OPTIMUM DISPATCH OF TURBINES IN A LOW HEAD HYDROPOWER PLANT FOR A GIVEN TOTAL FLOW RATE AND AVAILABLE VARIABLE HEAD - A CASE STUDY FOR MORAGAHAKANDA POWER STATION IN SRI LANKA

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Degree of Master of Science

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DECLARATION

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Dr. Logeeshan Velmanickam

Abstract

Hydropower is an extremely valuable resource. There are two types of reservoir based hydropower stations with different main goals. Some are following the demand of the utility and others follow irrigation water requirements. Therefore, always power generation is not at full capacity, and most of the time turbines are at partial loads as well.

By optimal hydropower generation and scheduling, great profits can be achieved for the company and also for the country. Therefore, the economical dispatch of hydropower generation provides great opportunities for electric utilities.

Moragahakanda hydropower station generates electrical power as an outcome of the irrigation water requirement, decided by the Water Management Secretariat (WMS) which is headed by the Mahaweli Authority of Sri Lanka (MASL). This research presents a strategy for the optimum distribution of dispatched power among its available generating units. The objective is the maximization of the total power generation for a low-head hydropower plant under varying head and for a given total water discharge rate.

Hydropower generation scheduling is a nonlinear programming problem. The nonlinearity is due to the generating characteristic of the hydro turbines, whose outputs are generally a nonlinear function of water discharge and net hydraulic head. This research work presents a method that takes into account both the nonlinear characteristic and the head loss effect.

This development is done based on the Moragahakanda hydropower station and, MATLAB software has been used for building mathematical models and solving the optimization problem.

Keywords: Hydropower generation, Optimal dispatch, Optimization method, Economical dispatch.

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LIST OF ABBREVIATIONS

Abbreviation	Description
DE	Differential Evolution
GA	Genetic Algorithms
LHP	Large Hydro Plants
HPP	Hydro Power Plant
MASL	Mahaweli Authority of Sri Lanka
PSO	Particle Swam Optimization
SCADA	Supervisory Control And Data Acquisition
SHP	Small Hydro Plants
WMS	Water Management Secretariat

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Background

Hydropower is one of the most important, renewable energy source. In hydroelectricity generation, its potential and kinetic energy are converted to electrical energy. It has become an electricity source since the late 19th century and still, hydroelectricity fulfils about 17 % of the electricity demand globally. And in Sri Lanka, hydropower generation is the oldest and, still one of the main electric power generation sectors which contribute around 20 % of average daily power generation.

Hydroelectric power generation is a good alternative, to fossil fuels with no direct emission of CO_2 or other atmospheric pollutants. Nevertheless, due to low operating and maintenance costs, providing a relatively consistent source of power and also comparative easiness of adjusting power output by controlling the flow through the turbines according to the demand, it is focused on building new hydropower plants or trying to maximize plant efficiency, more frequently nowadays.

Optimization is the major concept when considering maximizing the efficiency of power generation. More power plant operators who follow utility load demands have paid attention to maximizing power generation while minimizing water discharge. Therefore, many researchers have considered this strategy to achieve higher plant efficiencies. Nevertheless, some hydropower plants provide the irrigation water needs of downstream areas. For them, supplying the downstream water requirement is a very crucial task during all seasons of the year, especially in the dry season.

Therefore, in power plants having multiple generating conditions, it is necessary to implement an optimization method to manage hydropower generation for energy management and also for water management. And this will provide economic benefit to the whole country by reducing the cost incurred in fossil fuel-based power generation.

1.2 Moragahakanda hydropower plant

The Moragahakanda hydroelectric power station is the only hydro-power station operated by the Mahaweli Authority of Sri Lanka. This power plant is constructed near the Moragahakanda reservoir, and machines are operated under a low water head. The total capacity of the power generation of the plant is 25 MW. The electricity is generated using four Francis turbine and generator units. This consists of two 7.5 MW and other two 5 MW hydroelectric machines. The generated power is connected to the national grid, on a commercial basis, through the standard power purchase agreements.

The main purpose of the Moragahakanda project is fulfilling the water requirements of the Central, Northern, North Central and Eastern Provinces of Sri Lanka. The water released through hydro turbines is sent to Ambanganga and then to Giritale, Minneriya, Kavudulla and Kantale reservoirs through the old Elahera cannel for irrigation and domestic purposes of the people who are living in the downstream areas. Therefore, the amount of power to be generated by the Moragahakanda power plant is decided based on the downstream water requirement.

Figure 1.1: Moragahakanda reservoir and dam

1.3 Problem statement

The Moragahakanda reservoir is situated at 132 m height above the mean sea level and the dam height is around 58m. The maximum level of the reservoir is around 186m above the mean sea level. When the water level of the reservoir reaches 165 m above the mean sea level, the power plant can start power generation and, the operations of the generator are at their rated maximum when it reaches the highest level of 185 m. The downstream water requirement is decided by the Water Management Secretariat and informed weekly. Therefore, these operational and water requirements are needed to be fulfilled while the plant generates electricity.

Although the commercial operation of the Moragahakanda power station started on 8th January 2018, still there is no stabilized optimization method used for power generation. Presently, the required power generation is computed manually to match the required discharge water rate and operational limitations of the generator turbine units by referring to each unit's historical data, past experiences and running characteristic curves.

Hence, it is necessary to implement an optimal dispatch schedule for hydroelectric units in this plant which will provide the maximum amount of energy generation while accomplishing the downstream water requirement, under a given set of constraints.

1.4 Running characteristic curves

Two running characteristic curves have been provided by the turbine manufacturer of the Moragahakanda power station. One running characteristic curve has been supplied for 5MW capacity units (G1A and G1B) and another running characteristic curve has been supplied for 7.5MW capacity units (G2 and G3) as shown in appendix 1 and appendix 2. It was assumed that units of the same capacities are identical.

By using these curves, the variation of the efficiency can be obtained while power generation is varying with the net effective water head of the unit. The pink and red lines show the operating range at each head level of the relevant machine, without vibrations and mechanical instabilities, in these curves. Therefore, it is necessary to operate the unit within these boundaries.

1.5 Research goals and objective

The research objective of this study is the maximization of total hydropower generation, by building a mathematical model, for a low head hydropower plant, under varying head and varying flow conditions.

The proposed work is focused on the optimization of the Moragahakanda power station in Sri Lanka, as a case study.

1.6 Thesis overview

The second chapter of this thesis gives a comprehensive literature review on hydropower plants and different turbine types. It also discusses the importance of optimization of the plant dispatch and, different optimization methods and different solving techniques.

The third chapter focuses on how to model a hydropower plant and build the optimization problem under various bounds and constraints for dispatching the plant for the next scheduling period.

The fourth chapter describes the proposed model to accomplish the operational constraints of power generation and water discharge rate of each unit on analyzing the previous operating data. The fifth chapter focuses on solving the optimization problem with two real applications of the plant. Finally, in this chapter, the impact of the proposed optimization method has been discussed with two comparisons of the actual operating data.

CHAPTER 2

LITERATURE REVIEW

2.1 Types of hydropower plants

There are several types of hydropower plants operated in the world. Some hydropower plants have reservoirs and others are run-off river type. They can also be classified as Large Hydro Plants (LHP) and Small Hydro Plants (SHP). The power plants under 10 MW of total capacity are considered SHP. [1]

Reservoirs are built for the purpose of hydroelectricity, irrigation, flood control and water needs etc. Only a few power plants are constructed only for power generation requirements. Some other hydropower plants are built on irrigational reservoirs to get multiple economic benefits.

The hydropower plants which have been built to fulfil the electricity load demand, follow the requirement of the electricity demand and the other type of power plants are operated according to the irrigational water requirements. The second type of hydropower plant discharges water as a quantity while operating at the full capacities of the generator machines or as a flow rate during a period, to match irrigation requirements. [2, 5]

2.2 Types of turbine

There are two main types of turbines used for hydropower generation, classified as Impulse and Reaction types. In the design procedure of hydropower plants, the suitable turbine is selected according to the designed range of operating water head and flow rate of the water discharge. Among them, reaction-type turbines are used for low head and large flow rates. Propeller (including Kaplan) and Francis turbines can be considered as mostly used reaction type turbines. Figure 2.1 shows the classical Head/Flow rate characteristics of the hydropower turbines. [1, 2]

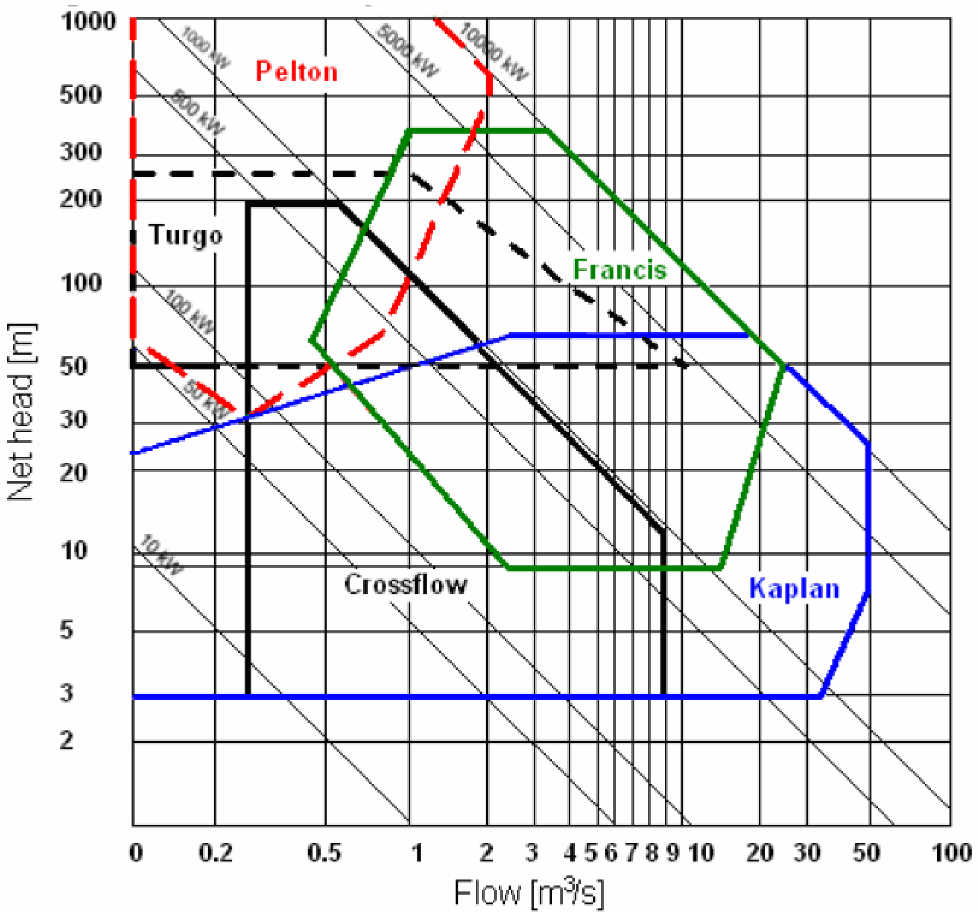

Figure 2.1: Classification of the hydro turbines [1]

Francis-type turbines can be used for a wide range of head variations. Nevertheless, they are commonly used in most power plants. According to the above figure, Francis turbines can be used for 10m to 400m head variation with more than 90% of efficiency. Moreover, the specially designed Francis turbines can be used for much higher water head levels.

Even though the turbines are designed for operating at their maximum rated efficiency point with 80% of rated flow, in practical situations, it is impossible to achieve maximum efficiencies when machines are operated in partial loads. The classical efficiency characteristic curves of different turbines are presented in Figure 2.2.

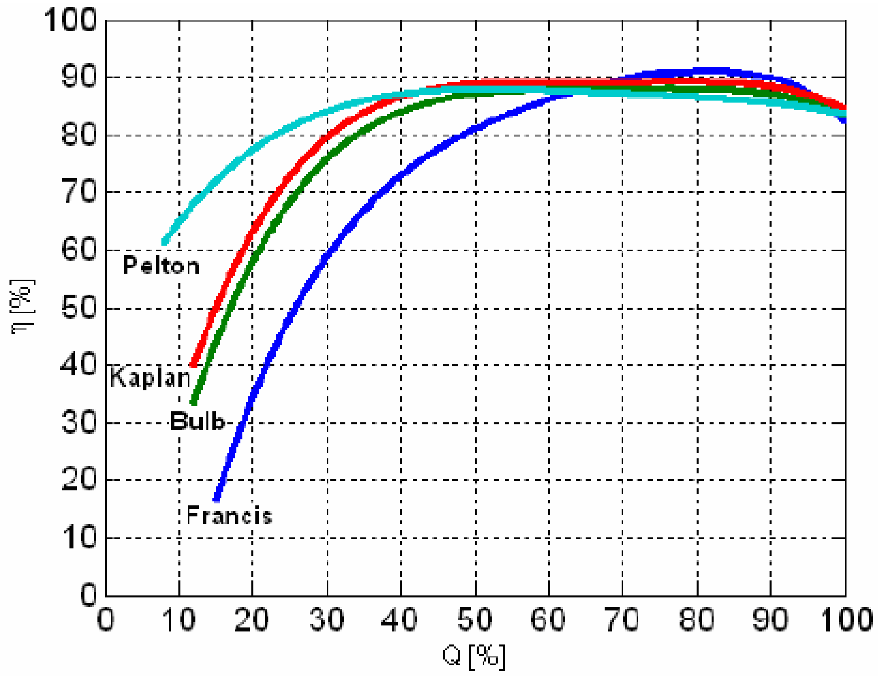

Figure 2.2: Efficiency /Flow characteristics for different turbine types [1]

Although Francis turbines show 90% peak efficiency, this turbine type is more sensitive to the flow rate variation. Francis turbine can be operated at its rated maximum speed, only with flow rates which are higher than 50% of the rated discharge. Otherwise, it shows the instabilities of the operations, due to the high vibrations and mechanical shocks. Therefore, it is necessary to carefully find the operating boundaries for the machines. [1]

Nevertheless, these curves have been defined for a fixed head. Hence, in building an optimization problem, it is important to find out the actual relationship between power generation and water discharge rate while varying the water head level.

2.3 Mathematical relationships for hydropower generation

For hydropower plants, a mathematical relationship can be obtained for hydropower generation from the rule of energy conversion. The following common linear mathematical relationship between power generation and water discharge is a basic equation that can be obtained from this concept for one particular head level. [3]

$$p = \eta . h \rho g . q \quad \text{-------------------------------- Eq (01)}$$

Where,

p is the power generation in MW;

ɳ is the efficiency of the turbine generator unit;

h is the effective net water head in m;

ᵽ is the density of water in kg/m^3;

g is the gravitational acceleration in m/s^2 and

q is the water discharge through the turbine in m^3/s.

The following equation can be obtained by further improving equation 1. [4]

$$p = ɳ_t \, . \, ɳ_g \, . \, h_\tau \, . \, ᵽ \, . \, g \, . \, q \text{ --------------------------------Eq (02)}$$

Where,

$ɳ_t$ is the efficiency of the turbine;

$ɳ_g$ is the efficiency of the generator and;

h_τ is the effective net head in m.

Effective net head h_t, can be denoted as below.

$$h_\tau = h_r - h_t - h_p \text{ --------------------------------Eq (03)}$$

Where,

h_r is the reservoir elevation in m;

h_t is the tailrace elevation in m and;

h_p is the penstock head losses in m.

On the other hand, the gross head is equal to the difference between the reservoir elevation and tailrace elevation, ($h_r - h_t$). Therefore, considering all hydraulic losses, equation 3 also can be explained below. [4].

$$Net\ \ head\ (h_\tau)\ =\ Gross\ head(h_r - h_t) - Hydraulic\ Losses(h_p)$$

---------------------------------Eq (04)

As shown in the above equations, the power generation of a hydropower station depends on a lot of variables. Many hydropower plants have sensors and SCADA systems for collecting and monitoring of these variables. [6, 7]

2.4 Hydropower losses

Above mentioned mathematical relationship by equation 1 is defined under the ideal hydropower generation conditions. Due to the impact of hydraulic power losses under actual conditions, the linear relationship between the power generation and water discharge rate is changed into a nonlinear relationship.

Considering hydraulic power losses in hydropower generation, following important facts can be examined. They are the elevation of the reservoir water level, the elevation of the downstream tailrace water level, penstock head losses, turbine efficiency and generator efficiency. The impact of each of these facts can be described as follow. [4]

2.4.1 Elevation of the reservoir water level

Reservoirs are constructed for purpose of the hydropower generation or irrigation water requirements as mentioned above. Nevertheless, the net head of the hydropower generation is directly calculated as a function of the elevation of the reservoir level and the elevation of the downstream water level as shown in equation 3.

Considering large reservoirs, during a short time duration, the variation of the reservoir level is very small. Therefore, the water level variation of the reservoir is considered negligible for the short-term period of the hydro plant scheduling problems. [4, 8]

2.4.2 Elevation of the downstream tailrace water level

In large hydropower plants, the elevation of the tailrace water level is raised according to the increase of the water discharge rate through the power plant. Therefore, it affects directly the decreasing of the net head of the generator unit according to equation 3. It is shown in the Figure 2.3 related to the Itaipu power plant.

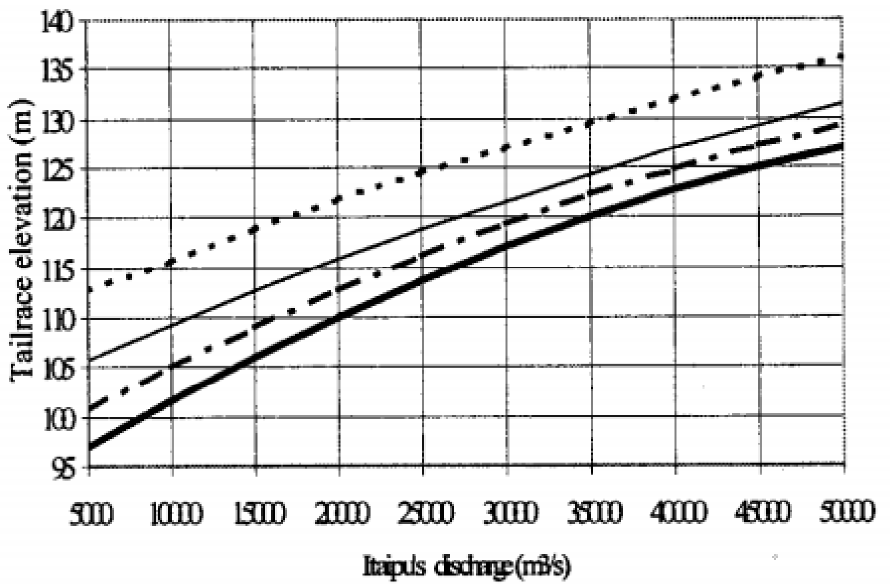

Figure 2.3: Tailrace elevation variation with water discharge in the Itaipu power plant [4]

2.4.3 Penstock head losses

These hydraulic head losses occur due to the friction between water and the walls inside of the penstock. This can be expressed as a quadratic function of a water discharge rate and can be denoted as the following equation.

$$p_h = (k \cdot q^2) \quad \text{-------------------------------Eq (05)}$$

Where,

p_h is penstock head loss in m;

k is a constant, obtained from the characteristic of the penstock (s^2/m^5) and;

q is the water discharge in m^3/s.

2.4.4 Turbine efficiency

The efficiency of the turbine can be expressed as the following relationship.

$$Turbine\ efficicency\ (\eta_t) = \frac{Mechanical\ energy}{Potential\ energy}$$

-------------------------------Eq (06)

In real conditions, parameters which are power generation, water discharge rate and water head level, are varying due to the efficiency figures varying with the changing head level in large power plants. Therefore, the variation of the three parameters turbine efficiency, net head and water discharge, are represented by one diagram named Hill curves as shown in Figure 2.4. [4]

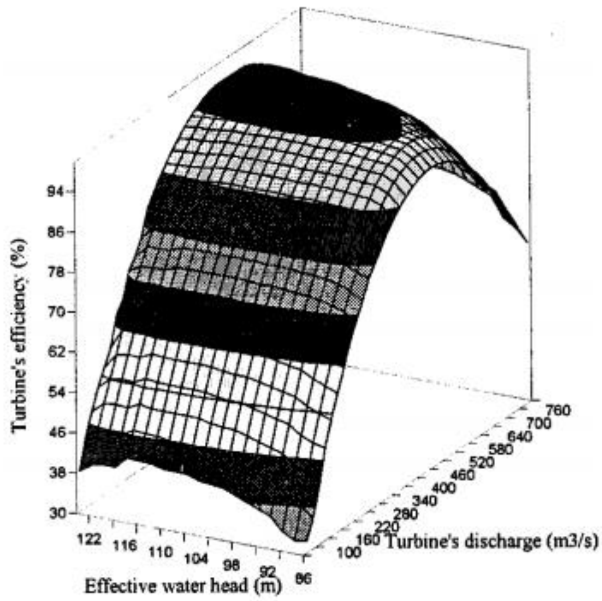

Figure 2.4: Turbine Hill curve

Turbine efficiency at a certain net water head level increases to a certain highest value and then it goes down again. Due to the varying head level, efficiency figures are also varying.

The characteristic curve of a Francis turbine is shown in Figure 2.5. It shows the variation of the turbine efficiencies, with the variation of the power generation and the effective water head. There is a point which maximizes the turbine efficiency called the Project point. It is also important to discharge the water, within the provided operational constraints, or otherwise, turbines may face harmful vibrations or cavitation as mentioned above.

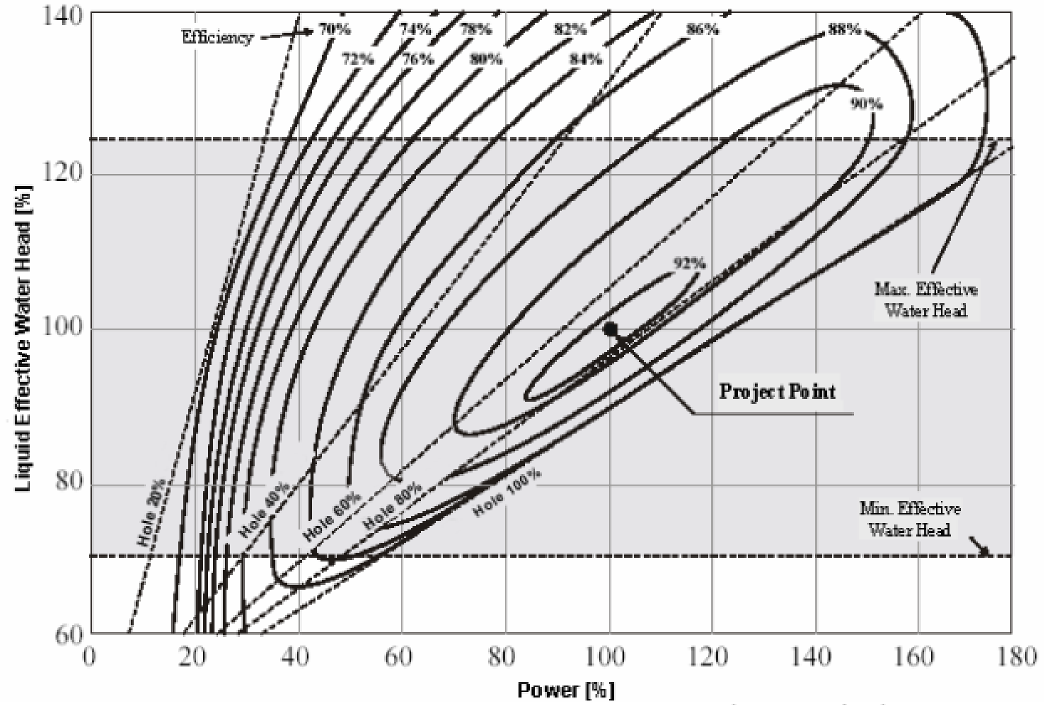

Figure 2.5: Performance curves of a Francis-type turbine [3]

2.4.5 Generator efficiency

The efficiency of the generator can be expressed as,

$$Generator\ efficiency\ (\eta_g) = \frac{Electrical\ energy}{Mechanical\ energy}$$

--------------------------------Eq (07)

As shown in Figure 2.6, the generator efficiency is a function of power generation of the generator. Therefore, it is also considered for the calculating process of the total power generation loss of a hydropower plant. [4]

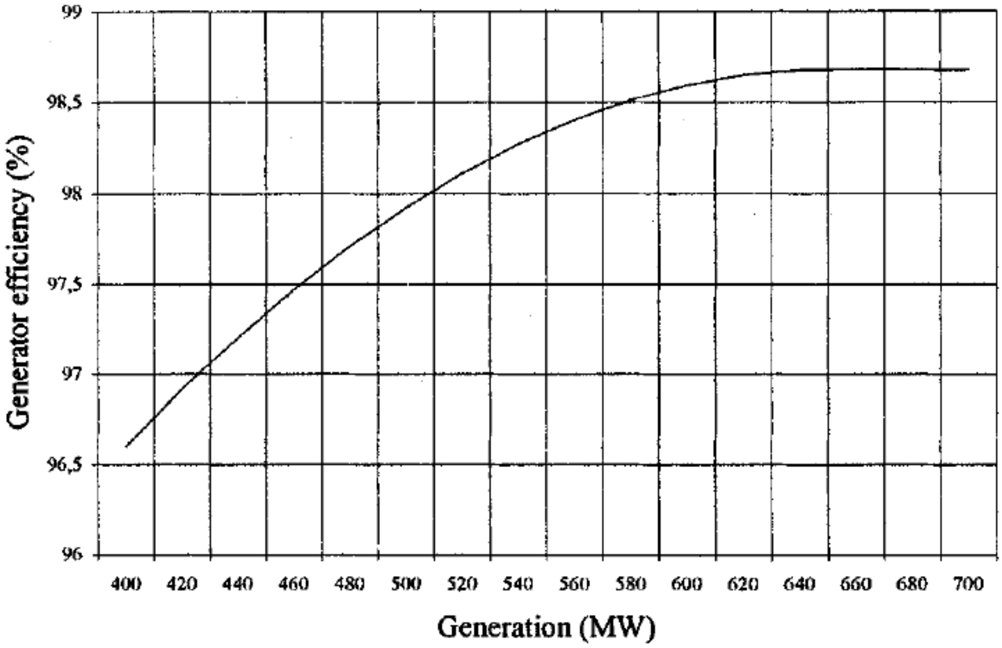

Figure 2.6: Typical generator efficiency curve

2.5 Nonlinear relationship of the hydro performance model

According to the impact of the above parameters, they create hydraulic losses in the water circuit of the hydropower plant. Therefore, a nonlinear behaviour is seen between the power generation and water discharge rate as shown in the blue colour curve in Figure 2.7. Therefore, at each particular head level, one nonlinear relationship can be obtained for power generation relating to water discharge rate. Hence when the water level is varying, a set of nonlinear curves can be obtained for one generator unit as shown in Figure 2.8. [8]

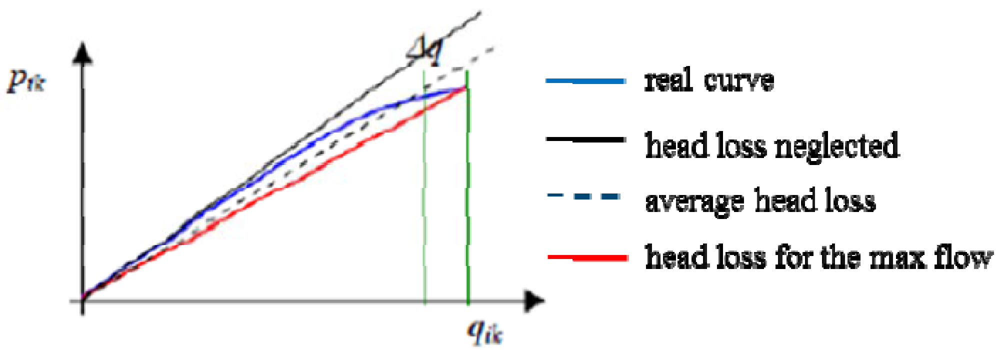

Figure 2.7: Power generation vs flow rate

In this case, researchers have introduced several approximations to obtain a suitable mathematical relationship. One of these methods is completely neglecting the head loss and developing a linear approximation. Hence the real curve is located under the approximate curve at all times, and erroneous results are more severe. [9]

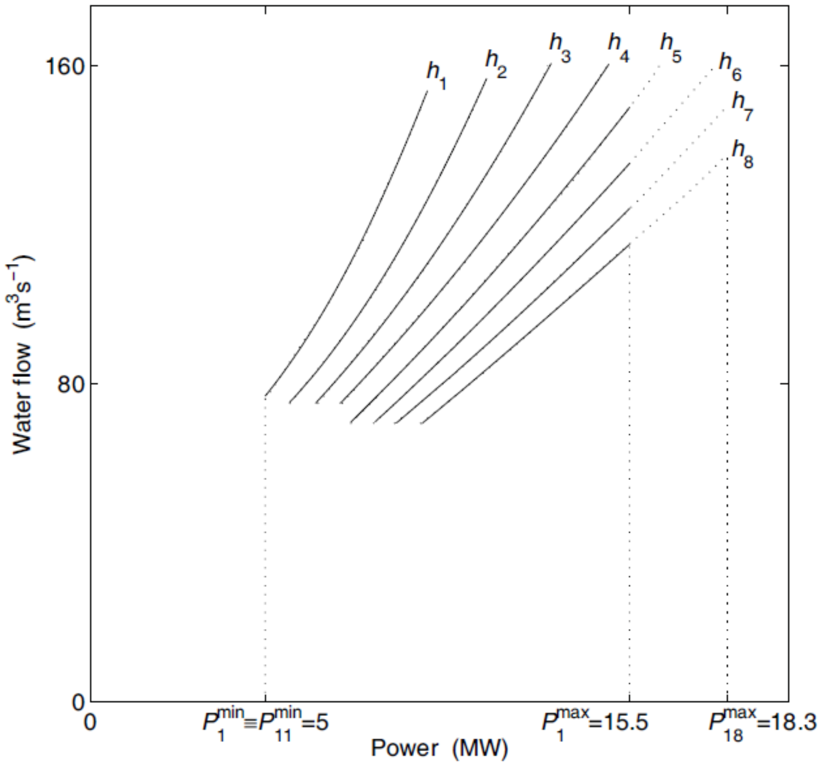

Figure 2.8: Set of characteristic curves with polynomial approximation [9]

One other method is getting the polynomial approximation by considering hydropower losses as well. This method is more accurate and, the mathematical relationship between power generation and water discharge rate at a particular head level is expressed using their 2nd order polynomial approximation. [5, 8]

$$q_{ji} = \alpha_{ji} + \beta_{ji}.p_{ji} + \frac{\gamma_{ji}}{2} p_{ji}^2 \text{ -------------------------Eq (08)}$$

Where,

q_{ji} is the water discharge rate through the turbine of the j^{th} machine in m^3/s at i^{th} head level.;

α_{ji} is the constant of the quadratic approximation of the j^{th} machine at i^{th} head level;

β_{ji} is the constant of the quadratic approximation of the j^{th} machine at i^{th} head level;

γ_{ji} is the constant of the quadratic approximation of the j^{th} machine at i^{th} head level and;

p_{ji} is the power generation of the j^{th} machine in MW at i^{th} head level.

2.6 Optimization of the hydropower plants

The optimum operation of the generating units is an important task in power plants. By optimization process, higher efficiency or more economic benefits are gained for the organization and also for the nation. For this task, minimization of the fuel cost or water discharge, or maximization of the power generation or economical profits, are considered as the main goals.

When hydropower plants are considered, they are valuable power plants which operate from a renewable energy source of water. By maximizing the hydropower generation, the deduction of the cost for power generation of petroleum is an attractive solution for fossil fuel as well. Moreover, due to advantages like no CO_2 or any greenhouse gas emission directly from the power plant, and the low maintenance and operation cost, are keeping more attention to the increasing number of hydropower plants. Therefore, building more hydropower plants and maximizing hydropower generation are the main discussions at present days. [5, 7]

Generator units of some power plants are unable to supply the rated maximum power output due to utility demand or irrigational water demand variations. Therefore, they have to be operated in partial loads frequently. On the other hand, because the reservoir water head level is decreasing dramatically in the dry season with a small rainfall, it is a crucial task to keep the continuous supply of power generation to match the demand as requested in this period. Therefore, the optimum operation of hydropower plants is a valuable strategy for energy management as well as for water management.

2.7 Optimization methods of hydropower plant

There are several methods that have been used for the optimization of hydropower plants. However, according to the main operating purpose of the HPP, the objective function differs from plant to plant. Hence, the optimization problem of the next scheduling period includes different boundaries and constraints.

Nevertheless, discharged water is considered the fuel in hydropower plants. Because a lot of plants are based on demand, the main objective of the optimization of these hydropower plants is obtaining the maximum plant efficiency under the minimum water discharge. Due to the efficiency being a function of the generated power and water discharge rate, maximizing the total plant power generation, by minimizing the cost of the power generation or in other words, by minimizing the water consumption of the plant, is considered in the optimization process of those hydropower plants. Operation constraints of the turbine generator machines and the constraints of the system, or environment constraints are taken into account while optimizing.

The capabilities of the power generation and the water discharge rate without mechanical instability or vibrations are considered operational boundaries of the machines. And the load demand of the system or water requirement for irrigation purposes is considered as the system or environment constraints. Some commonly used formulations by researchers for the optimization of hydropower plants are described below.

2.8 Classical economical dispatch optimization problem

The classical economical dispatch optimization problem can be defined as follow.

$$min\ C_T(P_i)$$

$$s.t. \sum_{i=1}^{n} P_i = P_d \quad \text{------------------------------Eq (09)}$$

$$P_{Li} \leq P_i \leq P_{Ui}$$

Where, C_T is the cost function and P_d is the total dispatched power. P_i is the generated power of each unit, n is the number of machines and P_{Li} and P_{Ui} are the lower and upper limits of the power generation of each turbine generator unit.

In this method, the relationship of the cost function with power generation needs to be obtained to solve the optimization problem. The cost function of each unit is the water consumption which is represented as a function of power generation in the hydropower station. Finding the cost function for a considered power plant is a very time-consuming process due to tracking and identifying the variation from time to time. Even though for same capacity turbine generator machines by the same manufacturer and same design, there are considerable variations in these relationships due to small variations in the manufacturing process, constructed elevations, and addition of the hydraulic losses under the real conditions. [6, 7]

2.9 Efficiency measurements optimization problem

Therefore, as a solution for the above issues, the efficiency of the energy conservation of each turbine generator unit is considered. The net efficiency of the power plant can be obtained by the following equation.

$$\eta_G = \frac{P_1 + P_2 + \cdots + P_{n-1} + P_n}{\frac{P_1}{\eta_1} + \frac{P_2}{\eta_2} + \cdots + \frac{P_{n-1}}{\eta_{n-1}} + \frac{P_n}{\eta_n}} \quad \text{------------------------------Eq (10)}$$

Where, for the *n* amount of the generator units, P_i is dispatched power of each unit and η_i is the efficiency of each unit.

Hence, the formulation of the optimization problem can be rewritten as follow. [6, 7]

$$max\ \eta_G$$

$$s.t. \sum_{i=1}^{n} P_i = P_d \quad \text{-------------------------------Eq (11)}$$

$$P_{Li} \le P_i \le P_{Ui}$$

Here, maximizing the total plant efficiency has been considered, and it can be done in two methods. The first one is using existing or factory-tested efficiency curves for each turbine generator unit as shown in Figure 2.5. By using these curves, the variation of efficiency with the power generation of each unit can be obtained at each head level. Even though the efficiency and power generation relationship can be obtained at each head level, there is no consideration of the machine ageing, and other factors like hydraulic losses, variations of temperature and vibration by using these curves which were obtained at a certain time under certain conditions. [7]

Therefore, to avoid the above drawbacks, as a second method, real-time efficiency measurements are considered by using real-time data which are obtained from the SCADA system. For this case, suitable sensors, measuring devices, and implementation of SCADA system for measuring and monitoring purposes are important tasks to obtain real-time efficiency data of each unit. [3]

2.10 Loss minimization optimization problems

The loss minimization optimization problems are also used for hydro systems and hydropower stations. If the dispatched power of the plants is Pi and the total demand is P_D, the relationship with the power loss (P_L) can be shown as the bellow equation for N number of power plants within a 't' time interval.

$$\sum_{i=1}^{N} P_i(t) = P_D(t) + P_L(t) \quad \text{------------------------Eq (12)}$$

For this case, the optimization problem can be written as follow.

$$min\,\{\sum_{t=1}^{T}[P_D^t - \sum_{i=1}^{N} P_i(t)]^2\} \quad \text{------------------------Eq (13)}$$

This objective function is subjected to the following constraints.

Power generation limit of hydropower plant

$$P_i^{min} \leq P_i(t) \leq P_i^{max}$$

Water discharge limit of hydropower plant

$$Q_i^{min} \leq Q_i(t) \leq Q_i^{max}$$

Reservoir volume limit of hydropower plant

$$V_i^{min} \leq V_i(t) \leq V_i^{max}$$

Hydro performance model equation for water discharge

$$Q_{ji}(t) = x_{ji}P_{ji}^2 + y_{ji}P_{ji} + zj_i \quad \text{-----------------------Eq (14)}$$

Where i = (1, 2, …, N) and x_i, y_i, and z_i are discharge coefficients.

For this case, the reservoir volume limits also have been considered. However, for large reservoirs, the water level variation during the small scheduling periods can be neglected due to the small variation in the volume of the reservoir. Therefore, in most research, the variation of the water level of the reservoir has been neglected for the optimization problem during the scheduling periods. [5]

All the above-mentioned optimization methods have been used to follow utility load demand by minimizing water consumption. Therefore, total water consumption is reduced while maximizing the total power generation output. But for hydropower plants which follow the irrigational water requirements and, discharge water as a flow rate, the total water discharge rate needed to be maintained at a certain flow rate within the given scheduling period.

2.11 Solving techniques for the optimization problem

As mentioned above, due to the impact of hydraulic power losses in the water circuit under the actual conditions, the mathematical relationship between power generation and water discharge rate is obtained as a nonlinear equation for each machine. Due to this nonlinearity of the optimization problem, different solving techniques and tools have been used to solve these problems.

Dynamic programming, semi-definite programming, and the concept of nonlinear network flow are the commonly used solution techniques for solving nonlinear optimization problems. Nevertheless, presently in solving hydro scheduling problems heuristic techniques such as simulated annealing techniques and evolutionary programming techniques are also used. Genetic Algorithms (GA), Particle Swarm Optimization (PSO), and the method of Differential Evolution (DE) are included in most algorithms. [5]

Nevertheless, some researchers have used different software to solve optimization problems. MATLAB software has also provided the ability to create and solve functions for optimization problems. [9]

CHAPTER 3

PROBLEM FORMULATION

This section describes the details of modelling the hydropower station and building the optimization problem for the next scheduling period. When creating the optimization problem, it is necessary to identify all controllable and independent variables and their boundaries/ constraints to define the objective function. Because this case study has focused on modelling the Moragahakanda power station, the maximization of the total power generation has been considered the objective function according to the research problem. Furthermore, this section provides details about obtaining the operational and environmental boundaries/ constraints for variables of power generation and water discharge flow rate.

3.1 Optimization modelling

As the first step, modelling the hydropower plant with all relevant variables under the actual condition is essential to identify and solve the optimization problem. It tries to achieve the maximum power generation output by finding the best unit combination as the optimum condition for small-term scheduling periods such as daily or weekly.

As mentioned in the introduction, even though the Moragahakanda hydropower station generates electrical power, it is mainly necessary to fulfil the downstream water requirements during the given period. Therefore, rather than most other power plants, this hydropower plant is independent of fulfilling the load demand of the utility. Thus, the only controllable variable for the Moragahakanda hydropower station is power generation output. Therefore, the main objective of this research is the economic dispatching of the hydropower plant by obtaining the maximum power generation, for every scheduled time duration, according to the water requirement of downstream areas.

The Moragahakanda Power station comprises two 5 MW and two 7.5 MW hydro turbine generator units. In the manufacturing stage, 5 MW units have been named G1A and G1B. And other 7.5 MW units have been denoted as G2 and G3, as shown in

Figure 3.1. These representations were used for this research study as well. Even though turbine-generator units with the same capacities have been constructed at the same elevation level, due to small variations in the manufacturing process and hydropower losses, adequate net head levels slightly differ from unit to unit. Hence, for modelling purposes, the net head levels were considered as h_1, h_2, h_3 and h_4 for units G1A, G1B, G2 and G3, respectively.

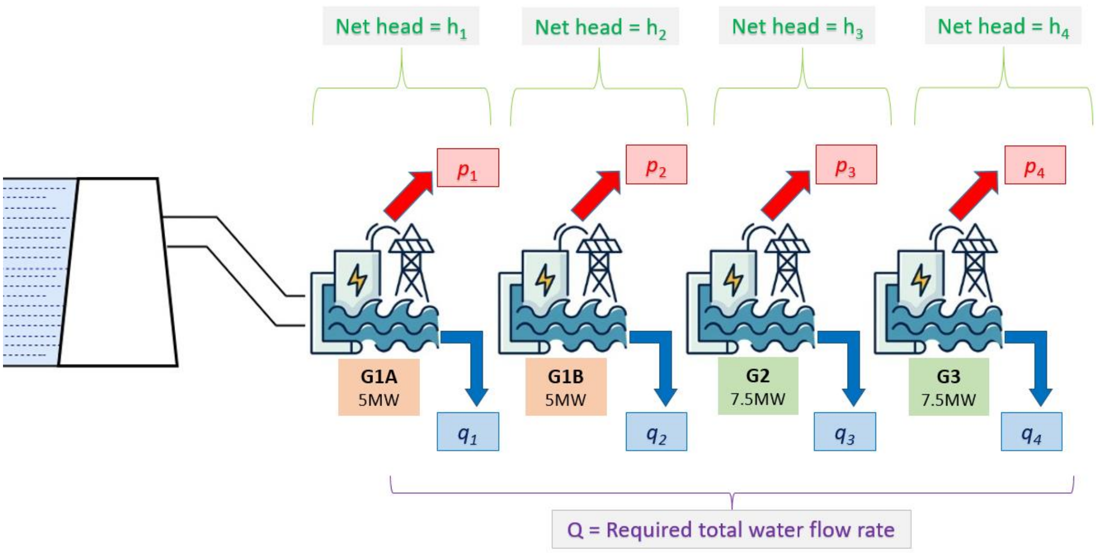

Figure 3.1: Modeling of the Moragahakanda power station

During a certain considered scheduling period, the power generation for the G1A machine was considered p_1 amount in MW then the water discharge flow rate through the turbine was considered q_1 m^3/s in this research work. Similarly, for other G1B, G2 and G3 units, power generation and water discharge flow rates have been considered p_2, p_3 and p_4 in MW, and as q_2, q_3, and q_4 in m^3/s respectively at that scheduling period.

Nevertheless, the water requirement of the downstream areas was considered Q m^3/s, and all water discharge flow rates from each unit should be fulfilled this given water requirement. Therefore, the total summation of water discharge flow rates should equal the Q m^3/s as represented in equation 18.

3.2 Notations

This optimization modelling used the following notations to identify all variables for this research study.

- p_1 - Power generation value for unit G1A at the head level of h_{j1}, with q_1 water discharge;
- p_2 - Power generation value for unit G1B at the head level of h_{j2}, with q_2 water discharge;
- p_3 - Power generation value for unit G2 at the head level of h_{j3}, with q_3 water discharge;
- p_4 - Power generation value for unit G3 at the head level of h_{j4}, with q_4 water discharge;
- p_{hjLi} – Minimum power generation of the i^{th} unit at the head level of h_j ;
- p_{hjUi} – Maximum power generation of the i^{th} unit at the head level of h_j ;
- q_{hjLi} – Minimum water discharge of the i^{th} unit at the head level of h_j ;
- q_{hjUi} – Maximum water discharge of the i^{th} unit at the head level of h_j ;
- Q - Total required downstream water discharge rate for the given time duration (m^3/s);
- a_{ihj} , b_{ihj} , c_{ihj} – Discharge coefficients of the i^{th} unit at the head level of h_j;

p_i – Power generation value in MW of the i^{th} unit;

q_i – Water discharge rate in m^3/s for the i^{th} unit.

Where $i = 1,2,3,4$ and if $i = 1,2$,h_j is in the range 24.6m to 45 m; if $i = 3,4$, h_j is in the range 21.7m to 42 m.

3.3 Objective function

The main objective of this study is to obtain the highest amount of total power generation as output by economically dispatching the four generator units of the power station. Therefore, the following equation can be obtained for the total power generation of the Moragahakanda power station.

$$Total\ power\ generation = (p_1 + p_2 + p_3 + p_4) \text{ -------------Eq (15)}$$

According to the above relationship, the objective function of the optimization problem for the Moragahakanda hydropower plant can be explained as follow for the next scheduling period under a given water requirement.

$$Maximizing\ total\ power\ generation = Max\ (p_1 + p_2\ + p_3\ +\ p_4)$$

$$=\ Max\ \sum_{i=1}^{4} p_i \text{ ----------------Eq (16)}$$

Therefore, this objective function consists of four variables, as p_1, p_2, p_3 and p_4.

3.4 Bounds and constraints

As mentioned in the literature review section, there are several operational boundaries and system or environmental constraints for considered variables that are included in an optimization problem under the actual solving conditions. Therefore, the boundaries and constraints can be represented below for the optimization problem of the Moragahakanda power station. They are,

Limitation of the power generation

$$p_{hjLi}\ \leq\ p_i(t) \leq\ p_{hjUi}$$

Limitation of the water discharge flow rate

$$q_{hjLi}\ \leq\ q_i(t) \leq\ q_{hjUi}$$

Hydro performance model for the power generation

$$p_i(t) =\ a_i q_i^2 +\ b_i q_i +\ c_i \text{ ------------------------Eq (17)}$$

Given the total water requirement of the downstream areas

$$Q\ \ =\ \ q_1 +\ q_2 +\ q_3 +\ q_4 \text{ ------------------------Eq (18)}$$

Hence, to solve this optimization problem, there are eight boundaries with one linear and four nonlinear equality constraints with zero inequality constraints.

3.5 Obtaining the boundaries and constraints

In the Moragahakanda power station, at each head level, different boundaries and constraints can be obtained for each unit. To identify the boundaries, basic running characteristic curves and operating limit curves for water discharge flow rates were used as described below section. These curves have been provided by the manufacturer of the turbine in the Moragahakanda power station.

3.5.1 Obtaining the power generation limits

The two running characteristic curves, supplied by the manufacturer of the turbine-generator units, were considered to obtain the operation boundaries of power generation at each level, for each Francis turbine-generator unit of the Moragahakanda power station.

As mentioned above, the one running characteristic curve, which is attached to appendix 1, has been supplied for 5MW capacity units (G1A and G1B), and another characteristic running curve which is attached to appendix 2, has been supplied for 7.5MW capacity units (G2 and G3). By using these curves, the variation of the efficiencies of the turbine unit can be obtained while power generation varies with the water head level of the unit. And also, the pink and red lines show the operating range of power generation at each head level without vibrations and mechanical instabilities. Therefore, this operational range was also used in identifying the boundaries at each head level for this research work.

As an example, at a certain scheduling period, if the effective net head of the unit G1A (5MW) is monitored as 36.5m in the SCADA system, then the power generation limitations can be obtained by using the 5MW running characteristic curve. According to this curve, the minimum and maximum power generations at 36.5m net head level for a 5MW unit are 2.3MW and 5.0MW, respectively. And if the net head level of the unit G2 (7.5MW) is considered 34m at a particular time, then the power generation limitations can be obtained using the 7.5MW running characteristic curve. According to this curve, the minimum and maximum power generations of 34m net head for 7.5MW units are 3.55MW and 7.5MW, respectively, as following Figure 3.2.

Therefore, these boundaries of power generation at the considered levels can be represented as follow.

$$2.30\ MW \leq p_1 \leq 5.0\ MW$$

$$3.55\ MW \leq p_3 \leq 7.5\ MW$$

Therefore, for this case, due to only one characteristic curve for the same capacity units, it was assumed that both the same capacity turbine generator units were identical.

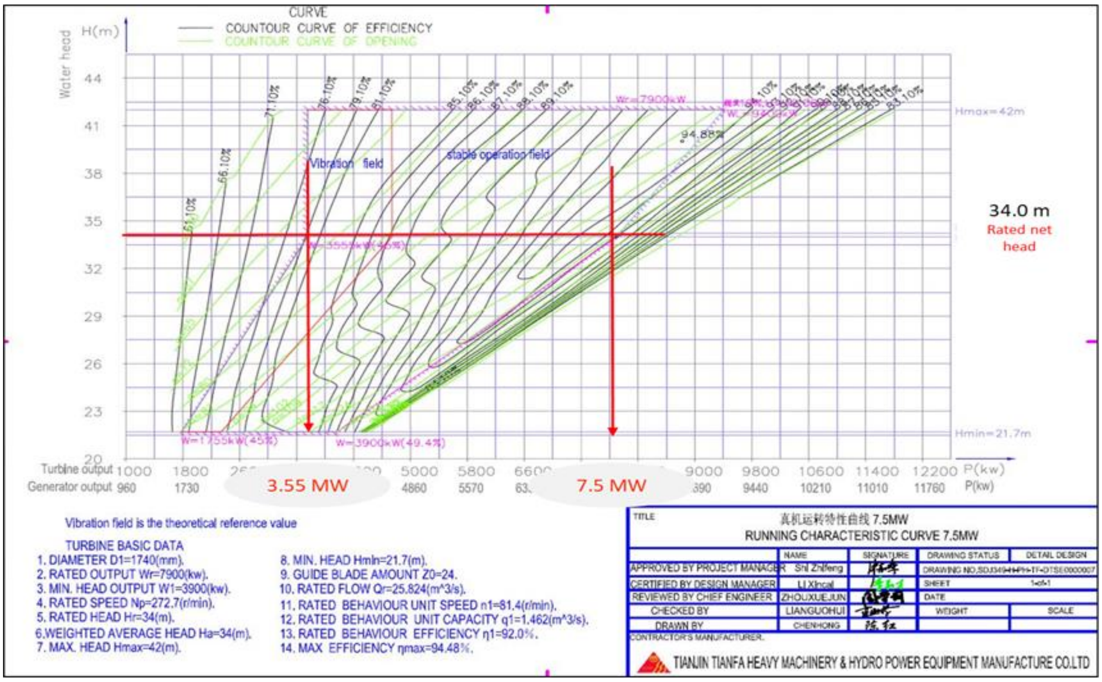

Figure 3.2: Obtaining the power generation limits from the characteristic curve of 7.5MW unit

3.5.2 Obtaining the water discharge flow rate limitations of the generation units

For this purpose, two operating limit curves, supplied by the turbine manufacturer of the Moragahakanda power station, were considered to obtain the boundaries of water discharge flow rates at each level for each turbine unit. Those are shown in Figure 3.3 and Figure 3.4.

These curves show the range of the water discharge flow rates through the turbine when the machines operate. The bounded area by the brown lines shows limitations at each water net head level. And also, due to the only one curve that has been supplied for both same capacity units, the same capacity units were considered identical to water discharge limitations.

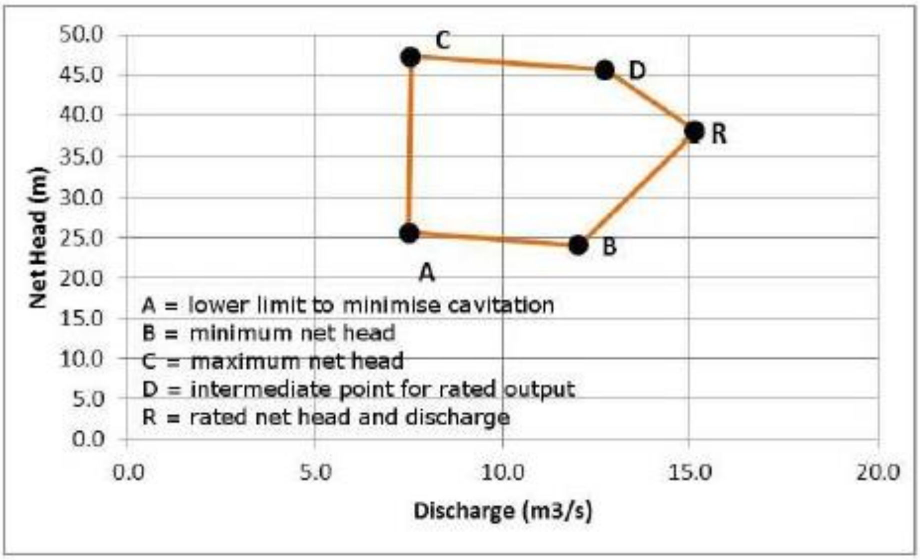

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Figure 3.3: Operating limits curve for Unit 5MW

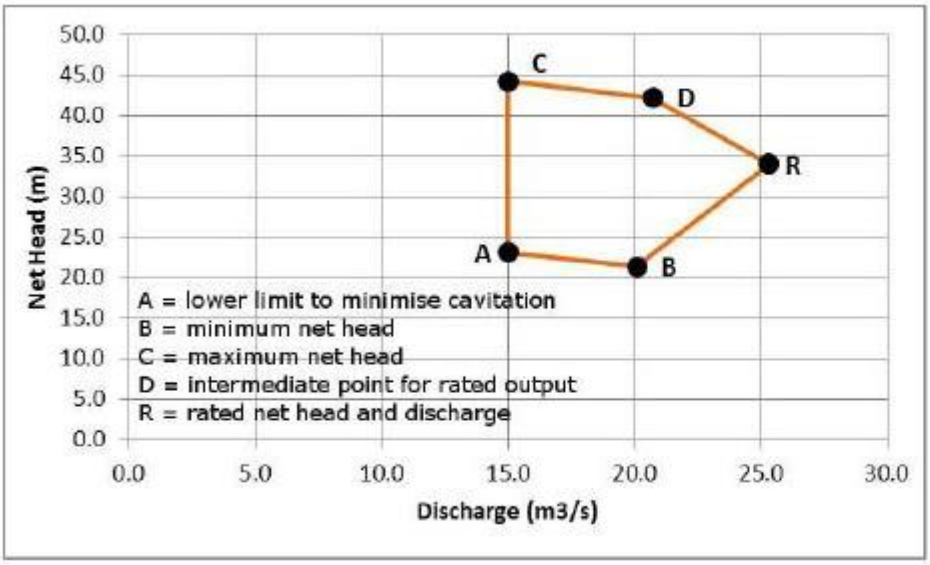

Figure 3.4: Operating limits curve for Unit 7.5MW

As an example, when the head level for the G1A unit (h_1) is 37m the minimum and maximum water discharge flow rates can be obtained as 7.5m^3/s and 15m^3/s, respectively. And also, when the head level for the G2 unit (*h2*) is 34m, the minimum and maximum water discharge flow rates can be obtained as 15m^3/s and 25m^3/s, respectively.

Therefore, these boundaries of the water discharge flow rates at the considered levels can be represented as follow.

$$7.5\text{m}^3/\text{s} \leq q_1 \leq 15\text{m}^3/\text{s}$$

$$15\text{m}^3/\text{s} \leq q_3 \leq 25\text{m}^3/\text{s}$$

3.5.3 Given total water discharge equality constraint

This is the main requirement of the Moragahakanda hydropower plant, fulfilling the downstream water requirement. The Water Management Secretariat estimates the downstream water requirement (Q) by considering the irrigation requirements of each scheme based on the hydrology, soil type, crop type, canal distribution system, and recent experience with water use efficiency.

Therefore, at a given time duration of the plant scheduling, the total flow rates of the water discharges throughout the turbine should equal the water requirement of the downstream areas. It can be represented in the following equation.

$$The\ total\ water\ requirement\ = Total\ water\ discharge\ of\ all\ units$$

$$Q \qquad = q_1 + q_2 + q_3 + q_4$$

Hence, this important linear equality constraint was also considered for building the optimization problem. The procedure for obtaining other nonlinear equality constraints for the hydro performance model of each unit has been described in the next chapter.

CHAPTER 4

HYDRO PERFORMANCE MODEL EQUATION FOR POWER GENERATION

As mentioned in the literature review, due to the variations in the head levels, it is difficult to obtain a relationship directly between the power generation and the water discharge flow rate for the turbine-generator units. Nevertheless, the hydropower losses also need to be considered to obtain a more realistic and accurate mathematical relationship under real conditions. Therefore, the above linear equation 01 cannot be used directly for this case.

Nevertheless, even though the manufacturer had provided two running characteristic curves for the Moragahakanda power plant, it is difficult to represent a clear mathematical relationship between power generation and water discharge flow rate at each head level. Therefore, as a solution, to get a more realistic mathematical relationship, a set of past historical data, which have been obtained from the past operations of the generator units, were collected and created into a database system.

4.1 Collecting a set of past operation historical data

Here, unit net head, power generation and water discharge flow rate data were collected from January 2018 to September 2020 according to each unit. The SCADA system implemented at the Moragahakanda power station was directly used for this purpose. By using this set of collected data, a database was created in Microsoft Excel software.

Using this collected data set, different head levels were selected with a range of power variations for each unit. Appendix 3 shows the selected set of data of power generation with water flow rate for each generator unit at the different operating net head levels.

4.2 Power generation vs. water discharge curves

Using these selected data sets, the variation of power generation vs water discharge flow rate curves were obtained for different heads level for units G1A, G1B, G2 and

G3, as shown in Figure 4.1, Figure 4.2, Figure 4.3 and Figure 4.4, respectively. In these figures, different colours indicate different head levels for each unit.

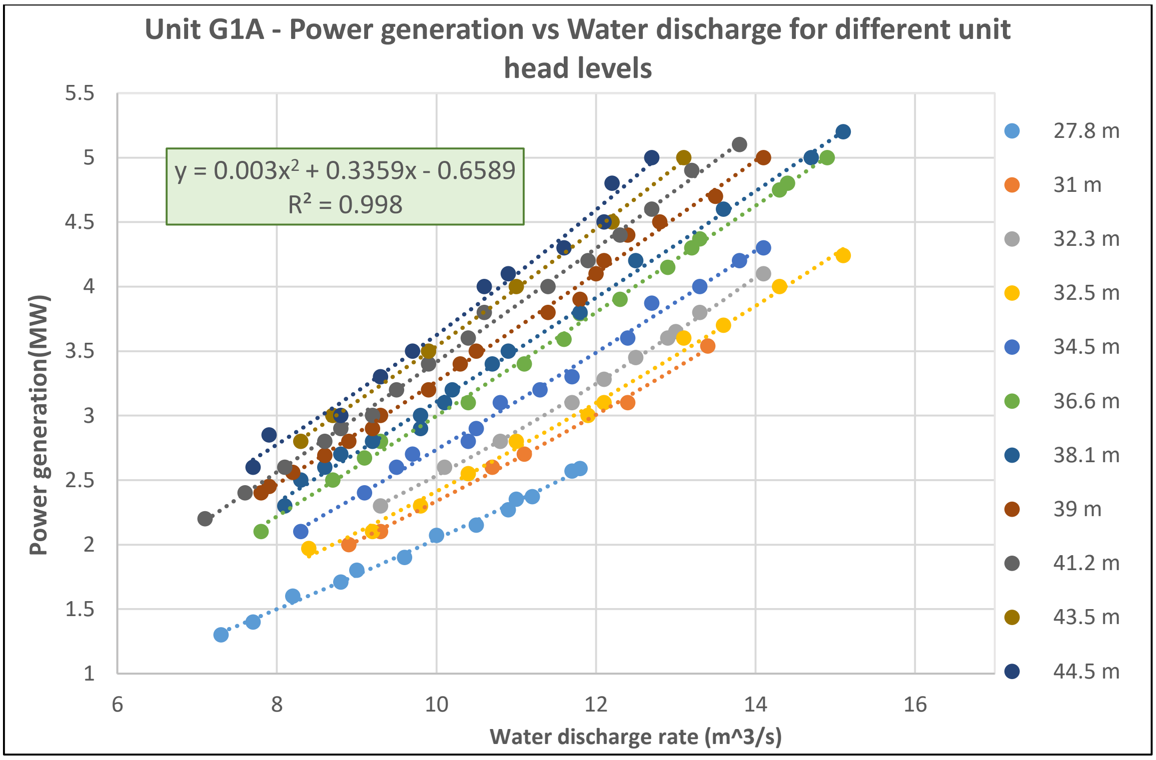

Figure 4.1: G1A-Power generation vs water discharge for different unit head levels

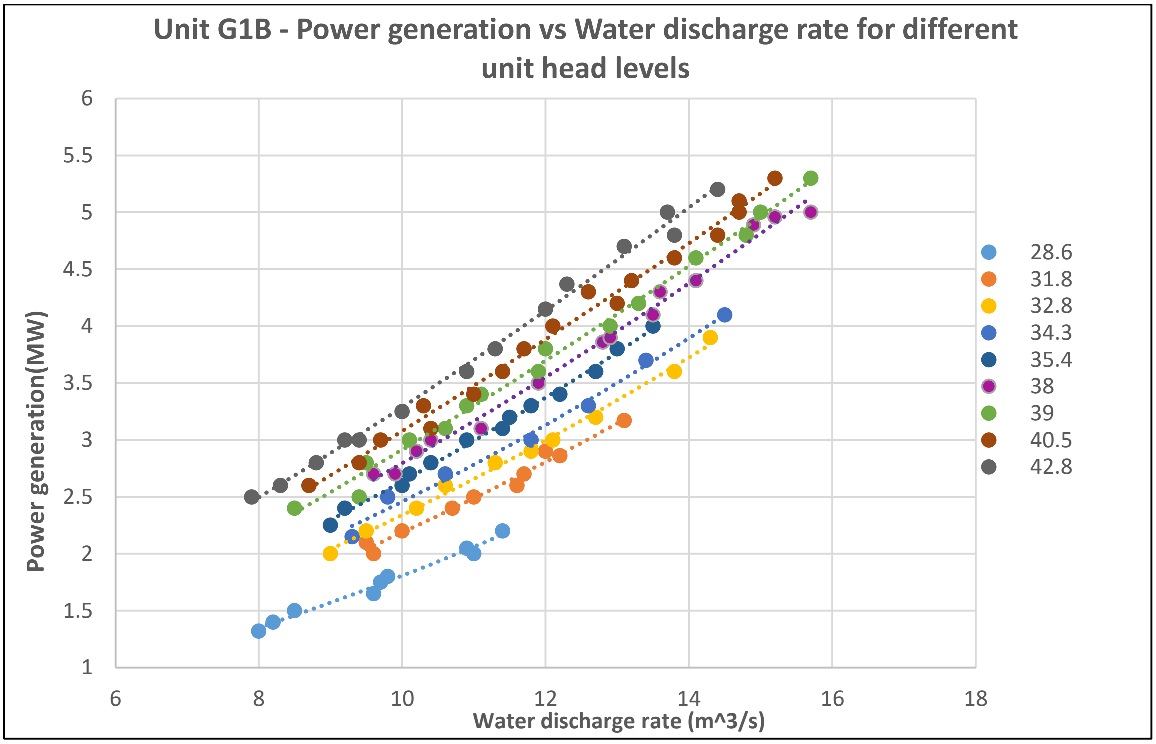

Figure 4.2: G1B-Power generation vs water discharge for different unit head levels

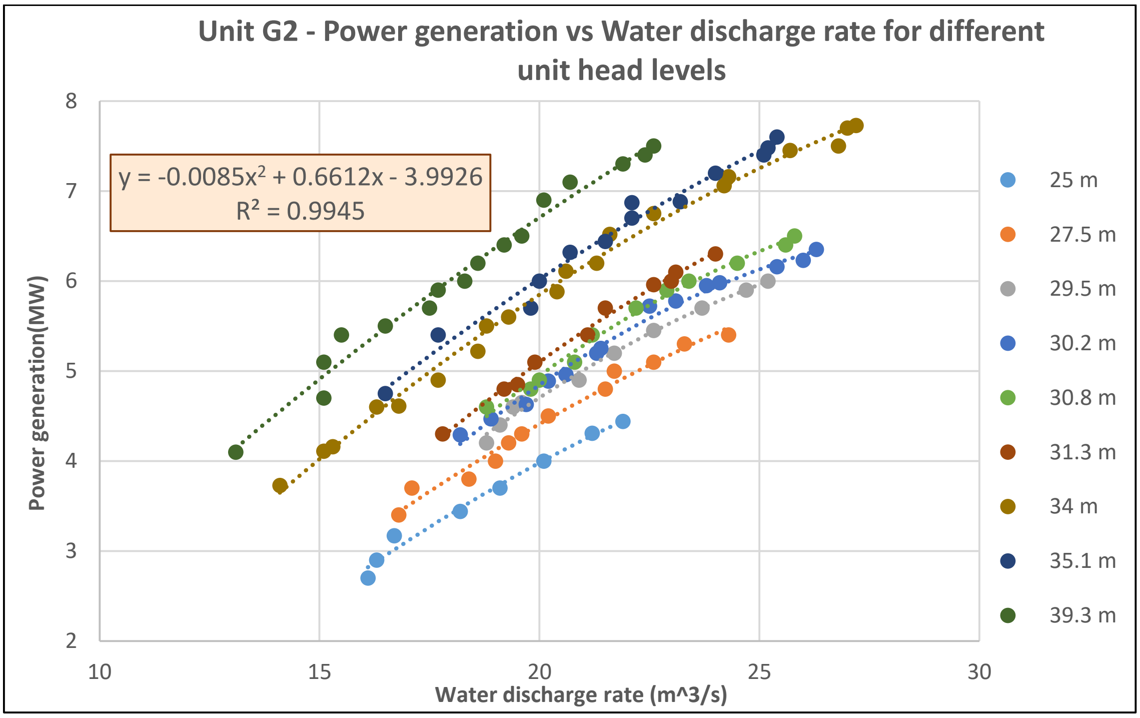

Figure 4.3: G2-Power generation vs water discharge for different unit head levels

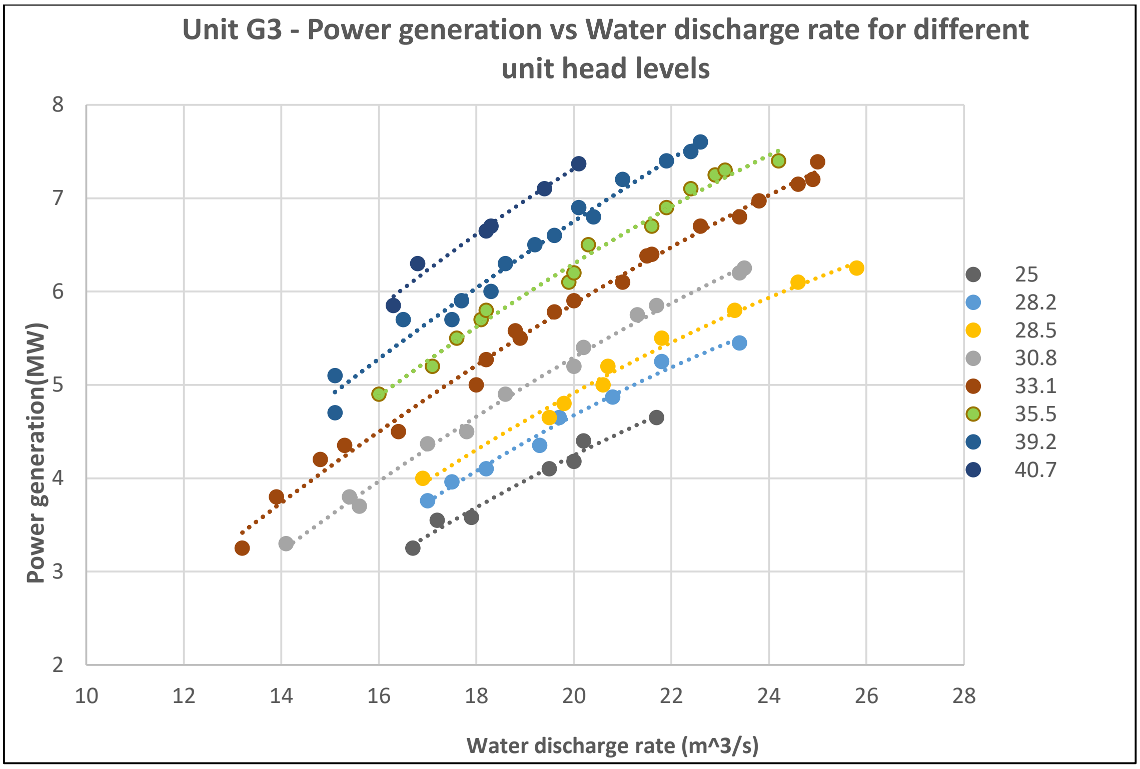

Figure 4.4: G3-Power generation vs water discharge for different unit head levels

For this research work, the 2nd order polynomial approximation was used to obtain the mathematical relationship between power generation and water discharge flow rate by considering hydropower losses as well, as described in the literature review. For each unit, it could be obtained around 0.99 R-square value for each water head level.

Even though the mathematical relationships between power generation and water discharge flow rate can be obtained from the drawn, selected discrete set of curves at one particular head level, the problem is the difficulty of obtaining the mathematical relationship between the two head levels directly. Nevertheless, the generator units can be operated at every operating head level within their minimum and maximum head levels.

Therefore, as a solution to this problem, a mathematical relationship of three-dimensional surface fitting was obtained for each unit using these data sets, considering three parameters; power generation, water discharge flow rate, and net head level.

4.3 Surface fittings

The MATLAB surface and curve fitting toolbox was used to obtain the 3D surface fitting for each unit. The window of the MATLAB surface fitting toolbox is shown in Figure 4.5.

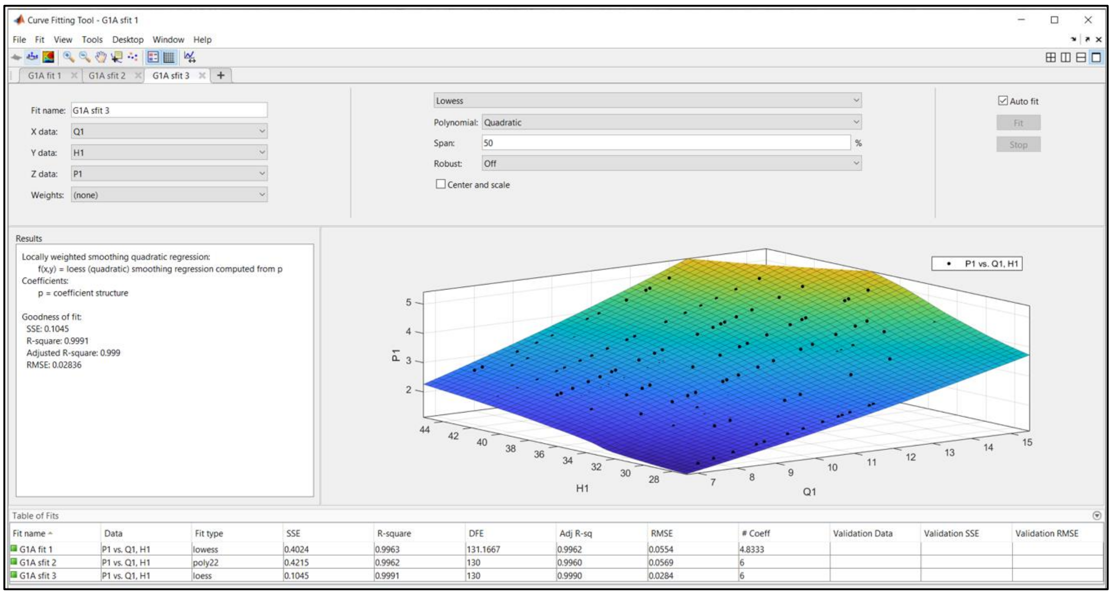

Figure 4.5: MATLAB surface and curve-fitting toolbox

Due to the desired mathematical relationships between the variables of power generation and water discharge flow rate, the water discharge flow rate value was used for the x-axis, for the y-axis, the net unit head of the unit was used and the power generation variable was used for the z-axis for obtaining the 3D surface.

In MATLAB curve fitting software, they have provided several surface smoothing functions. Therefore, to find out the goodness of the suitable surface fitting for this research study, a comparison was made among the Lowess quadratic function with 50% span, the Lowess quadratic function with 70% span and the Polynomial second quadratic function (x^2, y^2) as shown below the Table 4.1.

Table 4.1: Comparison of the goodness of the 3D surfaces of MATLAB surface fitting

Unit	Lowess Quad. with 50%	Lowess Quad. with 70%	Polynomial (x^2,y^2)
G1A	SSE: 0.1847 R-square: 0.9983	SSE: 0.2244 R-square: 0.9980	SSE: 0.4033 R-square: 0.9963
G1B	SSE: 0.4040 R-square: 0.9960	SSE: 0.6066 R-square: 0.9940	SSE: 1.2939 R-square: 0.9873
G2	SSE: 0.04079 R-square: 0.9997	SSE: 0.1051 R-square: 0.9993	SSE: 0.7812 R-square: 0.9947
G3	SSE: 0.2375 R-square: 0.9981	SSE: 0.4562 R-square: 0.9964	SSE: 0.7611 R-square: 0.9939

Therefore, the R square values for all fittings have been obtained at around 0.99. And also, because the selected approximation for the mathematical relationship between power generation and water discharge flow rate was the 2^{nd} order polynomial function, the 2^{nd} quadratic polynomial function was selected for obtaining the 3D surface fittings for each unit in this research work. The second-order polynomial relationships for 3D surfaces for each generator unit are shown in the following Figure 4.6, Figure 4.7, Figure 4.8 and Figure 4.9 for units G1A, G1B, G2 and G3 respectively.

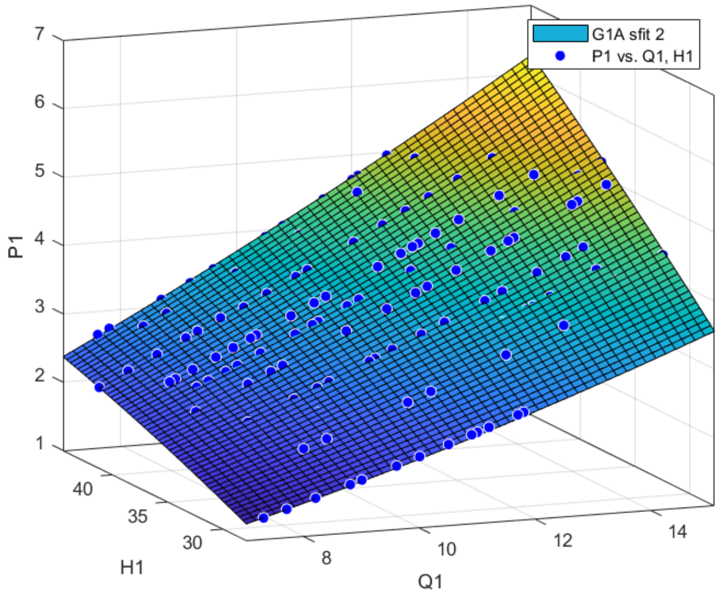

Figure 4.6: G1A surface fitting; P1: Power generation in MW, Q1: Water discharge flow rate in m^3/s, H1: Unit head in m

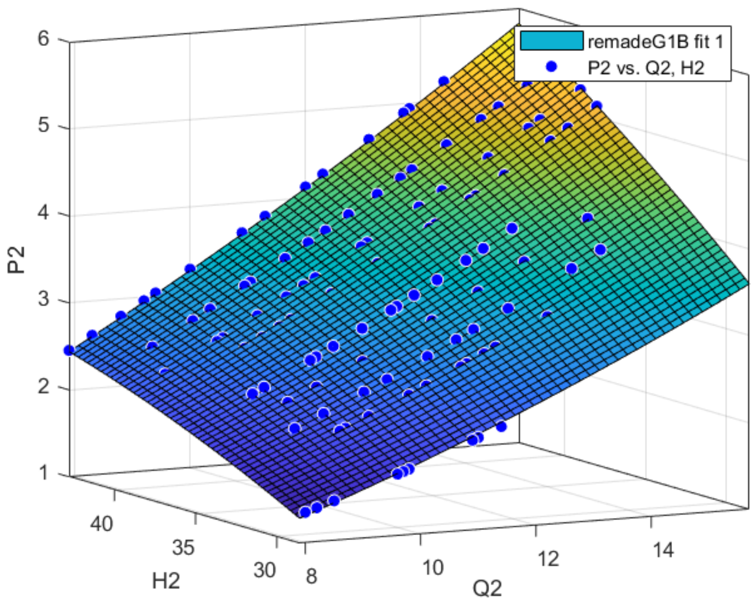

Figure 4.7: G1B surface fitting; P2: Power generation in MW, Q2: Water discharge flow rate in m^3/s, H2: Unit head in m

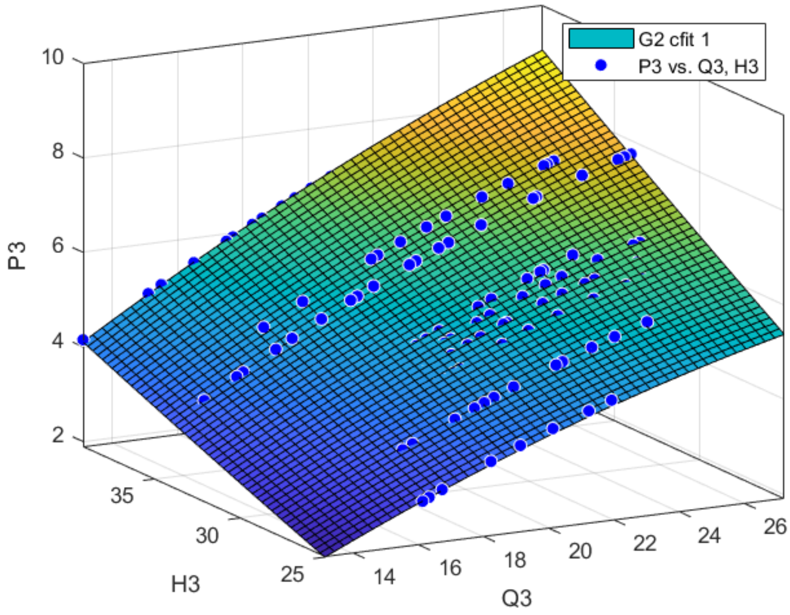

Figure 4.8: G2 surface fitting; P3: Power generation in MW, Q3: Water discharge flow rate in m^3/s, H3: Unit head in m

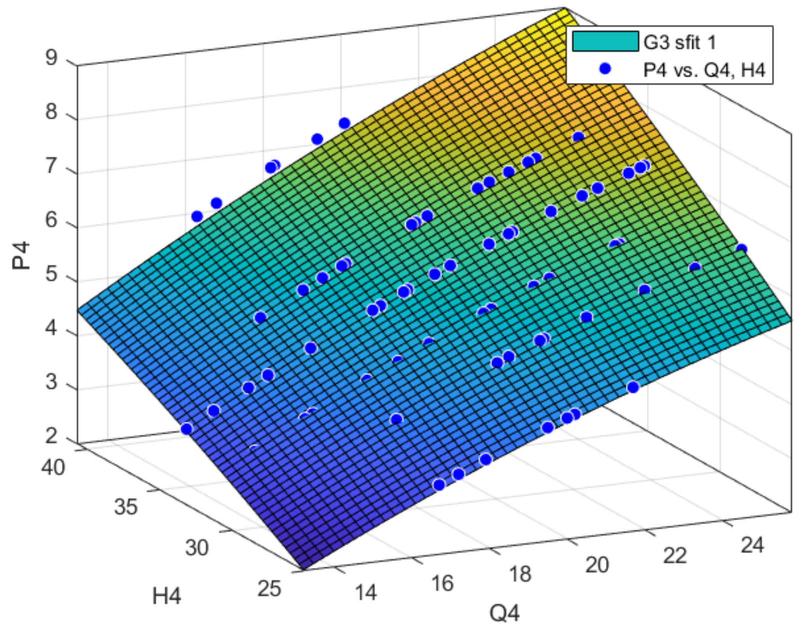

Figure 4.9: G3 surface fitting; P4: Power generation in MW, Q4: Water discharge flow rate in m^3/s, H4: Unit head in m

CHAPTER 5

MATLAB OPTIMIZATION

This research used MATLAB software with the R2020a version to solve this optimization problem. To solve optimization problems, inbuilt algorithm functions were created in MATLAB software to solve linear, nonlinear and integer problems. Therefore, this software can be used to create the necessary optimization problem, identify the variables and boundaries, apply the constraints, and obtain accurate results in a short-term period

After creating the optimization problem, the MATLAB software selects the suitable solving inbuilt algorithm automatically by itself. Because of the objective function's nonlinear characteristics and constraints, the Fmincon algorithm was used to solve this problem.

Here problem-based nonlinear programming was implemented to solve the problem. Hence following steps were used for creating MATLAB codes.

- Create the optimization problem
- Define the variables of the problem
- Introduce the constraints and boundaries
- Create the objective function
- Solve the problem

5.1 Possibilities of the dispatching of the units

In the Moragahakanda power station, if all generator units are available for operations, there are fifteen possibilities which can be operated as different combinations. It can be shown in the following Table 5.1.

Hence, as an example, if the required water discharge flow rate is 52 m^3/s, according to the following table, six combinations from Case 11 to Case 15 of plant dispatch are needed to be considered. Because the maximum water discharge flow rate is varying with the net water head level, the number of considerable possibilities is different with the given required water discharge and the net head of each unit at that scheduling period.

Table 5.1: Different possibilities of dispatching the plant with total water discharge amount

	Unit G1A (15 m3/s)	Unit G1B (15 m3/s)	Unit G2 (25 m3/s)	Unit G3 (25 m3/s)	Total maximum water discharge	
Case 1	✓	×	×	×	15m3/s	
Case 2	×	✓	×	×	15m3/s	≤ 15m3/s
Case 3	×	×	✓	×	25m3/s	
Case 4	×	×	×	✓	25m3/s	≤ 25m3/s
Case 5	✓	✓	×	×	30m3/s	≤ 30m3/s
Case 6	✓	×	✓	×	40m3/s	
Case 7	✓	×	×	✓	40m3/s	
Case 8	×	✓	✓	×	40m3/s	
Case 9	×	✓	×	✓	40m3/s	≤ 40m3/s
Case 10	×	×	✓	✓	50m3/s	≤ 50m3/s
Case 11	✓	✓	✓	×	55m3/s	
Case 12	✓	✓	×	✓	55m3/s	≤ 55m3/s
Case 13	✓	×	✓	✓	65m3/s	
Case 14	×	✓	✓	✓	65m3/s	≤ 65m3/s
Case 15	✓	✓	✓	✓	80m3/s	≤ 80m3/s

5.2 Results and analysis

The proposed technique was tested for two applications which were applied to the Moragahakanda power station for economically dispatching the machines. These two applications present the conditions of previous operations with desired irrigational water requirements and certain time durations. For this case, two scheduling periods were selected, previously, machines have been operated, to test for the economical dispatch of the Moragahakanda power station. Those dates are the 21st of March, 2018 and the 13th of July, 2019 and each case is described in the next section.

5.3 Application 01

The following details have been considered for the first example.

Water requirement : 56.5 m^3/s

Time duration : 21/ 03/ 2018 – 31/03/ 2018 (10 days)

The water requirement of the downstream area was 56.5 m^3/s and this requirement was continued within ten days. At the beginning of this scheduling period, the head levels were obtained by the SCADA system for the G1A unit as 37.58 m, for the G1B unit as 37.28 m, for the G2 unit as 33.70 m and for the G3 unit as 33.19 m. In this research study, the variations of the net head levels during the scheduling period were not considered as mentioned in the literature review for short-term scheduling periods. Therefore, this scheduling problem can be rearranged as below.

The net head level of the G1A unit (h_1)	=	37.58 m,
The net head level of the G1B unit (h_2)	=	37.28 m,
The net head level of the G2 unit (h_3)	=	33.70 m,
The head level of the G3 unit (h_4)	=	33.19 m.

At those net head levels, the minimum and maximum power generations according to the characteristic curves are,

$$2.3 \leq p_1 \leq 5.21\ (MW)$$

$$2.3 \leq p_2 \leq 5.15\ (MW)$$

$$3.51 \leq p_3 \leq 7.43\ (MW)$$

$$3.43 \leq p_4 \leq 7.6\ (MW)$$

By MATLAB surface fittings: the relationship between the power generation and water discharge at a particular head level, can be obtained as follow. (Nonlinear relationship)

$$p_1 = (-\,0.5865) + (-0.1788)\,q_1 + 0.03882\,h_1 + 0.006121 q_1^2 + 0.01178\ q_1 h_1 + (-0.0007517\,)h_1^2$$

$$p_2 = (-7.25) + (-0.3901\ )\,q_2 + 0.4711\,h_2 + 0.008771 q_2^2 + 0.0.01493\,q_2 h_2 + (-0.007055\,)h_2^2$$

$$p_3 = (-4.235) + 0.284\ q_3 + 0.02497\,h_3 + (-0.00546)\,q_3^2 + 0.007298\ \ q_3 h_3 + (0.0005395)\ h_3^2$$

$$p_4 = (-4.976) + 0.4302\ q_4 + 0.02469\ h_4 + (-0.006768)\ q_4^2 + 0.00497\ q_4 h_4 + (0.0004279)h_4^2$$

At that head levels, the minimum and maximum water discharge according to the characteristic curves are,

$$7.5 \leq q_1 \leq 14.73\ (m^3/s)$$

$$7.5 \leq q_2 \leq 14.42\ (m^3/s)$$

$$15 \leq q_3 \leq 24.88\ (m^3/s)$$

$$15 \leq q_4 \leq 24.67\ (m^3/s)$$

For this problem, the possibilities can be obtained in Table 5.2. Therefore, only three cases are needed to be checked for solutions, Case 13 to Case 15.

Table 5.2: Total maximum water discharges for each case of plant dispatching relevant to the application 1

	Unit G1A	Unit G1B	Unit G2	Unit G3	Total maximum water discharge (m^3/s)
Case 1	✓	×	×	×	14.73
Case 2	×	✓	×	×	14.42
Case 3	×	×	✓	×	24.88
Case 4	×	×	×	✓	24.69
Case 5	✓	✓	×	×	29.15
Case 6	✓	×	✓	×	39.61
Case 7	✓	×	×	✓	39.42
Case 8	×	✓	✓	×	39.3

Case 9	×	✓	×	✓	39.11
Case 10	×	×	✓	✓	49.57
Case 11	✓	✓	✓	×	54.03
Case 12	✓	✓	×	✓	53.84
Case 13	✓	×	✓	✓	64.3
Case 14	×	✓	✓	✓	63.99
Case 15	✓	✓	✓	✓	78.72

Considering Case 15,

The total water requirement (Q) is 56.5 m^3/s.

$$56.5 = q_1 + q_2 + q_3 + q_4$$

The summarized results are shown in Table 5.3, after solving this problem using MATLAB software.

Table 5.3: Solutions of six dispatchings according to application 1

Case	G1A	G1B	G2	G3	Total power Gen. MW	Solution	Power (MW)	Discharge (m^3/s)
Case 10	×	×	✓	✓	14.63	No Feasible Point Found	p_3: 7.43 p_4: 7.20	q_3: 24.24 q_4: 22.47
Case 11	✓	✓	✓	×	17.20	No Feasible Point Found	p_1: 5.10 p_2: 4.66 p_3: 7.43	q_1: 14.73 q_2: 14.42 q_3: 24.07

Case 12	✓	✓	×	✓	16.84	No Feasible Point Found	p_1: 5.15 p_2: 4.49 p_4: 7.20	q_1: 14.73 q_2: 14.42 q_4: 21.89
Case 13	✓	×	✓	✓	18.68	Optimal Solution	p_1: 5.04 p_3: 6.85 p_4: 6.79	q_1: 14.73 q_3: 21.92 q_4: 19.85
Case 14	×	✓	✓	✓	18.68	Optimal Solution	p_2: 4.95 p_3: 6.90 p_4: 6.83	q_2: 14.42 q_3: 22.09 q_4: 19.99
Case 15	✓	✓	✓	✓	18.99	Optimal Solution	p_1: 3.94 p_2: 4.95 p_3: 4.75 p_4: 5.35	q_1: 12.08 q_2: 14.42 q_3: 15.00 q_4: 15.00

As shown above, the maximum total power output can be obtained from Case 15 as 18.99 MW. Therefore, the plant dispatch for this outcome should be as follow.

Operated power of Unit G1A = 3.94 MW

Operated power of Unit G1B = 4.95 MW

Operated power of Unit G2 = 4.75 MW

Operated power of Unit G3 = 5.35 MW

Therefore, if the plant operates as the above dispatching schedule with these power outputs, it can obtain the optimum power generation from the power station by fulfilling the objective of this research.

5.4 Application 02

The following details have been considered for the second example.

Water requirement	: 49.4 m^3/s
Time duration	: 13/ 07/ 2019 – 20/ 07/ 2019

The water requirement of downstream was 49.4 m^3/s. This requirement needed to be continued within one week. At that time, the head levels can be obtained by the SCADA system as for the G1A unit as 29.5 m, for the G1B unit as 30.4 m, for the G2 unit as 26.57 m and for the G3 unit as 25.9 m. Therefore, this scheduling problem can be rearranged as below.

The net head level of the G1A unit (h_1)	=	29.5 m;
The net head level of the G1B unit (h_2)	=	30.4 m;
The net head level of the G2 unit (h_3)	=	26.57 m;
The net head level of the G3 unit (h_4)	=	25.9 m.

At those head levels, the minimum and maximum power generations according to the characteristic curves are,

$$1.63 \leq p_1 \leq 3.64\ (MW)$$

$$1.72 \leq p_2 \leq 3.81\ (MW)$$

$$2.47 \leq p_3 \leq 5.33\ (MW)$$

$$2.37 \leq p_4 \leq 5.13\ (MW)$$

By MATLAB surface fittings: the relationship between the power generation and water discharge at a particular head level, can be obtained as follow. (Nonlinear relationship)

$$p_1 = (-\,0.5865) + (-0.1788)\,q_1 + 0.03882\,h_1 + 0.006121 q_1^2 \\ + 0.01178\ q_1 h_1 + (-0.0007517\,) h_1^2$$

$$p_2 = (-7.25) + (-0.3901) q_2 + 0.4711 h_2 + 0.008771 q_2^2 + 0.0.01493 q_2 h_2 + (-0.007055) h_2^2$$

$$p_3 = (-4.235) + 0.284 q_3 + 0.02497 h_3 + (-0.00546) q_3^2 + 0.007298 q_3 h_3 + (0.0005395) h_3^2$$

$$p_4 = (-4.976) + 0.4302 q_4 + 0.02469 h_4 + (-0.006768) q_4^2 + 0.00497 q_4 h_4 + (0.0004279) h_4^2$$

At that head levels, the minimum and maximum water discharge according to the characteristic curves are,

$$7.5 \leq q_1 \leq 13.5 \ (m^3/s)$$

$$7.5 \leq q_2 \leq 13.2 \ (m^3/s)$$

$$15 \leq q_3 \leq 22.14 \ (m^3/s)$$

$$15 \leq q_4 \leq 21.88 \ (m^3/s)$$

For this problem, the possibilities can be obtained as following Table 5.4. Therefore, only three cases are needed to be checked for solutions, Case 13 to Case 15.

Table 5.4: Total maximum water discharges for each case of plant dispatching relevant to the application 2

	Unit G1A	Unit G1B	Unit G2	Unit G3	Total maximum water discharge (m^3/s)
Case 1	✓	×	×	×	13.5
Case 2	×	✓	×	×	13.2
Case 3	×	×	✓	×	21.8
Case 4	×	×	×	✓	22.0

Case 5	✓	✓	×	×	26.7
Case 6	✓	×	✓	×	35.3
Case 7	✓	×	×	✓	35.5
Case 8	×	✓	✓	×	35.0
Case 9	×	✓	×	✓	35.2
Case 10	×	×	✓	✓	43.8
Case 11	✓	✓	✓	×	48.5
Case 12	✓	✓	×	✓	48.7
Case 13	✓	×	✓	✓	57.3
Case 14	×	✓	✓	✓	57.0
Case 15	✓	✓	✓	✓	70.5

Considering Case 15,

The total water requirement (Q) is 49.4 m^3/s.

$$49.4 = q_1 + q_2 + q_3 + q_4$$

The summarized results are shown in Table 5.5, after solving this problem using MATLAB software.

Table 5.5: Solutions of six dispatchings according to the application 2

Case	G1A	G1B	G2	G3	Total power Gen. MW	Solution	Power (MW)	Discharge (m3/s)
Case 10	×	×	✓	✓	10.14	No Feasible Point Found	p_3: 5.00 p_4: 5.14	q_3: 22.14 q_4: 21.67
Case 11	✓	✓	✓	×	10.99	No Feasible Point Found	p_1: 3.27 p_2: 2.97 p_3: 4.76	q_1: 13.40 q_2: 13.42 q_3: 22.14
Case 12	✓	✓	×	✓	11.23	No Feasible Point Found	p_1: 3.27 p_2: 2.96 p_4: 5.00	q_1: 13.40 q_2: 13.42 q_4: 21.88
Case 13	✓	×	✓	✓	10.76	Optimal Solution	p_1: 3.28 p_3: 3.24 p_4: 4.24	q_1: 13.40 q_3: 16.61 q_4: 19.38
Case 14	×	✓	✓	✓	10.59	Optimal Solution	p_2: 1.71 p_3: 4.02 p_4: 4.86	q_2: 8.44 q_3: 19.36 q_4: 21.59
Case 15	✓	✓	✓	✓	9.90	Optimal Solution	p_1: 1.64 p_2: 1.71 p_3: 2.75 p_4: 3.80	q_1: 8.00 q_2: 8.44 q_3: 15.00 q_4: 17.94

As per the above table, the maximum total power output can be obtained from Case 13 as 10.8 MW. The plant dispatch for this outcome should be as follow.

Operated power of Unit G1A	=	3.28 MW
Operated power of Unit G1B	=	No operation
Operated power of Unit G2	=	3.24 MW
Operated power of Unit G3	=	4.24 MW

Therefore, if the plant operates as the above dispatching schedule with these power outputs, it can obtain the optimum power generation from the power station by fulfilling the objective of this research.

5.5 Validation

To validate, the proposed technique's purpose as acceptable, the actual previously set of data of the relevant scheduling time durations have been considered from the SCADA system of the Moragahakanda power station. The actual operating information at the beginning of the relevant scheduling period is represented in Table 5.6, Table 5.7 and Table 5.8 show the details of the solutions of the proposed technique with maximum total power generation with and without Unit 1A for application 01.

Table 5.6: Actual operating data of 21/03/2018 at 08:00hr from the SCADA system

	G1A	G1B	G2	G3	Total power Gen. MW
Power (MW)	×	5.06	5.58	7.53	18.17
Discharge(m^3/s)	×	14.84	18.76	22.9	56.50

Table 5.7: Best solution from the proposed optimization method for application 1

	G1A	G1B	G2	G3	Total power Gen. MW
Power (MW)	3.94	4.95	4.75	5.35	18.99
Discharge(m^3/s)	12.08	14.42	15.00	15.00	56.50

Table 5.8: Best solution from the proposed optimization method for application 1 without G1A

	G1A	G1B	G2	G3	Total power Gen. MW
Power (MW)	×	4.95	6.90	6.83	18.68
Discharge(m^3/s)	×	14.73	22.09	19.99	56.50

Hence, by comparing Table 5.6 and Table 5.7, a maximum total power generation amount has been obtained from the proposed scheduling method rather than the actual dispatching schedule relevant to the date of 21/03/2018. Nevertheless, suppose there was any pre-scheduled maintenance work on the mentioned date at Unit G1A. In that case, then the plant can be dispatched as in Case 14 of the proposed method, which is represented in Table 5.8 and also can be seen the maximum power generation amount is still higher than the actual dispatching schedule.

For the second application, the actual operating information at the beginning of the relevant scheduling period is represented in Table 5.9, Table 5.10 and Table 5.11 show the details of the solutions of the proposed technique with maximum total power generation with and without Unit 1A.

Table 5.9: Actual operating data on 13/07/2019 at 06:00hr from the SCADA system

	G1A	G1B	G2	G3	Total power Gen. MW
Power (MW)	×	2.54	3.89	3.94	10.36
Discharge(m^3/s)	×	11.89	19.02	18.48	49.39

Table 5.10: Best solution from the proposed optimization method for application 2

	G1A	G1B	G2	G3	Total power Gen. MW
Power (MW)	3.28	×	3.24	4.24	10.76

Discharge(m^3/s)	13.40	×	16.61	19.38	49.39

Table 5.11: Best solution from the proposed optimization method for application 2 without G1A

	G1A	G1B	G2	G3	Total power Gen. MW
Power (MW)	×	1.71	4.02	4.86	10.59
Discharge(m^3/s)	×	8.44	19.36	21.59	49.39

Here also, by comparing Table 5.9 and Table 5.10, a maximum total power generation amount has been obtained from the proposed scheduling method rather than the actual dispatching schedule relevant to the date on 13/07/2019. Nevertheless, suppose there was any pre-scheduled maintenance work on the mentioned date at Unit G1A. In that case, the plant can be dispatched as in Case 14 of the proposed method, which is represented in Table 5.11 and it also can be seen the maximum power generation amount is still higher than the actual dispatching schedule.

The extra energy obtained by using the optimization model can be calculated as below.

Table 5.12: Calculation of the extra energy from the optimization model

	Extra generated power from the model (MW)	Scheduling period (Days)	Extra energy from the model (MWh)	Extra energy from the model (kWh)
Application 2	0.82	10	196.8	196800
Application 3	0.40	7	96.0	96000

According to Table 5.12, 196.8 MWh and 96 MWh extra amount of energy can be gained by using the developed optimization model for above considered application 1 and application 2 respectively.

Therefore, according to the above comparisons, the proposed method for dispatching the power plant satisfies the desired optimization requirement.

CONCLUSION AND RECOMMENDATIONS FOR FURTHER STUDIES

Hydropower plant building has been more focused in recent years due to the use of more renewable energies. Because of that, the usage of optimizing techniques is considerably high with the profitable power generation in the hydro plant. Therefore, several optimization techniques are manipulated like the classical economical dispatch method, efficiency measurement method, loss reduction method etc.

Most power plants follow the electricity demand of the utility, therefore, many research works are focused on minimizing the water consumption of the power generation by maximizing the plant efficiency.

For this research, the Moragahakanda hydropower plant has been considered as a case study and it operates by following the irrigational water requirements. Presently by using past experience and operation boundaries of two running characteristic curves, the scheduling of the generator operations is decided. Due to the two reasons of total water discharge flow rate from the plant should be fulfilled the downstream water requirement given by WMS, and the variable water head level, the only controllable variable for the Moragahakanda power station is the power generation output. Therefore, this research has considered maximizing power generation of the total output of the low head hydropower plant which follows the irrigation water requirement. Hence, this approach is not considered the plant efficiency directly but also considered the hydro performance relationships of power generation concerning the water discharge through the turbine, because the only controllable variable is the power generation output.

On the other hand, the 3D surface model allows obtaining the relationship of the hydro performance model between power generation and water discharge flow rate at each head level for each turbine-generator unit. Due to the second-order polynomial approximation and this nonlinear behaviour of the relationship, the reasonable amount of hydraulic losses of the power generation also has been taken into the account for this study.

As a final point, depending on these results, this optimization model is accomplished by allowing for the dispatching of the generator units economically, with the total maximum power generations, considering the available water heads, and satisfying the water requirement of downstream areas.

Furthermore, a Graphical User Interface (GUI) can be implemented as a more sophisticated and user-friendly method to enter and click for the optimal results, because searching for optimal results for each scheduling period is some kind of time-consuming procedure. In this case, the downstream water requirement and available water heads of each unit are entered as inputs by the user while finally the optimal dispatching of the units is obtained as output.

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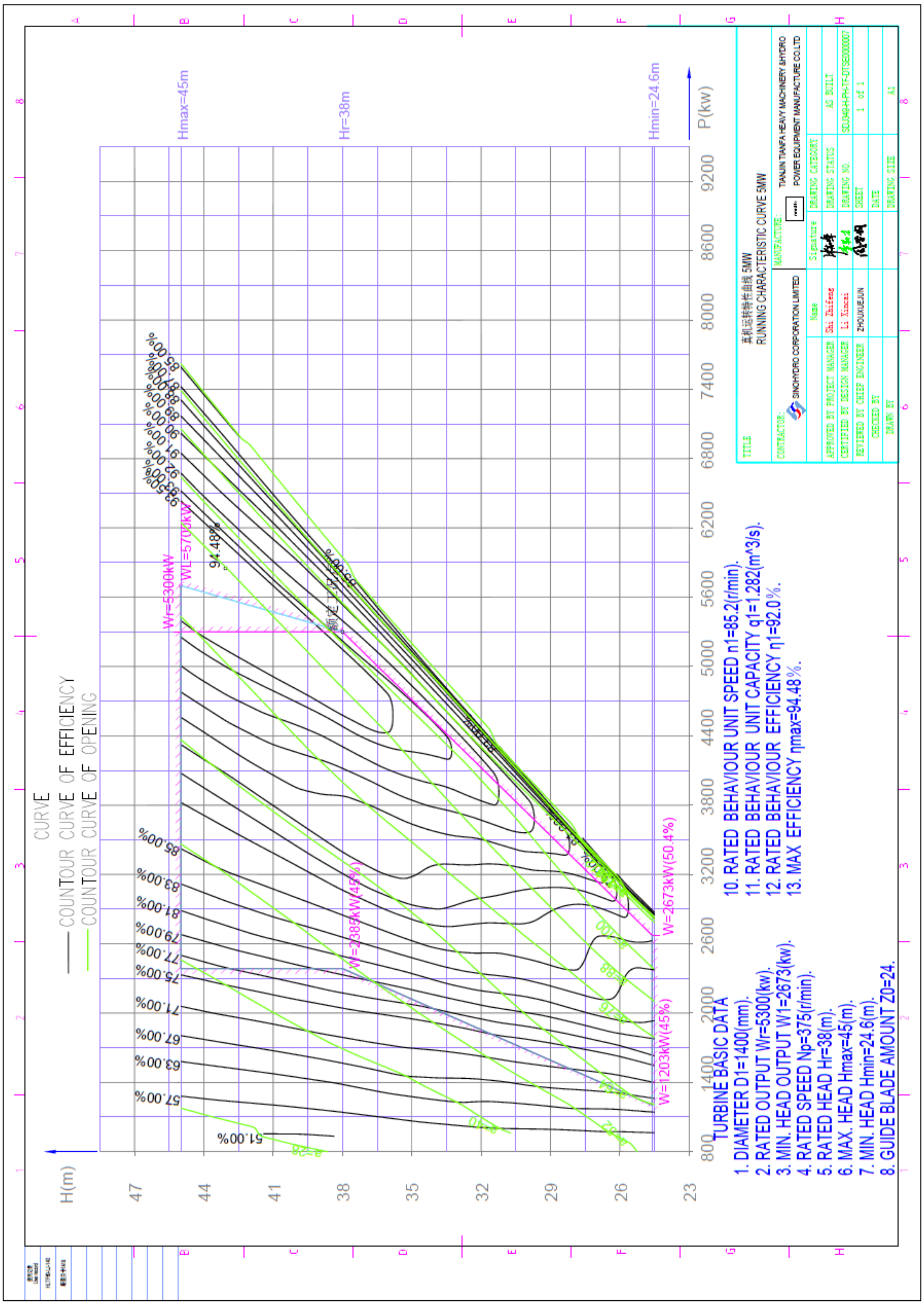
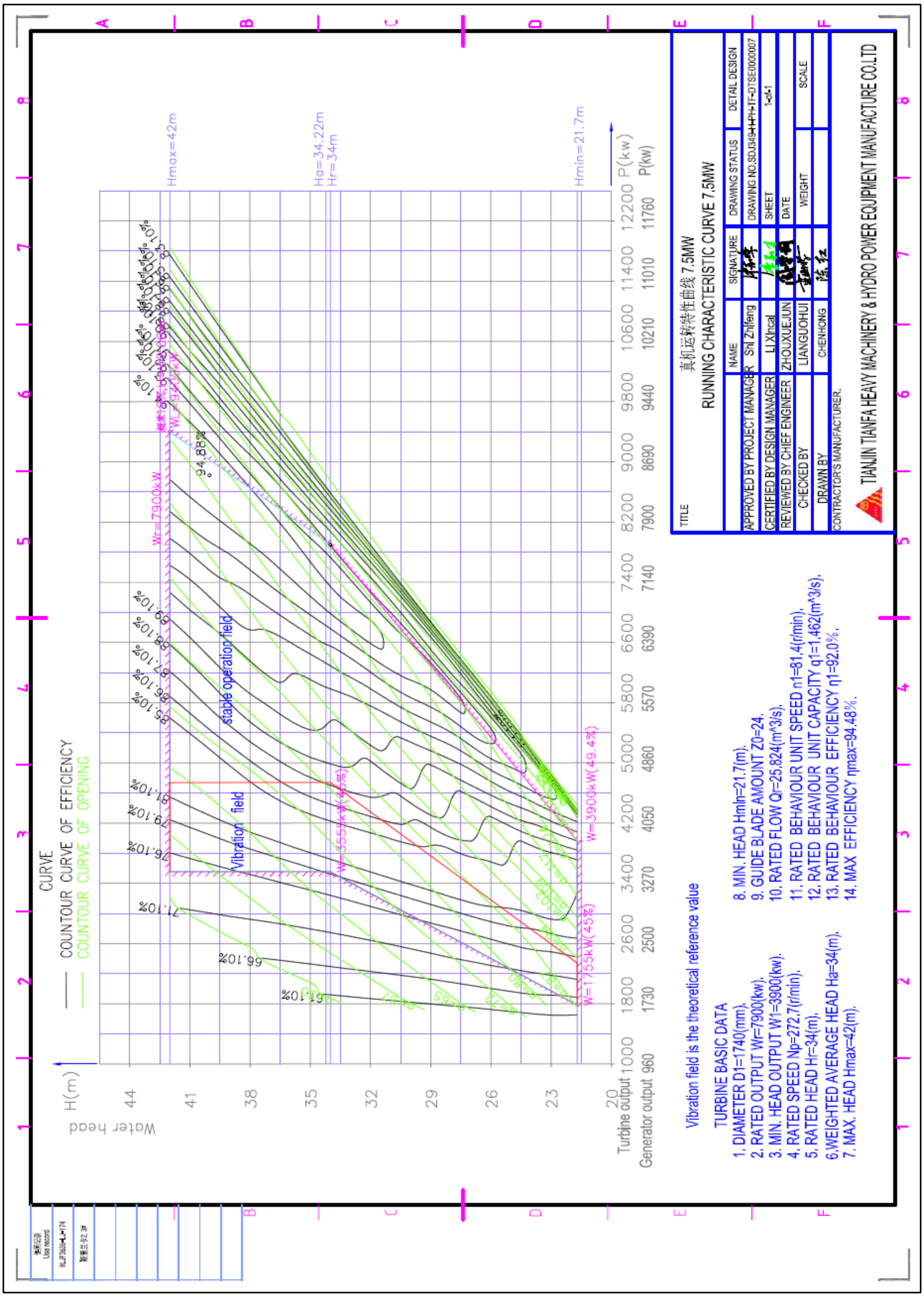
Appendix 1: Running characteristic curve for 5MW unit



Appendix 2: Running characteristic curve for 7.5MW unit



Appendix 3: Collected data set from the SCADA system

Initial Data Set - G1A

Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)
27.8	7.3	1.37	32.3			34.5	8.3	2.1	38.1	8.1	2.3	41.2	7.1	2.2	43.5	8.3	2.8
	7.7	1.45		9.3	2.3		9.1	2.4		8.3	2.5		7.6	2.4		8.7	3
	8.2	1.6					9.5	2.6		8.6	2.6		8.1	2.6		9.9	3.5
	8.8	1.71		10.1	2.6		9.7	2.7		8.8	2.7		8.6	2.8		11	4
	9	1.8		10.8	2.8		10.4	2.8		9.2	2.8		8.8	2.9		12.2	4.5
	9.6	1.87		11.7	3.1		10.5	2.9		9.8	2.9		9.2	3		13.1	5
	10	2.07		12.1	3.28		10.8	3.1		9.8	3		9.5	3.2			
	10.5	2.1		12.5	3.45		11.3	3.2		10.1	3.1		9.9	3.4			
	10.9	2.18		12.9	3.6		11.7	3.3		10.2	3.2		10.4	3.6			
	11	2.28		13	3.65		12.4	3.6		10.7	3.4		10.6	3.8			
	11.2	2.37		13.3	3.8		12.7	3.87		10.9	3.5		11.4	4			
	11.7	2.45		14.1	4.1		13.3	4		11.8	3.8		11.9	4.2			
	11.8	2.57					13.8	4.2		12.5	4.2		12.3	4.4			
							14.1	4.3		13.6	4.6		12.7	4.6			
										14.7	5		13.2	4.9			
										15.1	5.2		13.8	5.1			
31	8.9	2	32.5	8.4	1.97	36.6	7.8	2.1	39	7.8	2.4				44.5	7.7	2.6
	9.3	2.1		9.2	2.1					7.9	2.45					7.9	2.85
	10.7	2.6		9.8	2.3		8.7	2.5		8.2	2.56					8.8	3
	11.1	2.7		10.4	2.55		9.1	2.67		8.6	2.69					9.3	3.3
	12.4	3.1		11	2.8		9.3	2.8		8.9	2.8					9.7	3.5
	13.4	3.54		11.9	3		10.4	3.1		9.2	2.9					10.6	4
				12.1	3.1		11.1	3.4		9.3	3					10.9	4.1
				13.1	3.6		11.6	3.59		9.9	3.2					11.6	4.3
				13.6	3.7		11.8	3.79		10.3	3.4					12.1	4.5
				14.3	4		12.3	3.9		10.5	3.5					12.2	4.8
				15.1	4.24		12.9	4.15		11.4	3.8					12.7	5
							13.2	4.3		11.8	3.9						
							13.3	4.37		12	4.1						
							14.3	4.75		12.1	4.2						
							14.4	4.8		12.4	4.4						
							14.9	5		12.8	4.5						
										13.5	4.7						
										14.1	5						

Initial Data Set - G1B

Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)
28.6	8	1.32	31.8	9.5	2.1	38	9.6	2.7	39	8.5	2.4	40.5	8.7	2.6	42.8	7.9	2.5
	8.2	1.4		9.6	2		9.9	2.7		9.4	2.5		9.4	2.8		8.3	2.6
	8.5	1.5		10	2.2		10.2	2.9		9.5	2.8		9.7	3		8.8	2.8
	9.6	1.65		10.7	2.4		10.4	3		10.1	3		10.4	3.1		9.2	3
	9.7	1.75		11	2.5		11.1	3.1		10.6	3.1		10.3	3.3		9.4	3
	9.8	1.8		11.6	2.6		11.9	3.5		10.9	3.3		11	3.4		10	3.25
	10.9	2.05		11.7	2.7		12.8	3.86		11.1	3.4		11.4	3.6		10.9	3.6
	11	2		12	2.9		12.9	3.9		11.9	3.6		11.7	3.8		11.3	3.8
	11.4	2.2		12.2	2.86		13.5	4.1		12	3.8		12.1	4		12	4.15
				13.1	3.17		13.6	4.3		12.9	4		13	4.2		12.3	4.37
							14.1	4.4		13.3	4.2		12.6	4.3		13.1	4.7
							14.9	4.89		14.1	4.6		13.2	4.4		13.8	4.8
							15.2	4.96		14.8	4.8		13.8	4.6		13.7	5
							15.7	5		15	5		14.4	4.8		14.4	5.2
										15.7	5.3		14.7	5			
													14.7	5.1			
													15.2	5.3			
32.8	9	2	34.3	9.3	2.15	35.4	9	2.25									
	9.5	2.2		9.8	2.5		9.2	2.4									
	10.2	2.4		10.6	2.7		10	2.6									
	10.6	2.6		11.8	3		10.1	2.7									
	11.3	2.8		12.6	3.3		10.4	2.8									
	11.8	2.9		13.4	3.7		10.9	3									
	12.1	3		14.5	4.1		11.4	3.1									
	12.7	3.2					11.5	3.2									
	13.8	3.6					11.8	3.3									
	14.3	3.9					12.2	3.4									
							12.7	3.6									
							13	3.8									
							13.5	4									

Initial Data Set - G2

Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)
25	16.1	2.7	27.5	16.8	3.4	29.5	18.8	4.2	29.8	18.23	4.2	30.2	18.2	4.29	39.2	13.1	4.1
	16.3	2.9		17.1	3.7		19.1	4.4		19.05	4.4		18.9	4.47		15.1	5.1
	16.7	3.17		18.4	3.8		19.4	4.6		20.84	4.9		19.7	4.63		15.5	5.4
	18.2	3.44		19	4		19.6	4.65		24	5.8		20.2	4.89		16.5	5.5
	19.1	3.7		19.3	4.2		20.9	4.9		24.34	6.1		20.6	4.96		17.5	5.7
	20.1	4		19.6	4.3		21.7	5.2					21.3	5.2		17.7	5.9
	21.2	4.31		20.2	4.5		22.6	5.45					21.4	5.25		18.3	6
	21.9	4.44		21.5	4.8		23.7	5.7					22.5	5.72		18.6	6.2
				21.7	5		24.7	5.9					23.1	5.78		19.2	6.4
				22.6	5.1		25.2	6					23.8	5.95		19.6	6.5
				23.3	5.3								24.1	5.98		20.1	6.9
				24.3	5.4								25.4	6.16		20.7	7.1
													26	6.23		21.9	7.3
													26.3	6.35		22.4	7.4
																22.6	7.5

Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)
30.8	18.8	4.6	30.9	15.4	3.6	31.3	17.8	4.3	34	14.1	3.73	35.1	16.5	4.75
	19.8	4.8		16	3.8		19.2	4.8		15.1	4.11		17.7	5.4
	20	4.9		16.7	4		19.5	4.85		15.3	4.16		19.8	5.7
	20.8	5.1		17.3	4.3		19.9	5.1		16.3	4.6		20	6
	21.2	5.4		18.2	4.6		21.1	5.4		16.8	4.61		20.7	6.32
	22.2	5.7		19.1	4.8		21.5	5.7		17.7	4.9		21.5	6.44
	22.9	5.9		19.9	5		22.6	5.96		18.6	5.22		22.1	6.7
	23.4	6		21	5.3		23	6		18.8	5.5		22.1	6.87
	24.5	6.2		22.6	5.8		23.1	6.1		19.3	5.6		23.2	6.88
	25.6	6.4		23	6		24	6.3		20.4	5.88		24	7.2
	25.8	6.5		23.7	6.2					20.6	6.11		25.1	7.4
				25.3	6.4					21.3	6.2		25.2	7.48
										21.6	6.52		25.4	7.6
										22.6	6.75			
										24.2	7.06			
										24.3	7.16			
										25.7	7.45			
										26.8	7.5			
										27	7.7			

Initial Data Set - G3

Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)	Head (m)	Q (m^3/s)	P (MW)
28.2	17	3.76	30	17.8	4.33	30.8	14.1	3.3	40.7	16.3	5.85	33.1	13.2	3.25	25	16.7	3.25
	17.5	3.96		18.4	4.51		15.4	3.8		16.8	6.3		13.9	3.8		17.2	3.55
	18.2	4.1		18.6	4.62		15.6	3.7		18.2	6.65		14.8	4.2		17.9	3.58
	19.3	4.35		19	4.79		17	4.37		18.3	6.7		15.3	4.35		19.5	4.1
	19.7	4.65		19.6	4.8		17.8	4.5		19.4	7.1		16.4	4.5		20	4.18
	20.8	4.87		19.8	5.01		18.6	4.9		20.1	7.37		18	5		20.2	4.4
	21.8	5.25					20	5.2					18.2	5.27		21.7	4.65
	23.4	5.45					20.2	5.4					18.9	5.5			
							21.3	5.75					18.8	5.58			
							21.7	5.85					19.6	5.78			
							23.4	6.2					20	5.9			
							23.5	6.25					21	6.1			
													21.5	6.38			
													21.6	6.4			
													22.6	6.7			
													23.4	6.8			
													23.8	6.97			
													24.6	7.15			
													24.9	7.2			
28.5	16.9	4	30.3	18.3	4.8	34.7	16	5	39.2	15.1	4.7		25	7.39	35.5	16	4.9
	19.5	4.65		18.7	4.5		16.4	5.3		15.1	5.1					17.1	5.2
	19.8	4.8		18.79	5.3		18.6	5.6		16.5	5.7					17.6	5.5
	20.6	5		19.1	4.8		19.2	5.7		17.5	5.7					18.1	5.7
	20.7	5.2		19.5	5		19.9	6.3		17.7	5.9					18.2	5.8
	21.8	5.5		19.6	5		20.9	6.5		18.3	6					19.9	6.1
	23.3	5.8		19.7	5.3		21.5	6.6		18.6	6.3					20	6.2
	24.6	6.1		20.4	5.6					19.2	6.5					20.3	6.5
	25.8	6.25		20.8	5.6					19.6	6.6					21.6	6.7
				21.5	5.8					20.4	6.8					21.9	6.9
				21.9	5.9					20.1	6.9					22.4	7.1
				22.6	6					21	7.2					22.9	7.25
				22.9	6.1					21.9	7.4					23.1	7.3
				23.4	6.2					22.4	7.5					24.2	7.4
				23.9	6.2					22.6	7.6						
				24.6	6.4												

Appendix 4: MATLAB codes

1. Obtaining 3D surface fitting and regenerating the 2D relationship between power generation and water discharge rate for the G1A unit.

```
% For G1A unit
h1= 29.5;% Unit head(m)

% To load the excel data set for Unit G1A
Q1=xlsread('Power Generation and Discharge Curve1   remaden.xlsx','Data 1A','AC2:AC137');
P1=xlsread('Power Generation and Discharge Curve1   remaden.xlsx','Data 1A','AD2:AD137');
H1=xlsread('Power Generation and Discharge Curve1   remaden.xlsx','Data 1A','AE2:AE137');

% To get the surface fit for the G1A unit
[fitresult, gof] = createSFit2G1A(Q1,H1,P1)

% To regenerate the 2D polynomial curve for G1A unit at 29.5m head
q=1:0.2:15;
y=h1*ones(1,71);
p=fitresult(q,y);
pn=p.';
qn=q.';

[fitresult, gof] = createCFitG1A(qn, pn)
xlabel( 'q1');
ylabel( 'p1');
```

2. Obtaining the surface fitting for Lowess 50% function for unit G1A.

These are the auto generated codes from the curve fitting toolbox in MATLAB software for obtaining the surface fitting for Lowess 50% function for unit G1A unit.

```
function [fitresult, gof] = createSFit1G1A(Q1, H1, P1)
%CREATEFIT(Q1,H1,P1)
%  Create a fit.

%  Data for 'G1A fit 2' fit:
%      X Input : Q1
%      Y Input : H1
%      Z Output: P1
```

```
%  Output:
%      fitresult : a fit object representing the fit.
%      gof : structure with goodness-of fit info.

%  See also FIT, CFIT, SFIT.

%  Auto-generated by MATLAB on 04-Sep-2021 17:28:02


%% Fit: 'G1A fit 2'.
[xData, yData, zData] = prepareSurfaceData( Q1, H1, P1 );

% Set up fittype and options.
ft = fittype( 'loess' );
opts = fitoptions( 'Method', 'LowessFit' );
opts.Span = 0.5;

% Fit model to data.
[fitresult, gof] = fit( [xData, yData], zData, ft, opts );

% Plot fit with data.
figure( 'Name', 'G1A fit 2' );
h = plot( fitresult, [xData, yData], zData );
legend( h, 'G1A fit 2', 'P1 vs. Q1, H1', 'Location', 'NorthEast', 'Interpreter', 'none' );
% Label axes
xlabel( 'Q1', 'Interpreter', 'none' );
ylabel( 'H1', 'Interpreter', 'none' );
zlabel( 'P1', 'Interpreter', 'none' );
grid on
view( -51.1, -12.1 );
```

3. Create the optimization function and solve for case 15.

```
% To create optimization problem
hydro = optimproblem;

% To define variables and their boundaries
p1 = optimvar("p1","LowerBound",2.3,"UpperBound",5.300); % power generation limits for unit G1A at the head h1
p2 = optimvar("p2","LowerBound",2.3,"UpperBound",5.300); % power generation limits for unit G1B at the head h2
p3 = optimvar("p3","LowerBound",3.6,"UpperBound",7.500);
% power generation limits for unit G2 at the head h3
p4 = optimvar("p4","LowerBound",2.6,"UpperBound",6.600); % power generation limits for unit G3 at the head h4
```

```
q1 = optimvar("q1","LowerBound",7.5,"UpperBound",15); % water discharge limits from unit G1A
q2 = optimvar("q2","LowerBound",7.5,"UpperBound",15); % water discharge limits from unit G1B
q3 = optimvar("q3","LowerBound",15,"UpperBound",24.5); % water discharge limits from unit G2
q4 = optimvar("q4","LowerBound",15,"UpperBound",24.5); % water discharge limits from unit G3


Q = 50.26; % considered total water requirement (m3/s)

%Define the objective function
f = -1.* (p1 + p2 + p3+ p4); % (-1)is used for minimization problem
hydro = optimproblem("Objective",f);

% Nonlinear constraints = zero

% Linear constraints -- equality constraint
% Q =q1 + q2 + q3 + q4
hydro.Constraints.econs1 = Q == q1 + q2 + q3+ q4;

% by previous data (ideal condition)
% by MATLAB surface fitting

% Coefficients (with 95% confidence bounds):
 %G1A h1=37.63
      a1 =    0.00227   ;%
      b1 =      0.3507  ;%
      c1 =     -0.62    ;%

 %G1B h1=37.51
      a2 =    0.007047 ;
      b2 =     0.2157  ;
      c2 =    -0.05872 ;

  %G2 h1=33.75
      a3 =    -0.00546  ;
      b3 =      0.5304   ;
      c3 =      -2.778  ;

  %G3 h1=30.79
      a4 =    -0.006038 ;
      b4 =     0.5438  ;
      c4 =    -3.123 ;

% Linear model Poly22:
    hydro.Constraints.econs2 = p1 == a1.*q1^2 + b1.*q1 + c1;
```

```
    hydro.Constraints.econs3 = p2 == a2.*q2^2 + b2.*q2 + c2;
    hydro.Constraints.econs4 = p3 == a3.*q3^2 + b3.*q3 + c3;
    hydro.Constraints.econs5 = p4 == a4.*q4^2 + b4.*q4 + c4;

% Linear constraints -- No inequality constraint

% Define initial point
x0.p1 = 0;
x0.p2 = 0;
x0.p3 = 0;
x0.p4 = 0;

x0.q1 = 0;
x0.q2 = 0;
x0.q3 = 0;
x0.q4 = 0;

% Solve the optimization problem problem
show(hydro)

[sol, fval, exitflag] = solve (hydro, x0)

total_Power = -1*fval
```