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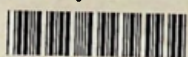
DEVELOPMENT OF A RATIONAL PROCEDURE FOR SHEAR DESIGN IN STRUCTURAL CONCRETE

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By

V. SURESHKUMAR

University of Moratuwa



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A THESIS

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SUBMITTED TO THE FACULTY OF ENGINEERING
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
DEGREE OF MASTER OF PHILOSOPHY

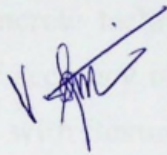
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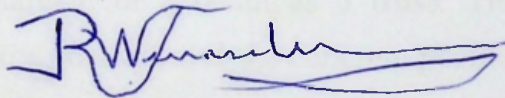
DECLARATION

I hereby, declare, that the work included in this thesis in part or whole,
has not been submitted for any other academic qualification at any
institution.



V. Sureshkumar
(Author)

Certified by



Dr. I. R. A. Weerasekera
Supervisor/ Senior Lecturer
Division of Building & Structural Engineering
Department of Civil Engineering
University of Moratuwa
Sri Lanka



ABSTRACT

Over the last century, the design for shear in reinforced concrete beams has been uncertain. The reason for this uncertainty is the lack of clear understanding of the true nature of load carrying mechanism. Researchers around the world have developed several shear design procedure as a remedy to this uncertainty. Most of them are empirical and some are rational which use various mathematical representations of load carrying mechanism. Almost all of these mathematical models are complicated and accuracy of is rather low compared to empirical methods.

With the advancement of concrete technology and introduction of new methods, the improvement of accuracy of shear design methods is rather unmatchable when compared with flexural design. However, some of the new shear design methods were adopted by codes of practice and the design formulae were modified by introducing various empirical factors based on test. So the shear design has now become very much empirical.

Some of the developed rational methods give good representation of the load carrying mechanism of beams. The method proposed here with the objective of developing a rational yet simple method, also assumes the load carrying mechanism of a beam as a truss. The tension carrying member represents the stirrups and the concrete portion is represented by compression member.

For this proposed method, the compression carrying concrete member is idealized as a cylindrical element applied with axial compression. The behavior of this idealized cylinder is modeled by using mathematical techniques like Isotropic and Anisotropic analysis. This developed method is compared with several key codes of practice using available test data for accuracy.

It is found that the proposed method is giving sound representation of the test results compared with most other codes of practice. According to the comparison, it is clear that this method is only second to Japanese code of practice.

ACKNOWLEDGEMENT

First and foremost I would like to express my gratitude and deep appreciation to My supervisor Dr. I.R.A Weerasekera, Senior Lecturer, Department of Civil Engineering, University of Moratuwa for his invaluable assistance, advice and guidance throughout this project. This association has been interesting and rewarding.

I also want to thank Prof. W. P. S. Dias, Head, Department of Civil Engineering, University of Moratuwa, for making available all resources and facilities for this research work.

The Senate Research Committee of University of Moratuwa should also be thanked for supporting and financing my research.

I appreciate very much invaluable support, encouragement and understanding shown by my parents and siblings.

Finally I would like to acknowledge with fraternal love my colleagues specially Mr. W.K.H.R.E. Wickramage and others who have assisted me in various ways whose contribution have led to the successful completion of this thesis.

Thank you

V. Sureshkumar

TABLE OF CONTENTS

DECLARATION	ii
ABSTRACT	iii
ACKNOWLEDGMENTS	iv
DEDICATION	v
TABLE OF CONTENTS	vi
LIST OF FIGURES	ix
LIST OF TABLES	xii
ABBREVIATION	xiii

DEDICATION

To

my parents

and

all those who are interested and committed in advancement of
science

CHAPTER 1 INTRODUCTION

1.1 GENERAL	1
1.2 RESEARCH OBJECTIVE	2
1.3 CONCEPT	3
1.4 OBJECTIVE	4
1.5 ORGANIZATION OF THE THESIS	5

CHAPTER 2 LITERATURE REVIEW

2.1 INTRODUCTION	7
2.2 ELASTIC SOLUTION-UNCRACKED REINFORCED CONCRETE BEAMS	7
2.3 PLASTIC SOLUTION-CRACKED RC BEAMS	8
2.3.1 Lower Bound Solution	14
2.3.2 Upper Bound Solution	19
2.4 TRUSS ANALOGY	25

TABLE OF CONTENTS

DECLARATION	ii
ABSTRACT	iii
ACKNOWLEDGMENTS	iv
DEDICATION	v
TABLE OF CONTENTS	vi
LIST OF FIGURES	ix
LIST OF TABLES	xii
ABBREVIATION	xiii

CHAPTER 1 INTRODUCTION

1.1	GENERAL	1
1.2	BEAM AS TRUSS	3
1.3	CONCEPT	4
1.4	CRACKED CONCRETE	5
1.5	OBJECTIVE	5
1.6	ORGANISATION OF THE THESIS	5

CHAPTER 2 LITERATURE REVIEW

2.1	INTRODUCTION	7
2.2	ELASTIC SOLUTION-UNCRAKED REINFORCED CONCRETE BEAMS	7
2.3	PLASTIC SOLUTION - CRACKED RC BEAMS	8
2.3.1	Lower Bound Solution	14
2.3.2	Upper Bound Solution	19
2.4	TRUSS ANALOGY	23

2.5	PLASTIC THEORY OF SHEAR FRICTION	30
2.5.1	Upper bound solution	30
2.5.2	Lower bound solution	34
2.6	SHEAR FRICTION BY LOOV	37
2.7	COMPRESSION FIELD THEORY	44
2.8	MODIFIED COMPRESSION FIELD THEORY	49
2.9	EXTENDED MODIFIED COMPRESSION THEORY	56
2.10	ANALYSIS OF DEI POLI, GAMBAROVA, AND KARAKOC	59
2.11	ANALYSIS OF REINECK	68
2.12	SUMMARY	73

CHAPTER 3 METHODOLOGY

3.1	INTRODUCTION	74
3.2	ISOLATION OF STRESSES	74
3.3	CONCEPT	76
3.3.1	Selection of Crack Angle	78
3.3.2	Selection of Cylinder Dimensions	79
3.4	SECTIONAL ANALYSIS	84
3.4.1	Cracked Concrete Analysis	87
3.4.2	Solving the Differential Equation	87
3.5	COHESIVE CRACKING	88
3.5.1	Nature of Fracture Zone	88
3.5.2	Stress Distribution along the Crack	88
3.6	SOLUTION PROCEDURE	93
3.7	PARTIALLY CRACKED SECTION	100
3.8	FULLY CRACKED SECTION	102
3.9	PROPOSED SHEAR DESIGN PROCEDURE	104
3.9.1	Selection of Number of Cracks	104
3.9.2	Selection of Failure Stress at Outer Surface	105
3.9.3	Finding the Capacity of the Cylinder	105
3.9.4	Step for the Proposed Calculation for Designing	106
3.10	SUMMARY	107

CHAPTER 4

COMPARISON WITH TEST DATA AND DISCUSSION

4.1	BEAMS WITH SHEAR REINFORCEMENT	108
4.1.1	Calculation Process Using the Proposed Model	108
4.1.2	Comparison V_c/V Ratio	113
4.1.3	Selection of Failure Stress	114
4.1.4	Calculated Shear Capacity of Available Beams with Stirrups	122
4.2	BEAMS WITHOUT SHEAR REINFORCEMENT	127
4.2.1	Calculation Process Using the Proposed Model	127
4.2.2	Selection of Appropriate Crack Angle	136
4.2.3	Calculated Shear Capacity of Available Beams without Stirrups	139
4.3	RESULTS	147
4.3.1	Beams with Stirrups	147
4.3.2	Beams without Stirrups	147
4.3.3	Comparison with Key Codes of Practice	147
4.4	SUMMARY	152
4.5	PROPOSED DESIGN PROCEDURE	152

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1	CONCLUSIONS	154
5.2	RECOMMENDATIONS	156
REFERENCES		157
APPENDIX A - ISOTROPIC ELASTIC ANALYSIS		160
APPENDIX B - ANISOTROPIC ELASTIC ANALYSIS		166
APPENDIX C- DEVELOPED COMPUTER PROGRAME FOR FINDING STRESS DISTRIBUTION ALONG THE RADIUS		170
APPENDIX D- DEVELOPED COMPUTER PROGRAME FOR FINDING SHEAR CAPACITY OF BEAM USING PROPOSED MODEL		180

LIST OF FIGURES

Figure 1.1: Traditional Beam Testing for Shear	3
Figure 2.1(a): Principal stress trajectories for simply - supported beam	8
Figure 2.1(b): Reinforcement layout to carry principal tensile stresses	8
Figure 2.2: Schematic stress - strain curves; (a): Structural steel; (b): Concrete with different compressive strengths; (c): Rigid - plastic idealization of concrete used in plasticity analysis	10
Figure 2.3 (a): Mohr-Coulomb envelope for sliding failure	12
Figure 2.3 (b): Modified Mohr-Coulomb failure criterion with tension cutoff showing failure under uniaxial tension, uniaxial compression and triaxial compression	13
Figure 2.4: Biaxial stress failure envelopes based on modified Mohr-Coulomb failure theory; (a): Tensile strength included; (b): Tensile strength neglected	13
Figure 2.5: Lower bound solution of beam under shear loading	17
Figure 2.6: Lower bound solution of beam under shear loading	17
Figure 2.7: Shear failure mechanism of beam subjected to concentrated loads	18
Figure 2.8: Strain energy of stirrup	18
Figure 2.9: Internal work of concrete	18
Figure 2.10: Yield line (crack) in concrete	19
Figure 2.11 (a): Internal forces in a cracked beam	24
Figure 2.11 (b): Pin-joined truss	24
Figure 2.12 (a): Pin-joined truss	25
Figure 2.12 (b): Pin-joined truss	25
Figure 2.13: Truss model for design; (a): Beam and reinforcement; (b): Truss model; (c): Statically determinate truss	27
Figure 2.14 (a): Pin-joined truss (Cut along A-A)	27
Figure 2.14 (b): Pin-joined truss (Cut along B-B)	28
Figure 2.15: (a) Disk subjected to shear, (b) Failure mechanism	36
Figure 2.16: Dissipation in a yield line for a modified Mohr-Coulomb material with a circular tension cut-off	36
Figure 2.17: Free body diagram	37
Figure 2.18: Possible shear failure planes	39

Figure 2.19: (a) Diagonal stresses and longitudinal equilibrium (b) Stirrup force	48
Figure 2.20: (a) Average strains in web element (b) Mohr's circle of strain	48
Figure 2.21: Comparison of principal stress and principal strain directions	53
Figure 2.22: (a) Principal stresses in concrete, (b) Tension in web reinforcement, (c) Mohr's circle of average concrete stress	54
Figure 2.23: Force transmission in between cracks and along the crack	54
Figure 2.24: (a) Inclined cracks due to shear, (b) Vertical crack due to axial tension, (c) Horizontal crack due to transverse tension	55
Figure 2.25: Parameters affecting the crack spacing	55
Figure 2.26: (a) Truss analogy; (b) Displacements at diagonal cracks; (c) Simplified representation of band between cracks; and (d) Free body diagram of band between cracks	67
Figure 2.27: (a) Truss analogy; (b) Force present at diagonal cracks; and (c) Equilibrium at stirrup	67
Figure 2.28: Free body diagram of end support region of beam (a) End support region and forces, (b) Forces due to friction	72
Figure 2.29: Forces and concrete stress in strut between cracks due to truss action	72
Figure 2.30: Stress in strut between cracks due to friction	73
Figure 3.1 (a): Internal forces in a cracked beam	74
Figure 3.1 (b): Pin-joined truss	75
Figure 3.2 (a): Pin-joined truss (Cut along A-A)	75
Figure 3.2 (b): Pin-joined truss (Cut along B-B)	75
Figure 3.3: Idealization of cylinder inside beam and application of axial compression	77
Figure 3.4: Shear carried by concrete as diagonal compression	77
Figure 3.5: Possible number of cracks, (PLAN VIEW)	78
Figure 3.6: Section of cylinder inside the beam; (a) Front view; (b) Sectional view; (c) Plan view	80
Figure 3.7: Variation of cylinder selection with decrease in crack angle. (a) Front Views; (b) Sectional views	82
Figure 3.8: Situation where width is critical, (a) Plan view; (b) Sectional view	83
Figure 3.9: Situation where depth is critical, Front view	83

Figure 3.10: Stress distribution according to isotropic analysis	85
Figure 3.11: Possible sections. (a) Uncracked; (b) Partially cracked; (c) Fully cracked	86
Figure 3.12: Nature of crack	88
Figure 3.13: Damage region and stress transfer. (Based on: Hillerborg 1983)	89
Figure 3.14: Nature of cracking inside the cylinder	90
Figure 3.15: Variation of Circumferential tensile stress with Radial crack width	91
Figure 3.16: Normalized Stress-Crack width curve	93
Figure 3.17: Variation in Circumferential Modules of Elasticity in Cracked Concrete	97
Figure 3.18: Variation of Reduced tensile strength with Reduced Modules of Elasticity	98
Figure 3.19: Circumferential stress distribution along the crack in Partial cracked zone	101
Figure 3.20: Figure 3.20: Possible number of cracks, (PLAN VIEW)	105
Figure 4.1: Variation of Results for Failure Stress is 10% of Concrete Tensile Strength	115
Figure 4.2: Variation of Results for Failure Stress is 30% of Concrete Tensile Strength	116
Figure 4.3: Variation of Results for Failure Stress is 50% of Concrete Tensile Strength	117
Figure 4.4: Variation of Results for Failure Stress is 70% of Concrete Tensile Strength	118
Figure 4.5: Variation of Results for Failure Stress is 90% of Concrete Tensile Strength	119
Figure 4.6: Variation of Results with Limiting Stress	121
Figure 4.7: Variation of Results for Minimum Possible Crack Angle	131
Figure 4.8: Variation of Results for 20° Crack Angle	132
Figure 4.9: Variation of Results for 30° Crack Angle	133
Figure 4.10: Variation of Results for 40° Crack Angle	134
Figure 4.11: Variation of Results for 50° Crack Angle	135
Figure 4.12: Variation of Results with Crack Angle	137
Figure 4.13: Comparison with Key Codes of Practice for Unsafe Category	149

Figure 4.14: Comparison with Key Codes of Practice for Accurate and Reasonable Category	150
Figure 4.15: Comparison with Key Codes of Practice for Under Estimation Category	151
Figure A.1: Stresses acting on element	161

LIST OF TABLES

Table 4.1: Results Variation for Various Failure Stress	120
Table 4.2: Available beam detail and calculated shear capacity (Beam with stirrups)	122
Table 4.3: Results Variation for Various Crack Angles	136
Table 4.4: Available beam detail and calculated shear capacity (Beam without stirrups)	139
Table 4.5: Comparison with Key Codes of Practice	148

ABBREVIATIONS

A	Cross sectional area
A_v	Cross sectional area of a stirrup
A_{sx}	Cross sectional area of longitudinal reinforcement
c	Radius of concrete cylinder
D	Compressive force acting on the cylinder
d_θ	Expansion in circumferential direction
$d_{\theta max}$	Maximum allowable expansion in circumferential direction
d	Depth of beam
d_{st}	Diameter of one leg of stirrup
E	Modulus of elasticity
E_c	Modulus of elasticity of concrete
E_{cr}	Reduced circumferential modulus of elasticity at the outer surface of cracked zone
E_θ	Circumferential modulus of elasticity of cracked concrete
f_c^*	Reduced concrete compressive strength
f_b	Stirrup-concrete bond stress
f_c'	Cubic compressive strength of concrete
f_c	Cylinder compressive strength of concrete
f_y	Yield strength of stirrup steel
f_{sx}	Yield strength of longitudinal reinforcement
f_t	Tensile strength of concrete
f_{tr}	Reduced tensile strength of concrete at the outer surface
f_θ	Stress in circumferential direction

h	Constant in hyperbolic stress-crack width equation for cracked concrete
k	Constant in hyperbolic stress-crack width equation for cracked concrete
L	Gauge length
L_c	Extent of radial cracking
m	Constant in hyperbolic stress-crack width equation for cracked concrete
N	Number of radial cracks
N^*	Axial forces in inclined bands
n	Number of stirrups crossing the crack plane
r	Radial distance
R	Radial distance to the inner boundary of elastic uncracked section
s	Stirrup spacing
t	Step size
u	Radial displacement
V^*	Shear forces in inclined bands
V	Shear capacity of beam
V_c	Shear forces carried by concrete
V_s	Shear forces carried by steel
V_t	Shear capacity of beam from test results
V_f	Calculated shear force from SFD
w	Crack width of radial crack
x'	Eccentricity of axial force in inclined band of concrete web
z	Lever arm of internal moment
θ	Inclination of crack with respect to the longitudinal axis

ε	Concrete strain
ε_r	Strain in the radial direction
ε_θ	Strain in the circumferential direction
$\varepsilon_{\theta \max}$	Maximum allowable tensile strain in the circumferential direction
ε_z	Strain in the z direction
ε_c	Concrete strain
ε_{sc}	Strain in solid concrete
ε_{2sc}	Principal compressive strain in solid concrete
ε_{xcr}	Longitudinal strain due to crack
ε_c	Longitudinal strain in solid concrete
ε''	Smeared strain due to crack slip
ε^*	Strain due to load
ε_x^*	Longitudinal strain due to load
ε_{sh}	Concrete shrinkage strain
ε_{cu}^*	Concrete strain at maximum compressive stress
ε_s	Stirrup strain
ε_{sy}	Yield strain of stirrups
$\Delta \varepsilon_s$	Anchorage slip of stirrup
σ_{nt}	Shear stress on crack faces
σ_{nn}	Normal stress on crack faces (compressive)
σ_2	Principal compressive stress in concrete strut
τ_u	Ultimate shear stress
$\Delta \tau$	Aggregate interlock contribution to the ultimate shear stress
φ	Crack inclination with respect to the longitudinal axis

ρ_{st}	Shear reinforcement ratio
ν	Poisson's ratio
ν_c	Poisson's ratio for concrete
σ_r	Normal stress in radial direction
σ_z	Normal stress in axial direction
σ_θ	Hoop stress acting in the circumferential direction
σ_{cr}	Reduced stress in the circumferential direction due to cracking
$\Delta\tau_{BND}$	Shear stress due to bending stiffness of band
$\Delta\tau_{INT}$	Shear stress due to aggregate interlock
Δ	A very small value for radius close to centre (to avoid ∞ stresses)
β	Crack angle
δ	Length between points where stresses are calculated
μ	Concrete - concrete friction coefficient
ψ	Friction angle ($\mu = \cot \psi$),
σ_f	Compressive stress normal to crack face
σ_{sw}	Stress in web reinforcement
τ_f	Shear friction across cracks
τ_{fo}	Friction term proportional to normal stress
τ_{f0}	Cohesion term of friction
' , "	First, Second derivative