (D74, D4), (D4, D75), (D75, D20), (D20, D58), (D58, D65), (D65, D0)]
Cluster 6	[(D0, D1), (D1, D11), (D11, D0)], [(D0, D33), (D33, D5), (D5, D0)]	[(D0, D33), (D33, D1), (D1, D5), (D5, D11), (D11, D0)]