

**THE DEVELOPMENT OF AN OPTIMIZED LONG-SPAN
CANTILEVER TRUSS SYSTEM FOR ROOFING
APPLICATION**

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Degree of Master of Science

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APPLICATION**

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Dissertation submitted in partial fulfilment of the requirements for the degree Master
of Science in Civil Engineering

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June, 2024

DECLARATION

I declare that this is my own work and this dissertation does not incorporate without acknowledgement any material previously submitted for a Degree or Diploma in any other University or institute of higher learning and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text.

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Date: June 12, 2024

Prof. (Mrs.) J. C. P. H. Gamage

ABSTRACT

The Development of an Optimized Long-Span Cantilever Truss System for Roofing Application

The structural efficiency of long-span cantilever trusses significantly relies on the vertical stiffness of the cantilever and its capacity of resistance against the vertical loads. When the cantilever span increases, the deflection of the end tip of the cantilevered truss under vertical loads becomes more pivotal as it reduces the vertical stiffness of the cantilever structure. Therefore, alternative member arrangements and geometry were altered innovatively to develop an optimum configuration of cantilever truss systems with a high capacity-to-weight ratio. This thesis illustrates the development approaches, numerical and analytical calculations of the decision-making process, and a detailed discussion of selected configurations and their performance.

Key Words:

Cantilever Roof Truss, efficiency, High Capacity, Optimization

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LIST OF ABBREVIATIONS

Abbreviation	Description
FRP	Fiberglass reinforced polymer
Zn/Al	Zinc Aluminium
thk.	Thickness
SDL	Superimposed dead load
LL	Live load
UDL	Uniform distributed load
CHS	Circular hollow section
BM	Bending moment
SF	Shear force
FEM	Finite element modelling
w.r.t	with respect to

LIST OF SYMBOLS

Symbol	Description	
ρ	Density	kg/m ³
γ	Unit weight	kN/m ³
E	Modulus of elasticity	MPa
G	Shear Modulus	MPa
f_y	Yield strength of steel	N/mm ²
f_u	The ultimate strength of steel	N/mm ²
ν	Poisson's ratio	[unitless]
α	Coefficient of linear thermal expansion	K ⁻¹
W	Total UDL load	kN
l	The span of the cantilever truss	m
D	Top and bottom chord depth	m
M	Cantilever moment at support	kNm
F_{chord}	Maximum axial force on chord member	kN
F_{web}	Maximum axial force on the web member	kN
$A_{\text{min, chord}}$	Minimum steel area required for chord member	cm ²
$A_{\text{min, web}}$	Minimum steel area required for web member	cm ²
A	The area of the steel section	cm ²
D	Outer diameter of CHS section	mm
t	Thickness	mm
P_{cr}	Euler's buckling load	kN
I	The second moment of the area	mm ⁴
L_e	Effective length	m
K	Effective length factor	[unitless]
σ_{cr}	Critical stress	N/mm ²
$(L_e/r)_{\text{cr}}$	Critical slenderness ratio	[unitless]
P	Member forces	N
L	Member length	m
P	The absolute value of member forces	N
$ P _{\text{max}}$	Maximum absolute value of member forces	N
A_{req}	Required area of steel section	cm ²
A_a	Area of initial proposed steel section	cm ²
A_b	Area of final selected steel section	cm ²
$(L_e)_{\text{cr, a}}$	Critical length for initially proposed section	m