Design of H_∞ Controller for Hybrid Electromagnetic Levitation System

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I. INTRODUCTION

The present research findings regarding the designing controllers for hybrid electromagnetic systems are limited to optimal working conditions, which is hard to fit with the real-world applications where systems encounter various disturbances and uncertainties. To address these challenges, a H_∞ robust controller has been successfully designed for a system consisting of a Hybrid Electromagnet (HEM) which performs disturbance rejection and deals with model uncertainties.

II. LITERATURE REVIEW

Robust control techniques offer different methods such as H_∞ control, H_2 control and µ synthesis. The H_2 control focuses on minimizing the effect of disturbances on the system by reducing the overall energy of the system's response. This is widely used when disturbances are well defined and to achieve optimal response with minimal variance. Therefore H_2 control is not inherently a robust control approach [1].

Reference [2], has shown a comparative analysis between H_∞ control and PI control applied to Brushless DC servo drive. Starting from the mathematical model, [2] has designed the controller with weights augmented plant. Key highlight is the robustness that has been guaranteed through H_∞ controller under different cases. In [3], authors have studied about the robust control on magnetic bearings for rotating machinery using both H_2 and H_∞ control. It has highlighted the importance of selecting proper weights in robust control to achieve desired performance over PID control techniques. According to [4], H_∞ control has been applied to magnetic rotor system by modelling the shaft using finite element analysis. Results have shown that the H_∞ controller has effectively stabilized the system under various transient disturbances which include initial displacements and external periodic inputs, thereby reducing sensitivity to such disturbances and enhancing stability.

The D-K iteration algorithm which is used in µ-synthesis, optimizes controller performance while ensuring robustness against parametric uncertainties. It can handle applications with higher uncertainties than previously discussed method. The key advantage of the mu synthesis is its ability to provide performance guarantees under structured model variations [1].

III. METHODOLOGY

A. Development of the state space model of the system

The design process started with developing the mathematical model of the system. This system is a Single-Input Multiple-Output (SIMO) system where voltage is the input and the air gap distance and current are the system outputs. Equation (1) shows the derived state space model of the system where x_1, x_2, x_3, m, R, and L are air gap distance (z), derivative of air gap distance, current (I), mass of the shaft, the coil resistance and coil inductance, respectively. The constants k_z and k_i were derived after linearizing (2) around the zero-power operating point (z_0) and the constant values are as shown in (3) and (4) where constants 'a' and 'b' were found using finite element analysis on the HEM system.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ \frac{k_z}{m} & 0 & \frac{k_i}{m}\\ 0 & 0 & \frac{-R}{L}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{1}{L}\end{bmatrix}V \tag{1}$$

$$F_{(i,z)} = {a(I+b)^2}\Big/{(z+c)^2} \tag{2}$$

$$k_z = {2ab^2}\Big/{(z_0+c)^3} \tag{3}$$

$$k_i = {2ab}\Big/{(z_0+c)^2} \tag{4}$$

B. Development of the robust controller

After developing the complete mathematical model of the system, possible robust control techniques was investigated. Considering the need of disturbance rejection and guaranteed robustness, H_∞ control method was implemented. The H_∞

controller needed to be designed such that, it minimizes the H∞ norm of the system. The mixed sensitivity approach was used where sensitivity (S) and complementary sensitivity (T) were considered to define the control problem. Equations (5) and (6) shows the general form of S and T.

$$S_{(s)}={}^{1}\!/_{1+GK} \tag{5}$$

$$T_{(s)}={}^{GK}\!/_{1+GK} \tag{6}$$

Then the nominal plant was augmented with the weights; W_S and W_T such that they captures disturbance rejection and enforces robustness respectively. W_S and W_T are used as tuning knobs to achieve the required performance level between sensitivity and complementary sensitivity. Moreover, W_S acts as a low pass filter and W_T as a high pass filter where M is the low frequency gain of W_S, ω_0/A is the high frequency gain of W_T and ω_0 is the cutoff frequency as in (7) and (8).

$$W_s = {}^{M\omega_0}\!/_{s+\omega_0} \tag{7}$$

$$W_T = {}^{S\omega_0}\!/_{As+\omega_0} \tag{8}$$

The designed H_∞ controller (K) after augmenting the nominal plant with weights, is a high order dynamic feedback controller.

IV. RESULTS AND DISCUSSION

The results include the results of the magnitude and phase bode plots of the nominal plant, the frequency response plots of W_S and W_T and the comparison between the singular value plot of the system without any controller and with the designed H∞ controller.

A. Magnitude and Phase Bode Plots of Nominal Plant

Fig. 1 shows the magnitude and phase frequency response of the system. According to the outputs, the system has a gain margin of 5.5e05 and a phase margin of infinity which proves that the nominal closed system is unstable.

B. Frequency Response of W_S and W_T

Fig. 2 shows that W_S acts as a low pass filter with a cutoff frequency as designed previously. W_T acts as the high pass filter as expected. These results prove that expected behavior of W_T being dominant at high frequencies and W_S being dominant at low frequencies has been achieved successfully. The frequency roll off happens at the cut off frequency which was decided during design process. The weights do not allow the system to reach its natural frequency where resonance takes place.

C. Singular value plot of closed loop plant with and without H∞ controller

Fig. 3 shows how the controller has minimized the low frequency gain of the system compared to original system output. The system with the controller has a gain margin of 2.12 while having a phase margin of 107.47 which clearly shows that system has reached its expected performance under H∞ controller.

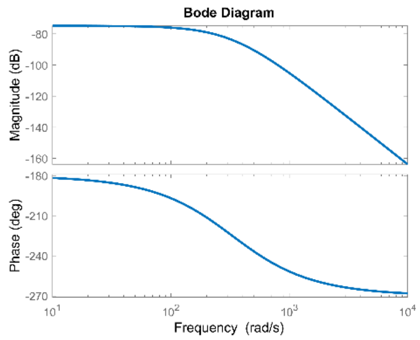

Fig. 1. Open loop frequency response of the system

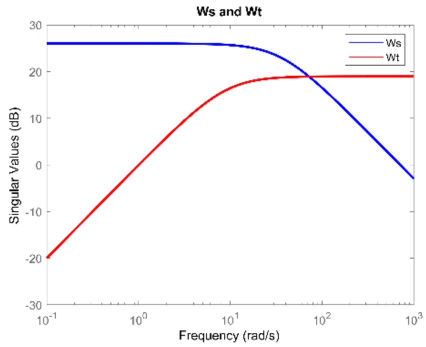

Fig. 2. Singular value plot of WS and WT

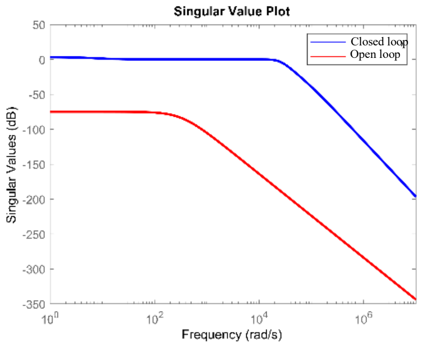

Fig. 3. Closed loop response with and without H∞ controller

V. CONCLUSION

Considering the results, it is clear that the designed H_∞ controller has been able to stabilize the system. The open loop frequency response is unstable with a higher gain margin and phase margin whereas the closed loop response with the controller has a much lower gain and phase margin.

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