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**INTEGRATED FORM-FINDING AND OPTIMIZATION
OF GRID-SHELL STRUCTURES USING COUPLED
ITERATIVE ALGORITHMS AND SPECIALIZED
GRAPH NEURAL NETWORKS**

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Master of Science (Major Component of Research)

Department of Civil Engineering
Faculty of Engineering

University of Moratuwa
Sri Lanka

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DECLARATION

I declare that this is my own work and this thesis/dissertation does not incorporate without acknowledgement any material previously submitted for a degree or diploma in any other University or Institute of higher learning, and to the best of my knowledge and belief, it does not contain any material previously published or written by another person except where the acknowledgement is made in the text. I retain the right to use this content in whole or in part in future works (such as articles or books).

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Date:

The above candidate has carried out research for the Master's thesis/under my supervision. I confirm that the declaration made above by the student is true and correct.

Name of Supervisor: Dr. H. M. S. T. Herath

Signature of the Supervisor:

Date: 05.12.2025

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ABSTRACT

A primary inefficiency in the design of complex grid-shell structures arises from the separation of form-finding and member sizing into distinct stages. This decoupled approach is problematic because the optimal structural form is directly dependent on the dimensions of its members, making the optimization process computationally demanding. This research addresses these challenges through two primary contributions. First, it introduces an enhanced iterative algorithm that couples form-finding, based on the Potential Energy Method, with member sizing optimization. This methodology is implemented in two novel MATLAB-based software tools, providing a flexible design environment that accommodates arbitrary geometries, materials, and loading conditions, which were validated against established analytical methods. Secondly, to significantly accelerate this workflow, the study explores the application of Graph Neural Networks for near-instantaneous prediction of optimal forms and member properties. A generalized Graph Neural Network model trained on a diverse dataset of mixed topologies struggled with generalization, showing limited accuracy. In contrast, a specialized Graph Neural Network model trained exclusively on dome-type grid-shells demonstrated outstanding predictive accuracy, with coefficient of determination values exceeding 0.99 for both final nodal coordinates and optimal member properties. This specialized model drastically reduces computational time compared to traditional iterative simulations. The findings demonstrate that while the developed software provides a robust tool for detailed analysis, a topology-specific Graph Neural Network approach offers a robust and viable strategy for real-time structural feedback in the conceptual design phase. This work lays the groundwork for integrating physics-informed machine learning into structural engineering to create more efficient and intuitive design tools.

Keywords: Grid-shells, Form-finding, Graph Neural Networks, Potential Energy Method, Size Optimization, Coupled Analysis

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LIST OF ABBREVIATIONS

Abbreviation	Description
2D	Two Dimensional
3D	Three Dimensional
ANN	Artificial Neural Network
AMP	Automatic Mixed Precision
CEM	Combinatorial Equilibrium Modeling
DDPG	Deep Deterministic Policy Gradient
FDM	Force Density Method
FEM	Finite Element Method
FF	Form-Finding
FSD	Fully Stressed Design
GAT	Graph Attention Networks
GCN	Graph Convolutional Network
GNN	Graph Neural Network
Hetero GAT	Heterogeneous Graph Attention Networks
LDA	Linear Discriminant Analyses
LIGNNs	Logic-Informed GNNs
M-LIGNN	Modification-LIGNN
ML	Machine Learning
MS	Member Sizing
MSE	Mean Squared Error
NBC	Naive Bayes Classifier
NLFDM	Non-Linear Force Density Method
P-LIGNN	Primary-LIGNN
PEM	Potential Energy Method
SO	Size Optimization
SVM	Support Vector Machine
VGS	Vector-based Graphic Statistics

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