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**A MULTISCALE FRAMEWORK FOR SIMULATING
LARGE-CURVATURE BENDING RESPONSE OF THIN
FIBRE COMPOSITE STRUCTURES WITH
INCREMENTAL SHELL STIFFNESS UPDATING**

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DECLARATION

I declare that this is my own work and this thesis/dissertation does not incorporate without acknowledgement any material previously submitted for a degree or diploma in any other University or Institute of higher learning and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text. I retain the right to use this content in whole or part in future works (such as articles or books).

Signature:

Date: 18/01/2026

The above candidate has carried out research for the ~~PhD/MPhil~~/Masters thesis/dissertation under my supervision. I confirm that the declaration made above by the student is true and correct.

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Signature of the Supervisor:

Date: 21/01/2026

Name of Supervisor: Dr. Sumudu Herath

Signature of the Supervisor:

Date: 21/01/2026

DEDICATION

*To every young researcher who stays up late, driven by wonder and hope, in search
of the truths the world has yet to see...*

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ABSTRACT

The nonlinear bending stiffness of thin carbon-fibre woven composites is critical in the design of deployable space structures, yet it is not represented in conventional analytical or numerical design frameworks. Although this nonlinear bending response has been experimentally observed and characterised, a generalised modelling strategy that can systematically incorporate it into structural-scale simulations remains limited. This thesis proposes a multi-scale numerical framework, supported by user-defined subroutines, to embed bending stiffness nonlinearity into finite element models and to predict the response of thin woven composite structures undergoing large deformations. At the meso-scale, the curvature-dependent bending stiffness was extracted from a previously developed and experimentally validated in-house representative unit-cell model, incorporating material plasticity and progressive damage. Building on this input, a macro-scale modelling approach was developed in this thesis using a UGENS user-defined subroutine to embed the curvature-dependent stiffness into structural simulations. The framework was applied to flat coupons and singly curved configurations, including tape springs, to assess the influence of bending stiffness degradation on global structural behaviour. Numerical predictions were validated against analytical solutions and experimental results reported in the literature. The proposed approach accurately reproduced the nonlinear bending moment response and the global deformation behaviour of both flat and curved structures, including complex response features such as bifurcations. Overall, the thesis demonstrates a practical and reliable modelling route for incorporating bending stiffness degradation in thin woven composite structures, enabling improved analysis and design of advanced space structures.

Keywords: Multiscale modelling, nonlinear bending response, stiffness reduction, subroutines, tape-springs, woven composites

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LIST OF ABBREVIATIONS

Abbreviation	Description
CBT	Column Bending Test
DoF	Degree of Freedom
EE	Embedded Element
FEA	Finite Element Analysis
FEM	Finite Element Modelling
GPR	Gaussian Process Regression
RP	Reference Point
RUC	Representative Unit Cell
RVE	Representative Volume Element
UD	Uni-Directional
UGENS	User-Defined General Section
UMAT	User-Defined Material
USDFLD	User Defined Field
XRCT	X-ray Computed Tomography

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