

# Study of Rock Anisotropic Effects on Mode II Fracture Toughness at Various Loading Rates

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## Abstract

Fracture toughness is an important parameter for evaluating the resistance of a material to crack initiation and propagation. Although Mode I and Mode II fracture toughness have been studied under quasi-static loading, their behavior under dynamic loading remains insufficiently understood. Previous research shows that compressive and tensile strength, as well as Mode I fracture toughness, are influenced by loading rate and often result in different fracture patterns. Based on this, it is expected that Mode II fracture toughness may also be sensitive to loading rate. Additionally, rock anisotropy, which affects crack propagation, may influence fracture behavior under varying loading conditions. This study first used Finite Element Method simulations with the J integral to evaluate geometry-related factors. Then, Mode II fracture toughness tests were conducted at different loading rates using the Short Core in Compression method. A servo-controlled hydraulic system and a Split Hopkinson Pressure Bar were used to apply quasi-static and dynamic loading, respectively. The effects of loading rate and anisotropy on Mode II fracture toughness and crack propagation were examined.

**Keywords:** Loading rate dependence, Mode II fracture toughness, Rock anisotropy, Short Core in Compression method

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## 1 Introduction

Understanding crack initiation and propagation under various geological and mechanical conditions is essential for the rational design of rock structures. These conditions include rock properties such as anisotropy and heterogeneity, and loading conditions such as confining pressure and loading rate. Recent studies [1] have shown that the strength of rock, including compressive and tensile strength, is dependent on loading rate, which is critical in operations like drilling, blasting, and seismic loading.

Among the mechanical properties of rock, fracture toughness is especially important for describing crack behavior under dynamic loading. While Mode I fracture toughness has been well studied, including its dependence on loading rate and material anisotropy [2-3],

studies focusing on Mode II fracture toughness under dynamic loading remain limited. This is due to experimental challenges such as specimen preparation and difficulty in observing shear fractures directly.

According to the ISRM Suggested Method, the punch-through shear (PTS) specimen is the standard for static determination of Mode II fracture toughness ( $K_{IIC}$ ), but its elaborate machining and challenging notch alignment limit experimental throughput. To overcome those limitations, the Short Beam in Compression specimen was developed [4]. In conducting dynamic tests, the specimen is required to be cylindrical to apply to the widely used experimental apparatus, the Split Hopkinson Pressure Bar (SHPB). Consequently, the Short Core in Compression (SCC) specimen proposed

by Jung et al [5]. has recently attracted increasing attention.

SCC specimen [6], in which a cylindrical granite specimen is notched with opposing semicircular cuts and loaded in uniaxial compression. Jung et al. showed that SCC specimens simplify notch fabrication by using conventional core-drilling and milling tools, integrate seamlessly with both uniaxial and triaxial rigs, promote a more uniform stress distribution around notches, thereby reducing scatter in measured  $K_{IIc}$ , and facilitate direct observation of shear crack initiation.

This study investigates the effects of loading rate and anisotropy on Mode II fracture toughness. Tests were performed on anisotropic granite using a servo-controlled hydraulic system for quasi-static loading and a Split Hopkinson Pressure Bar system for dynamic loading. The SCC method was selected due to its ease of specimen preparation and compatibility with dynamic testing systems.

Before testing, Finite Element Method (FEM) simulations were performed, and the J-integral was used to evaluate correction factors for fracture toughness calculations. Mode II fracture toughness tests were then conducted on granite in three orthogonal directions, defined as Hardway, Grain, and Rift, to examine anisotropic behavior.

## 2 Mode II Fracture Toughness Test for Anisotropic Rock under Various Loading Rates

### 2.1 Preparation of Rock Sample

SCC specimens were prepared as shown in Figure 1(a). A cylindrical rock specimen (30 mm in diameter and 60 mm in length) was modified by adding two notches, each 15 mm in length and 1.5 mm in thickness. During loading, shear failure occurs between the notches, enabling evaluation of Mode II fracture toughness.

The optimal notch distance to height ratio  $C/H$  was determined to be 0.2 based on three-dimensional simulations for  $C/H$  values of 0.5, 1, 2, and 2.5. This value minimized tensile stress concentrations and complex stress distributions near the loading surfaces.

Inada granite was chosen as the test material because of its well-defined anisotropic properties. Its basic properties are as follows: density of 2631 kg/m<sup>3</sup>, Young's modulus of 45 GPa, and Poisson's ratio of 0.2. The granite contains three microcrack planes: Hardway, Grain, and Rift, with increasing microcrack density in that order. The orientation of each plane was determined based on P-wave velocity measurements (Table 1). Specimens were cored and notched according to these orientations (Figure 2).

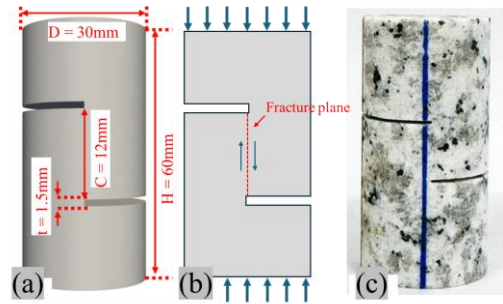


Figure 1: Schematic of SCC specimen: (a) geometry and dimensions, (b) representative cross-sectional view, (c) photograph of tested specimen

Table 1 : P-wave velocities of Inada granite

Microcrack plane	P wave velocity (m/s)
Hardway	4243
Grain	4098
Rift	3846

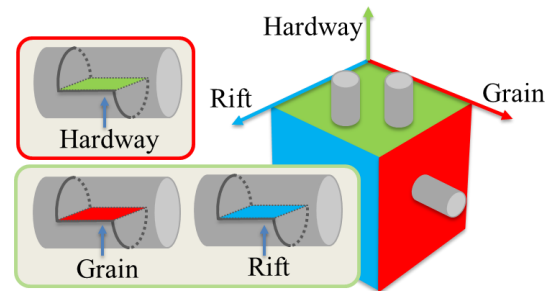


Figure 2: Coring direction and notch orientation relative to microcrack planes

### 2.2 Determination of Stress Intensity Factor in the SCC Method

Mode II fracture toughness,  $K_{IIc}$ , is defined using the following equation:

$$K_{II} = Y(C/H) \tau \sqrt{\pi a} \quad (1)$$

where  $Y(C/H)$  is the geometry correction factor,  $\tau$  is the nominal shear stress, and  $a$  is the notch depth.

FEM analysis was conducted using LS-DYNA as shown in Figure 3. A 2-D plane strain model was used, and  $K_{IIC}$  was calculated using the J integral. To address challenges due to non-coplanar crack propagation, small fictitious cracks (length  $h_c = 0.2, 0.6,$  and  $1.0$  mm) were introduced at the notch tips. With an axial stress of  $1.0$  MPa and  $C/H = 0.2$ , the correction factor  $Y(C/H = 0.2)$  was determined to be  $0.536$  as presented in Figure 4.

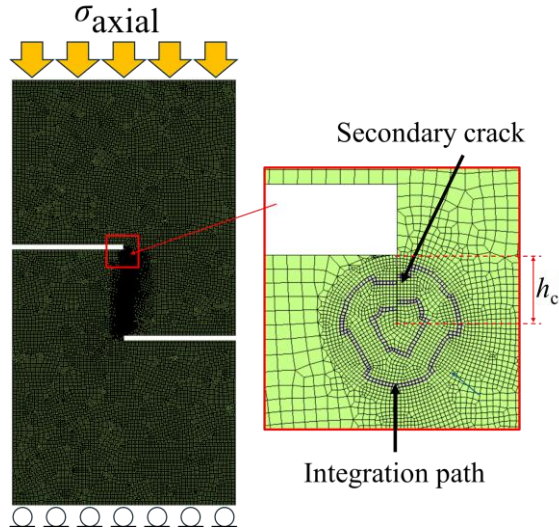


Figure 3: Finite element model of SCC specimen with secondary cracks ( $C/H = 0.2$ ,  $h_c = 1$  mm,  $\sigma_{axial} = 1.0$  MPa)

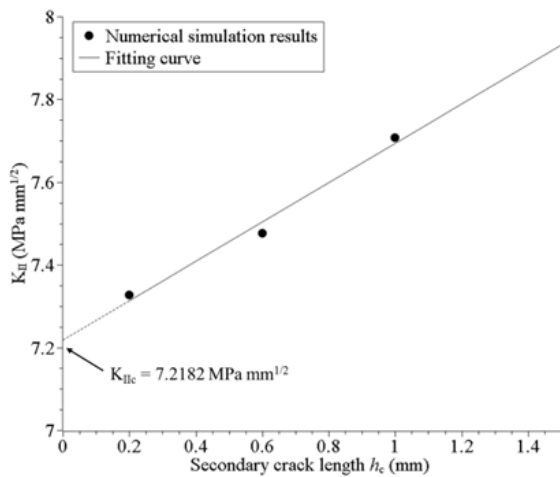


Figure 4: Effect of secondary crack length ( $h_c$ ) on calculated  $K_{IIC}$  values ( $C/H = 0.2$ ,  $\sigma_{axial} = 1.0$  MPa)

### 2.3 Mode II Fracture Toughness Tests under Various Loading Conditions

Mode II fracture toughness tests were conducted using a servo-hydraulic testing machine (Figure 5) and a Split Hopkinson Pressure Bar (Figure 6).

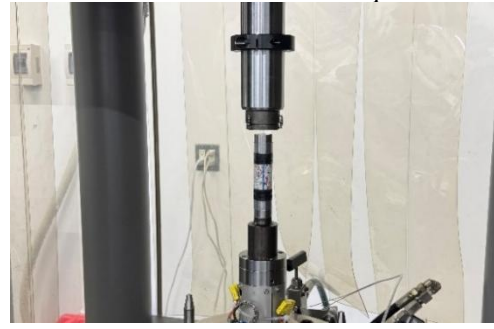


Figure 5: Test setup for Mode II fracture toughness under quasi-static loading using a servo-hydraulic testing machine

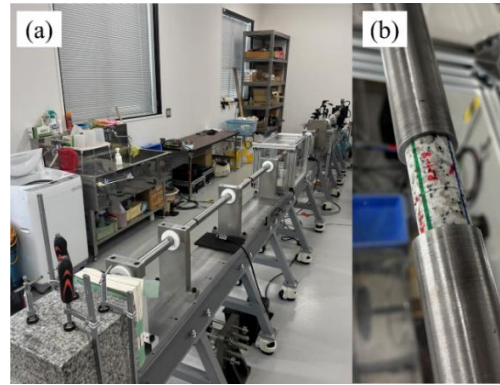


Figure 6: Test setup for Mode II fracture toughness under dynamic loading using SHPB system: (a) system overview, (b) rock specimen

Hardway and Rift orientations were selected for comparison to examine differences in anisotropy. In quasi-static tests, a loading rate of approximately  $0.001$  mm/sec was applied. Three specimens were tested for each orientation. For dynamic loading, impact velocities of  $7.5$  and  $15$  m/s were applied using the SHPB system. A high-speed camera (Phantom TE2010) was used to capture fracture events at  $560,000$  frames per second.

### 3 Experimental Results

Figure 7 presents  $K_{IIC}$  values for SCC specimens under different loading rates. Under quasi-static loading,  $K_{IIC}$  for the Hardway was  $1.82$  MPa $\sqrt{m}$ , nearly double that of the Rift at  $0.95$  MPa $\sqrt{m}$ . Under dynamic conditions, the difference between the two decreased significantly. For example, the difference was only  $1.88\%$  at approximately  $250$  GPa/s $\sqrt{m}$  and  $7.78$  percent at  $550$  GPa/s $\sqrt{m}$ .

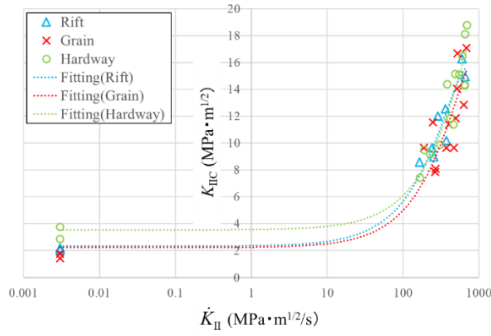


Figure 7:  $K_{IIC}$  values of Inada granite SCC specimens under various loading rate conditions

**Table 2: Fitting curve for each microcrack plane**

Microcrack plane	Fitting curve
Hardway	$y = -1.0 \times 10^{-5}x^2 + 0.027x + 3.5$ ( $R^2 = 0.95$ )
Grain	$y = -1.0 \times 10^{-5}x^2 + 0.029x + 2.2$ ( $R^2 = 0.82$ )
Rift	$y = -2.0 \times 10^{-5}x^2 + 0.036x + 2.3$ ( $R^2 = 0.91$ )

## 4 Discussion

### 4.1 Variation of Anisotropy Coefficient with Loading Rate

To evaluate anisotropic effects, an anisotropy coefficient  $\alpha$  was defined. It was calculated by comparing  $K_{IIC}$  values between two orientations at the same loading rate using the fitted curves. For example,  $\alpha$  for Hardway versus Grain is defined as  $K_{IIC}^{Hardway}$  divided by  $K_{IIC}^{Grain}$  along the value at the same loading rate on each fitting plane.

Comparisons were made for all combinations: Hardway vs. Grain, Hardway vs. Rift, and Grain vs. Rift. The results (Figure 8) show that  $\alpha$  approaches 1 as the loading rate increases, indicating that anisotropic effects diminish under dynamic loading.

In the case of Grain vs. Rift, a less than 1.0 value of  $\alpha$  was observed. This may be due to similar P wave velocities in these two directions, suggesting low anisotropic contrast. Further experiments with more anisotropic rock types are recommended.

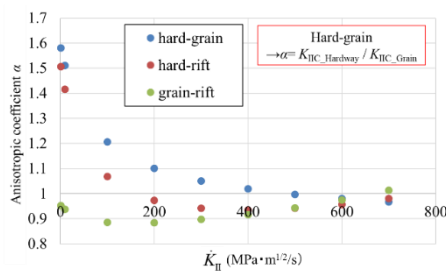


Figure 8: Variation of anisotropy coefficient  $\alpha$  with loading rate

## 5 Conclusion

This study investigated the Mode II fracture toughness of anisotropic granite under quasi-static and dynamic loading using the SCC method.  $K_{IIC}$  increased with the loading rate. Under static loading, significant differences were observed between fracture planes, but these differences decreased under dynamic conditions. These results indicate that both anisotropy and loading rate affect Mode II fracture behavior. Further studies using other rock types with clearer anisotropies are necessary to confirm these findings.

## References

- [1] K. Xia and W. Yao, "Dynamic rock tests using split Hopkinson bar system: A review," *Int. J. Rock Mech. Min. Sci.*, vol. 50, pp. 59–74, 2012.
- [2] S. W. Oh, G. J. Min, S. W. Park, M. S. Kim, Y. Obara, and S. H. Cho, "Anisotropic influence of fracture toughness on loading rate dependency for granitic rocks," *Eng. Fract. Mech.*, vol. 221, Art. no. 106677, Sep. 2019.
- [3] F. Dai and K. W. Xia, "Laboratory measurements of the rate dependence of the fracture toughness anisotropy of Barre granite," *Int. J. Rock Mech. Min. Sci.*, vol. 60, pp. 57–65, 2013.
- [4] T. Backers and O. S. Stephansson, "ISRM Suggested Method for the Determination of Mode II Fracture Toughness," *Rock Mechanics and Rock Engineering*, vol. 45, 11/01 2012, doi: 10.1007/s00603-012-0271-9.
- [5] Y. Xu, W. Yao, G. Zhao, and K. Xia, "Evaluation of the short core in compression (SCC) method for measuring mode II fracture toughness of rocks," *Engineering Fracture Mechanics*, vol. 224, p. 106747, 2020/02/01/ 2020.
- [6] Y. Jung, E. Park, and H. Kim, "Determination of Mode II toughness of granite by using SCC test," in *Proc. ISRM Int. Symp. EUROCK, Ürgüp, Turkey, 2016*