Optical Injection Locking and Characterization for Optical Communication

Gresha Srimani Samarakkody

(168051B)

Thesis submitted in partial fulfillment of the requirements for the degree
Master of Philosophy

Department of Electronic and Telecommunication Engineering

University of Moratuwa
Sri Lanka

May 2020

Declaration

I declare that this is my work, and this thesis does not incorporate without acknowledgement any material previously submitted for a degree or diploma in any other university or institute of higher learning, and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text.

Also, I hereby grant to University of Moratuwa the non-exclusive right to reproduce and distribute my thesis, in whole or in part, in print, electronic, or any other medium. I retain the right to use this content in whole or part in future works (such as articles or books).

Signature: Date:

The candidate, whose signature appears above, carried out research for the MPhil dissertation under my supervision.

Signature: Date:

Abstract

The rise of demand for mobile and broadband communication has caused a worldwide requirement of sophisticated transmitters. High volume and low-cost manufacturing of semiconductor optoelectronic devices are playing an integral role in allowing for the deployment of high-speed communication links. Requirements of side mode suppression, reducing nonlinearities, reducing relative intensity noises and chirps, enhancing the bandwidth are key parameters that are leading for the enhancement of quality and the performance of fiber optic communication system. Optical injection locking (OIL) is one technique to overcome these limitations arising in developing better optical transmitters. Improved device performances can be achieved by locking the high-quality phase and frequency of the master laser to a low-quality slave laser. In this research, a proper optical injection locking system will be developed and characterized by the perspective of improving certain parameters in optical communication systems. A simulation model will be developed and then the characterization will be done for the locking range, bandwidth, amplitude, and phase noise transfer function. The anticipated output will be helpful for several applications like developing optical synchronization in phase-sensitive amplifiers which is useful for improving the transparent length and the operating margins of the optical network system. Also, it will build up a pathway to increase the speed of the optical communication system by the increment of the bandwidth.

Index terms— Optical injection locking, locking range, bandwidth, intensity and phase noise.

Acknowledgements

Firstly, I would like to express my sincere gratitude to my M.Phil supervisor Prof. L. W. P. R. Udayanga for the continuous support provided throughout the research, for his patience, motivation and immense knowledge. His guidance and support helped me in all the time of research and writing of this thesis. I could not have imagined having a better advisor and mentor for my M.Phil study.

Besides my supervisor, I would like to thank my co-supervisor: Dr. Y. A. Kumarayapa, and the progress review panel members Dr. V. R. Herath and Dr. T. N. Samarasinghe, for their insightful comments and encouragement, but also for the hard question which incented me to widen my research from various perspectives.

I thank my fellow research group members D. R. Munasinghe and S. E. Alahakoon for the stimulating discussions, for the sleepless nights we were working together before deadlines, and for all the fun we have had in the last two years. Also I thank my friends in the Department of Electronics and Telecommunication Engineering, University of Moratuwa.

Finally, I must express my very profound gratitude to my parents, to my two sisters and two brother in laws for providing me with unfailing support and continuous encouragement throughout my years of study and through the process of researching and writing this thesis. This accomplishment would not have been possible without them.

Thank You...!

This is for you all....!

This research was conducted with the support of the grant (RG/2017/EA & ICT/03) of National Science Foundation of Sri Lanka (NSF).

Contents

List of Figures

List of Abbreviations

Abbreviation	Description
OIL	Optical Injection Locking
RIN	Relative Intensity Noise
ASE	Amplified Spontaneous Emission Noise

Chapter 1

INTRODUCTION

1.1 Lasers and Laser Operation

1.1.1 Laser and Laser Types

The LASER is an acronym used for Light Amplification by Stimulated Emission of Radiation, where it is also referring as laser in day-to-day life. As the name implies, the stimulated emission of radiation is the process that remains behind the output light beam (photon beam) generation of the lasers. Albert Einstein developed the first theoretical foundation related to the lasers in 1917 with the use of Plank's law of radiation [1].

Lasers play a unique role among other light sources as they possess various types of special features, including,

- Narrow linewidth
- Monochromatic output beam
- Coherence of the output beam
- Directionality of the output beam
- An ability of delivering light with a higher intensity on to a small area of incident.

When it is taking from the application point of view, the laser's bandwidth and the relaxation oscillation frequency are the identified output characteristics that are contributing to the quality enhancement of the laser's output beam [2]. Hence, the elimination of nonlinear distortions, reduction of relative intensity noise (RIN), and chirps are the significant factors that affects the enhancement of the output quality of a laser diode as these factors effecting on the bandwidth and relaxation oscillation frequency [3].

In general, the laser classification took place according to the fundamental properties of the lasers. Hence the lasers are categorized into four groups based on the nature of the active media as [1],

- Solid
- Liquid
- Gas
- Semiconductor

The other effective way of laser classification is categorizing lasers into two categories based on the number of frequencies generated by the output laser beam namely,

- Single-mode laser
- Multimode laser

1.1.2 Semiconductor Lasers

The semiconductor laser is also known as a p-n junction laser that creates a coherent beam with a fixed wavelength the carrier recombination process [1]. The development of semiconductor lasers had begun with the first observation of stimulated emission in GaAs in 1962 [4]. The invention of double heterostructures, which resulted in stable, efficient, compact, and small in size laser diodes, black-made a significant contribution to the advancement of laser diode technology [5].

The semiconductor lasers are very popular and widely incorporated in various applications because of its inherent properties such as [1, 6],

- Compactness
- Excitation by injection
- Continuous oscillation at the room temperature
- Wide wavelength coverage
- Wide gain bandwidth
- Direct modulation capability

- High coherence
- Generation of ultra-short optical pulses
- High reliability
- Cost-effectiveness
- Mass reproducibility
- Monolithic integration
- Easy scalability

When the semiconductor lasers are considering from the point of view of their applications, there are several drawbacks among the advantages. Semiconductor lasers are lagging behind in three of below mentioned factors [6],

- Temperature characteristics
- Noise characteristics
- Divergent output beam

Since semiconductor lasers offer numerous advantages in the functional, performance and, economical domains, they are widely utilizing for a wide range of applications. Optical data storage and processing, optical measurements, and sensing, optical communication and optical energy applications are some of the popular fields covering by semiconductor lasers. Some of the in-demand semiconductor laser applications are CD players, laser printers, image scanners, bar code readers, and transmitters in fiber optic communication systems [6].

One of the special characteristics that is possessed in the semiconductor lasers to become popular in optical communication and on many of the other applications is its ability to be modulated at higher frequencies.

1.1.3 Structure of a Semiconductor Laser Diode

A laser diode consists of several elements that play unique and significant roles in the laser beam generation process,

- Gain medium
- Pump source
- Mirror (with 100% reflectivity)
- Partial mirror (with <100% reflectivity)

The gain medium of the laser is storing inside the cavity and it may in the forms of solid, liquid, or gas. The gain medium should possess the appropriate optical properties, concentration, level of purity, controlled in size and shape.

The pump source supplies the required energy from the outside to move particles to the excited or high energy quantum states to generate a higher number of stimulated photons.

Only the photons which are moving in the direction perpendicular to the mirror contributing to the laser operation as those are the ones that generate multiple photons inside the laser cavity. A mirror with 100% reflectivity helps to generate a higher number of photons through the process of stimulated emission. A partial mirror allows some photons to leave the cavity to generate the laser beam while the other photons incident on the partial mirror gets reflected back into the lasing cavity [7].

1.1.4 Semiconductor Laser Operation

Semiconductor laser consists of a P-I-N diode and cavity which works together to generate a monochromatic laser beam [8].

The energy gap between the conduction band and the valance band of a semiconductor material is almost equal to $1eV$. Therefore, thermal excitation energy and photon energy fall inside the visible spectrum. These characteristics lead the semiconductors to become the most suitable material for optical devices [9].

The whole process related to the semiconductor laser is associated with the electrons in the conduction band and the holes in the valence band when referred in terms of the thermal excitation energy [9]. Hence the internal operation 0f a semiconductor laser can be explained using the three quantum mechanical processes, namely,

- Absorption
- Spontaneous Emission
- Stimulated Emission.

A movement of electron in the valence band to the conduction band with absorbing energy equal to bandgap energy is named as absorption.

The movement of an excited electron to the ground state or a lower energy level from its current state by emitting energy in the form of a photon is referring as spontaneous emission [10].

Stimulated emission was discovered by Albert Einstein in 1917. In stimulated emission, excited electron moves to the ground state with the effect of a stimulated photon. Consequently, it generates another photon which has the same phase, frequency, polarisation and direction as the stimulated photon. The fundamental underpinning of the laser operation depends on the stimulated emission [8]. When considering the electrons in the energy levels,

- Electron in excited state $>$ Electrons in ground state –$>$ Emission is higher
- Electrons in excited state $<$ Electrons in ground state –$>$ Absorption is higher

In order to maintain the laser operation, stimulated emission has to be maintained throughout, while ensuring higher electron density should be maintained in the excited state which refers to as the population inversion. Hence, to enhance the lasing process, a non-equilibrium system driver is to be employed. Due to that the rate of stimulated emission is less than the rate of absorption in practical scenarios. Supply of external energy by pumping in either forms of optical or electrical helps in this process by letting electrons to move into the excited state. To dominate the stimulated emission, a higher photon density should be maintained inside the laser cavity as the stimulated emission rate is proportional to the photon density. The same is achieved by cavity mechanisms in general. Therefore, the lasing requirements can be summarized as [8],

- Stimulated Emission $>$ Absorption –$>$ Non-equilibrium Excitation

- Stimulated Emission > Spontaneous Emission –> Higher Photon Density

With the effect of stimulated emission, it generates floods of back and forth, moving waves which result in constructive as well as destructive waves.

The stimulated emission and spontaneous emission support to originate the optical modes. The amplification of optical mode is done by the gain medium and partially reflected mirrors. Amplitude reflectivity at the mirror surface (r) is given by [11],

$$r = \frac{n_{air} - n_{semi}}{n_{air} + n_{semi}} \tag{1.1}$$

Where,

n_{semi} = the dielectric constant of semiconductor

n_{air} = the dielectric constant of the air.

The power reflectivity (R) can be found by,

$$R = r^2 \tag{1.2}$$

In general, 0.3 is the power reflectivity of a typical semiconductor laser. The semiconductor lasers are acting as amplifiers and in some scenarios they are also acting as an amplifier with a feedback. In general semiconductor laser maintains a unit round trip gain, which means optical wave power should hold the same value after moving back and forth through the laser cavity [8].

Following terms provide some important parameters related to laser operation:

- Carrier Life Time : Average time taking by the minority carriers for recombination [12]
- Spontaneous Emission Factor : The ratio between the rate of spontaneous emission coupled with the laser modes and total rate of spontaneous emission [12].

- Damping Factor : Ratio between input impedance and output impedance [12].
- Resonance Frequency : The frequency which provides the maximum power output [12].
- Threshold Current : The minimum current required to generate the laser beam. Below the threshold current, it is working as a LED [11].

1.2 Optical Injection Locking

Optical injection locking (OIL) is an approach to synchronize the frequency and phase of a master laser with the slave laser by injecting a small amount of optical power into the cavity of the slave laser [13]. Also OIL can be described as a method of controlling the frequency offset between two lasers [14].

1.3 Historical Background of Injection Locking

The Origin of the concept of the OIL system is the pendulum clock which was invented by Christian Huygens in 17^{th} century. The story about the concept of synchronization runs that far into history. Even though that is the general history, when it comes to published works, injection locking of electrical systems takes the first place which was a work of R. Adler. Adler conducted this research work by injection locking an electrical oscillator with an external frequency source [15].

After inventing the coherent light sources, injection locking was applied to the light sources with the application of Adler's theory to the lasers by Pantell in 1965 [16]. In 1966, the first demonstration of OIL was conducted by the Stove and Strier using the HeNe laser [17]. With the development of new lasers and with the use of different materials, OIL was applied and tested with newly developed lasers and for their applications. Again injection locking became a highlighted research area with the demonstration of OIL using semiconductor laser by Kobayashi and Kimura in 1980 [18]. This demonstration motivated researchers towards the applications of OIL such as optical communication. Optical phase modulation was demonstrated by Kobayashi and Kimura by applying direct modulation of the slave laser current [18]. Later in 80's attention of researcher's had moved towards the impacts of OIL while focusing on the applications of OIL [19]. The theoretical works related to the OIL were started in 80's and the very first theoretical work related to OIL was published by Lang in the year 1982 [20]. Then a researcher named Henry had published a research work by introducing the rate equations for semiconductor lasers with including the linewidth enhancement factor [21].

From 90's onwards the rise of demand for mobile and broadband communication had created a worldwide demand for sophisticated transmitters. High volume and low-cost manufacturing of semiconductor optoelectronic devices had started to play an integral role opening the pathways for the deployment of high-speed communication links. Requirements of side mode suppression, reduction of nonlinearities, reduction of relative intensity noises and chirps, enhancing the bandwidth are key parameters that were leading to the enhancement of quality and the performance of a fiber optic communication system. Optical injection locking (OIL) became one of the techniques to overcome above limitations aroused while developing better optical transmitters. As improved device performances can be achieved by locking the high-quality phase and frequency of the master laser on to a low-quality slave laser, this research was focused on the studying of characteristics of OIL system performances.

1.4 Basic Operation of OIL

There are two different configurations to achieve the OIL namely, transmission style configuration and reflection style configuration. The conceptual models for the two configurations are depicted in the figures below.

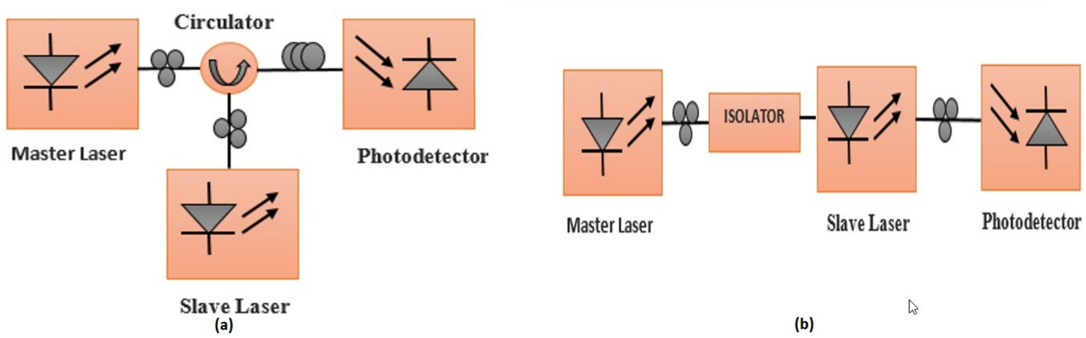

Figure 1.1: Configurations of OIL (a) Reflection Style, (b) Transmission Style

In the reflection style configuration (Fig. 1(a)), the output is taken at the same facet as where the injected light is the input. An optical circulator is placed to ensure that only the output beam goes into the photodetector or any subsequent setup. In the configuration of the transmission style (Fig. 1(b)), the output field of the master laser is injected through one facet of the slave laser and the output is obtained from the other facet. An isolator is placed between the master laser and the slave laser to eliminate the back reflections. In both configurations, polarization controllers are employed to match the polarization between the master and the slave lasers [19].

Through the OIL the output field of the master laser, under the single-mode condition had been fed into the active region of the slave laser. If it satisfied the locking conditions which are based on detuning frequency and injection strength, it synchronizes the slave laser with the master laser [22]. With the optical injection locking, the slave laser gradually obtains the frequency modulated noise spectrum and lineshape of the master laser [23].

The basic condition to remain in the process of injection locking is,

$$\left| \frac{\Delta f}{\Delta f_L} \right| \leq 1 \tag{1.3}$$

Where,

Δf = The detuning frequency (the difference of frequencies between master laser and slave laser)

Δf_L = Maximum detuning frequency to attain locked state.

The other requirement to be fulfilled is that maintaining the phase difference between two lasers at a constant value [24]. Still, when the linewidth enhancement factor obtains a larger value, locking is hard to be achieved [25].

Generally, narrow band lower power laser is selected for the master laser while broadband high power laser is chosen for the slave laser in an OIL system [26].

1.5 Properties and Characteristics of OIL

When describing the injection locking concept, the injection ratio and detuning frequency play a major role [12]. Locking range and bandwidth can be taken as the main characteristics to be considered while describing the OIL system.

1.5.1 Injection Ratio

The injection ratio is the power ratio between the injected power of the master laser and the output power of the free-running slave laser. The same is given by [27],

$$Injection\ Ratio = \frac{Injected\ Power\ from\ the\ Master\ Laser}{Output\ Power\ of\ the\ Free\ Running\ Slave\ Laser} \tag{1.4}$$

Depending on the scenarios injection ratio can be defined as either an internal injection ratio or external Injection ratio [12]. The external injection ratio can be defined as, measured power ratio between the injected power and the lasing power of the free-running slave laser outside the slave laser cavity [28]. The internal injection ratio is the measured power ratio between the injected power and the lasing power of the free-running slave laser inside the slave laser cavity [29].

In addition, the injection ratio can be defined as the ratio of number of photons between master laser and free running slave laser because of the ratio between power is proportional to the ratio between number of photons [27]. Hence the injection ratio can be expressed as,

$$Injection\ Ratio = \frac{Number\ of\ Photons\ Injected\ by\ the\ Master\ Laser}{Number\ of\ Photons\ Emitted\ by\ the\ Free-running\ Slave\ Laser} \tag{1.5}$$

1.5.2 Detuning Frequency

Detuning frequency is the frequency difference between master laser and slave laser [25]. The detuning frequency is defined as,

$$\Delta f = f_{MasterLaser} - f_{SlaveLaser} \tag{1.6}$$

Where,

$f_{MasterLaser}$ = Frequency of master laser

$f_{SlaveLaser}$ = Frequency of slave lasers

Frequency detuning can be represented as wavelength detuning as the frequency and wavelength are holding an inverse relationship between them [29]. Hence the wavelength detuning can be represented as,

$$\Delta\lambda = \lambda_{MasterLaser} - \lambda_{SlaveLaser} \tag{1.7}$$

Where,

$\lambda_{MasterLaser}$ = Wavelength of master laser

$\lambda_{SlaveLaser}$ = Wavelength of slave laser

Detuning frequency adjustments or frequency tuning is done using the temperature tuning method [25].

According to the Kramers-Kronig relationship, frequency detuning of the optical injection-locked system results in a change in the gain of the side-modes and, it causes a shifting in the frequency of the side modes [29].

1.5.3 Locking Range

Locking range plays a crucial role when analyzing the behaviors and characteristics of OIL. The Locking range can be defined as the span of frequencies that results in a locked state [30]. When the frequency difference between the master laser and the slave laser is considerably larger, the slave laser gets unlocked from the master laser and it starts emitting its natural cavity resonance frequency. Hence it provides a measures of the stability of the injection locked system [31].

Fig. 2 shows the boundaries of the locking range. The area between the upper boundary and the lower boundary indicates the stable locking range. The unstable locking range is depicted by the area outside the upper boundary and lower boundary.

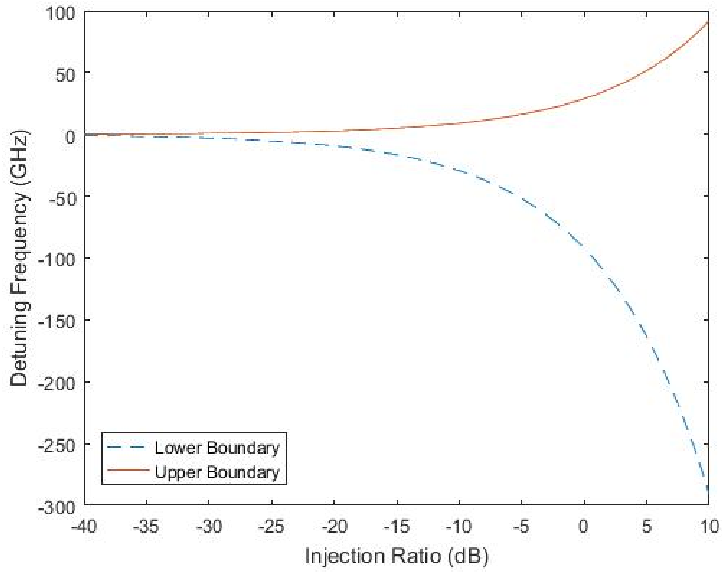

Figure 1.2: Locking Range of Injection Locked Semiconductor Laser

The slave laser's frequency achieved master laser's frequency if and only if the detuning frequency lay within the locking range conditions while the injection ratio is high [32]. Hence the locking range becomes an important factor to be considered while designing an injection locking system.

Locking range basically depends on two primary parameters namely [27],

- Injection ratio
- Detuning fequency

Hence that locking range can be denoted as a function of the injection ratio and the detuning frequency [12].

The slave laser's locking properties depend on the decoupling of phase and amplitude [31], which is indicated by the linewidth enhancement factor. When the linewidth enhancement factor is not equal to zero, the locking rage boundaries hold an asymmetry [12]. In general external light injection cause for a carrier density decrement inside the slave laser cavity and as a result of that, it reduces the gain of the slave laser. As the linewidth enhancement factor showing a coupling between the gain and the phase of the laser, gain variation resulted in a redshift (shift to longer wavelengths) in the free-running wavelength of the slave laser cavity. Hence slave laser locked to a longer wavelength than its initial wavelength and it generates an asymmetry in the locking range [33].

The linewidth enhancement factor can be measured by a few methods such as interferometric measurements, RF modulation measurements, injection locking method, and amplified spontaneous emission method [34]. Locking is difficult to achieve for large linewidth enhancement factors as they possess higher noises [25]. Hence the stability of the locking range strongly depends on the value of the linewidth enhancement factor. When the linewidth enhancement factor is equal to zero, it provides the boundaries of the unstable locking range and when the linewidth enhancement factor holds a value that is greater than zero, it falls into the stable locking range [22].

The width of the locking range shows a proportionality with the injection ratio of the injection locked semiconductor laser [27]. Hence a stable locking range can be widened by increasing the injected power and shifting the detuning frequency into lower value [25]. The locking range gets narrower when the current level is higher while the linewidth enhancement factor possesses a higher value. The locking range also gets wider when the power level is higher [25].

With the effect of noises, emitted power, as well as the emitted frequency of the master laser, will be fluctuated. The master laser's frequency fluctuations resulted in fluctuations in the stable locking range [22].

Beat phenomena and intensity fluctuations that cause a nonlinear effect can be generated in the area of the unstable locking range [35].

1.5.4 Bandwidth

The width of some frequency or wavelength is simply defined as bandwidth or spectral width of a laser [10].

Relaxation oscillation frequency and modulation bandwidth hold an approximate proportionality. Hence, the relaxation oscillation frequency is an identified factor which is limiting the modulation bandwidth of a semiconductor laser [36].

One method to increase the modulation bandwidth is increasing the driving current as it is increasing the resonance frequency. But it consists of some limitations such as causing laser's facet damages as well as excessive heating of the laser. Thus the increment of bias current can be done up to some level only. The modulation bandwidth enhancement can be achieved by avoiding these problems by using the injection locking technique [36].

The optical injection locking is a well-known method that is used for the enhancement of the modulation bandwidth [22]. When the master laser field is injected into the slave laser cavity, it causes a change in the photon density and the carrier density inside the slave laser cavity. As a result of that, for an instant slave lasers frequency shift to another frequency.

The explanation for the modulation bandwidth enhancement of the optical injection locked semiconductor lasers can be provided by considering the photon density fluctuations inside the laser cavity [22]. The relationship between relaxation oscillation frequency and the number of photons in the laser cavity can be expressed by,

$$Relaxation\ Oscillation\ Frequency \propto \sqrt{Photon\ Density\ Inside\ the\ Laser\ Cavity} \tag{1.8}$$

This photon density depends on the bias injection current of the semiconductor laser [22].

The output frequency variation is occurring as a result of the beatings between shifts cavity resonance frequency and the slave laser's new frequency which is equal to the master laser's frequency. The new frequency does not have sufficient gain to sustain in that frequency and with the time it indicates an oscillation decay in the new frequency. Finally, the slave laser's output restored to the previous value it held which is equal to the master lasers injected frequency [37].

Enhancement of modulation bandwidth is occurring due to the interference between the initial frequency of the slave laser at free running state and the shift frequency of the slave laser when it is at the locking state [22]. This shift of resonance frequency can be obtained using the equation below,

$$\Delta\omega_{Shift} = \frac{1}{2}\alpha G_n \delta_n \tag{1.9}$$

Where,

α = The linewidth enhancement factor

G_n = The gain coefficient

δ_n = Change of the carrier density

Fig. 1.3 depicts the slave laser's frequency variation with the effect of optical injection locking. The $\Delta\omega_{SL}$ is the slave laser's free running frequency. With the effect of injection locking slave laser's frequency obtain ω_{Shift} frequency for an instant with holding an amount of shift which is equal to the $\Delta\omega_{Shift}$. But, after that instant of time slave laser attains the master laser's frequency which is indicating by ω_{ML} by shifting from $\Delta\omega_{Shift}$ amount. This $\Delta\omega_{Shift}$ indicates the frequency detuning between master laser and slave laser.

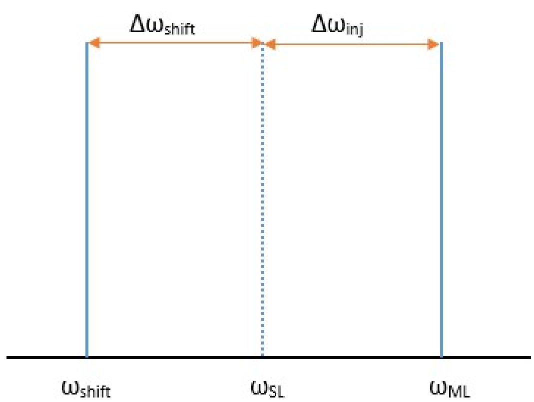

Figure 1.3: Effect on the laser frequencies with OIL

.

1.6 Semiconductor Lasers for OIL

The Semiconductor lasers are the most widely used lasers for optical injection locking as they possess low mirror reflectivity [37]. The injection current is controlling the output power of the semiconductor laser while the injection current modulation controlling the laser frequency [22]. When considering the injection locked semiconductor lasers, they show low noise characteristic as well as it reduces chirps and enhances the stability of the output as semiconductor laser carrier density effect on the refractive index [38].

1.7 Advantages of OIL Semiconductor Lasers

Due to the various advantages of the optical injection-locked semiconductor laser, the attention is moving towards it over free running semiconductor laser.

Some of the leading advantages are,

- Noise reduction
- Bandwidth enhancement
- Linewdith reduction
- Side mode suppression
- Chirp reduction
- Reduction of non linear distortions

1.7.1 Noise reduction

The word noise implies random fluctuations of various output parameters. Laser noises can categorize into two categories based on the place where it generates such as [39],

- Internal noises
- External noises

The types of noises in lasers are,

- Intensity noise
- Phase noise
- Carrier noise

The intensity noise implies the intensity fluctuations due to different reasons. Phase noise indicates the phase or frequency fluctuations while carrier noise implies the carrier fluctuations [39].

When considering the transmission aspect, noise reduction in a laser is very important for the higher distance transmission with maintaining the signal quality. Not only for optical communication, but also for several other laser applications, noise reduction is very important

The injection-locking resulted in a reduction in the impact of noise in the laser output than the free-running laser output [40]. When the semiconductor laser is strongly injection locked for a given frequency detuning value, then the system remains in the stable locking range, where the noise reduction is improve while the injection parameter is increasing through a wide range [41].

The mode partition noise in optical communication is another considerable noise generated in the link due to the combined effort of the mode hopping in the optical source and inter-modal distortion in the fiber cable. Mode hopping generates random wavelength variation which resulted in a variation of the group velocity. The variation of group velocity resulted in a mode partition noise. OIL helps to overcome the mode partition noise by using the side mode suppression [27].

1.7.2 Bandwidth Enhancement

Among several other methods, blackthe OIL technique is a leading method utilizing to enhance the laser bandwidth. According to the literature, bandwidth enhancement can be achieved in the OIL technique through several parameters such as increment of the bias current, injection ratio increment and photon decay rate increment [42]. Bandwidth can be enhanced to several times of its free-running relaxation oscillation bandwidth, with the use of strong OIL [43].

The bandwidth of a semiconductor laser depends on the relaxation oscillation frequency. This has been deeply analyzed and explained in the section bandwidth in this thesis.

When considering optical communication, it is based on the conversion of electrical modulation into optical modulation. The efficiency of this conversion plays a major role when considering this as a communication perspective. Hence, bandwidth enhancement in a directly modulated laser is a major requirement when it comes to optical communications. In optical communication through the bandwidth enhancement, it allows users to perform the communication by using a higher bit rate [44]. Bandwidth enhancement is not only important for optical communication but also it is important for several other laser applications. Hence OIL is one identified method to enhance the bandwidth [42].

1.7.3 Linewidth Reduction

The spectral linewidth of an output beam of a laser is simply defined as the linewidth of a laser.

Laser linewidth reduction can be achieved with the use of the particularly interesting technique of strong OIL. Linewidth reduction can be achieved through OIL as it increases the photon density inside the laser cavity [45]. If the detuning frequency is controlled carefully and locking bandwidth is large enough to accommodate the frequency fluctuations, the linewidth reduction can be easily achieved in the OIL [23].

Linewidth narrowing becomes an important aspect when spectral purity is blacka must for the laser applications [45].

1.7.4 Sidemode Suppression

Single-mode of a laser can be simply defined as a laser that operates in a single resonator mode. Hence it results in the generation of the monochromatic output laser beam [46]. There exists a small amount of optical power in the various resonating modes while having one dominating mode in the single-mode lasers. If there is a possibility to suppress side modes, the whole optical power will be dominated by the main mode. Through this process, the efficiency of the laser can be increased [26].

The side mode suppression can be achieved by the optical injection locking as it assures the single-mode by reducing the mode partition noise [35].

The injected signal by the master laser act as an additional pump for the one mode of the slave laser. Due to that it enhances the intensity of that mode. If it enhances the intensity of that mode to a considerable amount, other modes will deplete and they will appear below the threshold. This the way it causes side mode suppression [26].

1.7.5 Chirp Reduction

Change of directly modulated semiconductor laser's frequency over, its resonance frequency in accord of the applied modulation is simply defined as a chirp [47]. When the instantaneous frequency indicates a rise with time, it is said to be up chirp and when the instantaneous frequency decreases with time, it is said to be down chirp. Undesired chirps are generated by the direct intensity modulation of a laser. When the laser is directly intensity-modulated, it causes a carrier density variation inside the laser cavity which has resulted in a refractive index change. Due to the refracted index change, it induces a frequency chirp [48].

As chirp is an undesired signal it causes broadening of the optical spectrum width of the signal and it degrades the system performances [49]. Hence it is better to reduce the chirps by using some techniques. OIL is an identified technique to reduce chirps [47].

There are two parameters that affect the chirps reduction of OIL semiconductor lasers. They are detuning frequency and the linewidth enhancement factor [50]. According to the literature, chirps can be 100% suppressed through OIL, when the detuning frequency between master and slave lasers becomes zero [47].

1.7.6 Reduction of Nonlinear Distortions

When it focuses on semiconductor laser applications, linearity is a highly desirable factor [51]. Generally, non-linearities are generated due to the interaction between photons and electrons [27]. Hence laser output beam should obey linearity with the drive current.

Parasitic frequencies get strengthened due to the energy transfer from nonlinear effects. These parasitic components limit the performance of laser output and the overall system [52]. Some examples of such nonlinear distortion effect are the limitation of the number of channels and transmission distance due to the inter-channel interference, which generates due to the effect of nonlinear distortion [53].

Researchers found several methods to overcome these nonlinearities. Some of such methods are electro-optical feedback and feed-forward compensation. OIL also helps in reducing these nonlinear distortions [27]. When focusing on the OIL to improve the characteristics of the laser, it is found that OIL substantially suppress the nonlinear distortions [54].

1.8 Applications of OIL

Semiconductor lasers are widely used in laser applications such as optical communication, signal processing, sensor applications and many others due to semiconductor laser's valuable characteristics like lower power consumption, and compact structure for integration. As OIL is another way to improve the laser characteristics, it opens up several applications with overcoming existing matters related to semiconductor lasers [55].

1.8.1 Optical Communication Applications

The diode lasers play a major role in any long-haul fiber optics-based communication systems as directly modulated transmitters to obtain higher data rates [56].

The OIL is one method to overcome the matters that arise with the laser transmitter in optical communication systems. Therefore, using this method towards many important applications of optical communication can be influenced. Since the phase is synchronized with the master laser, the method can be exploited and, can be used in coherent homodyne detection which is one of the most advanced detection techniques. Moreover, this will allow increasing the direct modulation bandwidth with advanced modulation formats, as information can be encoded in amplitude and phase, or alternatively on both in-phase and quadrature components of a carrier [57].

1.8.2 Biomedical Laser Applications

Biomedical applications can be identified as one leading force behind the development of diode laser technology [58]. Lasers are used in several biomedical applications and they can be categorized into few categories as follows,

- Medicine for diagnostic use
- Intermediate for surgical use
- High power lasers for biomedical research purposes.

Characteristics such as high partial and temporal accuracy, high directionality, excellent hemostatic effect, postoperative complications, and reduction of pain driven the biomedical laser applications [59].

As OIL is a technique that enhances laser properties, it applies to biomedical applications as well [60].

1.8.3 Sensor Applications

When considering the coherent detection systems made out of semiconductor lasers, the narrow spectral width and stability of the frequencies became major concerning factors [61]. The noises originating from the optical sources are considerable for the sensor applications as the dominant noise source in sensor application is RIN [40].

Injection Locked lasers are used for high-performance sensor applications as they are having the ability to reduce intrinsic noise as well as to improve the system characteristics [40].

1.8.4 Industrial Laser Applications

Industrial laser applications also another identified application that contributes to the development of the laser diode technology [58]. Lasers are used in many industries for purposes such as laser cutting, laser welding, laser drilling, laser marking, and laser cladding. Hence improvements in laser properties will enhance the industrial process efficiency. The laser property enhancement can be achieved through the OIL technique even in the industrial laser applications [62].

1.9 Types of Noise in Semiconductor Lasers

Noise can be defined as an undesirable signal. Hence it degraded the system performances.

The internal operation of the semiconductor lasers resulted in undesired phase and intensity fluctuations. These fluctuations will reduce laser application system performances. Semiconductor lasers consist of several noises and the dominant source of noise is spontaneous emission noise [63]. Among all types of noise the other considerable noises are,

- Spontaneous emission noise
- Shot noise
- Thermal noise

1.10 Objectives of the Research Work

- Evaluate the exiting methods of optical injection locking.
- Develop and simulate a suitable model for optical injection locking. Characterize the system for its locking range for different input power level.
- Characterize the system for different levels of amplitude and phase noise transfer from the master laser.
- Characterize the system for its bandwidth enhancement by transferring noises from the master laser.

1.11 Novelty of the Research Work

While analysing the literature, different types of analysis associated with the locking range and bandwidth for optical injection locking was found. In addition, a number of noise analyses are presented in relation to the optical injection locking characteristics. However, the literature indicates that noise analysis was not performed for the locking range or bandwidth of a semiconductor laser. Hence, this is the first study where it is carried out an analysis to identify the noise effect on the locking range and the bandwidth of optical injection locked semiconductor laser.

1.12 Structure of the Dissertation

This research was conducted under two phases.

The first phase has been carried out to identify the influence of noises on the locking range of OIL semiconductor lasers.

The second phase was carried out to identify the influence of noises on the bandwidth of the OIL semiconductor lasers.

1.13 Organization of Dissertation

In Chapter 2, the theoretical background of the OIL semiconductor laser is presented. The development of rate equation from free-running state to OIL state is expounded. The later part of the chapter is illustrated the noise theories of the semiconductor lasers.

In Chapter 3, the noise merged locking range derivation is presented with presenting a description to Langevin noise sources.

In Chapter 4, the locking range characterization is presented. Starting from the influence of the intensity noise effect, the chapter presents the phase noise effect and the collective effect of intensity and phase noise on the locking range of OIL semiconductor lasers.

In Chapter 5, presents the theoretical background of the bandwidth of OIL semiconductor lasers. The latter part presents the development of noise inserted bandwidth model.

In Chapter 6, it presented a characterization of bandwidth for intensity noise and phase noise separately and collectively.

Finally, Chapter 7 summarized the whole research work and discussion was carried out for the future works.

Chapter 2

THEORIES OF OIL

2.1 Rate Equation Theories for Free Running Semiconductor Lasers

The rate equations are used to describe the dynamic behavior of a laser diode [33]. Since rate equations provide the time dependence of the atomic population inside the laser cavity [64], the rate equation governing the complex field of free-running laser is given by [12, 43],

$$\frac{dA^{'}(t)}{dt} = \frac{1}{2}(1+i\alpha)G_0\big(N^{'}(t) - N_{th}\big)A^{'}(t) \tag{2.1}$$

Where,

$A^{'}(t)$ - the electric field strength of free running semiconductor laser

α - the linewidth enhancement factor

G_0 - the linear gain coefficient

$N^{'}(t)$ - the carrier number of free running semiconductor laser

N_{th} - the threshold carrier number

By substituting $G_0(N^{'}(t) - N_{th}) = G_0(N^{'}(t) - N_{tr}) - \frac{1}{\tau_P}$ on equation (2.1), it can be written as [12],

$$\frac{dA^{'}(t)}{dt} = \frac{1}{2}(1+i\alpha)\left\{G_0\big(N^{'}(t) - N_{tr}\big) - \frac{1}{\tau_P}\right\}A^{'}(t) \tag{2.2}$$

Where,

N_{tr} - the transparency carrier number

$G_0(N^{'}(t) - N_0)$ - the modal gain

τ_P - the photon lifetime

When the normalized field is $A^{'}(t)$, the complex field equation can be spilt into the field intensity and phase by assuming $A^{'}(t) = \sqrt{S'(t)} \exp\left(i\theta'(t)\right)$, Where $S^{'}(t)$ is the intensity of the free running semiconductor laser field and $\theta^{'}(t)$ is the phase of the free running semiconductor laser field [12].

The rate equation governing the intensity (photon number) of the free running laser is given by the [12],

$$\frac{dS^{'}(t)}{dt} = G_0\big(N^{'}(t) - N_{tr}\big)S^{'}(t) - \frac{S^{'}(t)}{\tau_P} + \beta\frac{N^{'}(t)}{\tau_N} \tag{2.3}$$

The rate equation governing the carrier number of the free running laser is given by the [12],

$$\frac{dN^{'}(t)}{dt} = \eta_i\frac{I(t)}{q} - \frac{N^{'}(t)}{\tau_N} - G_0\big(N^{'}(t) - N_{tr}\big)S^{'}(t) \tag{2.4}$$

Where,

τ_N - the carrier lifetime

β - the spontaneous emission factor

η_i - the carrier injection efficiency

$I(t)$ - the injected electrical current

q - the charge of an electron

When considering the first term of the right-hand side in Eq.2.3, it denotes the rate of photons generated by stimulated emission. The second term denotes the photon decay rate while the last term denotes the rate of photons generated by spontaneous emission. First-term in the right-hand side of the Eq. 2.4 denotes the carrier injection rate while the second term denotes the spontaneous recombination rate and the last term provides the stimulated emission rate [12].

2.2 Rate Equation Theories for Optical Injection Locked Semiconductor Lasers

The rate equation governing the complex field of injection locked semiconductor laser is given by [12],

$$\frac{dA(t)}{dt} = \left\{\frac{1}{2}(1+i\alpha)G_0(N(t)-N_{th}) - i2\pi\Delta f\right\}A(t) + K_C A_{inj} \tag{2.5}$$

Where,

$A(t)$ - the electric field magnitude of slave laser after OIL

$N(t)$ - the number of carriers in the slave laser cavity after OIL

Δf - the detuning frequency

K_C - the internal coupling rate

A_{inj} - the complex field of the master laser injected into the slave laser cavity

When the normalized fields are given by $A(t) = \sqrt{S(t)}\exp(i\theta(t))$ and $A_{inj} = \sqrt{S_{inj}}\exp(i\theta_{inj})$, where $S(t)$ is the number of photons in the slave laser cavity, S_{inj} is the number of photons injected by master laser, $\theta(t)$ is the phase of the electric field of slave laser and θ_{inj} is the phase of the electric field of master laser. Hence the Eq. 2.6 can be split into rate equations of intensity(photon number), carrier number and phase of the injection-locked semiconductor lasers, which are given by the [12],

$$\frac{dS(t)}{dt} = G_0\Big(N(t)-N_{tr}\Big)S(t) - \frac{S(t)}{\tau_P} + 2K_C\sqrt{S_{inj}S(t)}\cos(\theta(t)-\theta_{inj}) \tag{2.6}$$

$$\frac{dN(t)}{dt} = \eta_i\frac{I(t)}{q} - \frac{N(t)}{\tau_N} - \frac{S(t)}{\tau_P} - G_0(N(t)-N_{th})S(t) \tag{2.7}$$

$$\frac{d\theta(t)}{dt} = \frac{\alpha}{2}G_0(N(t) - N_{th}) - 2\pi\Delta f - K_C\sqrt{\frac{S_{inj}}{S(t)}}\sin\left(\theta(t) - \theta_{inj}\right) \tag{2.8}$$

Steady state (nothing change with the time) solutions for the above equations can be obtained by setting the derivatives of Eq.2.7, Eq. 2.8 and Eq. 2.9 to zero [33]. The photon number inside the slave laser cavity, carrier number inside the slave laser cavity and the phase of the slave laser at the steady state is denoted by S_0, N_0 and θ_0 respectively.

The locking range equation derived by using the rate equations governing the injection-locked semiconductor lasers can be denoted as [10, 19],

$$-K_C\sqrt{\frac{S_{inj}}{S_0}}\sqrt{(1+\alpha^2)} \leq 2\pi\Delta f \leq K_C\sqrt{\frac{S_{inj}}{S_0}} \tag{2.9}$$

2.3 Noise Theories

Noise can be considered as an undesirable signal which is consists of randomness. Due to its randomness, it is impossible to reconstruct and also it is hard to predict its future values [65].

The intensity and phase fluctuations are introduced by the noises inherent within the cavity of the semiconductor laser during its lasing operation [66].

There exist two fundamental reasons for these fluctuations namely,

- Spontaneous emission noise
- Shot noise

Mode partition noise is another considerable noise which is originated at a semiconductor laser that arises in multimode laser due to the intensity fluctuations in each mode [67].

The spontaneous emission can be considered as the major dominant source of noise in the semiconductor lasers which is generated due to the occurrence of spontaneous emission events [66, 67]. It is referred to as quantum noise as it is a quantum mechanical origin of the spontaneous emission [63, 66]. These spontaneously generated photons are amplified at the gain medium before it escapes from the gain medium. Hence it is called as Amplified Spontaneous Emission (ASE) noise [68].

Spontaneous emission noise resulted in an instantaneous intensity fluctuations, which generates a delayed phase change [10]. Hence the intensity and phase fluctuations originated as a result of this spontaneous emissions that occurred in the coherent optical field established within the laser cavity [66]. Each spontaneously emitted photon causes for fluctuations in the electric field by its intensity as well as the phase-in random manner [66].

The spontaneous emission noise reduces the efficiency of the output beam [69]. When considering the injection locking process, as the spontaneous emission cause for fluctuation in the relative phase, it resulted in a momentarily unlocked the injection-locked laser [24].

Simply shot noise is defined as superposition generated at randomly distributed time intervals [69]. Shot noise is quantum noise which is generated due to the carrier recombination and generations. Shot noise introduced by the lasers denotes the noise generated by the pump sources [68]. Shot noise is negligible as it is very low compared to the spontaneous emission noise [67].

The sources of shot noise are [70],

- Process like absorption or emission of photons
- Processes like insertion or removal of an electron into or from the conduction band.

The shot noise causes for intensity fluctuations in the output laser beam [68]. Carrier fluctuations indirectly causes for phase fluctuations as a second mechanism through the coupling between optical gain, carrier density and refractive index in the optical cavity [29].

2.3.1 Intensity Noise

The laser output power is quantified by the intensity noise where the fluctuations of the photons are considered as the intensity noise [67]. To obtain a value it will take the average value of the intensity. Also, the amplitude noise is referred to as the intensity noise [71].

Intensity noise is measured as RIN which normalizes the optical intensity into the average value of the intensity [25].

Properties of the intensity noise are effecting for some devices like absorption spectroscopy, bandwidth and the transmission range of the optical free-space data links [72].

2.3.2 Phase Noise

Phase noise is referred to as frequency noise [71]. Carrier fluctuations inside the laser cavity induce a second mechanism of phase fluctuations due to the coupling between carrier density, optical gain, and the refractive index [73].

Phase noises are generated due to the fluctuations in the relative phases between the master laser and slave laser in the injection-locked laser system. Generally, these phase fluctuations are originated with the effect of spontaneous emissions [24].

2.3.3 Langevin Noise Sources

The major characteristic of a Langevin noise source is its complete randomness. Langevin noise sources are used to describe the noises which obeys the Brownian motion.

Brownian motion can be applied to a particle which immense in a fluid medium and they consist of random motions [10].Hence the Langevin equations are using to describe the Brownian motions as they are having frictional forces and random forces [40].

The cross-correlation between two Langevin noise sources can be derived by considering all particle movements into and out of the reservoirs which effect both reservoirs simultaneously. If the particles flow into and out of the reservoir is correlated, the correlation will behold a positive value. If the particle movements are anti-correlated the correlation will hold a negative value [74].

Generally, the Langevin noise source is denoted by the form of $F_x(t)$ and it has a completely random nature. The best analogy to describe $F_x(t)$ is a random number generator which generates a new number between + ∞ and - ∞ at every Δt second in the limit of Δt goes to 0. When considering in mathematical terms, it describes a memoryless process which means, the value of the $F_x(t)$ at time t does not correlate with any previous value [25, 74].

Chapter 3

LOCKING RANGE MODEL

3.1 Noise Derivation

When $F_S(t)$ and $F_\theta(t)$ denotes the Langevin noise source of intensity (photon number) and Langevin noise source of phase (frequency) respectively, $F_s(t)$ and $F_\theta(t)$ is just as often positive as it is negative. Hence, the average value over time of $F_s(t)$ and $F_\theta(t)$ is zero. Thus, it can be denotes as,

$$\left\langle F_S(t) \right\rangle = \left\langle F_\theta(t) \right\rangle = 0 \tag{3.1}$$

Correlation function generally implies the relationship between two functions which indicates the effect of one function on the other function. The correlation between any two Langevin noise sources can be generalized as,

$$\left\langle F_i(t) F_j(t - t^{'}) \right\rangle = U_{ij}\delta(t^{'}) \tag{3.2}$$

Where,

F_i and F_j = two Langevin noise sources

U_{ij} = the correlation strength between two noise sources.

The autocorrelation implies the relationship of one functional value with another value of the same function at a different time location. The cross-correlation compares one function with another function. By following the methods in [25] the auto and cross-correlations of the Langevin noise sources of intensity and phase are derived as,

$$\left\langle F_s(t)^2 \right\rangle = G_0(N(t) - N_{tr})S(t) + \frac{S(t)}{\tau_P} + 2K_C\sqrt{S_{inj}S(t)}\cos\left(\theta(t) - \theta_{inj}\right) \quad (3.3)$$

$$\left\langle F_\theta(t)^2 \right\rangle = \frac{\alpha}{2}G_0(N(t) - N_{th})S(t) + K_C\sqrt{\frac{S_{inj}}{S(t)}}\sin\left(\theta(t) - \theta_{inj}\right) \quad (3.4)$$

$$\left\langle F_S(t)F_\theta(t) \right\rangle = 0 \quad (3.5)$$

The detailed derivation for the auto and cross-correlations of the above equations has been presented in the appendix section. Concerning the derived final equation for the auto and cross-correlations presented above, attaining the positive autocorrelation for the intensity noise as well as for the phase noise presenting a dependency of the noises on another same type noises at different time locations. The cross-correlation which is equal to zero indicates no dependency between intensity and phase noises.

3.2 Noise Inserted Locking Range

The Langevin noise sources $F_s(t)$ and $F_\theta(t)$ are the driving sources for the photon number and phase has been taken to determine the effect of intensity and phase noises on the locking range. These two noise sources are assumed to be white noises and also assumed that they are small enough to make use of the differential rate equations.

Noise included rate equations of photon number and phase for the injection-locked semiconductor laser can be modified with adding the Langevin noise sources as follows,

$$\frac{dS(t)}{dt} = G_0(N(t) - N_{tr})S(t) - \frac{S(t)}{\tau_P} + 2K_C\sqrt{S_{inj}S(t)}\cos\left(\theta(t) - \theta_{inj}\right) + F_S(t) \quad (3.6)$$

$$\frac{d\theta(t)}{dt} = \frac{\alpha}{2}G_0(N(t) - N_{th}) - 2\pi\Delta f - K_C\sqrt{\frac{S_{inj}}{S(t)}}\sin(\theta(t) - \theta_{inj}) + F_\theta(t) \quad (3.7)$$

The rate equation model considers all noises arises with semiconductor lasers such as spontaneous emission noise, shot noise, and thermal noise followed by the Langevin approach.

The rate equation of carrier number is neglecting here as there is no effect from the carrier rate equation towards the locking range equation.

The locking range of the noise embedded injection-locked semiconductor laser can be obtained by solving the noise embedded rate equations of injection-locked semiconductor laser as given by the Eq. 3.6 and Eq. 3.7, with considering the steady-state values of an injection-locked laser (See Appendix for the derivation). Hence the derived noise embedded locking range equation of injection-locked semiconductor lasers can be presented as,

$$-K_C\sqrt{\frac{S_{inj}}{S_0}}\sqrt{(1+\alpha^2)} - \frac{\alpha F_S(t)}{2S_0} + F_\theta(t) \leq 2\pi\Delta f \leq K_C\sqrt{\frac{S_{inj}}{S_0}} + F_\theta(t) \quad (3.8)$$

The injection ratio (R_{inj}) of injection-locked semiconductor laser can be denoted as[5],

$$R_{inj} = \frac{S_{inj}}{S_0} \quad (3.9)$$

The noise embedded locking range equation for the injection-locked semiconductor lasers are modified with substituting Eq. 3.9 on Eq. 3.8 as follows,

$$-K_C\sqrt{R_{inj}}\sqrt{(1+\alpha^2)} - \frac{\alpha F_S(t)}{2S_0} + F_\theta(t) \leq 2\pi\Delta f \leq K_C\sqrt{R_{inj}} + F_\theta(t) \quad (3.10)$$

] When considering the Eq. 3.10 lower boundary is denoting by $\frac{-K_c}{2\pi}\sqrt{\frac{S_{inj}}{S_0}}\sqrt(1+\alpha^2)$ while $\frac{K_c}{2\pi}\sqrt{\frac{S_{inj}}{S_0}}$ denotes the upper boundary.

Chapter 4

CHARACTERIZATION OF LOCKING RANGE

The virtual representation of the real-world system can be defined as modeling. After identification of the system components as mathematical relationships, it is easy to simulate the model as a virtual representation. This system will help to identify the system behavior for a wide range of conditions. Modeling and simulation can be taken as an effective way of analyzing a system that is hard to find practical experimentation facilities. MATLAB can be considered as better software to model the OIL scheme as it is,

- Efficient system simulation software for communication, computer vision and radar system technology.
- MATLAB helps to generate and customize embeddable C codes.
- When preparing a MATLAB code for implementation, guidance and tools are available.

Hence, the characterization of the noise embedded locking range of the OIL system has been conducted with the use of MATLAB software.

4.1 Parameters for the Analysis

The values used for the characterization of noise embedded locking range of OIL semiconductor lasers have been obtained from the Lau's thesis [19] and they are given below,

α	The linewidth enhancement factor	3
K_C	The internal coupling rate	$183 \times 10^9 s^{-1}$
S_0	The photon number in the cavity at the steady state	1.6×10^5

4.2 Locking Range Characterization

4.2.1 Intensity Noise Analysis

The intensity noise merged locking range equation can be obtained by ignoring the phase noise in the Eq. 3.10 and it can be denoted as,

$$-K_C\sqrt{\frac{S_{inj}}{S_0}}\sqrt{(1+\alpha^2)} - \frac{\alpha F_S(t)}{2S_0} \leq 2\pi\Delta f \leq K_C\sqrt{\frac{S_{inj}}{S_0}} \quad (4.1)$$

When comparing the noiseless locking range equation which is denoted by Eq. 2.5 and intensity noise added locking range equation which is denoted by the Eq. 4.1, the intensity noise does not influence on the upper boundary of the locking range while it is influencing on the lower boundary of the locking range of OIL semiconductor lasers.

The intensity noise indicates the master laser's photon number fluctuations, which is occurring while injecting the master laser field towards the slave laser. These fluctuations of photons can hold a positive value as well as a negative value. Intensity noise holds a positive value, while the master laser injected a higher number of photons than the average number of photons towards the slave laser. When the master laser's injected number of photons towards the slave laser cavity is less than the average number of photons, then intensity noise holds a negative value. For the easiness of the analysis, the intensity noise can be presented by comparing the photon number change in the slave laser cavity with the steady-state number of photons in the slave laser cavity, which is denoting the rate of change of relative intensity noise [43, 75, 76].

To observe this boundary movement with the effect of the intensity noise clearly, the locking range was simulated for different intensity noise levels. The analysis has been conducted under two categories namely boundary variation with the effect of positive intensity noise and boundary variation with the effect of negative intensity noise.

As the derivation was done under the assumption that the linewidth enhancement factor, which is the ratio between the change of the real component of the refractive index and the change of imaginary components of refractive index approaches zero, this shift affects only on the lower boundary [10]. Due to this assumption, the derivation neglects the change in refractive index resulting in output with no effect on the upper boundary of the locking range.

4.2.1.1 Influence of Positive Intensity Noise on the Locking range of OIL Semiconductor Laser

The effect of positive intensity noise on the locking range was analysed under two scenarios, where the ranges of the injection ratios are different. The Fig 4.1 depicts the considering two injection ratio ranges in a same plot.

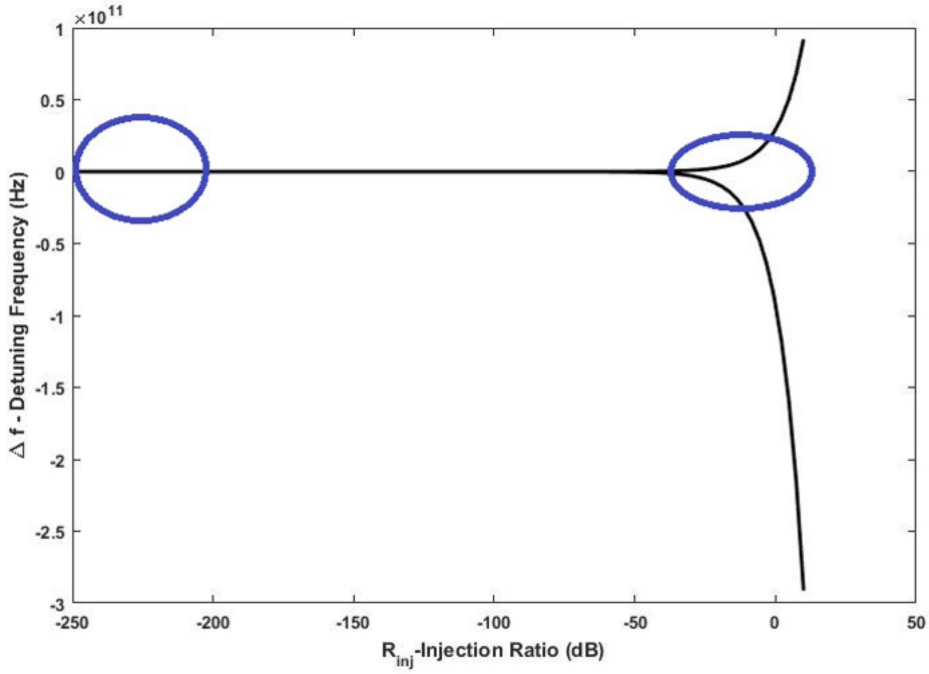

Figure 4.1: The ranges of injection ratios considering for the analysis

The figures below show the width variations with the effect of positive intensity noise. The Fig. 4.2(a) denotes the locking range variations with the effect of positive intensity noise for higher injection ratios, while Fig. 4.2(b) denotes the locking range variation with the effect of positive intensity noise for the lower injection ratios.

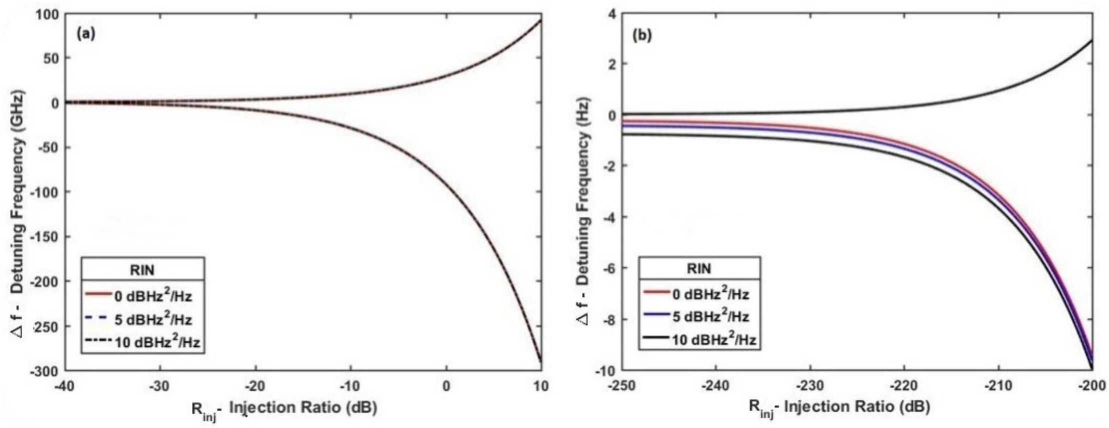

Figure 4.2: Effect of positive intensity noise on the locking range (a) for higher injection ratios; (b) for lower injection ratios

As per Fig. 4.2, with the increment of positive intensity noise, the lower boundary shifts downwards through the detuning frequency axis while the upper boundary remains in the same position. It resulted in a widening of the locking range [77].

The explanations can be provided for the above broadening with the use of quantum mechanical operation, which is taking place inside the slave laser cavity. The positive intensity noise indicates an increment of the rate of the injected number of photons from master laser towards the slave laser cavity, and it resulted in a photon concentration increment inside the slave laser cavity. Hence the carrier concentration inside the slave laser cavity decreases. As the carrier concentration and refractive index having an inverse relationship, the refractive index inside the slave laser cavity increases with the effect of carrier concentration decrement. The refractive index increment causes a decrement of the resonance frequency of the slave laser, as a refractive index and resonance frequency holding an inverse relationship. Due to the relationship between resonance frequency and detuning frequency of an injection-locked semiconductor laser, the resonance frequency decrement resulted in a downwards shift of locking range boundary with the positive intensity noise effect [25, 30, 77].

4.2.1.2 Influence of Negative Intensity Noise on the Locking Range of OIL Semiconductor Laser

The effect of negative intensity noise on the locking range is analysing under two scenarios, where the ranges of the injection ratios are different. The Fig 4.3 depicts the considering two injection ratio ranges in a same plot.

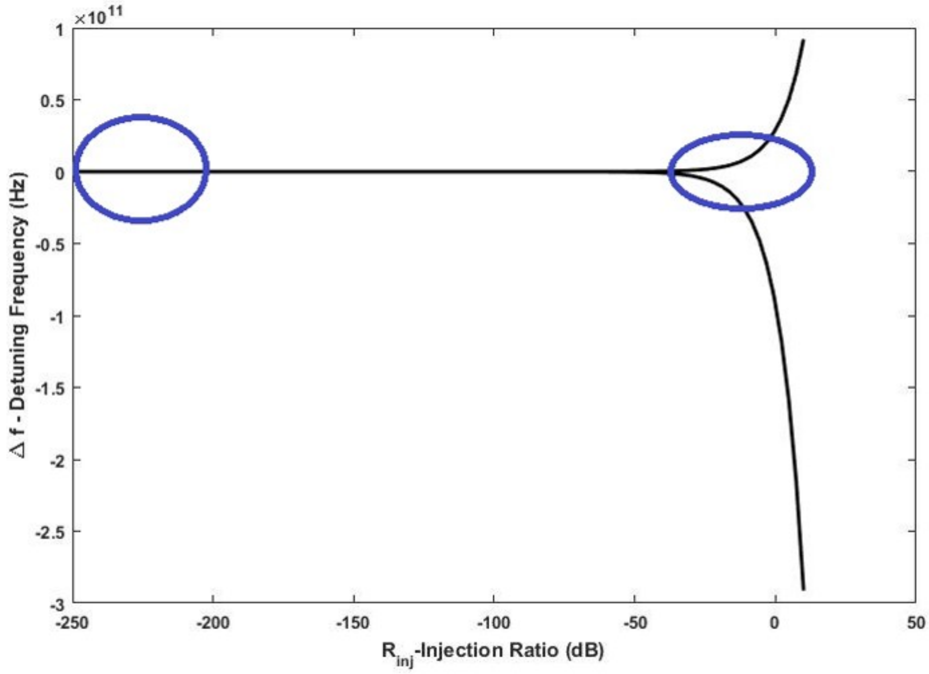

Figure 4.3: The ranges of injection ratios considering for the analysis

The locking range variation with the influence of negative intensity noise is depicted in Fig. 4.4(a) for higher injection ratios, while Fig. 4.4(b) depicts it for the lower injection ratios.

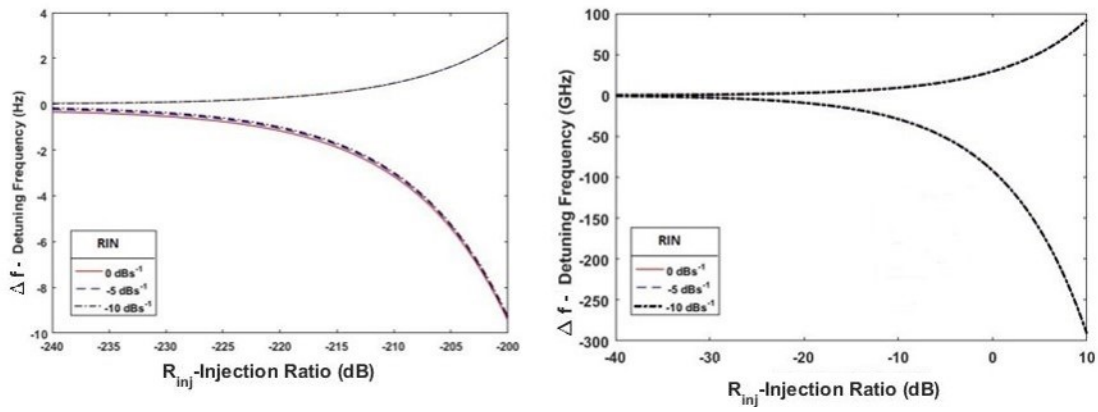

Figure 4.4: Effect of negative intensity noise on the locking range (a) for higher injection ratios; (b) for lower injection ratios

When considering the Fig. 4.4(a) and Fig. 4.4(b), the negative intensity noise is considerable only for lower injection ratio. For higher injection ratios, the negative intensity noise effect is negligible.

The explanations for the negative intensity noise also can be presented using the quantum mechanical processes that are taking place with the effect of the noise. Negative intensity noise represents the reduction of the photon injection rate from the average photon injection rate by the master laser towards the slave laser. This photon injection rate decrement resulted in a decrement of the photon concentration inside the slave laser cavity. When photon concentration decreases, it resulted in an increment in carrier concentration. With the carrier concentration increment, it causes a refractive index decrement. The refractive index decrement resulting in an enhancement of the resonance frequency of the slave laser which is influencing the detuning frequency shift. This detuning frequency shift causes a locking range boundary to shift towards the upward direction. But this intensity noise effect is only working on the lower boundary. With the assumption of zero linewidth enhancement factor, which is taken into account while developing the theoretical model, the intensity noise effect is only working on the lower boundary. Hence, with the negative intensity noise effect, the locking range width getting narrower.

4.2.1.3 Locking Range Width Variation with the Influence of Intensity Noise

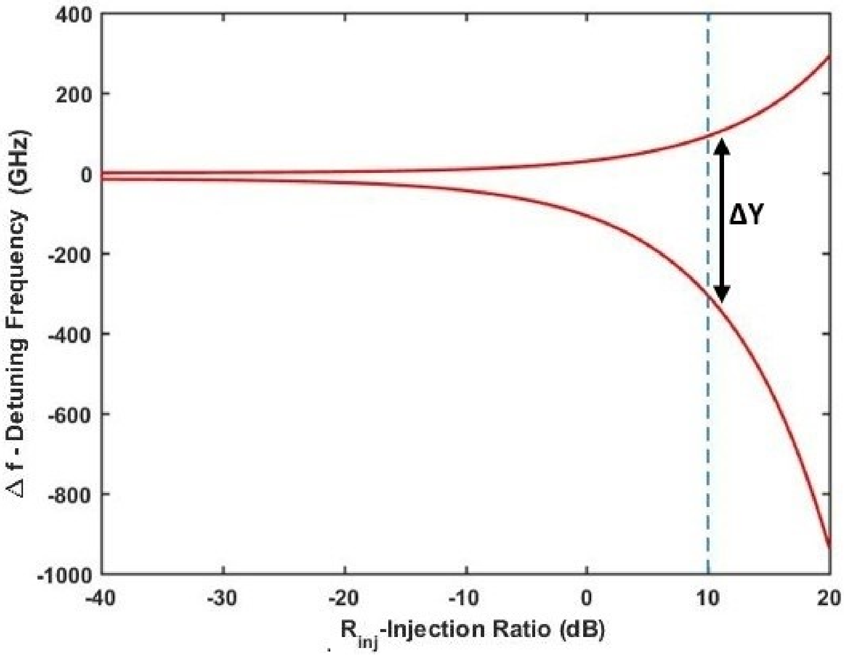

Figure 4.5: Span of detuning frequencies for given injection ratio marked on the locking range

Fig. 4.5 depicts the width of the locking range for a given injection ratio. The locking range width can be obtained by calculating the difference between the upper boundary and lower boundary for a given injection ratio.

The upper boundary and lower boundary equations can be present as,

$$UpperBoundary = K_C\sqrt{R_{inj}} \tag{4.2}$$

$$LowerBoundary = -K_C\sqrt{R_{inj}}\sqrt{(1+\alpha^2)} - \frac{\alpha F_S(t)}{2S_0} \tag{4.3}$$

When ΔY denotes the span of detuning frequencies for a given injection ratio, to find out the ΔY value, the difference between upper boundary and the lower boundary has to be found for a given injection ratio. Hence through the calculations ΔY can be presented as,

$$\Delta Y = K_C\sqrt{R_{inj}}\left(1+\sqrt{(1+\alpha^2)}\right) + \frac{\alpha F_S(t)}{2S_0} \tag{4.4}$$

The Fig. 4.5 simulation simply interprets the dependency of the value of the opening ΔY on the injection ratio (R_{inj}). Fig. 4.6 and Fig. 4.7 has been obtained to find the opening width dependency on the intensity noise.

By using the Eq. 4.4, it can gain the plots for ΔY vs positive intensity noise for different injection ratios. Fig. 4.6 shows the ΔY variation with the per second relative intensity noise. As relative intensity noise is very high to response on the skewness towards the large values the interpretation of $dBHz^2/Hz$ is provided.

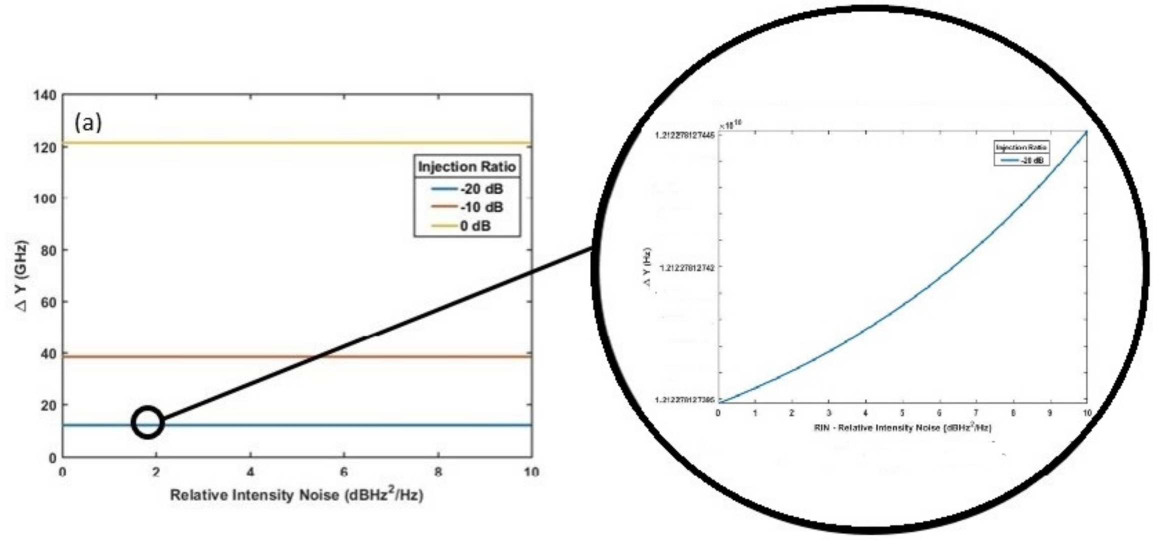

Figure 4.6: Positive intensity noise effect on the opening width of the locking range

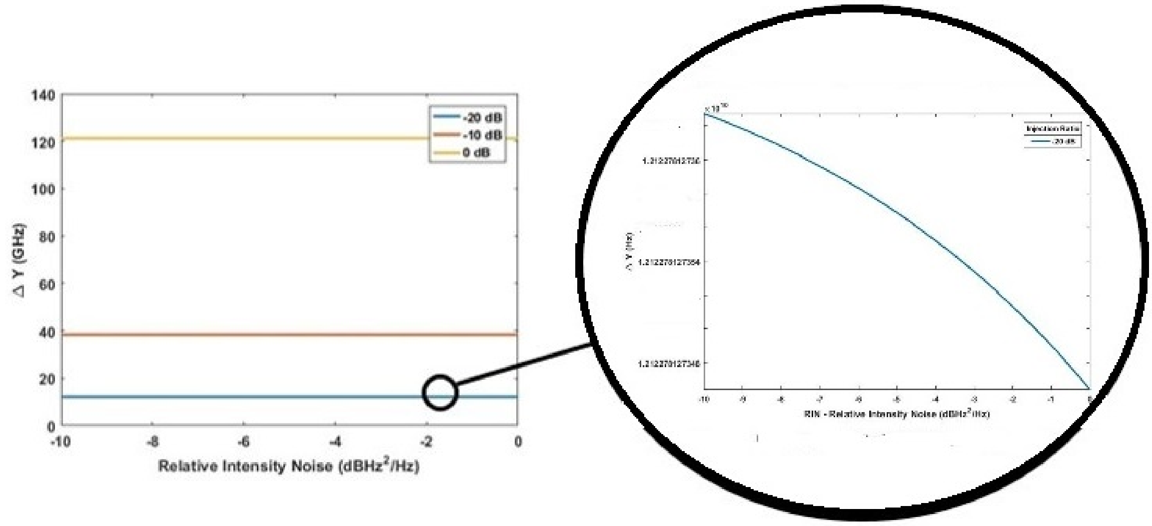

Figure 4.7: Negative intensity noise effect on the opening width of the locking range

According Fig. 4.7, with the influence of negative intensity noise effect, the locking range width possesses a negative relationship with the RIN, and it also shows different y-intercepts for different injection ratios.

In other words, the intersection point with the Y-axis shows a dependency on the injection ratio, but the magnitude of the gradient which is equal to $\alpha/2$ is very low even for a considerable amount of intensity noise. Therefore, it can be stated that the width of the locking range mainly depends on the injection ratio.

As per Fig. 4.5, Fig. 4.6. and Eq. 4.7, there exist different width variations for different injection ratios as well as for different noise levels. Hence it is worth to analyze the width variation concerning the initial width which is not having the noise effect to take a better idea regarding the noise effect on the locking range.

The ΔF_0 is the difference between the upper boundary and lower boundary while intensity noise is zero. ΔF_X is the difference between the upper boundary and lower boundary when there exists an intensity noise. Then the ratio in between ΔF_X and ΔF_0 can represent as,

$$\frac{\Delta F_X}{\Delta F_0} = \frac{K_C\sqrt{R_{inj}}\left(1+\sqrt{(1+\alpha^2)}\right) + \frac{\alpha F_S(t)}{2S_0}}{K_C\sqrt{R_{inj}}\left(1+(1+\sqrt{\alpha^2})\right)} \tag{4.5}$$

This variation can be obtained as a percentage change with respect to the initial width, it can be denoted as,

$$\left(\frac{\Delta F_X}{\Delta F_0}\right)\% = \frac{K_C\sqrt{R_{inj}}\left(1+\sqrt{(1+\alpha^2)}\right)+\frac{\alpha F_S(t)}{2S_0}}{K_C\sqrt{R_{inj}}\left(1+(1+\sqrt{\alpha^2})\right)}100\% \tag{4.6}$$

When it is presented in the form of dBs,

$$\frac{\Delta F_x}{\Delta F_0}_{dB} = 10\log\left\{\frac{K_C\sqrt{R_{inj}}\left(1+\sqrt{(1+\alpha^2)}\right)+\frac{\alpha F_S(t)}{2S_0}}{K_C\sqrt{R_{inj}}\left(1+(1+\sqrt{\alpha^2})\right)}\right\} \tag{4.7}$$

The width variation with the intensity noise as a ratio has been presented in Fig. 4.8,

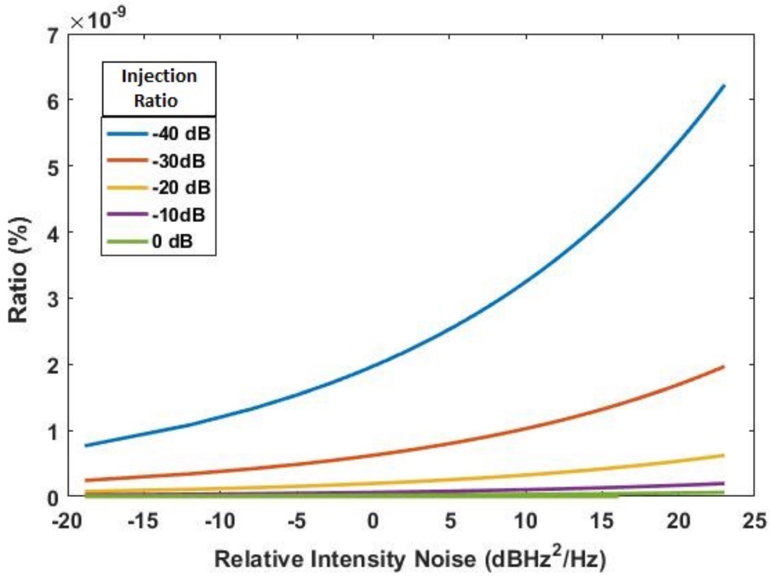

Figure 4.8: The width variation with the influence of intensity noise with respect to the initial width as a ratio

The plot in Fig. 4.8 having a curve, whose gradient is decreasing while the intensity noise is increasing from negative to positive. The different injection ratios follow different curves. It says that the opening (Detuning frequency difference) size of a locking range depends on the intensity noise and injection ratio. When considering the opening width of the locking range, including the noise is greater compared to the opening width of the locking range without adding the noises, and it is getting higher with the increment of positive intensity noise.

For the easiness of the analysis, the ratio can be represented using dB. Hence, Fig. 4.9 and Fig. 4.10 depicts the variation of the ratio in dB with the positive and negative relative intensity noise, respectively.

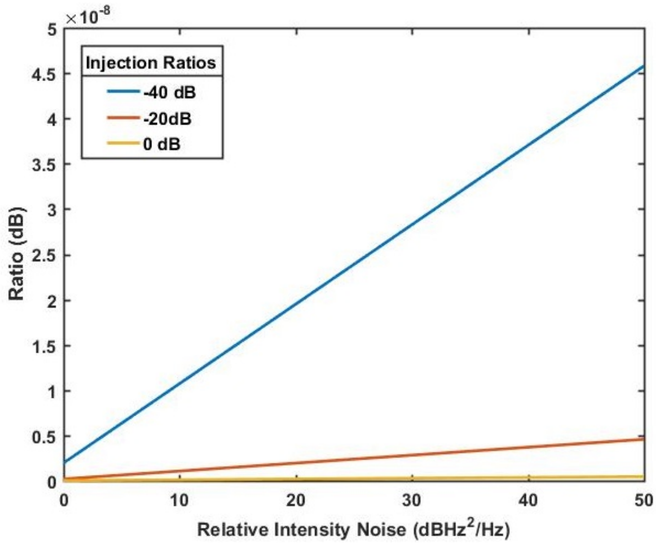

Figure 4.9: Variation of the Ratio in dB with the positive RIN

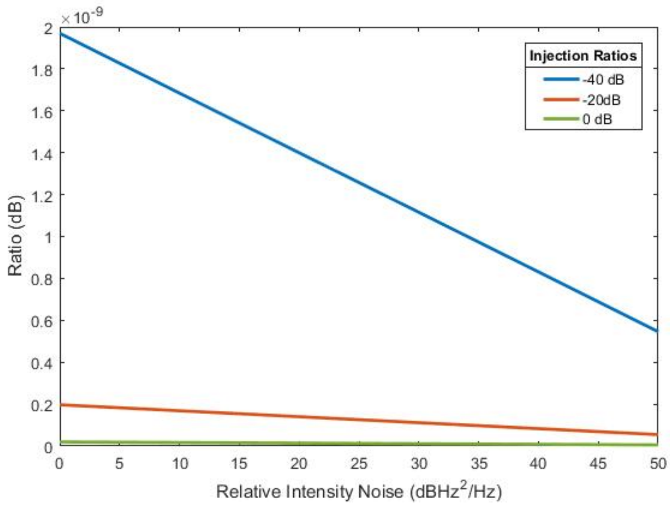

Figure 4.10: Variation of the Ratio in dB with the negative RIN

The variation presented in Fig. 4.9 indicates positive gradients while variation presented in Fig. 4.10 present negative gradients. Both figures illustrate higher gradients for lower injection ratios. Hence, the change in width in the presence of intensity noise is very high, when compared to the initial width (width without the influence of the noises) for the lower injection ratios. According to Fig.

4.9, the initial width of the locking range is lower for low injection ratios. As the intensity noise effect is higher for the lower locking range widths, the width variation ratio is higher for the lower injection ratios than the higher injection ratios.

Hence with the effect of intensity noise at a given injection ratio, the state of injection locking can be achieved for the different ranges of detuning frequencies than the range of detuning frequencies obtained from the without noise state. Hence the slave laser which was in the unlocking state can be moved to a locking state and also the slave laser which was in the locking state can be moved to an unlocking state with the influence of the intensity noise.

4.2.2 Phase Noise Analysis

The effect of phase noise on the locking range can be analyzed by setting the intensity noise $F_s(t)$ to zero in the Eq. 3.10 and it is given by the Eq. 4.8.

$$-K_C\sqrt{R_{inj}}\sqrt{(1+\alpha^2)} + F_\theta(t) \leq 2\pi\Delta f \leq K_C\sqrt{R_{inj}} + F_\theta(t) \tag{4.8}$$

The effect of phase noise on the locking range is analysing under two scenarios, where the ranges of the injection ratios are different. The Fig 4.11 depicts the considering two injection ratio ranges in a same plot.

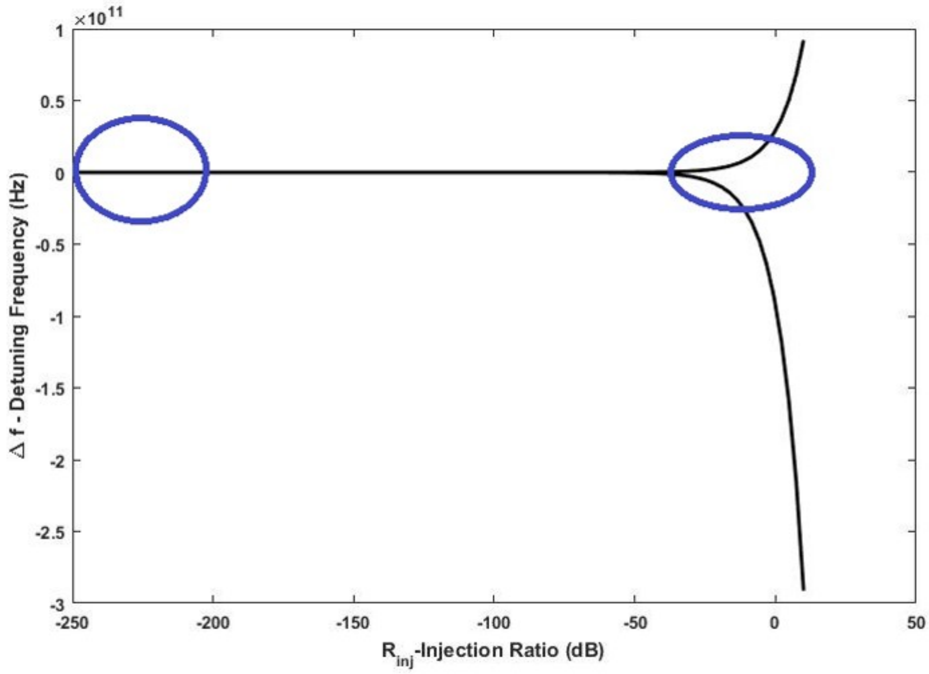

Figure 4.11: The ranges of injection ratios considering for the analysis

The Fig. 4.12 depicts the phase noise effect on the locking range of injection locked semiconductor laser for higher injection ratios and lower injection ratios separately.

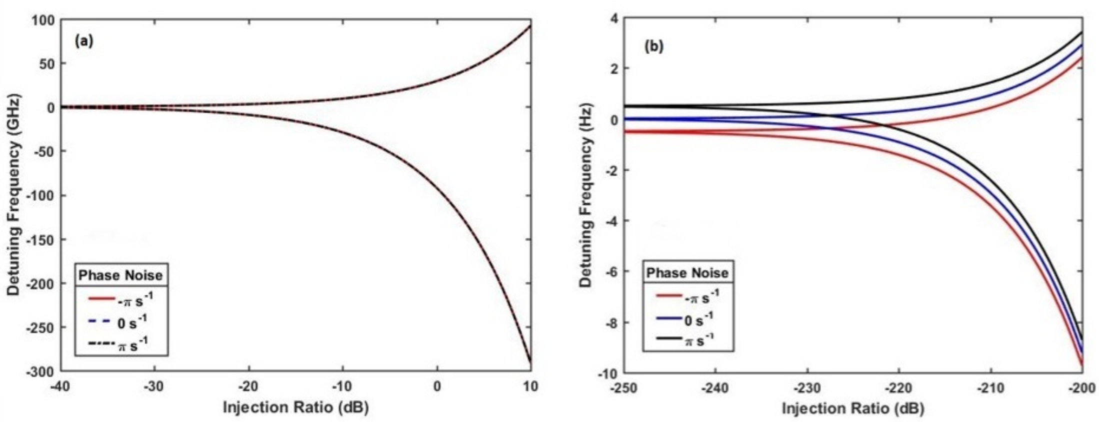

Figure 4.12: Effect of phase noise on the locking range

The change of the phase for this analysis is considered between the range of $-\pi$ to π radiance per second. The range $-\pi$ to π has been selected as it is the possible phase changing range that can be occurred between master and slave laser.

According to Fig. 4.12, the impact of phase noise for higher injection ratios is negligible, as depicted in Fig. 4.12(a). Referring to the lower injection ratios as illustrated in Fig. 4.12(b), the effect of phase noise for the locking range is considerable. With the effect of phase noise, the locking range is shifting upwards or downwards through the detuning frequency axis. The upper bound and lower bound of the locking range are shifting from the same amount of detuning frequency. Hence, both boundaries move upwards or downwards without affecting the width of the locking range for a given injection ratio. The boundaries are moving upwards when the added phase noise is positive and the boundaries are moving downwards when the added phase noise is negative. The amount of the shift depends on the magnitude of the phase noise.

With the effect of phase noise, the locking occurs at another detuning frequency range, where the locking range width remains in the same value as the previous value for a given injection ratio. Hence, the slave laser which is in the locking state can be shifted to the unlocking state and the slave laser which is in the unlocking state can be shifted to the locking state with the effect of phase noise.

Explanation for the scenario can be provided by considering the quantum mechanical operation which is occurring inside the slave laser cavity with the effect of phase noise. When there exists a phase noise, there will be out-of-phase as well as in-phase photons that are propagating with the output field of the master laser. Hence, the photons injected into the slave laser cavity consist of both coherent and incoherent photons with respect to the slave lasers output field [78]. As a result of such variation of photons that occur inside the slave laser cavity, it will lead to variation in the carrier density as well as in the refractive index. This will induce a variation or shift in the resonance frequency resulting in a variation or shift of detuning frequency range. Accordingly, the effect of phase noise on both boundaries will be equal and hence both the boundaries get shifted by equal amounts.

4.2.2.1 Finding the Effect of Phase Noise on the Boundary Shift

As the upper boundary and lower boundary shift upwards or downwards through the detuning frequency axis, with the effect of phase noise, it is necessary to identify shift effect on the locking range. Hence it is worth to obtain the amount of shift as a function of phase noise.

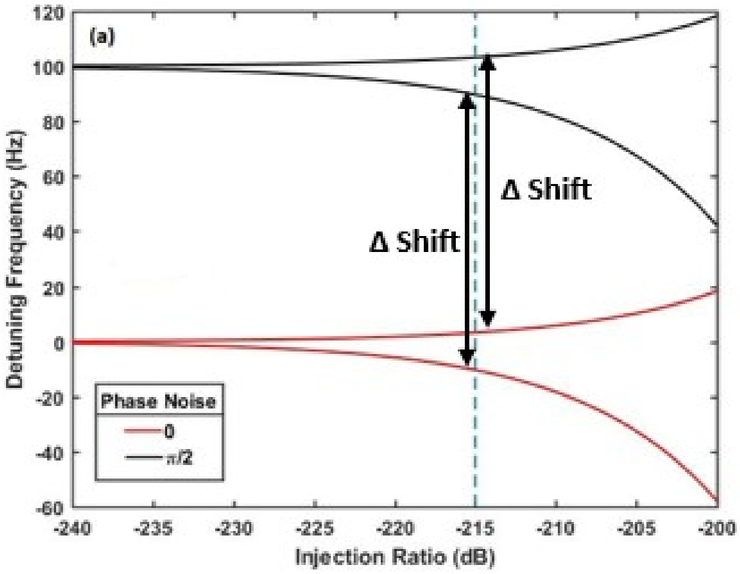

Figure 4.13: ΔShift indication in the locking range

With the effect of phase noise, the upper boundary and lower boundary shifting from the same amount, and the amount of shift can be denoted by the $\Delta Shift$ as shown in Fig. 4.13.

$\Delta Shift$ can be obtained by calculating the shift of the upper boundary or shift of lower boundary. Hence the equation for $\Delta Shift$ can be obtained by considering the new upper boundary equation, which is including the phase noise and it is denoted by the Eq. 4.9 as follows,

$$UpperBoundary = K_C \sqrt{R_{inj}} + F_\theta(t) \tag{4.9}$$

By using the Eq. 4.9, the equation for boundary shift with compared to the initial state of the locking range, which consists of zero noise can be calculated. Hence, this shift can be obtained by finding the difference between the upper boundary equations of with phase noise and without adding the phase noise and it is denoted using the Eq. 4.10.

$$\Delta Shift = \Delta \ Upper \ \ Boundary \ \ Shift = F_{\theta}(t) \tag{4.10}$$

According to the Eq. 4.10, the plot of boundary shift with compared to the phase noise can be obtained. This boundary shift is directly proportional to the phase noise, and it only depends on the phase noise. This dependency is denoted in Fig. 4.14 below.

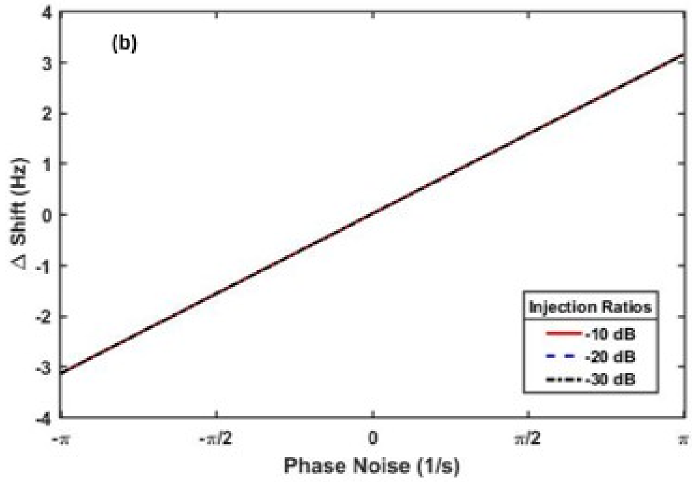

Figure 4.14: Effect of phase change on the boundary shift

As indicated by Fig. 4.14, for all injection ratio the graph follows a linear variation with a constant gradient which is equal to 1. This gradient is the same irrespective of the injection ratio. Hence, it can be stated that the amount of shift is independent of the injection ratio and mainly depending on the amount of incoherent phase noise. Also, the locking range shift depends on the amount of phase noise.

4.2.3 Combined Effect of Intensity and Phase Noise on the Locking Range

When considering the Eq. 3.10, it indicates the combined effect of intensity and phase noise on the locking range of injection-locked semiconductor laser.

$$-K_C\sqrt{R_{inj}}\sqrt{(1+\alpha^2)} - \frac{\alpha F_s(t)}{2S_0} + F_{\theta}(t) \leq 2\pi\Delta f \leq K_C\sqrt{R_{inj}} + F_{\theta}(t) \tag{4.11}$$

When both amplitude and phase noises are added simultaneously, the locking range is moving upwards or downwards through the detuning frequency axis depending on the amount of added phase noise and the opening width (difference in between the upper boundary and lower boundary) of the locking range become wider or narrow with the effect of intensity noise. Hence the total width variation depends on the combined effect of the intensity and phase noise. Accordingly, when both intensity and phase noises act on the master laser field simultaneously, the locking range can be wider or narrower while moving the boundaries along the detuning frequency axis. It is denoted in the Fig. 4.15.

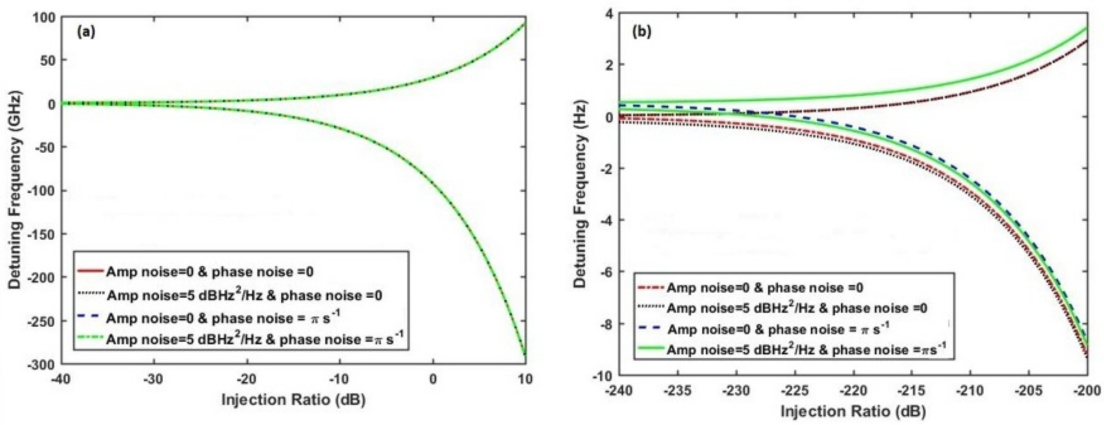

Figure 4.15: The collective effect of amplitude and phase noise on the locking range (a)for higher injection ratio; (b) for lower injection ratio

By referring to the Fig. 4.15, the effect of phase noise and the intensity noise at very low injection ratio is considerable while the noise effect is negligible for higher injection ratios.

Hence, with the combined effect of both noises the locking range would appear on a new place on the detuning frequency axis and it will hold a new locking range width for a given injection ratio. Hence, with the noise effect, the system which is locking state can be moved to an unlocking state, and the system which is in an unlocking state can be moved to a locking state.

Chapter 5

BANDWIDTH MODEL

5.1 Rate Equation Theories for Bandwidth Model of OIL Semiconductor Lasers

The equation for the field magnitude of OIL semiconductor laser has been presented in the chapter 2 by using the Eq. 2.5 and it can be also presented as [19],

$$\frac{dA(t)}{dt} = \frac{1}{2}G_0[N(t) - N_{th}]A(t) + K_C A_{inj} \cos\phi(t) \tag{5.1}$$

Where $A(t)^2 = S(t)$ and ${A_{inj}}^2 = S_{inj}$. When $\phi(t) = \theta(t) - \theta_{inj}$ and the $\eta_i \frac{I(t)}{q} = J$, where J is providing the current, the Eq. 2.8 can be modified as [19],

$$\frac{dN(t)}{dt} = J(t) - \frac{N(t)}{\tau_N} - \frac{{A(t)}^2}{\tau_P} - G_0[N(t) - N_{th}]{A(t)}^2 \tag{5.2}$$

The Eq. 2.9 also can be modified by substituting the $A(t)^2 = S(t)$ and $\phi(t) = \theta(t) - \theta_{inj}$. Then it is in the form of,

$$\frac{d\phi(t)}{dt} = \frac{\alpha}{2}G_0[N(t) - N_{th}] - K_C \frac{A_{inj}}{A(t)} \sin\phi(t) - 2\pi\Delta f \tag{5.3}$$

5.1.1 Dynamic Solutions of the Rate Equations of OIL semiconductor lasers

The derivation of the dynamic solutions can be initiate by linearizing the Eq. 5.1- Eq. 5.3 by applying small-signal time-harmonic perturbations to the steady-state solutions. The small-signal time-harmonic perturbation addition to the steady-state solutions can be denoted as [19],

$$A(t) = A_0 + \Delta A \exp(st) \tag{5.4}$$

$$N(t) = N_0 + \Delta N \exp(st) \tag{5.5}$$

$$\phi(t) = \phi_0 + \Delta\phi \exp(st) \tag{5.6}$$

$$J(t) = J_0 + \Delta J \exp(st) \tag{5.7}$$

Where $A(t), N(t), \theta(t)$ and $J(t)$ denotes the state variables of field magnitude, carrier number, phase and current, while S_0, N_0, θ_0 and J_0 denotes their steady state solutions respectively. $\Delta S, \Delta N, \Delta\theta$ and ΔJ denotes the small signal magnitudes. The s denotes the complex time harmonic angular frequency [19].

When the linearized equations are presented in matrix form,

$$\begin{bmatrix} X_{AA}+s & X_{A\phi} & X_{AN} \\ X_{\phi A} & X_{\phi\phi} & X_{\phi N} \\ X_{NA} & 0 & X_{NN}+s \end{bmatrix} \begin{bmatrix} \Delta A \\ \Delta\phi \\ \Delta N \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \Delta J \end{bmatrix} \tag{5.8}$$

Where,

$$X_{AA} = K_C A_{inj} \cos(\phi_0)/A_0$$

$$X_{A\phi} = K_C A_{inj} \sin(\phi_0)$$

$$X_{AN} = -\tfrac{1}{2} G_0 A_0$$

$$X_{\phi A} = -K_C A_{inj} \sin(\phi_0)/{A_0}^2$$

$$X_{\phi\phi} = K_C A_{inj} \cos(\phi_0)/A_0$$

$X_{\phi N} \quad = -\frac{1}{2}\alpha G_0$

$X_{NA} \quad = 2A_0(\frac{1}{\tau_P} - 2K_C\frac{Ainj}{A_0}\cos{(\phi_0)})$

$X_{NN} \quad = \frac{1}{\tau_N} + G_0{A_0}^2$

The above matrix terms, which are in the form of X_{mn}, provide the change of m with the perturbations of n.

According to the bandwidth theories, the bandwidth can be calculated using the equation below [19],

$$Bandwidth = \omega_R - \frac{\gamma}{2} \tag{5.9}$$

Where,

$\omega_R \quad$ = The resonance frequency of the semiconductor laser

$\gamma \quad$ = The damping rate of the relaxation oscillations of semiconductor laser

The resonance frequency of an injection-locked semiconductor laser is depending on the two major factors, namely, the detuning frequency and the injection ratio [19]. As the bandwidth holds a direct relationship with the resonance frequency, the detuning frequency, and the injection ratio, also affecting the bandwidth.

As per Lau's thesis, the resonance frequency can be obtained with the use of the determinant calculation of the frequency response, which is given by the Eq. 5.8. The determinant can be obtained by [19],

$$D(s) = s^3 + As^2 + Bs + C \tag{5.10}$$

Where,

$$A = X_{AA} + X_{\phi\phi} + X_{NN} \tag{5.11}$$

$$B = X_{AA}X_{\phi\phi} + X_{AA}X_{NN} + X_{\phi\phi}X_{NN} - X_{A\phi}X_{\phi A} - X_{AN}X_{NA} \tag{5.12}$$

$$C = X_{AA}X_{\phi\phi}X_{NN} + X_{A\phi}X_{\phi N}X_{NA} - X_{A\phi}X_{\phi A}X_{NN} - X_{AN}X_{NA}X_{\phi\phi} \tag{5.13}$$

This equation consists of three roots, where two of them are having complex conjugate roots while other root is real and negative [19].

When the modulation frequencies are in the ranges of GHz, THz or above, C can be neglected as it is very small compared to other terms in the Eq. 5.10 [19].

$$C << s^3, As^2, Bs \tag{5.14}$$

When finding the solutions for Eq. 5.10, it can obtain pure driving frequency of [19],

$$D(i\omega) \approx i\omega(i^2\omega^2 + i\omega A + B) = i\omega(-\omega^2 + i\omega A + B) \tag{5.15}$$

This determinant takes the form of classic damped oscillator which is in the form of [19],

$$D(i\omega) \approx i\omega({\omega_R}^2 - \omega^2 + i\gamma\omega) \tag{5.16}$$

As the Eq. 5.15 follows the same form as Eq. 5.16 [19],

$${\omega_R}^2 = B \tag{5.17}$$

$$\gamma = A \tag{5.18}$$

When considering the term B, which is given by the Eq. 5.12, first three terms indicates the diagonal matrix terms which are corresponding to the state variables that are having an effect on itself. These terms are typically weak and the first three terms are negligible while last two non-diagonal terms are accounting for the term B under the typical conditions. Hence the resonance frequency can be obtained as follows [19],

$$\omega_R{}^2 \approx B \approx -X_{AN}X_{NA} - X_{A\phi}X_{\phi A} \tag{5.19}$$

According to the right-hand side of the Eq. 5.19, there exist two terms that are effecting on the resonance frequency. When considering the first term in the right-hand side which is equal to $-X_{AN}X_{NA}$ can be expanded as,

$$-X_{AN}X_{NA} = G_0\frac{1}{\tau_P}A_0{}^2 - 2K_C G_0 A_0 A_{inj}\cos(\phi_0) \tag{5.20}$$

The relaxation oscillation frequency for the free running semiconductor laser can be presented by using an equation as,

$$\omega_{R0} = \sqrt{\frac{G_0 A_{fr}{}^2}{\tau_P}} = \sqrt{\frac{G_0 A_0{}^2}{\tau_P}} \tag{5.21}$$

Where A_{fr} denotes the free-running field magnitude which is nearly equal to the A_0. By substituting Eq. 5.21 on Eq. 5.20 we can obtain an equation for $-X_{AN}X_{NA}$ as,

$$-X_{AN}X_{NA} = \omega_{R0}{}^2\left\{1 - \frac{2\tau_P K_C A_{inj}\cos(\phi_0)}{A_0}\right\} \approx \omega_{R0}{}^2 \tag{5.22}$$

In the Eq. 5.22, the term $\frac{2\tau_P K_C A_{inj}\cos(\phi_0)}{A_0}$ is holding a very small value with compared to 1. Hence $-X_{AN}X_{NA} \approx \omega_{R0}{}^2$.

The second term of Eq.5.19 can be present as,

$$-X_{A\phi}X_{\phi A} = \left\{\frac{K_C A_{inj} \sin\phi_0}{A_0}\right\}^2 = \Delta{\omega_R}^2 \tag{5.23}$$

Where, $\Delta{\omega_R}^2$ denotes the term of resonance frequency enhancement. Hence, with the use of Eq. 5.19, Eq. 5.22 and Eq. 5.23, can obtain an equation to denote ω_R as,

$${\omega_R}^2 \approx {\omega_{R0}}^2 + \Delta{\omega_R}^2 \tag{5.24}$$

By considering on the steady state condition of Eq. 5.3,

$$0 = \frac{\alpha}{2}G_0[N(t) - N_{th}] - K_C\frac{A_{inj}}{A(t)}\sin\phi(t) - 2\pi\Delta f \tag{5.25}$$

By using Eq. 5.23 and Eq. 5.25,

$$\Delta\omega_R = \left\|2\pi\Delta f - \frac{\alpha}{2}G_0[N(t) - N_{th}]\right\| \tag{5.26}$$

The second term in the Eq. 5.26 denotes the shift of cavity mode by α. The Eq. 5.26 presents that difference between master lasers frequency and the slave lasers frequency at free running state is effecting on the resonance frequency enhancement [43].

When considering the damping factor, it describes the resonance peak height. By considering the Eq. 5.8 and Eq. 5.18, the damping factor can be defined as,

$$\gamma = A = X_{AA} + X_{NN} + X_{\phi\phi} \tag{5.27}$$

$$\gamma = \frac{2K_C A_{inj} \cos(\phi_0)}{A_0} + \frac{1}{\tau_N} + G_0 {A_0}^2 \tag{5.28}$$

The damping term for the free running semiconductor laser can be written as [19],

$$\gamma_0 = \frac{1}{\tau_N} + G_0 {A_0}^2 \tag{5.29}$$

By substituting Eq. 5.29 on Eq. 5.28,

$$\gamma = \gamma_0 + \frac{2K_C A_{inj} \cos(\phi_0)}{A_0} \tag{5.30}$$

When considering the steady state condition of the Eq. 5.1 $\frac{2K_C A_{inj} \cos(\phi_0)}{A_0} = G_0[N_0 - N_{th}]$ and by substituting it on Eq. 5.30,

$$\gamma = \gamma_0 + G_0[N_0 - N_{th}] \tag{5.31}$$

By adding the reduction of gain below the threshold into the free-running damping term of the semiconductor laser, it presents the damping term of the injection-locked semiconductor laser [19].

5.2 Noise Merged Bandwidth Model

To obtain the noise effect on the bandwidth, the derivation has to be started with adding the Langavin noise source of field magnitude noise $F_A(t)$, carrier noise $F_N(t)$ and the phase noise $F_\phi(t)$. As the field magnitude and the photon number holding a relationship $A(t)^2 = S(t)$, the relationship between field magnitude noise and amplitude noise (noise originated due to the photon variations) can be presented as,

$$F_A(t)^2 = F_S(t) \tag{5.32}$$

5.2.1 Noise Theories Relates to the Bandwidth Model

As the $F_S(t)$, $F_\phi(t)$ and $F_N(t)$ denotes the Langevin noise sources and as their probabilities to hold positive value as well as negative value is equal, the average value over time for $F_S(t)$, $F_\phi(t)$ and $F_N(t)$ are zero. Hence, it can be denoted as,

$$\Big\langle F_S(t) \Big\rangle = \Big\langle F_\phi(t) \Big\rangle = \Big\langle F_N(t) \Big\rangle = 0 \tag{5.33}$$

The derived auto and cross correlation functions can be denoted as,

$$\Big\langle F_s(t)^2 \Big\rangle = G_0(N(t) - N_0)S(t) + \frac{S(t)}{\tau_P} + 2K_C\sqrt{S_{inj}S(t)}\cos(\phi(t)) \tag{5.34}$$

$$\Big\langle F_\phi(t)^2 \Big\rangle = \frac{\alpha}{2}G_0(N(t) - N_{th})S(t) + K_C\sqrt{\frac{S_{inj}}{S(t)}}\sin(\theta(t) - \theta_{inj}) \tag{5.35}$$

$$\Big\langle F_N(t)^2 \Big\rangle = \frac{N(t)}{\tau_N} + \frac{S(t)}{\tau_P} + G_0(N(t) - N_0)S(t) \tag{5.36}$$

$$\Big\langle F_N(t)F_S(t) \Big\rangle = -\Big\{G_0(N(t) - N_0)S(t) + \frac{S(t)}{\tau_P}\Big\} \tag{5.37}$$

$$\left\langle F_N(t)F_\phi(t)\right\rangle = 0 \tag{5.38}$$

$$\left\langle F_S(t)F_\theta(t)\right\rangle = 0 \tag{5.39}$$

Detailed derivation can be found in the appendix section.

5.2.2 Noise Inserted Bandwidth Model

The rate equations concerning the derivation of the noise added bandwidth model is obtained by adding the Langevin noise sources to the Eq. 5.1 - Eq. 5.3. Noise (Langevin Noise Sources) added rate equations for the field magnitude, phase and carrier number is presented below,

$$\frac{dA(t)}{dt} = \frac{1}{2}G_0[N(t) - N_{th}]A(t) + K_C A_{inj}\cos\phi(t) + F_A(t) \tag{5.40}$$

$$\frac{dN(t)}{dt} = J(t) - \frac{N(t)}{\tau_N} - \frac{A(t)^2}{\tau_P} - G_0[N(t) - N_{th}]A(t)^2 + F_N(t) \tag{5.41}$$

$$\frac{d\phi(t)}{dt} = \frac{\alpha}{2}G_0[N(t) - N_{th}] - K_C\frac{A_{inj}}{A(t)}\sin\phi(t) - 2\pi\Delta f + F_\phi(t) \tag{5.42}$$

To obtain the dynamic solutions, small-signal time-harmonic perturbations which are denoting by the Eq. 5.4 to Eq. 5.7 can be added to the Eq. 5.20 to Eq. 5.22. The obtained linearized equations can be presented by using a matrix as shown below,

$$\begin{bmatrix} Y_{AA}+s & Y_{A\phi} & Y_{AN} \\ Y_{\phi A} & Y_{\phi\phi} & Y\phi N \\ Y_{NA} & 0 & Y_{NN}+s \end{bmatrix} \begin{bmatrix} \Delta A \\ \Delta\phi \\ \Delta N \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \Delta J \end{bmatrix} \tag{5.43}$$

Where,

$$Y_{AA} = \frac{K_C A_{inj}\cos(\phi_0)}{A_0} + \frac{F_A(t)}{A_0}$$

$$Y_{A\phi} = K_C A_{inj}\sin(\phi_0)$$

$$Y_{AN} = -\tfrac{1}{2}G_0 A_0$$

$$Y_{\phi A} = \frac{-K_C A_{inj}\sin(\phi_0)}{{A_0}^2}$$

$$Y_{\phi\phi} = \frac{K_C A_{inj}\cos(\phi_0)}{A_0}$$

$$Y_{\phi N} = -\tfrac{1}{2}\alpha G_0$$

$$Y_{NA} = 2A_0\frac{1}{\tau_P} - 4K_C A_{inj}\cos(\phi_0) - 4F_A(t)$$

$$Y_{NN} = \frac{1}{\tau_N} + G_0{A_0}^2$$

The matrix terms presented above is in the form of Y_{mn}, which denotes the change of m with the perturbations of n.

The determinant of the Eq. 5.43 plays a major role, when it is finding the resonance frequency of the injection locked semiconductor laser. The determinant for the Eq. 5.43 can be presented as,

$$D'(s) = S^3 + A's^2 + B' + C' \tag{5.44}$$

Where,

$$A' = Y_{AA} + Y_{\phi\phi} + Y_{NN} \tag{5.45}$$

$$B' = Y_{AA}Y_{\phi\phi} + Y_{AA}Y_{NN} + Y_{\phi\phi}Y_{NN} - Y_{A\phi}Y_{\phi A} - Y_{AN}Y_{NA} \tag{5.46}$$

$$C' = Y_{AA}Y_{\phi\phi}Y_{NN} + Y_{A\phi}Y_{\phi N}Y_{NA} - Y_{A\phi}Y_{\phi A}Y_{NN} - Y_{AN}Y_{NA}Y_{\phi\phi} \tag{5.47}$$

As similar to Eq. 5.10, this equation is consists of three roots. Two of them are complex conjugate roots while other root is negative and real.

The GHz and THz are the frequency ranges which are utilizing for the most of today's modulation frequency applications. Hence C' can be neglected as it is holding a relatively smaller value compared to the other three terms in the equation. It can be denoted as,

$$C' << s^3, A'S^2, B's \tag{5.48}$$

Pure driving frequencies can be found by finding the solutions for the Eq. 5.44, as denoted in the equation below.

$$D'(i\omega) \approx i\omega(i^2\omega^2 + i\omega A' + B') = i\omega(-\omega^2 + i\omega A' + B') \tag{5.49}$$

When considering the Eq. 5.49, the determinant takes the form of the classical damped oscillator, which can be represented by the form of Eq. 5.16. Hence the resonance frequency and the damping rate can be presented as,

$$\omega_{ROIL}^2 = B' \tag{5.50}$$

$$\gamma_{OIL} = A' \tag{5.51}$$

Where,

ω_{ROIL} = The resonance frequency of noise added injection locked semiconductor laser.

γ_{OIL} = The damping rate of the relaxation oscillations of the optical injection locked semiconductor laser

When considering the Eq. 5.46, first three terms in the right-hand side of the equation denotes the diagonal matrix terms which are representing the state variables. These state variables show the effect of itself and they are negligible as they are having weak responses [19]. Hence, only the last two non-diagonal terms are accounting for the B' and it can be represented as,

$$\omega_{ROIL}^2 \approx B' \approx -Y_{AN}Y_{NA} - Y_{A\phi}Y_{\phi A} \tag{5.52}$$

When referring to the Eq. 5.52, two terms from the matrix given by the Eq. 5.43 are effecting the resonance frequency of the optical injection-locked semiconductor laser. The first term of the right-hand side which is given by $-Y_{AN}Y_{NA}$ can be expanded as,

$$-Y_{AN}Y_{NA} = \frac{G_0 A_0}{2}\Big(\frac{2A_0}{\tau_P} - 4K_C A_{inj}\cos(\phi_0) - 4F_A(t)\Big) \tag{5.53}$$

When substituting the relaxation oscillation frequency equation of free running laser which is presenting by the Eq. 5.21 on Eq. 5.53,

$$-Y_{AN}Y_{NA} = \omega_{R0}^2\Big(1 - \frac{2K_C\tau_P A_{inj}\cos(\phi_0)}{A_0} - \frac{2\tau_P F_A(t)}{A_0}\Big) \tag{5.54}$$

Generally the term $\frac{2K_C\gamma_P A_{inj}\cos(\phi_0)}{A_0}$ is holding a negligible value compared to 1. Hence the term $-Y_{AN}Y_{NA}$ can be expressed as,

$$-Y_{AN}Y_{NA} = \omega_{R0}^2\Big(1 - \frac{2\gamma_P F_A(t)}{A_0}\Big) \tag{5.55}$$

If $F_A(t) << A_0$, then the term $\frac{2\gamma_P F_A(t)}{A_0}$ became zero. Hence the term $-Y_{AN}Y_{NA}$ can be represent as,

$$-Y_{AN}Y_{NA} \approx \omega_{R0}^2 \tag{5.56}$$

When considering the second term of Eq. 5.52, it can be presented as,

$$-Y_{A\phi}Y_{\phi A} = \left\{\frac{K_C A_{inj}\sin(\phi_0)}{A_0}\right\}^2 = \Delta\omega_{ROIL}{}^2 \tag{5.57}$$

By considering the steady state condition of Eq. 5.42,

$$0 = \frac{\alpha G_0[N_0 - Nth]}{2} - K_C\frac{A_{inj}}{A_0}\sin(\phi_0) - 2\pi\Delta f + F_\phi(t) \tag{5.58}$$

With substituting the Eq. 5.57 on Eq. 5.58, can obtain an equation for the resonance frequency variation as,

$$\Delta\omega_{ROIL} = \left\| \frac{\alpha G_0[N_0 - Nth]}{2} - 2\pi\Delta f + F_\phi(t) \right\| \tag{5.59}$$

Hence, when $F_A(t) << A_0$ the resonance frequency can be denoted as,

$${\omega_{ROIL}}^2 \approx {\omega_{R0}}^2 + {\Delta\omega_{ROIL}}^2 \tag{5.60}$$

But if $F_A(t)$ is considerable with compared to A_0, then the resonance frequency becomes,

$${\omega_{ROIL}}^2 \approx \omega_{R0}^2\Big(1 - \frac{2\tau_P F_A(t)}{A_0}\Big) + {\Delta\omega_{ROIL}}^2 \tag{5.61}$$

The damping term, which is presenting the resonance frequency height can be expressed as,

$$\gamma_{OIL} = A' = Y_{AA} + Y_{\phi\phi} + Y_{NN} \tag{5.62}$$

The Eq. 5.62 can be expanded as,

$$\gamma_{OIL} = \frac{2K_C A_{inj}\cos(\phi_0)}{A_0} + \frac{F_A(t)}{A_0} + \frac{1}{\tau_N} + G_0{A_0}^2 \tag{5.63}$$

By substituting damping term equation for free running semiconductor lasers which is denoting by the Eq. 5.29 on above equation,

$$\gamma_{OIL} = \gamma_0 + \frac{2K_C A_{inj}\cos(\phi_0)}{A_0} + \frac{F_A(t)}{A_0} \tag{5.64}$$

By considering the steady state condition of the Eq. 5.40,

$$0 = \frac{1}{2}G_0[N_0 - N_{th}] + K_C A_{inj}\cos(\phi_0) + F_A(t) \tag{5.65}$$

By substituting values of Eq. 5.65 on Eq. 5.64,

$$\gamma_{OIL} = \gamma_0 + G_0[N_{th} - N_0] - \frac{F_A(t)}{A_0} \tag{5.66}$$

As the bandwidth (BW) of OIL semiconductor laser can be presented as a sum of resonance frequency and half of the damping factor, the bandwidth can be expressed as,

$$BW = \sqrt{\omega_{R0}^2\Big\{1 - \frac{2\gamma_P F_A(t)}{A_0}\Big) + \Delta\omega_{ROIL}{}^2} - \frac{1}{2}\left(\gamma_0 + G_0[N_{th} - N_0] - \frac{F_A(t)}{A_0}\right) \tag{5.67}$$

The Eq. 5.67 is valid only if $F_A(t)$ is holding a considerable value with compared to A_0. If $F_A(t) << A_0$ bandwidth becomes,

$$BW = \sqrt{\omega_{R0}^2 + \Delta\omega_{ROIL}{}^2} - \frac{1}{2}\left(\gamma_0 + G_0[N_{th} - N_0] - \frac{F_A(t)}{A_0}\right) \tag{5.68}$$

Chapter 6

CHARACTERIZATION OF BANDWIDTH

When considering the laser applications, the bandwidth plays a major role as it is a considerable factor in most of the laser applications. Hence it is very important to conduct a bandwidth characterization to identify the impact of noises on the bandwidth.

The values for the bandwidth characterization was taken from the Lau's thesis and they are [19],

α	The linewidth enhancement factor	3
G_0	The linear gain coefficient	$5667S^{-1}$
N_{th}	The threshold carrier number	2.214×10^8
N_0	The carrier number at the free running state	5×10^8
τ_N	The carrier lifetime	$1 \times 10^{-9}s$
τ_P	The photon lifetime	$\frac{1}{333} \times 10^{-9}s$
K_C	The internal coupling rate	$183 \times 10^9 S^{-1}$
S_0	The photon number in the cavity at the steady state	1.6×10^5

6.1 Effect of Intensity Noise

The intensity noise and the field magnitude noise have a direct relationship, and this relationship is denoted by the Eq. 5.32. Hence the intensity noise effect on the bandwidth can be identified by considering the field magnitude noise.

When considering the noise included bandwidth equation, it consists of the terms of field magnitude noise and phase noise. When identifying the intensity noise effect on the bandwidth, the phase noise term has to be ignored. The phase noise only effects on the resonance frequency of the OIL semiconductor laser. Hence, the term $\Delta\omega_{ROIL}$ can be modified as,

$$\Delta\omega_{ROIL1} = \left| \frac{\alpha G_0[N_0 - N_{th}]}{2} - 2\pi\Delta f \right| \tag{6.1}$$

The resonance frequency of the OIL semiconductor laser becomes as follows,

$$\omega_{ROIL1} = \sqrt{\frac{G_0{A_0}^2}{\tau_P}\left\{1 - \frac{2\tau_P F_A(t)}{A_0}\right\} + \left\{\frac{\alpha G_0[N_0 - N_{th}]}{2} - 2\pi\Delta f\right\}^2} \tag{6.2}$$

The damping term with the intensity noise can be presented as,

$$\gamma_{OIL1} = \left\{\frac{1}{\tau_N} + G_0{A_0}^2 + G_0[N_{th} - N_0] - F_A(t)\right\} \tag{6.3}$$

The equation for the bandwidth with only including the intensity noise effect can be written as,

$$BW_{intensity} = \omega_{ROIL1} - \frac{1}{2}\gamma_{OIL1} \tag{6.4}$$

The bandwidth equation which is denoted by the Eq. 5.67, it shows a bandwidth dependency on the detuning frequency of the OIL semiconductor laser. As this research section based on the OIL laser's bandwidth characteristics, it should be considered the place where it achieved 100% injection locking. The 100% locking is achieved when the detuning frequency is zero. Hence, this intensity noise analysis conducted firstly for the zero detuning frequency.

As per the Eq. 6.2 the resonance frequency consists of dependency on the intensity noise. It is shown in the Fig. 6.

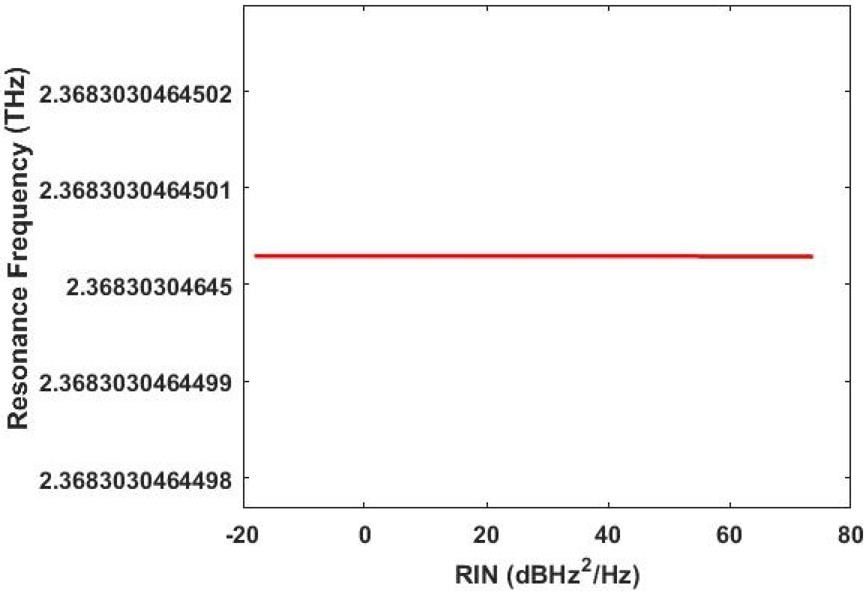

Figure 6.1: Effect of RIN on the resonance frequency of the OIL semiconductor laser

Referring to Fig. 6.2, the resonance frequency holds a constant value for the presented relative intensity noises. But there exist slight decrements with the increment of the RIN, which is negligible for the considering intensity noise range.

With considering the Eq. 6.3, the effect of intensity noise on the damping term can be identified. Fig. 6.2 shows the dependency of the damping term on the relative intensity noise which is plotted with the use of Eq. 6.3.

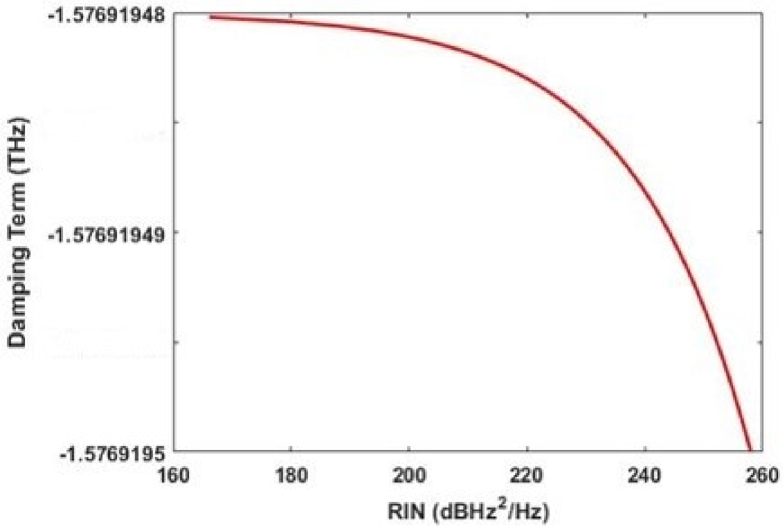

Figure 6.2: Effect of RIN on the damping term of the OIL semiconductor laser

According to Fig. 6.2, it shows damping term decrement with the increment of relative intensity noise. This decrement is occuring in the fractional values when it is taking in the THz region, which indicates a variation in the ranges of KHz or MHz.

The effect of intensity noise on the bandwidth is denoted by the Eq. 6.4, and the relationship is depicted using Fig. 6.3 below.

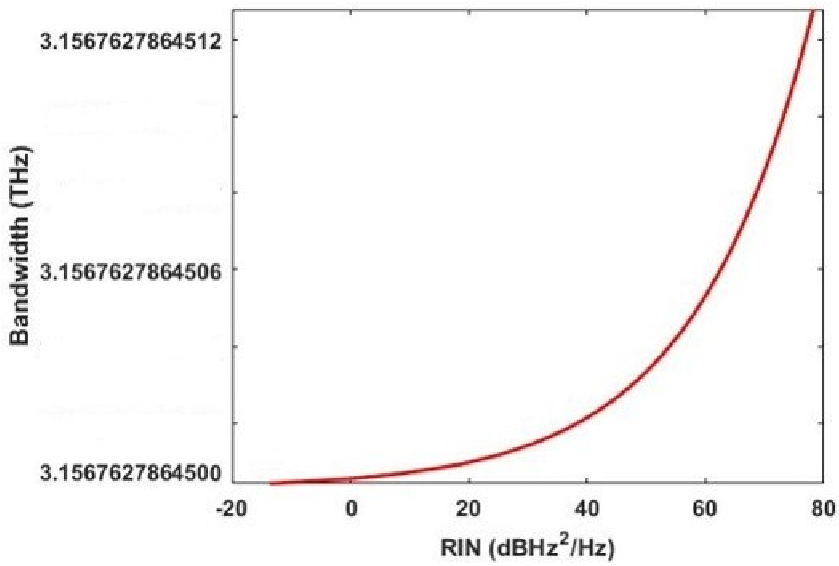

Figure 6.3: Effect of RIN on the bandwidth of the OIL semiconductor laser

As per Fig. 6.3, it depicts the increment of bandwidth with the effect of intensity noise. This increment is occuring in the fractional values when it is taking in the THz region, which indicates a variation in the range of Hz. The bandwidth can be denoted as a result of the combined effects of the resonance frequency and the damping term. According to the figures presented for the resonance frequency and damping term, the resonance frequency effect is negligible. Hence, the total intensity noise effect for the bandwidth is caused by the damping effect, which is possessing an inverse relationship.

This process can be explained using quantum mechanical processes. The photon concentration inside the slave laser's cavity increases as a result of OIL, and this resulted in bandwidth enhancement. But according to the literature, the resonance frequency has no considerable effect from this photon density increment from the injected photon field towards the slave laser cavity. Hence, this bandwidth enhancement cannot be explained using the resonance frequency of the OIL semiconductor laser [43]. The injected photon field effects the damping term, and this resulted in the bandwidth enhancement in the OIL semiconductor laser. Hence, with the intensity noise effect, it increases the photon density inside the slave laser cavity, which is resulting in the damping term decrement. As the damping term and bandwidth holds an inverse relationship, with the damping term decrement, it causes an increment of bandwidth with the intensity noise increment.

.

Up to this point the bandwidth analysis considered only for the zero detuning frequency as it is the state where the two lasers hold perfect locking. The analysis of intensity noise effect on the bandwidth for different detuning frequency is also important. Hence, the figure below shows the bandwidth variation with the intensity noise effect for different injection ratios.

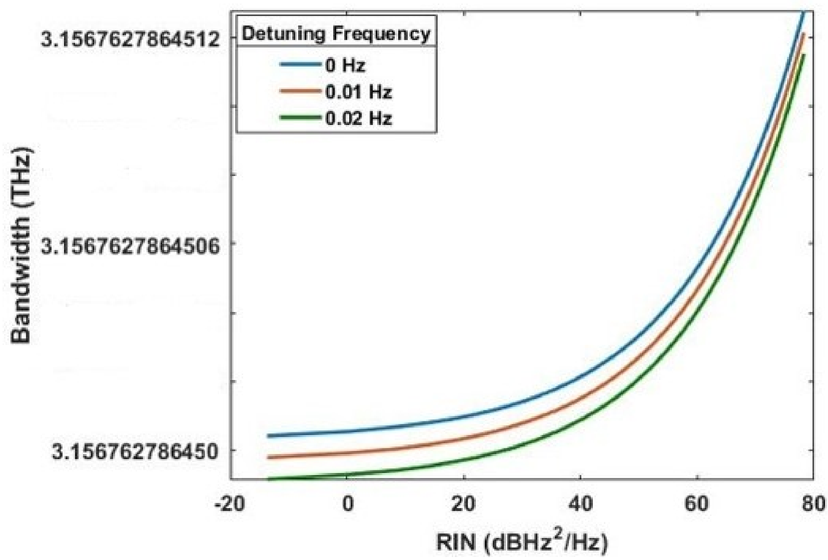

Figure 6.4: Effect of RIN on bandwidth for different injection ratios

The decrement of the bandwidth with the detuning frequency increment can be observed in Fig. 6.4. When considering the detuning frequencies, the perfect detuning can only be achieved at the zero detuning. With the increment of detuning frequency value, the status of the optical injection locking deviates from locking state to an unlocking state. With the OIL the semiconductor laser's bandwidth gets enhanced, when it is moving from locking status to the unlocking regime the bandwidth reduction can be observed. As a result of this, the bandwidth decrement can be observed with the detuning frequency enhancement in the above figure.

With the increment of RIN, the bandwidth gets enhanced while it reduces the bandwidth variation with the effect of detuning frequency. This can be explained by the reduction of the effect of additional photons while it is moving from locking to unlocking state.

6.2 Effect of Phase Noise

When considering the phase noise, the field magnitude noise effect which is denoting the intensity noise effects has to be neglected. As per the Eq. 5.59, the phase noise is only effecting the resonance frequency shift while there exists no effect for the damping term from the phase noise.

The resonance frequency with the phase noise effect can be denoted as,

$$\omega_{ROIL2} = \sqrt{\frac{G_0 {A_0}^2}{\tau_P} + \left\{ \frac{\alpha G_0 [N_0 - N_{th}]}{2} - 2\pi \Delta f + F_\phi(t) \right\}^2} \tag{6.5}$$

Under this scenario, the model neglects the magnitude noise effect. Hence, the damping term modified as,

$$\gamma_{OIL2} = \frac{1}{\tau_N} + G_0 {A_0}^2 + G_0 \{N_{th} - N_0\} \tag{6.6}$$

The bandwidth model with considering only the phase noise effect can be represented as,

$$BW_{Phase} = \omega_{ROIL2} - \frac{1}{2}\gamma_{OIL2} \tag{6.7}$$

As per the Eq. 6.7, the bandwidth shows a dependency on the detuning frequency of the OIL semiconductor laser. Firstly, the analysis was conducted for the zero detuning frequency, which caused the perfect injection locking.

This analysis is conducting for the phase noise range of $-\pi$ to π per second phase variation.This has been selected as it is the possible phase changing range when considering the OIL scenario.

With considering the Eq. 6.5, the effect of phase noise on the resonance frequency can be denoted by,

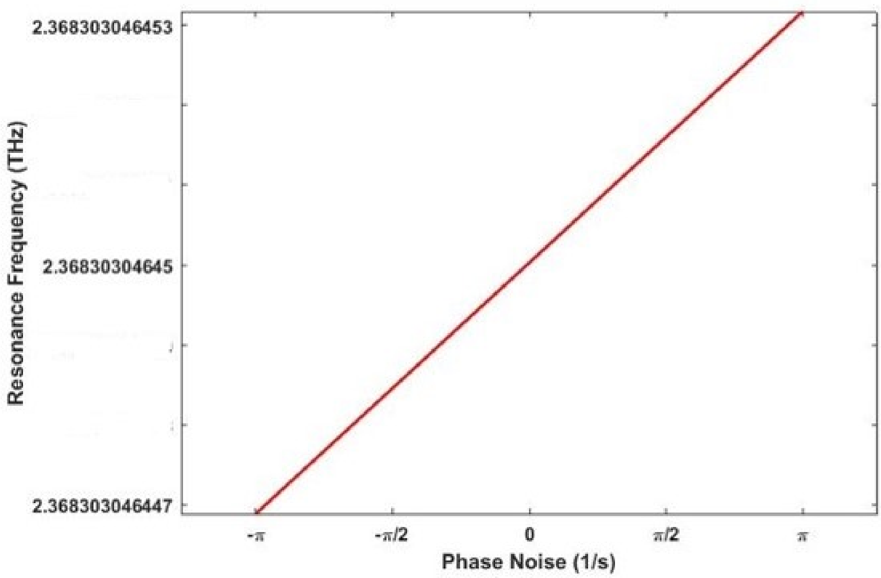

Figure 6.5: Effect of phase noise on the resonance frequency of the OIL semiconductor laser

With referring to Fig. 6.5, the resonance frequency causes for an enhancement when it moves from $-\pi s^{-1}$ phase noise to πs^{-1} phase noise. In general, the phase variation is referred to as frequency variation, which is resulted in a resonance frequency variation [79]. Hence this resonance frequency variation is induced as a result of the phase variation

There exist no effect from the phase noise on the damping term. Hence the bandwidth totally depends on the resonance frequency variation which is induced by the phase noise.The bandwidth variation with the effect of phase noise can be presented as,

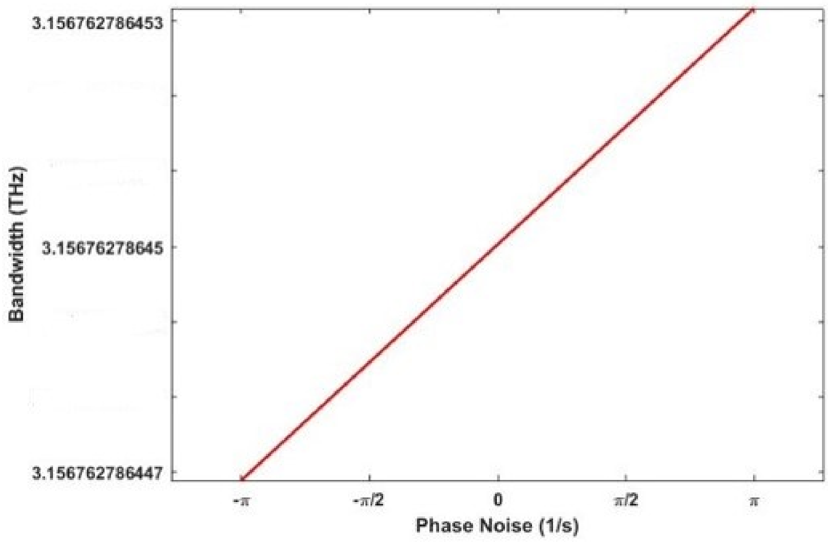

Figure 6.6: Effect of phase noise on the bandwidth of the OIL semiconductor laser

The bandwidth also follows the same variation of resonance frequency with the effect of phase noise. When the phase is causes a positive phase change, the bandwidth gets widened, and when it obtained a negative phase change, the bandwidth gets narrower. This scenario can be explained by using the quantum mechanical processes which are occurring inside the slave laser cavity. When it causes for a phase change, it induces a variation in the in-phase number of photons and out of phase number of photons with the master laser's injected photon beam. Depending on the injected photon from the master laser, it varies the coherency of the slave laser's photon field. This photon density variation resulted in a variation in the carrier density, and the carrier density variation resulted in a refractive index change inside the salve laser cavity. As a result of all of these, the resonance frequency shift will be induced. Hence, it causes a bandwidth variation.

As the detuning frequency has a dependency on the bandwidth it is worth studying the behavior with the effect of phase noise.

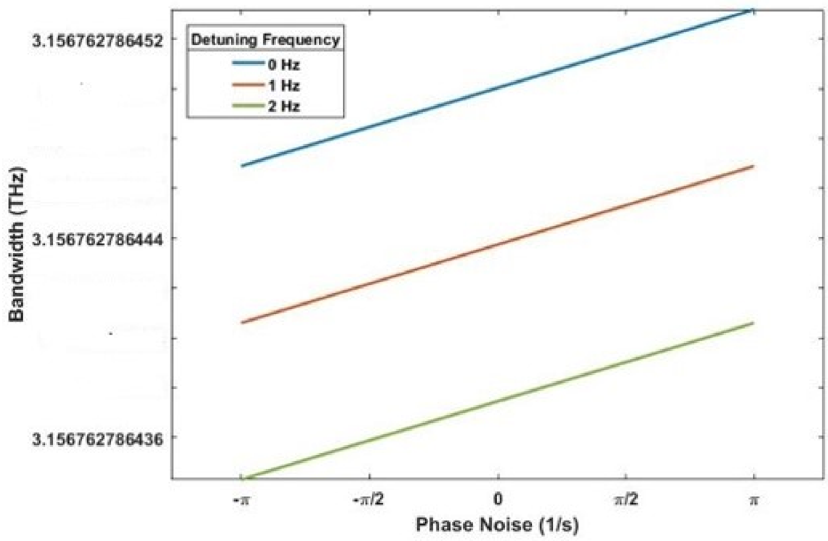

Figure 6.7: Effect of phase noise on the bandwidth for different detuning frequencies

According to Fig. 6.7, when considering the zero detuning frequency the bandwidth holds different values. With the phase noise effect, it follows the same shape variations which are parallel to each other. The difference between parallel lines depends on the detuning frequency. When increasing the detuning frequency, it reduces the bandwidth as the system is moving from perfect locking state to unlocking state direction.

6.3 Collective Noise Effect

The collective effect of intensity and phase noise on the resonance frequency is presented by the Eq. 5.61. Hence the figure below depicts the combined effect on the resonance frequency,

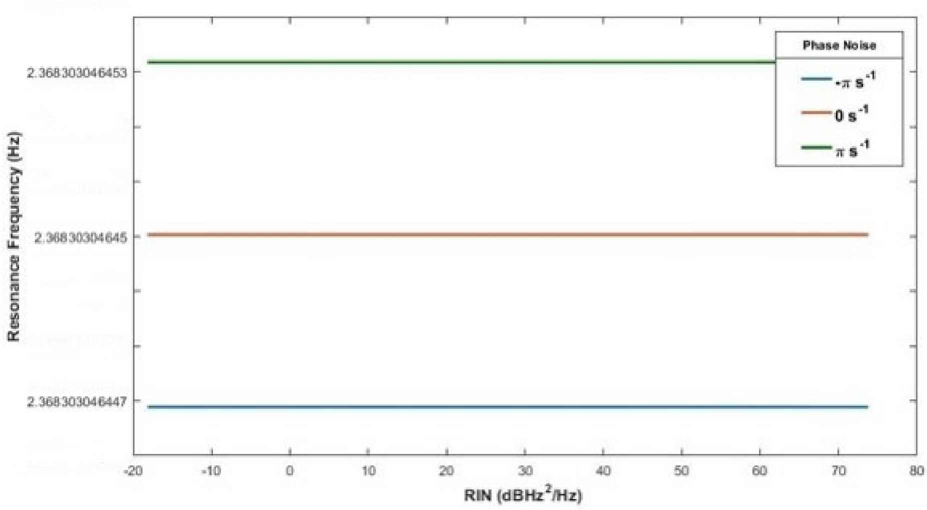

Figure 6.8: Collective effect of intensity and phase noises on the resonance frequency of OIL semiconductor laser

When considering the above graph, it obeys a similar patter as Fig. 6.1. Hence it depicts that the collective effect is led by the intensity noise effect and the phase noise effect is negligible.

Damping term has no effect from the phase noise. Hence, it is 100% depends on the intensity noise and collective effect on the damping term is equal to Fig. 6.2.

The collective effect of intensity and phase noise on bandwidth is depicts by the figure below,

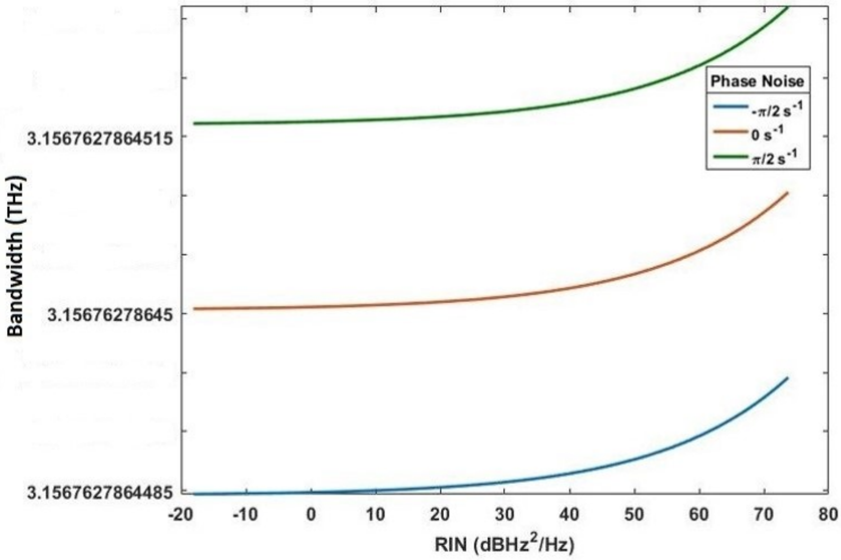

Figure 6.9: Collective effect of intensity and phase noise on the bandwidth of OIL semiconductor laser

When referring the Fig. 6.9, it indicates the combined effect of intensity and phase noise. With the effect of phase noise, the effect from the intensity noise reduced from considerable amount.

Chapter 7

CONCLUSION AND FUTURE WORK

This chapter briefs the whole research while reviewing the major findings. The last part provides the future works to be carried out when extending this research. This research has been conducted under two categories namely locking range characterization and bandwidth characterization.

7.1 Findings Relates to the Locking Range

Under the locking range characterization, firstly derived the equation for the locking range with modifying the rate equations with adding the Langevin noise sources. The derivation presented the locking range dependency on the intensity noise and the phase noises, while it denoted the locking range independency from carrier noises.

According to the simulation results obtained from the developed theoretical model, identified that the intensity noise causes a widening or narrowing of the locking range width. With the influence of positive intensity noise, the locking range gets widened by the movement of lower boundary downwards through the detuning frequency axis. The negative intensity noise leads to an upwards shift of a lower boundary through the detuning frequency axis. As a result, it narrowing the locking range width. The intensity noise does not influence on the upper boundary of the locking range. Hence the total effect is generated by the lower boundary shift. But, these effects are valid only for the lower injection ratios. For higher injection ratios intensity noise make a negligible effect. Through the deeper analysis conducted using the intensity noise effect identified that the intensity noise is higher for smaller locking range widths than the higher locking range widths.

With the effect of phase noises, it cases for a locking range movement through the detuning frequency axis. With the effect of positive phase noise, the locking range shift upwards through the detuning frequency axis, while negative phase noise shifting the locking range downwards through the detuning frequency axis. The boundaries are shifting from the same amount through the detuning frequency axis with the influence of phase noise. Hence, the phase noise do note resulted in a locking range width variation. But, the phase noise effect is observable for the lower injection ratios, and for higher injection ratios the phase noise effect is negligible.

The collective effect can be observed while applying both noises. Hence, with the influence of intensity and phase noises, the locking range can be resulted in widening or narrowing while moving the locking range boundaries upwards or downwards through the detuning frequency axis.

As per the obtained results, a system that is in the locking range can be moved to an unlocking state and vise versa with the influence of the noises. As the effect of noise is negligible for higher injection ratios and considerable for lower injection ratios, these noise effects are only applicable for low injection ratio applications.

7.2 Findings Relates to the Bandwidth

The bandwidth characterization has started with deriving an equation for bandwidth with adding the Langevin noise terms. This derivation denotes bandwidth independence of carrier noise while it indicates a dependency of intensity and phase noises.

When considering the bandwidth with the effect of intensity noise, it indicates an increment of bandwidth with the intensity noise. This intensity noise increment is lead by the damping term variation, which is occurring as a result of photon number change inside the slave laser cavity. The resonance frequency variation does not make a considerable contribution towards the bandwidth variation of the OIL semiconductor laser. As the detuning frequency contributing to the bandwidth of OIL semiconductor laser, the research carried out an analysis to identify the noise effect for different detuning frequencies. According to the result obtained from that analysis, with the increment of detuning frequency, the noise effect on bandwidth gets reduced.

The research identified that with the phase noise effect the bandwidth gets enhanced for positive phase noises while it reduces for negative phase noises. There exist no effect from the damping effect from the phase noise. Hence, this effect is influenced by the resonance frequency changes, which are governing by the phase noises. When considering the detuning frequency effect with the phase noise, detuning frequency enhancement reduces the bandwidth without influencing on the phase noise effect.

When considering the collective effect of intensity and phase noise on the bandwidth of OIL semiconductor laser bandwidth gets enhanced with the influence of RIN while this variation moved upwards or downwards through the detuning frequency axis with the effect of phase noise. Hence, depending on the phase noise and intensity noise, the bandwidth is causing a considerable change.
.

As this phase and intensity noise affecting bandwidths in the THz region, these effects are considered for the optical communication-related OIL applications.

7.3 Future Work

The theoretical part of the research is already finished. But further analysis can be carried out for the bandwidth analysis as the bandwidth analysis falls on the THz region. THz is the current trend in laser technology as it is utilizing for optical communication and some other applications as well.

This research is completed with the use of theoretical model. To prove the result further practically implementation stage is very important. Hence research can be extended to prove the result using experimental setup.

References

[1] S. C. Singh, H. Zeng, C. Guo, and W. Cai, *Nanomaterials: Processing and Characterization with Lasers*, fifth edition ed. Wiley-VCH Verlag GmbH Co. KGaA, 2012.

[2] W. D. Sacher, E. J. Zhang, B. A. Kruger, and J. K. S. Poon, "High-speed laser modulation beyond the relaxation resonance frequency limit," *Optics Express*, vol. 18, no. 7, pp. 7047–7054, March 2010.

[3] R. N. Clarke, "Expanding mobile wireless capacity: The challenges presented by technology and economics," *Telecommunication Policy*, vol. 38, no. 8-9, pp. 693–708, September 2014.

[4] M. I. Nathan, "Semiconductor lasers," *Applied Optics*, vol. 05, no. 10, pp. 1514–1528, October 1966.

[5] H. G. Shiraz, *The principle of semiconductor laser diodes and amplifiers.* Imperial College Press, 2004.

[6] T. Suhara, *Semiconductor laser fundamentals.* Marcle Dekker, Inc., 2004.

[7] G. P. Agrawal and N. K. Dutta, *Semiconductor Lasers*, second edition ed. Kluwer Acadamic Publishers, 2001.

[8] D. K. Koltzkin, *Introduction to Semiconductor Lasers for Optical Communications.* Springer New York Heidelberg Dordrecht London, 2014.

[9] J. M. Pol, "Semiconductor laser dynamics: Compound cavity, polarization and transverse modes," Ph.D. dissertation, 2002.

[10] j. Henry, C. H., "Theory of the linewidth of semiconductor lasers."

[11] B. V. Zeghbroeck, *Principles of Semiconductor Devices.* University of Colorado, 2007.

[12] K. Iniewski, *Integrated Microsystems.* FL: CRC Press, Taylor Francis Group, 2011.

[13] D. Parekh, “Optical injection locking on vertical cavity surface emitting lasers: Physics and applications,” Ph.D. dissertation, Berkley, 2012.

[14] A. Ng’oma, *Radio over Fiber Technology for Broadband Wireless Communication Syetems*. Netherland, 2005.

[15] R. Adler, “A study of locking phenomena in oscillators,” *Proceedings of IRE*, vol. 34, pp. 351–357, 1946.

[16] R. H. Pantell, “The laser oscillator with an external signal,” *Proceedings of the IEEE*, vol. 53, no. 5, pp. 474–477, 1965.

[17] H. L. Strover and W. H. Steier, “Locking of laser oscillators by light injection,” *Applied Physics Letters*, vol. 8, pp. 91–93, 1966.

[18] S. Kobayashi and T. Kimura, “Coherence on injection phase-locked algaas semiconductor laser,” *Electronics Letters*, vol. 16, pp. 668–670, 1980.

[19] E. K. Lau, “High speed modulation of optical injection locked semiconductor lasers,” Ph.D. dissertation, Berkley, 2006.

[20] R. Lang, “Injection locking properties of a semiconductor laser,” *IEEE Journal of Quantum Electronics*, vol. QE-18, pp. 976–983, 1982.

[21] C. H. Henry, N. A. Olsson, and N. K. Dutta, “Locking range and stability of injection locked 1.54 m ingaasp semiconductor lasers,” *IEEE Journal of Quantum Electronics*, vol. QE-21, pp. 1152–1156, 1985.

[22] J. Ohtsubo, *Semiconductor Lasers*, second, enlarged edition ed. Springers Publishers, 2008.

[23] F. Mogensen, H. Olesen, and G. Jacobsen, “Fm noise suppression and linewidth reduction in an injection-locked semiconductor laser,” *Electronics Letters*, vol. 21, no. 16, pp. 696–697, August 1985.

[24] M. C. Espana-Boquera and A. Puerta-Notario, “Noise efeects in injection locked lasers simulation: Phase jumps and associated spectral component,” *Electronics Letters*, vol. 32, no. 9, pp. 818–819, April 1996.

[25] N. Schunk and K. Petermann, “Noise analysis of injection-locked semiconductor injection laser,” *IEEE Journal of Quantum Electronics*, vol. 22, no. 5, pp. 642–650, May 1986.

[26] W. W. chow, "Phaser locking of lasers by an injected signal," *Optics Letters*, vol. 7, no. 9, pp. 417–419, September 1982.

[27] E. K. Lau, L. J. Wong, and M. C. Wu, "Enhanced modulation characteristics of optical injection-locked lasers: A tutorial," *IEEE Journal of Selected Topics in Quantum Electronics*, vol. 15, no. 3, pp. 618–633, May/June 2009.

[28] P. Wang, J. Xiong, T. Zhang, D. Chen, P. Xiang, J. Zheng, Y. Zhang, R. Li, L. Huang, T. Pu, and X. Cheng, "Frequency tunable optoelectronic oscillator based on a directly modulated dfb semiconductor laser under optical injection," *Optics Express*, vol. 23, pp. 20 450–20 458, 2015.

[29] C. Wang, K. Schires, M. Osinski, P. J. Poole, and F. Grillot, "Thermally insensitive determination of the linewidth broadening factor in nanostructured semiconductor lasers using optical injection locking," *Nature Research*, vol. 6, no. 27825, pp. 1–8, June 2016.

[30] E. K. Lau, H. K. Sung, and M. C. Wu, *Fundamentals of optical injection locking*, C. H. Lee, Ed. CRC Press, Taylor Francis Group, 2013.

[31] I. Petitbon, P. Gallion, G. Debarge, and C. Chabran, "Locking bandwidth and relaxation oscillations of an injection locked semiconductor laser," *Electronics leters*, vol. 22, no. 17, pp. 889–890, August 1986.

[32] A. C. Bordonalli, M. J. Fice, and A. J. Seeds, "Optical injection locking to iptical frequency combs for superchannel coherent detection," *Optics Express*, vol. 23, no. 2, pp. 1547–1557, January 2015.

[33] X. Zhao, "Optical injection locking on vertical cavity surface emitting lasers: Physics and applications," Ph.D. dissertation, 2008.

[34] G. Liu, X. Jin, and S. L. Chuang, "Measurements of linewidth enhancement factor of semiconductor lasers using an injection locking technique," *IEEE photonics technology leters*, vol. 13, no. 5, pp. 430–432, May 2001.

[35] F. Mogensen, H. Olesen, and G. Jacobsen, "Locking conditions and stability properties for a semiconductor laser with external light injection," *IEEE Journal of Quantum Electronicss*, vol. 21, no. 7, pp. 784–793, July 1985.

[36] J. Wang, M. K. Haldar, L. Li, and F. V. C. Mendis, "Enhancement of modulation bandwidth of laser diodes by injection locking," *IEEE Photonics Technology Letters*, vol. 8, no. 1, pp. 34–36, January 1996.

[37] J. Ohtsubo, *Semiconductor Lasers: Stability, Instability and Chaos*. Springer International Publishing, 2017.

[38] L. Li, "Small signal analysis of optical fm signal simplification by an injection locked type semiconductor laser amplifier," *Journal of Light Wave Technology*, vol. 10, no. 04, pp. 477–481, April 1992.

[39] R. Paschotta, H. R. Telle, and U. Keller, *Solid State Lasers and Applications*. CRC Press, 2007.

[40] H. Simos, A. Bogris, D. Syvridis, and W. Elsasser, "Intensity noise properties of mid-infrared injection locked quantum cascade lasers: I. modeling," *IEEE Journal of Quantum Electronics*, vol. 50, no. 2, pp. 98–105, February 2014.

[41] J. Liu, M. X. J. Chen, H. F., and T. B. Simpson, "Modulation bandwidth, noise and stability of a semiconductor laser subject to strong injection locking," *IEEE Photonic Technology Letters*, vol. 9, pp. 1325–1327, 1997.

[42] E. K. Lau, H. K. Sung, and C. W. Ming, "Frequency response enhancement of optical injection locked lasers," *IEEE Journal of Quantum Electronics*, vol. 44, pp. 90–101, 2008.

[43] A. Murakami, K. Kawashima, and K. Atsuki, "Cavity resonance shift and bandwidth enhancement in semiconductor lasers with strong light injection," *IEEE Journal of Quantum Electronics*, vol. 39, no. 10, pp. 1196–1204, October 2003.

[44] P. Bardella, W. W. Chow, and I. Montrosset, "Design and analysis of enhanced modulation response in integrated coupled cavities dbr lasers using photon-photon resonance," *Photonics*, vol. 03, pp. 1–14, 2016.

[45] W. Ma, B. Xiong, C. Sun, X. Ke, J. Wang, Z. Hao, L. Wang, Y. Han, H. Li, J. Yu, and J. Luo, "Linewidth narrowing of mutually injection locked semiconductor lasers with short and long delay," *Applied Science*, vol. 09, pp. 9 071 436–907 150, 2019.

[46] K. Kobayashi and I. Mito, "Single frequency and tunable laser diodes," *Journal of Lightwave Technology*, vol. 6, pp. 1623–1633, 1988.

[47] M. Bhattacharya, R. Mukherjee, and T. Chattopadhyay, "Frequency chirping in semiconductor lasers through injection locking," *IETE Journal of Research*, vol. 45, pp. 341–344, 1999.

[48] P. Krehlik, "Characterization of semiconductor laser frequency chirp based on signal distortion in dispersive optical fiber," *Opto Electronics Reviews*, vol. 14, pp. 119–124, 2006.

[49] R. Hui and M. O'Sullivan, *Fiber Optics Measurement Techniques.* Elsevire Academics Press, 2009.

[50] S. N. Hosseinimotlagh, M. Nikravesh, H. Molaei, and M. Pezeshkian, "Studies on the effective physical parameters forchirp reduction in optical injection locked semiconductor lasers," *American Journal of Physics and Apllications*, vol. 2, pp. 56–60, 2014.

[51] K. Stubkjaer and M. Danielse, "Nonlinearities of gaalas lasers-harmonic distortion," *IEEE Journal of Quantum Electronics*, vol. 16, pp. 531–537, 1980.

[52] G. Yabre and J. L. Bihan, "Reduction of nonlinear distortion in directly modulated semiconductor lasers by coherent light injection," *IEEE Journal of Quantum Electronics*, vol. 33, pp. 1132–1140, 1997.

[53] J. H. Seo, Y. K. Seo, and W. Y. Choi, "Nonlinear distortion suppression in directly modulated dfb lasers by sidemode optical injection." 2001 IEEE MTT-S International Microwave Sympsoium Digest, 2001, pp. 555–558.

[54] X. J. Meng, T. Chau, and M. C. Wu, "Improved intrinsic dynamic distortions in directly modulated semiconductor lasers by optical injection locking," *IEEE Transactions on Microwave Theory and Techniques*, vol. 47, pp. 1172–1176, 1999.

[55] W. Han, N. Zhu, L. Xie, M. Ren, B. Zhang, L. Li, and G. Zhang, "The optical injection locking technique and its applications for wdm-pon spare function," *Proceedings of SPIE - The International Society for Optical Engineering*, vol. 7278, 01 2009.

[56] Y. Chi, D. Hsieh, C. Tsai, H. Chen, H. Kuo, and G. Lin, "450-nm GaN laser diode enables high-speed visible light communication with 9-Gbps QAM-OFDM," *Optics Express*, vol. 23, no. 10, pp. 13 051–13 056, May 2015.

[57] R. U. Weerasuriya, "Data retiming for multi- wavelength regeneration," Ph.D. dissertation, Cork, 2011.

[58] E. Udd and W. B. Spillman Jr, *Fiber Optics Sensors: An Introduction for Engineers and Scientists.* John Wiley Sons, Inc:Hoboken, NJ, 2011.

[59] M. Cutroneo, L. Torrisi, and C. Scolaro, "Laser applications in biomedical field." University of Messinna, Preprint 2012, pp. 144–154.

[60] N. Kavanagh, D. Goulding, and F. C. G. Gunning, "Stable injection locking with slotted fabry–perot lasers at 2 m," *Journal of Physics: Photonics*, vol. 1, p. 015005, 2018.

[61] E. M. Strzelecki and . C. L. A. Cohen, D. A., "Investigation of tunable single frequency diode lasers for sensor applications," vol. 6, 1988, pp. 1610–1618.

[62] C. J. H. Pagett, "Injection locking of a diode laser for use in an ultra-cold strontium experiment," 2015.

[63] G. P. Agrawal, "Noise in semiconductor lasers and its impact on optical communication systems," *Laser Noise*, vol. 1376, pp. 224–236, 1990.

[64] K. Thyagarajan and A. Ghatak, *Lasers : Fundamentals and Applications*, second edition ed. Springers, 2010.

[65] S. Miller and D. Childers, *Probability and Random Processes.* Elsivier Academic Press, 2004.

[66] G. P. Agrawal, "Noise in semiconductor lasers and its impact on optical communication systems," vol. 1376. San Diego, USA: Proceedings of SPIE conference, March 1991, pp. 26–33.

[67] G. P. Agawal, *Fiber-Optic communication systems*, 4th ed. A john wiley sons, Inc. Publication, 2010.

[68] N. P. Barnes and B. M. Walsh, "Amplified spontaneous emission-application to nd:yag lasers," *IEEE Journal of Quantum Electronics*, vol. 35, pp. 101–109, 1999.

[69] E. N. Gilbert and H. O. Pollak, "Amplitude distribution of shot noise," *bell System Technical Journal*, vol. 39, pp. 333–350, 1960.

[70] D. Marcuse, "Computer simulation of laser photon fluctuations: Theory of single-cavity laser," *IEEE Journal of Quantum Electronics*, vol. 20, no. 10, pp. 1139–1148, October 1984.

[71] I. Joindot, "Measurements of relative intensity noise (rin) in semiconductor lasers," *Journal of Physics III*, vol. 2, no. 9, pp. 1591–1603, September 1992.

[72] T. Gensty and W. Elsäßer, "Semiclassical model for the relative intensity noise of intersubband quantum cascade lasers. optics communications," *Optics Communications*, vol. 256, no. 1–3, pp. 171–183, 2005.

[73] C. Wang, K. Schires, M. Osiński, P. J. Poole, and F. Grillot, "Thermally insensitive determination of the linewidth broadening factor in nanostructured semiconductor lasers using optical injection locking," *A nature research journal*, vol. 6, no. 27825, June 2016.

[74] L. A. coldren, S. W. Corzine, and M. L. Masanovic, *Diode Lasers and Photonic Integrated Circuits*, second edition ed. A John Wiley Sons, 2012.

[75] S. E. Hashemi, "Relative intensity noise (RIN) in high-speed VCSELs for short reach communication," Master's thesis, Goteborg, Sweden, 2012.

[76] S. Shin, "Characterization and comparison of optical source relative intensity noise and effects in optical coherence tomography," Master's thesis, Urbana, 2010.

[77] G. S. Samarakkody, D. Munasinghe, S. Alahakoon, A. Kumarayapa, and R. Weerasuriya, "The behavioral study of an optical injection-locked semiconductor laser under the influence of intensity and phase noise." Proceedings of 2019 IEEE 5th International Conference on Wireless and Telematics, July 2019, pp. 1–6.

[78] B. Redding, M. A. Choma, and H. Cao, "Spartial coherence of random laser emission," *Optics Letters*, vol. 36, no. 17, pp. 3404–3406, September 2011.

[79] H. L. T. Lee, R. J. Ram, O. Kjebon, and R. Schatz, "Bandwidth enhancement and chirp reduction in dbr lasers by strong optical injection." Proceedings of Conference on Lasers and Electro-Optics, 2000, pp. 99–100.

APPENDIX A

Auto correlation and correlation of Langevin noise sources

The Langevin noise sources considered in this thesis are the Langevin noise source of intensity, the Langevin noise source of carriers and the Langevin noise source of phase, which are indicating by $F_S(t)$, F_N and $F_\theta(t)$ respectively. The cross-correlation provides the effect of one noise source on other noise sources. The cross-correlation between $F_s(t)$ and $F_\theta(t)$ is denoted by the $\langle F_S(t)F_\theta(t)\rangle$.

The derivation for the correlation between $\langle F_S(t)F_\theta(t)\rangle$ can be found by summing up all particle flowing towards the particle reservoir and all particles flowing outwards from the particle reservoir. Intensity and phase effect on both reservoirs at the same time.

This $\langle F_S(t)F_\theta(t)\rangle$ will be holding a positive value if and only if the particle flowing towards the reservoir and particles flowing outwards from the reservoir is correlated. The $\langle F_S(t)F_\theta(t)\rangle$ will hold a negative value when particle flowing towards the reservoir and particles flowing outwards from the reservoir is anti-correlated.

In most of the cases, one reservoir gains a particle while other reservoir losing a particle. If i reservoir gains a particle when j reservoir losses a particle, it can be indicated as [74],

When $\langle F_i(t)\rangle > 0$,$\langle F_j(t)\rangle < 0$ occurs.

Hence the product of Langevin noise sources of i and j always holds a negative value. The reason for holding a negative value is, adding a particle whole positive value while losing a particle holds a negative value.

When considering the Langevin noise sources, the cross correlation strengths can be represented by using the in and out particle flows. Hence this can be represented as [74],

$$\left\langle F_iF_j\right\rangle = -\left[\Sigma R_{ij} + \Sigma R_{ji}\right] \tag{A.1}$$

Where ΣR_{ij} denotes the particle flow from i to j, while ΣR_{ji} denotes the particle flow from j to i.

By considering the particle flows inbetween two particle reservoirs, it can be present the cross correlation between the photons, carriers and the phase as,

$$\left\langle F_N(t)F_S(t)\right\rangle = -\left\{G_0(N(t)-N_0)S(t)+\frac{S(t)}{\tau_P}\right\} \tag{A.2}$$

$$\left\langle F_N(t)F_\phi(t)\right\rangle = 0 \tag{A.3}$$

$$\left\langle F_S(t)F_\theta(t)\right\rangle = 0 \tag{A.4}$$

The auto correlation can be denotes as,

$$\left\langle F_iF_i\right\rangle = \Sigma {R_i}^{+} + \Sigma {R_i}^{-} \tag{A.5}$$

Where, $\Sigma {R_i}^{+}$ and $\Sigma {R_i}^{-}$ rate of particles flow in and out respectively.

Hence the autocorrelations of the intensity, carriers and phase can be presented as,

$$\left\langle F_s(t)^2\right\rangle = G_0(N(t)-N_0)S(t)+\frac{S(t)}{\tau_P}+2K_C\sqrt{S_{inj}S(t)}\cos\left(\phi(t)\right) \tag{A.6}$$

$$\left\langle F_\phi(t)^2 \right\rangle = \frac{\alpha}{2} G_0(N(t) - N_{th})S(t) + K_C \sqrt{\frac{S_{inj}}{S(t)}} \sin\left(\phi(t)\right) \tag{A.7}$$

$$\left\langle F_N(t)^2 \right\rangle = \frac{N(t)}{\tau_N} + \frac{S(t)}{\tau_P} + G_0(N(t) - N_0)S(t) \tag{A.8}$$

APPENDIX B

Noise Inserted Locking Range Derivation for the OIL Semiconductor Lasers

The rate equations for the photon number and phase inside the OIL slave laser cavity can be denoted as,

$$\frac{dS(t)}{dt} = G_0\Big(N(t) - N_{tr}\Big)S(t) - \frac{S(t)}{\tau_P} + 2K_C\sqrt{S_{inj}S(t)}\cos\phi(t) + F_S(t) \quad \text{(A.9)}$$

$$\frac{d\theta(t)}{dt} = \frac{\alpha}{2}G_0\big(N(t) - N_{th}\big) - 2\pi\Delta f - K_C\sqrt{\frac{S_{inj}}{S(t)}}\sin\phi(t) + F_\theta(t) \quad \text{(A.10)}$$

By subsituting $\Delta N = N(t) - N_{th}$ and $G_0(N(t) - N_{th})S(t) = G_0(N(t) - N_{tr})S(t) - \frac{S(t)}{\tau_P}$ on Eq. B.1,

$$\frac{dS(t)}{dt} = G_0\Delta N(t)S(t) + 2K_C\sqrt{S_{inj}S(t)}\cos\phi(t) + F_S(t) \quad \text{(A.11)}$$

By setting $\frac{dS(t)}{dt} = 0$ in Eq. A.11, can attain the steady state equation and by rearranging it can obtain the equation,

$$\Delta N = \frac{-2K_C}{G_0}\sqrt{\frac{S_{inj}}{S_0}}\cos\left(\phi_0\right) - \frac{F_S(t)}{G_0 S_0} \quad \text{(A.12)}$$

By substituting $\Delta N = N(t) - N_{th}$ on Eq. A.10, the equation can be rearrange as,

$$\frac{d\theta(t)}{dt} = \frac{\alpha}{2}G_0\Delta N - 2\pi\Delta f - K_C\sqrt{\frac{S_{inj}}{S(t)}}\sin\phi(t) + F_\theta(t) \tag{A.13}$$

To attain the steady state equation, it set $\frac{d\theta(t)}{dt} = 0$ into the Eq. A.13 and by rearranging it can be attain the equation,

$$2\pi\Delta f = \frac{\alpha}{2}G_0\Delta N - K_C\sqrt{\frac{S_{inj}}{S_0}}\sin(\phi_0) + F_\theta(t) \tag{A.14}$$

By substituting Eq. 5.12 on the Eq. 5.14 and simplifying can attain the equation,

$$2\pi\Delta f = -K_C\sqrt{\frac{S_{inj}}{S_0}}\Big\{\alpha\cos(\phi_0) + \sin\phi_0\Big\} - \frac{\alpha F_S(t)}{2S_0} + F_\theta(t) \tag{A.15}$$

The $\alpha cos(\phi_0) + \cos(\phi_0)$ can be denoted as,

$$\alpha cos(\phi_0) + \cos(\phi_0) = \sqrt{(1+\alpha^2)}\sin(\arctan(\alpha) + \phi_0) \tag{A.16}$$

The Eq. A.16 attains the maximum value when α achieve the maximum value and $sin(\arctan(\alpha) + \phi_0) = 1$. By substituting maximum values on Eq. A. 15 can obtain the lower boundary equation as,

$$2\pi\Delta f = -K_C\sqrt{\frac{S_{inj}(1+\alpha^2)}{S_0}} - \frac{\alpha F_S(t)}{2S_0} + F_\theta(t) \tag{A.17}$$

The Eq. A.16 attains the minimum value, when $\alpha = 0$ and $sin(\arctan(\alpha) + \phi_0) = -1$. By substituting these values on Eq. A.15, can obtain the upper boundary equation which is denoted by the,

$$2\pi\Delta f = K_C\sqrt{\frac{S_{inj}}{S_0}} + F_\theta(t) \tag{A.18}$$