Advanced Finite Element Formulations for Accurate and Efficient Topology Optimization of 2D Continuum Structures

J. A. N. N. Jayaweera

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MSc by Research

Department of Civil Engineering
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Thesis/Dissertation submitted in partial fulfillment of the requirements for the degree

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DECLARATION

I declare that this is my own work and this thesis/dissertation does not incorporate without acknowledgement any material previously submitted for a degree or diploma in any other University or Institute of higher learning, and to the best of my knowledge and belief, it does not contain any material previously published or written by another person except where the acknowledgement is made in the text. I retain the right to use this content in whole or in part in future works (such as articles or books).

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The candidate above has carried out research for the Master's thesis/dissertation under my supervision. I confirm that the declaration made above by the student is true and correct.

Name of Supervisor: H. M. S. T. Herath

Signature of the Supervisor: Date: 05.12.2025

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ABSTRACT

Finite element analysis is crucial for topology optimization. Most implementations use 4-noded membrane elements (Q4) despite low accuracy from membrane locking. This highlights the need for alternatives balancing accuracy and computational cost. This work presents algorithms to implement advanced finite elements into topology optimization. The advanced elements include Pian-Sumihara (PS) and Allman elements, and quadratic membrane element (Q8). Three problem types are addressed: volume-constrained compliance minimization, stress-constrained volume minimization, and stress- and volume-constrained compliance minimization. Performance of different finite elements is analyzed and compared using multiple criteria: accuracy, convergence characteristics, and computational cost. For compliance minimization with uniform regular meshes, PS delivers the most accurate layouts. Its solutions closely resemble Mitchell's truss theory and follow load paths effectively. PS converges to the lowest compliance with smooth black-and-white designs. Q8 matches closely but yields slightly higher objective values. For distorted mesh problems, Q8 shows superior performance. For stress-constrained volume minimization, PS elements achieved optimal results with 0.226 volume fraction compared to Q4's 0.245. Allman and Q8 values are 0.241 and 0.237. Similar behavior occurred for volume- and stress-constrained compliance minimization. PS showed the most optimal design, while Q8 produced similar designs. Allman and Q4 failed to reach optimal designs, showing gray regions. Average computation times relative to Q4 vary: 1.06-1.19× for PS, 1.39-1.74× for Q8, and 1.55-2.54× for Allman. Q4 is fastest but provides lower accuracy. With adequate h-refinement, Q4 can achieve results similar to advanced elements. PS elements are recommended for uniform regular meshes requiring high accuracy. Q8 elements are preferred for distorted mesh problems. Allman elements suit moderate accuracy requirements.

Keywords: Topology optimization, Finite element analysis, 2D continuum mechanics, Stress constraints, Compliance minimization

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LIST OF ABBREVIATIONS

Abbreviation	Description
DOF	Degrees Of Freedom
FEA	Finite Element Analysis
FEM	Finite Element Method
KS	Kreisselmeier-Steinhauser
MMA	Method of Moving Asymptotes
OC	Optimality Criteria
PS	Pian-Sumihara element
Q4	4 noded bilinear quadrilateral membrane element
Q8	8 noded quadratic quadrilateral membrane element
SCTO	Stress Constrained Topology Optimization
SIMP	Solid Isotropic Material with Penalization
TO	Topology Optimization

LIST OF APPENDICES