MODELLING OF MAGNETIC HYSTERESIS OF A THREE PHASE TRANSFORMER UNDER DYNAMIC CONDITIONS

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Degree of Master of Science in Electrical Engineering

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Dissertation submitted in partial fulfillment of the requirements for the
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Declaration of the Candidate and Supervisor

I declare that this is my own work and this dissertation does not incorporate without acknowledgement any material previously submitted for a Degree or Diploma in any other University or institute of higher learning and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text.

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Name of the supervisor: Prof. J.P. Karunadasa

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Abstract

The phenomenon of magnetic hysteresis effect of the core material of a transformer plays an important role when representing the transformer in a simulated environment.

Input-output behavior of a transformer is critically correlated with the magnetic properties of the core material which is influenced by the magnetic hysteresis effect. To analyze the magnetizing characteristics of the core material of a transformer, it is required to develop a mathematical model for simulating the hysteresis phenomenon.

As the outcome of this research, reliable representation of magnetizing characteristics of a three-phase transformer core, without considering any core material related parameters, a test based mathematical model is simulated. Mathematical model, which capable of producing magnetic characteristics for any given voltage level is designed by conducting two tests separately for a single phase transformer and three limbs of a three phase transformer.

In the scope of this research, magnetizing characteristic model is developed using Matlab Simulink. Test data required to develop the proposed model is obtained using single phase and three transformer. Results obtained from the developed model is validated with the practically obtained outputs. Practical tests executed on transformer show close connectivity with simulated results evidencing the extensiveness of the designed model.

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Acknowledgement

I would like to express my truthful appreciation to those who were behind me in completing my research project.

I am deeply appreciative to my supervisor Pro. J.P. Karunadasa, for his continuous support, supervision, guidance and encouragement all over the period.

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Last but not least, I would express my heartiest gratitude to my husband, my parents and my siblings for their continuous support, motivation and encouragement which assisted me to complete this research study in challenging situations.

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List of Abbreviations

B	Magnetic Flux
H	Magnetic Field
JA	Jiles-Atherton
FE	Finite Element
PM	Preisach Model

CHAPTER 1

1 INTRODUCTION

1.1 Background

The most common way to symbolize magnetic properties of a ferromagnetic material is by plotting flux density B for various magnetizing forces H. The event of flux density lagging behind the magnetizing force in a magnetic material is called Magnetic Hysteresis. Magnetic hysteresis plays a significant role in certain phenomena associated with engineering applications.

Various types of magnetic materials are used in many applications. From the existing magnetic materials, ferromagnetic materials are mainly accountable for the generation of hysteresis loops. In case of transformer core, the material use exclusively in transformer core is ferromagnetic material. Therefore the effect of magnetic hysteresis should be vitally studied in order to model the transformer.

Most commonly used material of the transformer core is grain oriented silicon-ion. Ion is a very good ferromagnetic material which is very sensitive to be magnetized. Transformers are highly efficient but do convert some of the energy to hysteresis loss in the core. That means, when magnetic flux pass through the transformer core, it behaves like a magnet.

There are number of domains in the structure of ferromagnetic materials. Domains are very small particles in the material structure. They are similar to small permanent magnets positioned random manner in the structure material. These domains are positioned in the material structure in a random way. Therefore the net resultant field of the said material is zero. Once an external magnetic field or mmf is applied to that material, these randomly directed domains got positioned themselves in parallel to the axis of applied mmf. After eliminating this external mmf, most of domains again come to random positions, but some of them still remain. Because of these constant domains, the material becomes considerably magnetized permanently. This is called Spontaneous Magnetism. In order to neutralize this magnetism, some opposite mmf is needed to apply.

In the transformer, the applied magneto motive force is not constant. For every cycle, there will be an additional work done due to this domain reversal. Hence, there will be a consumption of electrical energy which is called Hysteresis loss of the transformer.

1.2 Magnetizing Characteristic Curve

Hysteresis is characterized as a lag of magnetic flux density B behind the magnetic field strength H. That is, when an initially unmagnetized ferromagnetic material is put in to a fluctuating magnetic field, the flux density formed in the material fluctuates. Therefore, to analyze magnetic characteristics of a ferromagnetic material, B-H characteristic curve is required.

The correlation between the magnetic field strength H and the magnetic flux density B can be expressed as the magnetic permeability.

$$\mu = \frac{\mathbf{B}}{\mathbf{H}}$$

$$\mu\mathbf{r} = \frac{\mu}{\mu\mathbf{0}}$$

μ – Permeability

B – Flux Density

H – Magnetic Field Strength

μ_r – Relative Permeability

μ_o – Vacuum Permeability

The ratio of flux density to field strength (B/H) in ferromagnetic materials is not constant but do vary with flux density.

By plotting flux density (B) against field strength (H), magnetizing characteristic curve (B-H curves) can be obtained.

A typical magnetizing characteristic curve is shown in Figure 1.

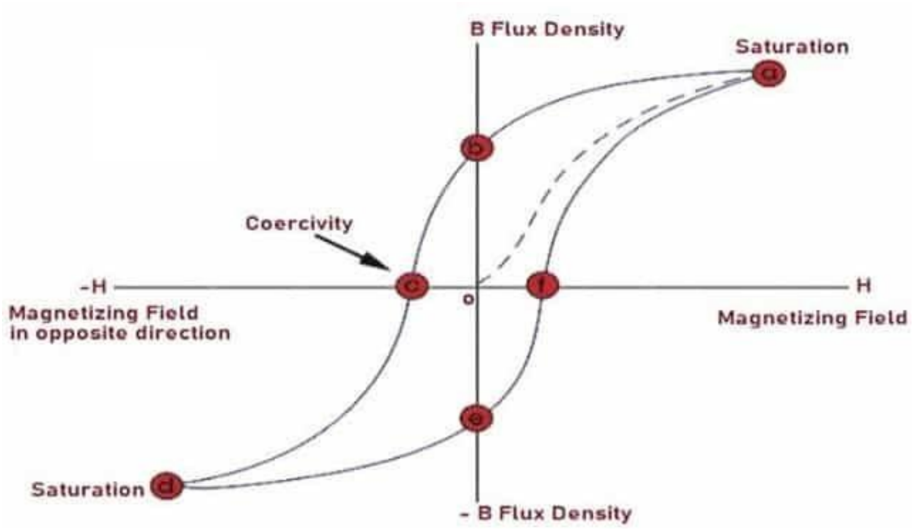


Figure 1: Magnetizing Characteristic Curve

Magnetizing characteristic curve shows the behavior of a ferromagnetic material graphically as a relationship between flux density and magnetic field strength. Begin with an unmagnetized core, both flux density and magnetizing field will be at zero on the magnetizing curve.

Until reaches a certain value, flux density B increases proportionately to the field strength. The point on the graph where the flux density reaches its maximum limit is called Magnetic Saturation.

If the magnetizing current increased in a positive way to some extent, the magnetic field strength increases linearly with flux density.

Similar way, if the magnetizing current reduced to zero, the magnetic field flowing around the core would reduce to zero. But the magnetic flux in the core do not reduce to zero as a result of the residual magnetism existing in the core.

Therefore today modelling of hysteresis is one of the most inspiring and energizing field of study in many engineering aspects. Since inaccuracy of hysteresis decreases the performance of control systems, an accurate modelling of hysteresis is well needed for performance evaluation of electrical systems. In order to overcome this problem, it is requisite to model hysteresis behavior with their parameters easily and accurately identified which are appropriate for real time system designs.

1.3 Problem Statement

The available methods for determination of magnetic hysteresis of a transformer needs many physical measurements, material specific data and internal dimensions of the transformer core. As most of the models are based on heavy reliance of material properties and characteristics, it is difficult to apply practically. Also the variation of magnetic saturation with different operating levels cannot be predicted easily. The general characteristic of most existing hysteresis models has their complexity.

Most commonly used Jiles-Atherton model requires five core specific input parameters relevant to magnetic properties of the core material. These parameters,

- Not available as data.
- Cannot obtain from direct measurements.
- Involve parameter identification techniques on a material sample.
- Five constants are inadequate to predict loops precisely.
- Physical asymmetries along magnetic paths are not considered.

Therefore in order to place a mathematical model into practical use, compromise between simplicity and accuracy is necessary.

1.4 Objectives of Study

The main objective of this project is to develop a mathematical model for flux-current hysteresis behavior of individual three limbs of a core type transformer based on magnetic data obtained through appropriate tests.

1.5 Motivation

In this proposed mathematical model for magnetic hysteresis effect of the three phase transformer, the core material properties are not incorporated. For a given material, using test based data, by extracting its characterization, plot hysteresis curves. Due to a test based mathematical model, relatively less complex equations are used to incorporate the model. Therefore the computational efficiency is higher than the conventional magnetic hysteresis models.

1.6 Methodology

Major activities of this research can be ordered as follows.

- Literature review/ Background study.
    - Investigating the work that has been already carried out under hysteresis modelling of magnetic materials.
    - Identifying the most optimum methods derived for hysteresis modelling.
- Test for flux-current characteristic of a single phase transformer for different amplitude of flux.
- Develop a mathematical model for flux-current characteristic incorporating hysteresis
- Design tests to obtain data for flux-current characteristic of individual cores of a three phase transformer.
- Model flux-current characteristic of individual limbs of the three phase transformer.
- Develop complete model for three phase transformer for input-output behavior.
- Validate the model with practical test results.

CHAPTER 2

2 LITERATURE REVIEW

Magnetic hysteresis model is required for many electrical engineering applications in which calculations of magnetic fields are made. For this purpose many types of hysteresis models have been proposed in the literature with their advantages and disadvantages. Review of some of the models are presented in this chapter with their technical specifications.

2.1 Jiles-Atherton (JA) Model

JA model is one of the most popular model in literatures to determine the hysteresis effect. JA model is stimulated by the physics of ferromagnetism and converts the physical phenomena of hysteresis into a first order differential equation with five constants called JA parameters [6][7].

JA model involves a conversion from M-H (M-Magnetic moment) loop to a B-H loop characteristics using the following equation.

$$B = \mu_0 (H+M)$$

$$[k \,.\, \delta - \alpha (M_{an} - M + k \,.\, \delta \,.\, c)] \; \frac{dM}{dH} = Man - M + k.\delta.c. \frac{dMan}{dHe}$$

Following hysteresis parameters should be given as input data to determine the hysteresis curve.

- Magnetization saturation
- Thermal energy parameter
- Domain flexing constant
- Domain anisotropy constant
- Inter-domain coupling parameter

This JA model has few inheritable disadvantages that the five constants are inadequate to predict inward loops accurately and the accuracy is low at lower induction levels.

In order to overcome this problem, modification has done to original JA model's one of the five parameters by Sajid Hussain and David A. Lowther [4]. They have suggested a new method that considers one of JA parameters to be a function of magnetic field intensity H or magnetic flux density B. According to that, in the existing model, parameter k is modified as function of H or B. Parameter k is a function of average density of pinning sites.

All five JA parameters are identified from the saturated B-H loop using non-linear least square method. By keeping four parameters constant, modified parameter k is identified. Parameter k is fitted to a mathematical expression as a function of H or B. Magnetic fields are computed using this model. Computed results are compared against measurements.

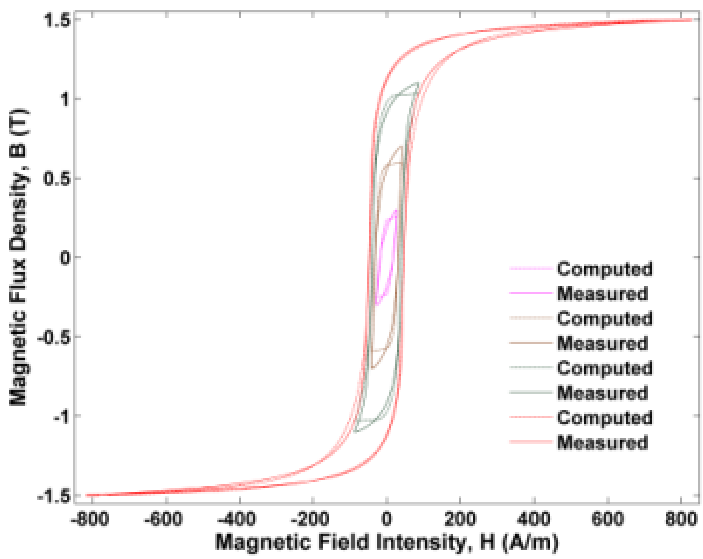


Figure 2: Modified JA Model Measured and Modelled Inner B-H Loops

Modelled results using this modified JA model are much similar to the measured results. New parameter enhances the accuracy of the model in an acceptable limit.

Major drawback identified in this model is complexity. JA input parameters are not easy to find out. These parameters cannot be determined using direct measurements. To identify these parameters extensive fitting is required.

2.2 Preisach Hysteresis Model

This is a commonly used model in literatures to simulate hysteresis loops. In Preisach hysteresis model, magnetic material is assumed to be consisted of an infinite number of magnetic particles, whose statistical distribution determines the permeability and models the hysteresis loop. A particle is characterized by rectangular hysteresis loop. Total magnetization is calculated by the summation of each individual particle [8].

Magnetic polarization J is calculated by integrating a distribution function using the following equation.

$$J = J_s \iint P\,(H_M, H_C)\, \gamma_{\alpha\beta}\, dH_\alpha\, dH_\beta$$

In literature many researchers have proposed various types of models to obtain magnetic hysteresis using Preisach model. In the study proposed by P. Pruksanubal, A. Binner and K.H. Gonschorek, distribution function of ferromagnetic material is well determined to represent the hysteresis property [1].

Distribution functions are formed by combining conventional continuous statistical distributions, Gaussian, Cauchy and Lognormal. These three distributions are combined to form different distribution functions and simulate the hysteresis loops. Using these three distribution functions, six different relationships are obtained.

For the six cases, distribution functions are produced and hysteresis loops are simulated using obtained functions. Then comparison is done with simulated hysteresis loops and the measured data.

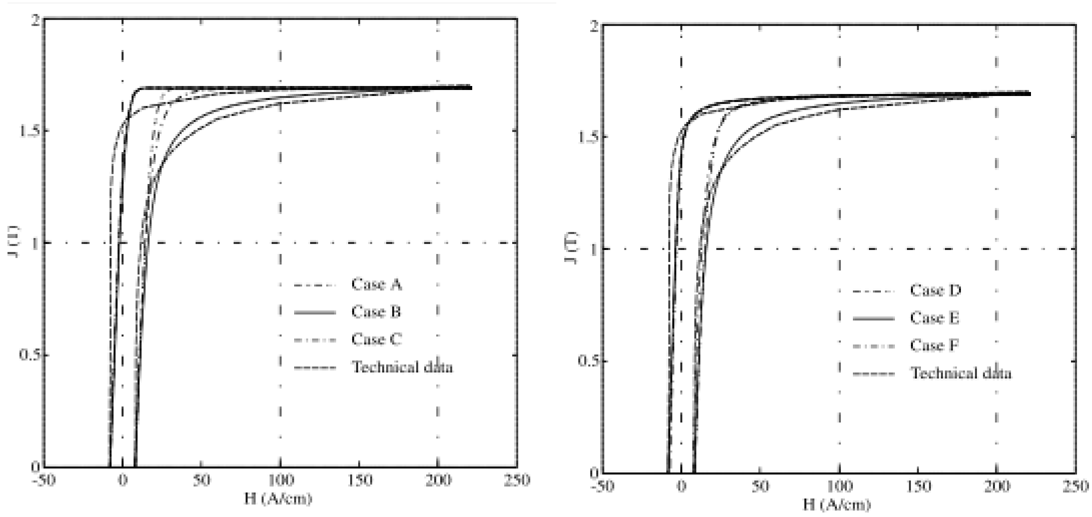


Figure 3: Comparison between Hysteresis Loops of Simulated and Measured Data of Grain Oriented Electrical Sheet

The hysteresis loops fit fairly well with some alterations. Limitations identified with Preisach model is that the identification procedures for the PM demands lots of material specific data. Also it needs more computational and memory resources. These issues make the use of PM computationally very expensive.

2.3 Bergqvist's Lag Model

In the paper proposed by David Ribbenfjard and Goran Engdahl, dynamic hysteresis model is obtained using Bergqvist's lag model (for static part) and Bertotti's model (for dynamic part) [3].

This model assumes that the magnetic material contains finite number of pseudo particles. Total magnetization is taken from weighted sum of individual magnetization of all particles. Pinning strength of pseudo particles is considered to model hysteresis behavior.

Hysteresis curves are simulated by varying frequencies of alloy Metglass and the transformer steel Surahammer to verify the model. Once curves are simulated, comparison is done with simulation results and measured data of the materials.

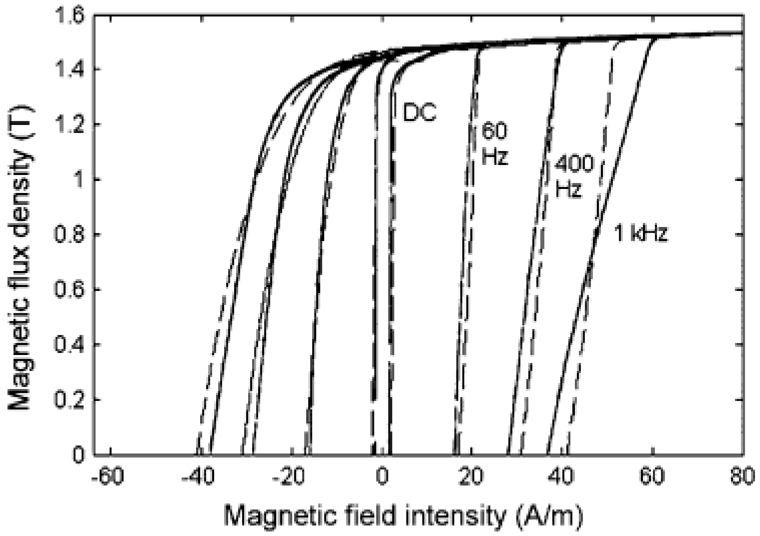


Figure 4: Hysteresis Curve Upper Quadrants of Metglass

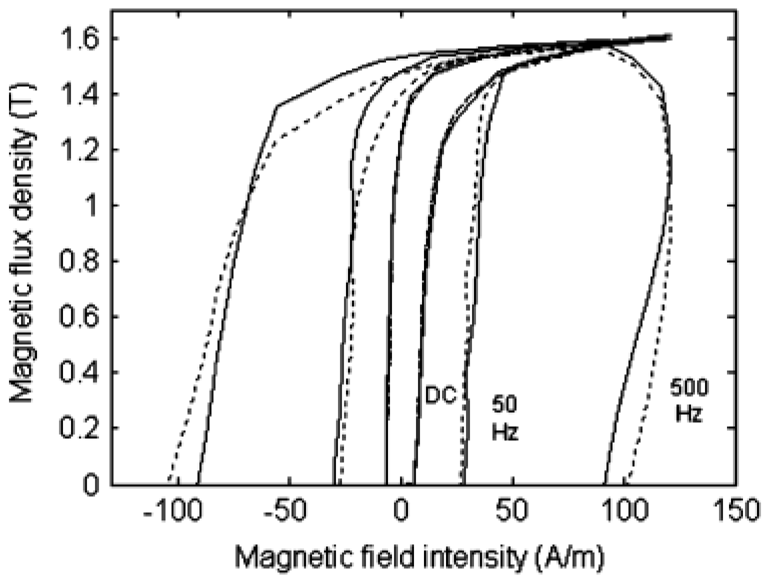


Figure 5: Hysteresis Curve Upper Quadrants of Surahammer

As this model based on existing general models, it is possible in simulating a variety of magnetic materials. Limitations identified in this model is, obtained hysteresis curves are not closely match with the measured curves at higher frequencies.

2.4 An Analytical Model

In the paper presented by Septimiu Motoasca, Andrei Nicolaide, Elena Helerea and Gheorghe Scutaru, an analytical method is used to model hysteresis loop using hysteresis cycle representation [2]. Hysteresis curve represented with series of arcs of circles and section of line is expressed using analytical geometry.

As this proposed model is based on analytical geometry, firstly equations for sections of line and arcs of circle are formed. Then arcs and lines are linked in nearly considerable points. These sections of line and arcs of circle are connected to model divisions of main hysteresis curve.

For modelling of the hysteresis curve, points H_s, B_s, N_s, H_2, B_2, B_r, H_c, H_6 and B_6 are considered as shown in the Figure 6.

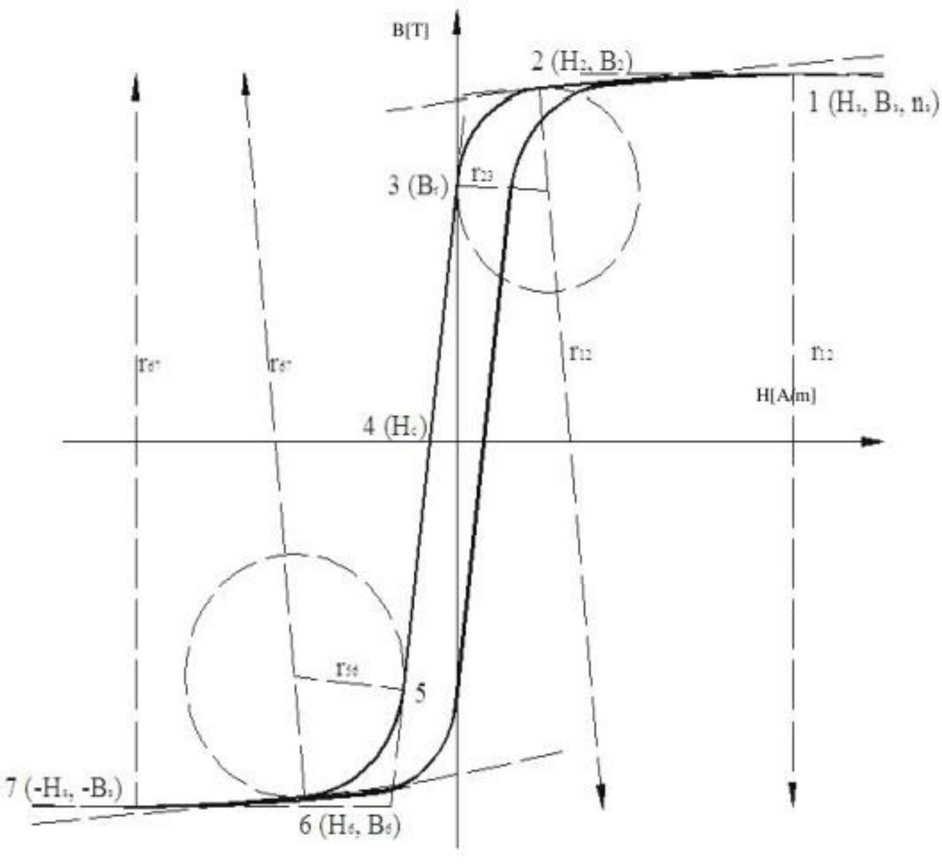


Figure 6: Points Used to Model of Hysteresis Cycle

Using the graphic representation, values for nine parameters are obtained. These parameter values are given as input data in LabVIEW program to model the hysteresis curve.

Steel sheet type material M800-65A has used to compare the modelled hysteresis curves. Measurements have obtained using Epstein frame method.

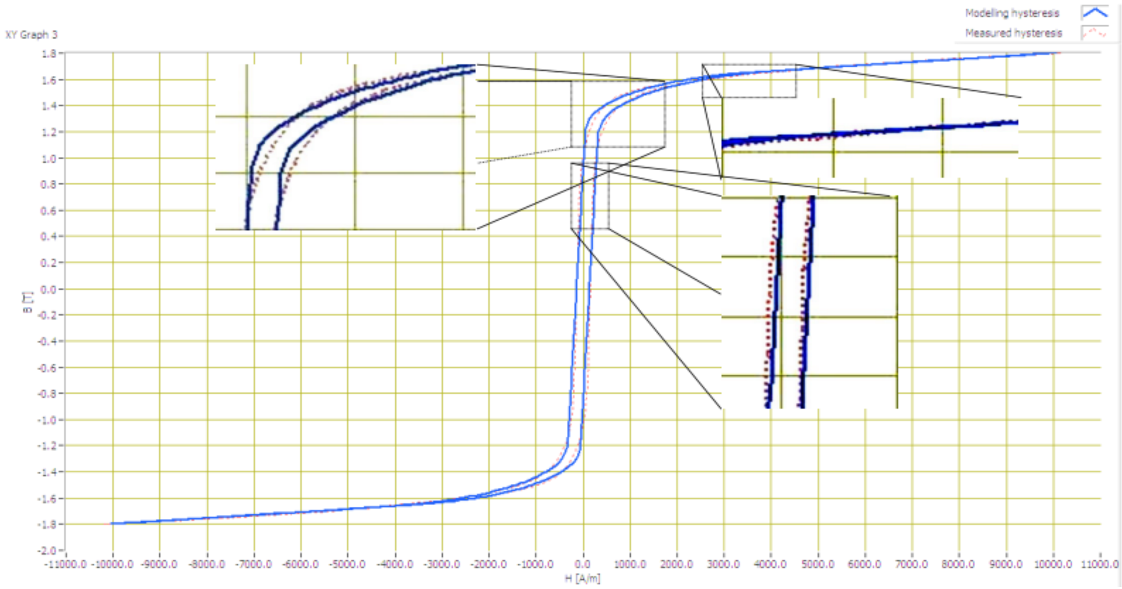


Figure 7: Comparison of Measured and Modelled Hysteresis Curves

The proposed model approaches in an acceptable way with the measured hysteresis curve. But this method needs the nine parameters as input data to obtain the curve. Therefore this requires an experimental approach. Another limitation of this analytical model is, it cannot predict the behavior of the material for different excitation levels.

CHAPTER 3

3 TESTING AND DATA ANALYSING

3.1 Research Overview

To investigate the behavior of a three phase transformer under dynamic conditions involving voltage unbalances, the magnetizing characteristics of its individual cores need to be modelled. This model should be necessarily incorporated the hysteresis characteristics of the individual cores and the variations in the characteristics with the amplitude of core-flux. Through properly designed tests, obtain necessary data to model the hysteresis behavior of a core type transformer and validate the model with test results.

3.2 Testing Methodology

This model is based on simulation process using test based data. The data is collected for both single phase transformer and three phase transformer. Using test results, mathematical model is developed. In the event of three phase transformer, mathematical model is developed for its individual limbs separately. Obtained model is validated with practical test results.

3.3 Tests Conducted

3.3.1 Test on Single Phase Transformer

This test was conducted on the primary side of the single phase transformer while keeping the secondary side open. The open circuit test is performed in order to find no-load current, voltage and power which no-load parameters to determine.

Steps carried out to determine the magnetizing characteristic curves.

- Performed standard open circuit test on a single phase transformer.
- Captured magnetizing current waveform and voltage waveform at rated voltage of transformer using oscilloscope.
- Varied input voltage values and captured magnetizing current waveforms and voltage waveforms for different voltage levels (75% of rated voltage, 50% of rated voltage and 25% of rated voltage).

- Using the obtained values, extracted graphs for magnetic flux versus magnetizing current ($N\Phi$ Vs I_m).

3.3.1.1 Test Setup

The secondary side of the transformer was kept open and ammeter, voltmeter and wattmeter were connected to the primary side of the transformer as in Figure 8. The nominal rated voltage was applied to the primary winding.

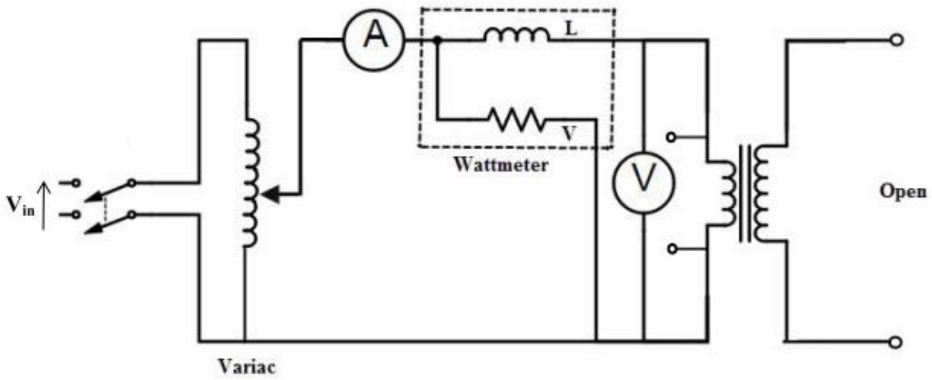


Figure 8: Single Phase Transformer Open Circuit Test Setup

W_0 – Wattmeter reading | V_0 – Voltmeter reading | I_0 – Ammeter reading

The no-load shunt parameters can be calculated as,

- No-load power factor, $\cos \Phi_0 = W_0/V_0I_0$

After the power factor is obtained, the no-load components can be determined using the following equations.

- Magnetizing component, $I_m = I_0 \sin \Phi_0$
- Core loss component, $I_c = I_0 \cos \Phi_0$
- Magnetizing reactance, $X_0 = V_0/I_m$
- Core loss, $R_c = V_0/I_c$

The magnetic flux is calculated as,

- Induced voltage, $e = N_1 \frac{d\Phi}{dt}$

$$\int e\, dt = N_1\Phi \;\ldots\ldots\ldots. (1)$$

- $e = V_0 - (r_1 I_0 + L_1 \frac{dIin}{dt})$ ………. (2)

Therefore magnetic flux is calculated as,

- $N1\Phi = \int[V_0 - (r_1 I_0 + L_1 \frac{dI0}{dt})]$

N_1 – No. of turns of primary winding

Φ – Magnetic flux

r_1 – Winding resistance

L_1 – Winding inductance

Using these equations magnetizing characteristic curve ($N\Phi$ Vs I_m) can be obtained.

3.3.1.2 Obtained Results

For the single phase transformer open circuit test, rated voltage waveform and magnetizing current waveform were captured using the oscilloscope as an initial approach.

Figure 9: Single Phase Transformer - Rated Voltage Waveform (Yellow) and Current Waveform (Green) Plotted in Oscilloscope

By investigating the plotted waveforms, significant noise was identified in both current and voltage waveforms as shown in Figure 10 and Figure 11.

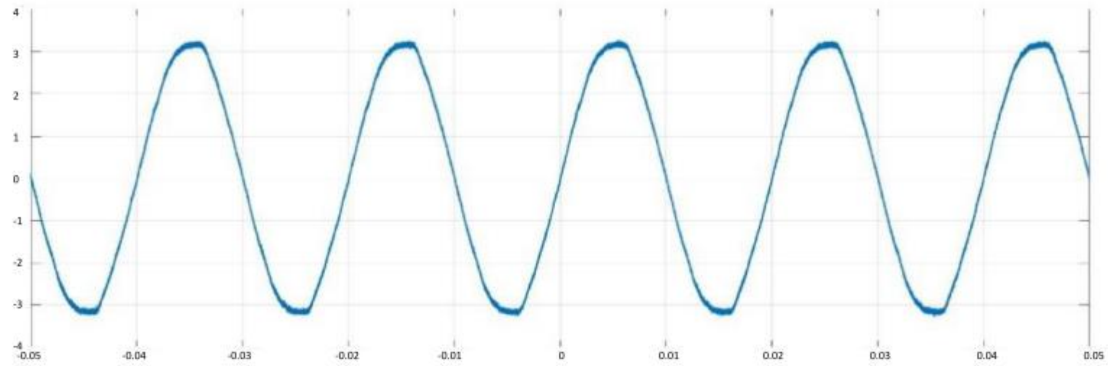


Figure 10: Rated Voltage Waveform with Noise Obtained from Matlab Plotting

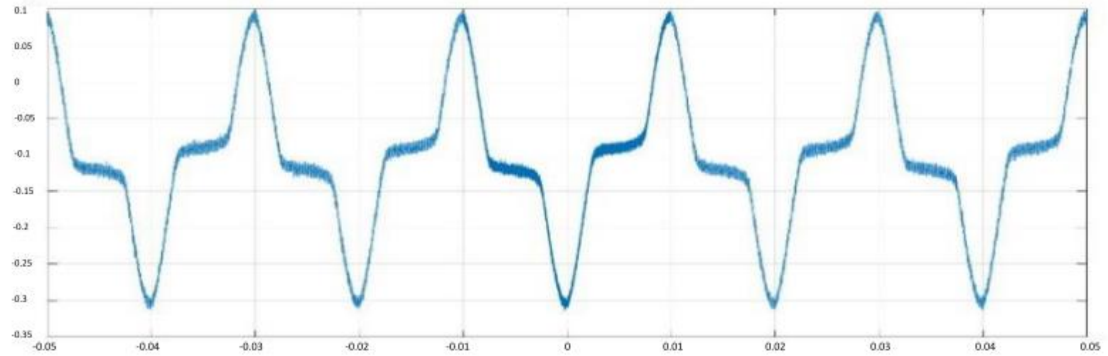


Figure 11: Rated Current Waveform with Noise Obtained from Matlab Plotting

Obtained magnetizing curve with noise for rated voltage is shown in Figure 12.

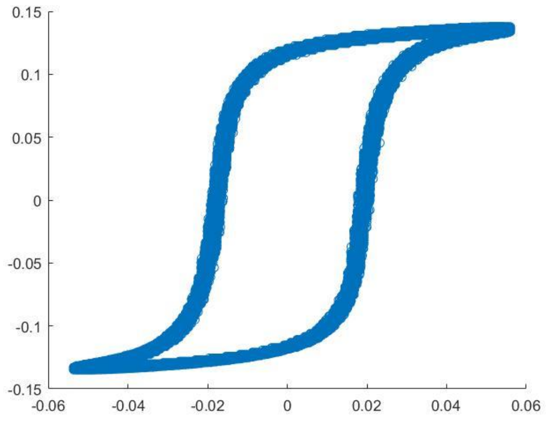


Figure 12: Magnetizing Characteristic Curve at Rated Voltage with Noise

Using noise reduction technique in Matlab, curves were smoothened and reduced the noise to an acceptable level.

After the noise reduction, obtained magnetizing curve at rated voltage is shown in Figure 13.

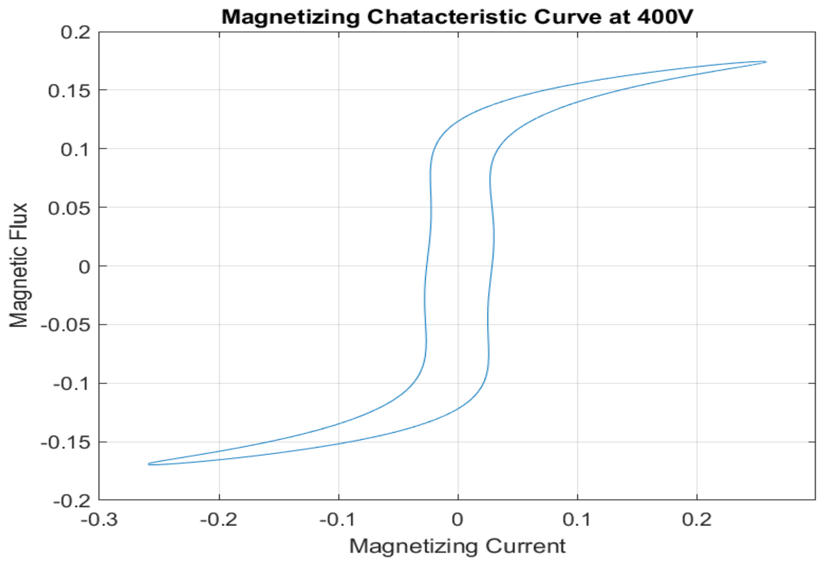


Figure 13: Noise Filtered Magnetizing Characteristic Curve at Rated Voltage

Following the above mentioned steps, magnetizing characteristic curves for 75% of rated voltage, 50% of rated voltage and 25% of rated voltage were obtained.

- At 75% of rated voltage (300V)

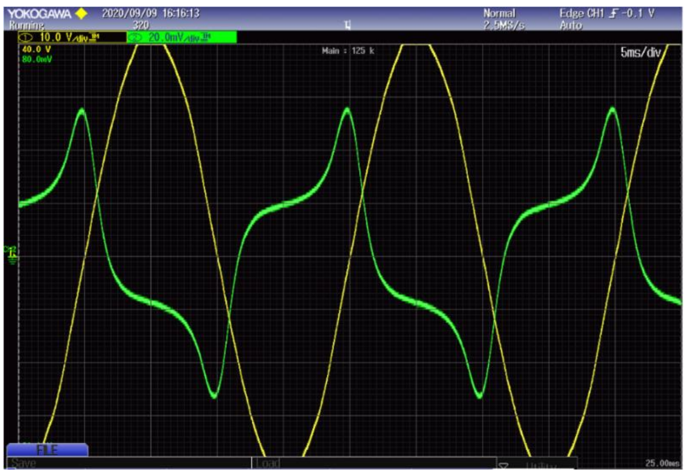


Figure 14: Single Phase Transformer - 75% of Rated Voltage Waveform (Yellow) and Current Waveform (Green) Plotted in Oscilloscope

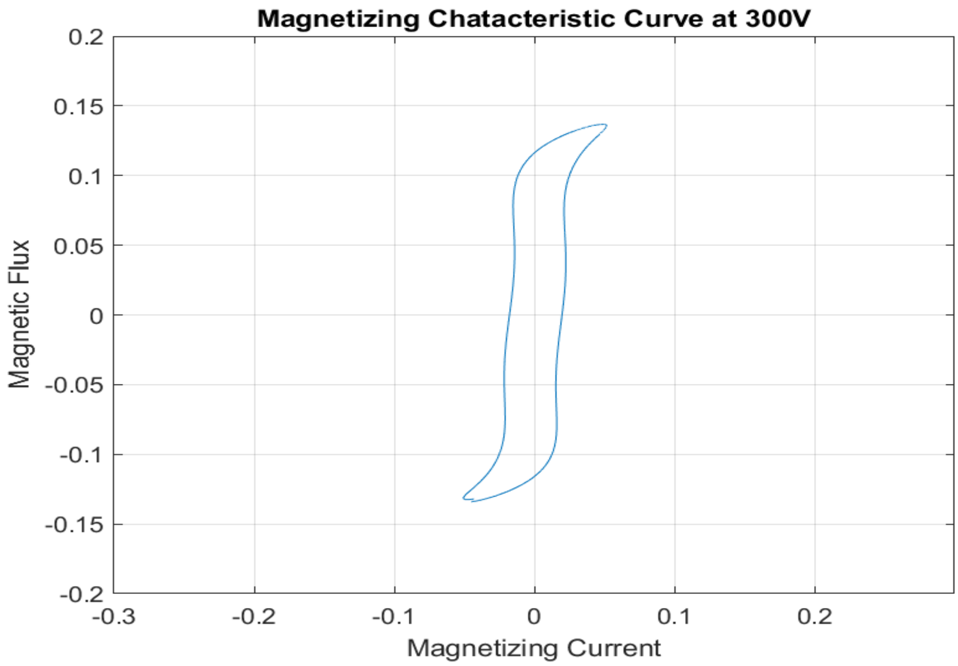


Figure 15: Noise Filtered Magnetizing Characteristic Curve at 75% of Rated Voltage

- At 50% of rated voltage (200V)

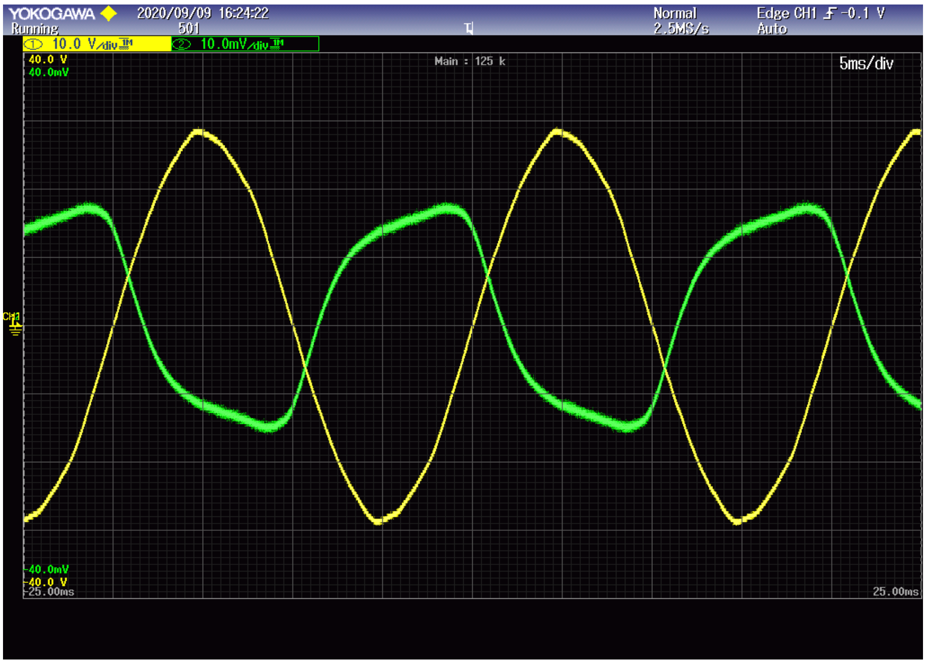


Figure 16: Single Phase Transformer - 50% of Rated Voltage Waveform (Yellow) and Current Waveform (Green) Plotted in Oscilloscope

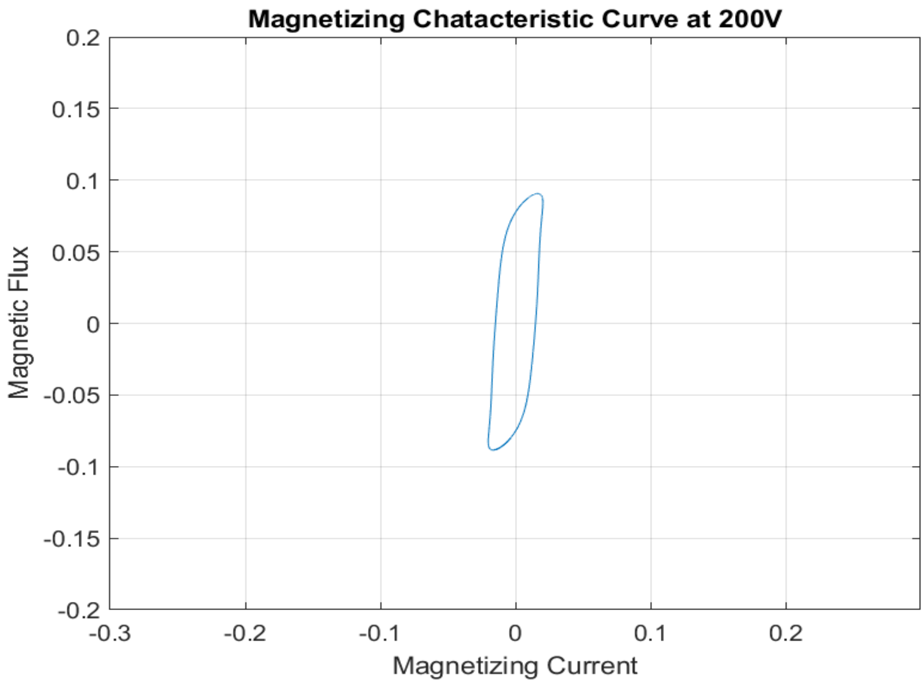


Figure 17: Noise Filtered Magnetizing Characteristic Curve at 50% of Rated Voltage

- At 25% of rated voltage (100V)

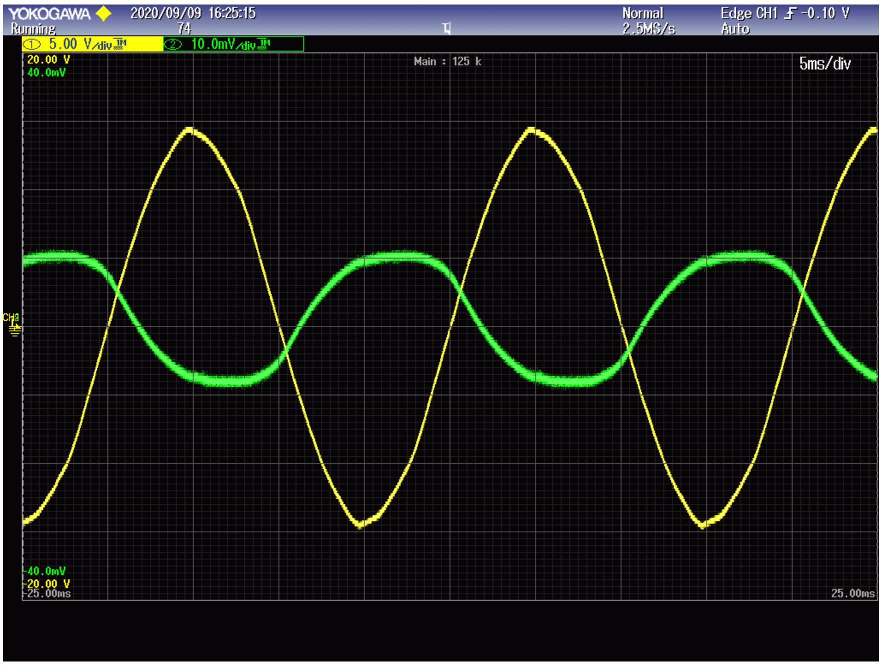


Figure 18: Single Phase Transformer - 25% of Rated Voltage Waveform (Yellow) and Current Waveform (Green) Plotted in Oscilloscope

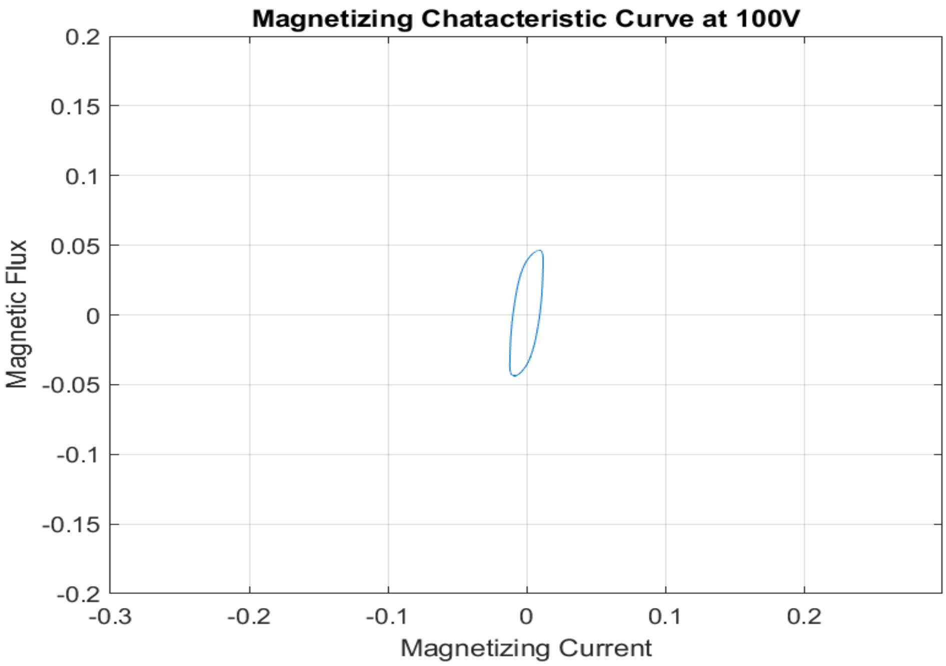


Figure 19: Noise Filtered Magnetizing Characteristic Curve at 25% of Rated Voltage

Plotted magnetizing characteristic for all voltage levels, 100V, 200V, 300V and 400V are shown in the Figure 20.

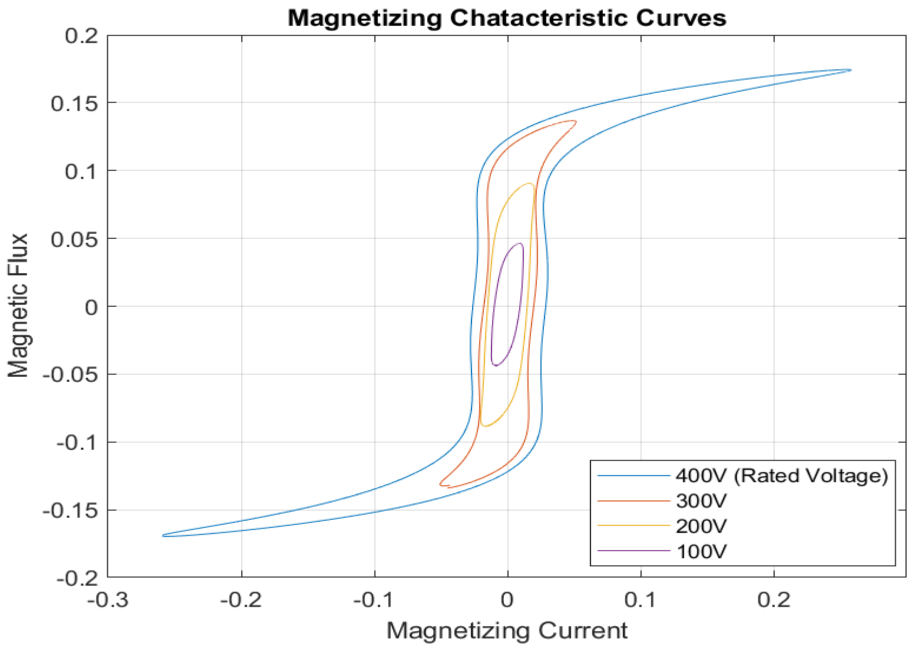


Figure 20: Noise Filtered Magnetizing Characteristic Curves

3.3.2 Test on Three Phase Transformer

In order to obtain magnetic characteristics of a three phase transformer, a piecewise approached was used. Magnetic characteristics of cores were obtained separately for the three lengths of transformer core. For this limb wise modelling, two tests were designed.

Tests were conducted using a 10 kVA, three phase (400/230V), Dyn11 transformer.

Arrangement of transformer legs with lengths and other parameters are illustrated in the Figure 21.

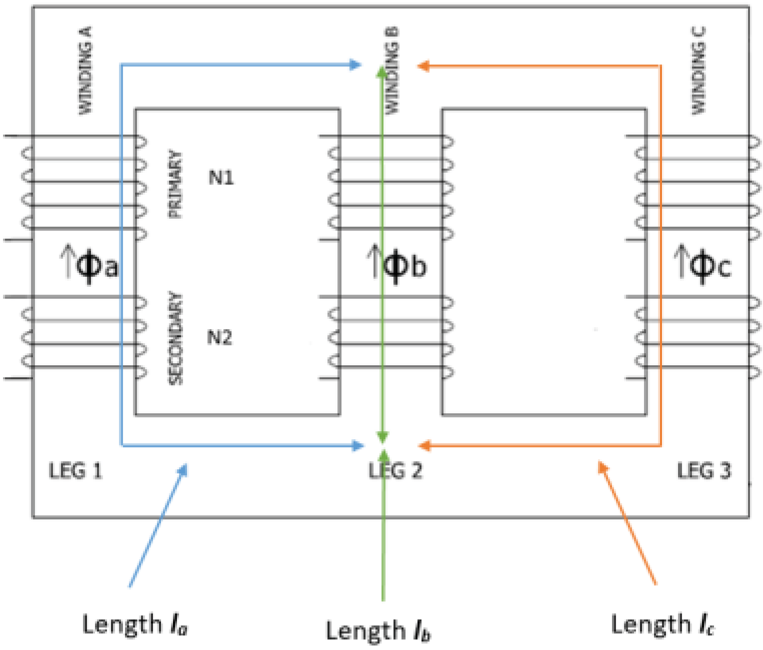


Figure 21: Details of Three Limbs of the Three Phase Transformer

3.3.2.1 Test Setup

3.3.2.1.1 Test 01

As explained above, core has been divided in to three lengths to derive equations. This test was performed to obtain magnetizing characteristics of core length of limb 1 and limb 3. For this case, net flux was only allowed to circulate in transformer limb 1 (l_a) and limb 3 (l_c).

Therefore the test was performed by giving two inverted currents to the primary windings of limb 1 and limb 3 at rated voltage, as shown in Figure 22. Currents were given at the rated voltage of the transformer, so that the rated current would flow in the intended core path. Until it became the status of no induced voltage in the secondary winding of limb 2 (e_2), magnitude of applied currents were adjusted.

Then the waveforms of secondary winding voltage and primary winding current of limb 1 and limb 3 were obtained. This test was repeated for other three voltage levels (75% of rated voltage, 50% of rated voltage and 25% of rated voltage).

Figure 22 shows the test setup of test 01.

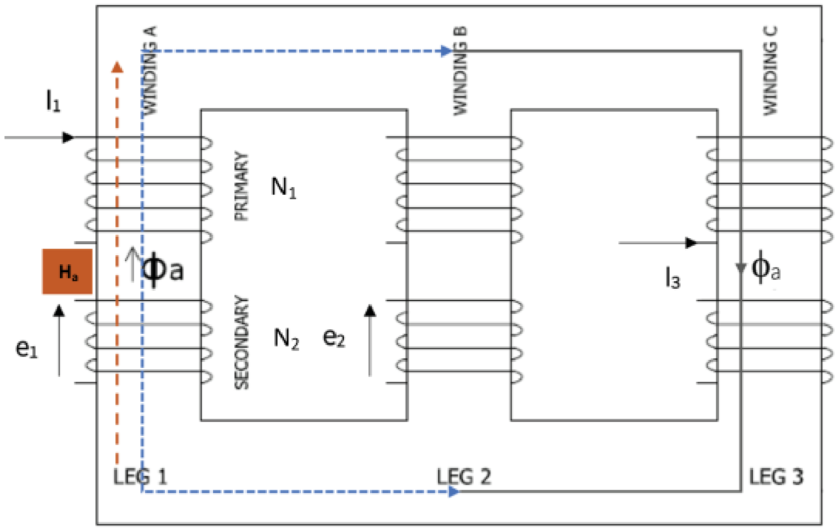


Figure 22: Test Setup of Test 01

Applying Faraday's Law to the limb 1 and limb 3,

$$e_1 = N_2 \frac{d\Phi}{dt} \text{ ……….. } (1)$$

Applying Ampere's Circuital Law to outer core flux path,

$$H_a l_a = N_1 I_1 \text{ ……..… } (2)$$

$$H_c l_c = N_1 I_3 \text{ ……….. } (3)$$

From the above expressed equations, following relationships were acquired for lengths l_a and l_c of limb1 and limb 3.

$$N1\Phi_a = \frac{N1}{N2}\int e_1\,dt \text{ .... } (4)$$

$$\frac{Hala}{N1} = I_1 \text{ .... } (5)$$

$$\frac{Hclc}{N1} = I_3 \text{ .... } (6)$$

The magnetizing characteristic for limb 1 was formed using equations (4) and (5) by means of the obtained results of limb 1 secondary winding voltage (e_1) and primary winding current (I_1).

In a similar manner, the magnetizing characteristic for limb 3 was formed using equations (4) and (6) by means of the obtained results of secondary winding voltage of limb 1 (e_1) and primary winding current of limb 3 (I_3).

Figure 23 shows the test results of limb 1 and limb 3.

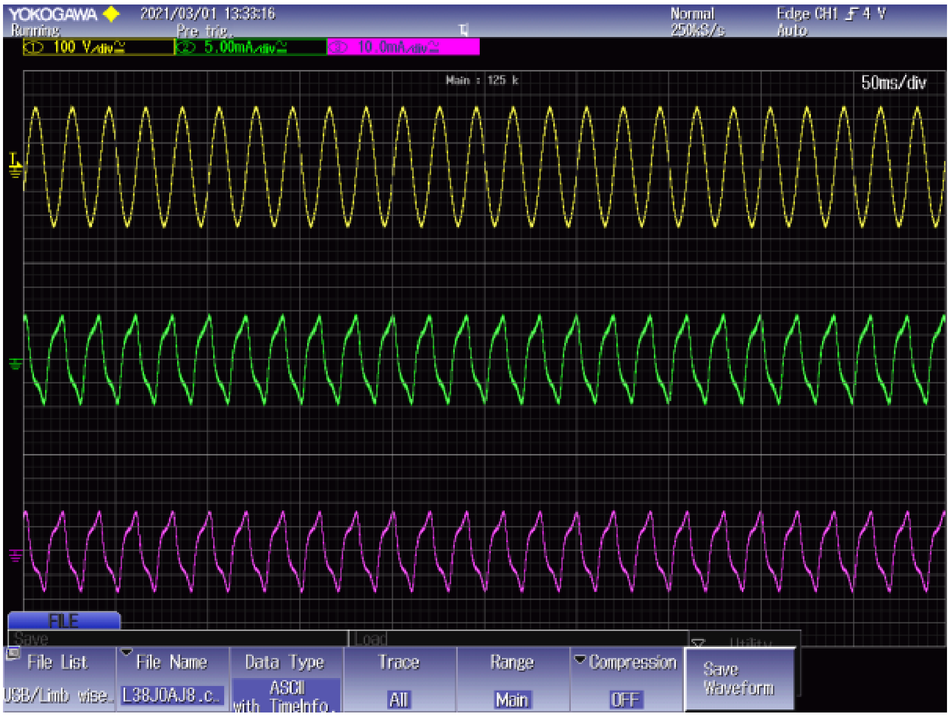


Figure 23: Three Phase Transformer Rated Voltage Waveform (Yellow), Magnetizing Current of Leg 1 (Green) and Magnetizing Current of Leg 3 (Purple)

Likewise voltage waveforms and magnetizing current waveforms were obtained for other three voltage levels using oscilloscope.

Using the test results, captured graphs for magnetizing characteristic of leg 1 and leg 3 for all four voltage levels are shown in Figure 24 and Figure 25 respectively.

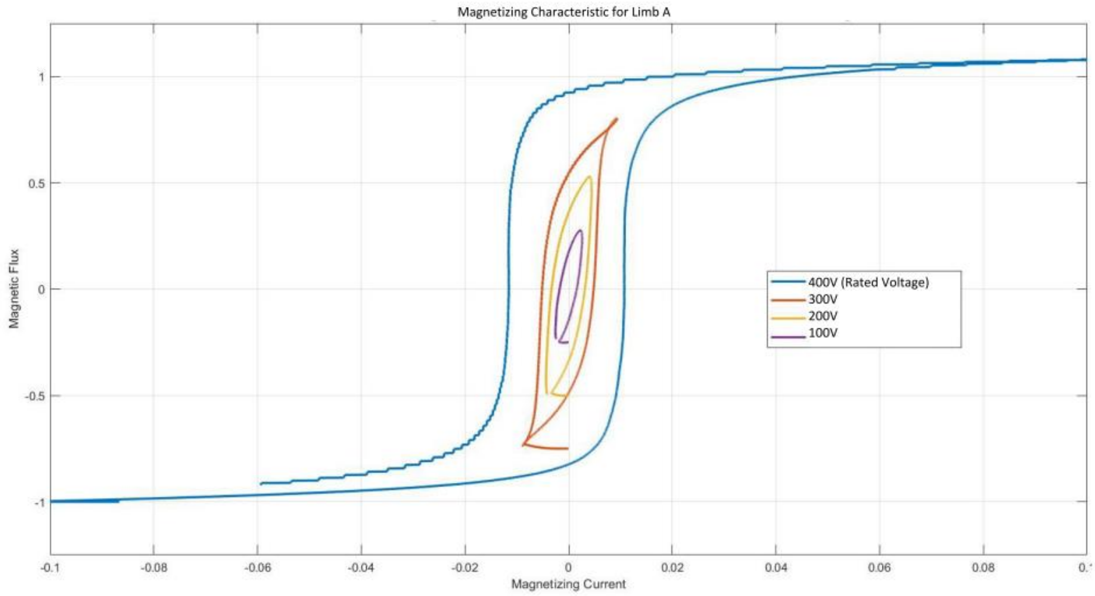


Figure 24: Magnetizing Characteristic of Leg 1

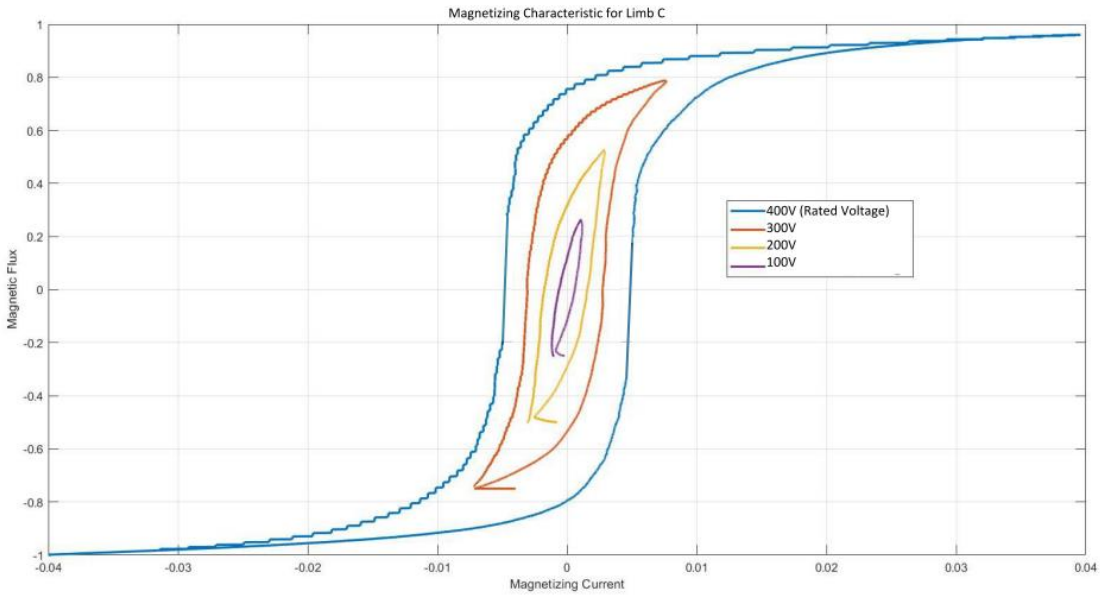


Figure 25: Magnetizing Characteristic of Leg 3

3.3.2.1.2 Test 02

This test was performed to obtain magnetic characteristics of core length l_b. The test was conducted by giving two inverted currents to the primary winding of limb 1 and limb 2 at rated voltage, as shown in Figure 26. Until it became the status of no induced voltage in the secondary side of the limb 1 (e_1) and the limb 2 (e_2) at its rated voltage, magnitude of these applied currents were varied. Therefore the only flux path was along the limb 2 and limb 3.

Using this test, waveforms of secondary winding voltage and primary winding current of limb 2 were obtained.

Figure 26 shows the test setup of test 02.

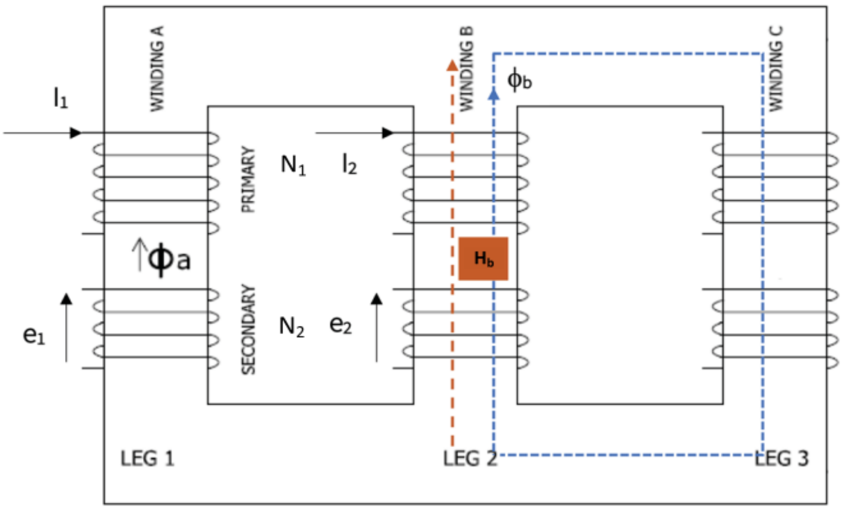


Figure 26: Test Setup of Test 02

Applying Faraday's Law to the leg 2,

$$e_2 = N_2 \frac{d\Phi}{dt} \text{ .... } (1)$$

Applying Ampere's Circuital Law to outer core flux path,

$$H_b l_b = N_1 I_1 (I_2 - I_1) \text{ .... } (2)$$

From equations (1) and (2), for core lengths l_b, following equations were acquired.

$$N1\Phi_b = \frac{N1}{N2}\int e_2\, dt \text{ .... (3)}$$

$$\frac{Hblb}{N1} = (I_2 - I_1) \text{ .... (4)}$$

For limb 2, magnetizing characteristic was modelled using equations (3) and (4) by means of the measured values of secondary winding voltage of limb 2 (e_2), primary winding current of limb 1 (I_1) and primary winding current of limb 2 (I_2).

Figure 27 shows the test results of leg 2.

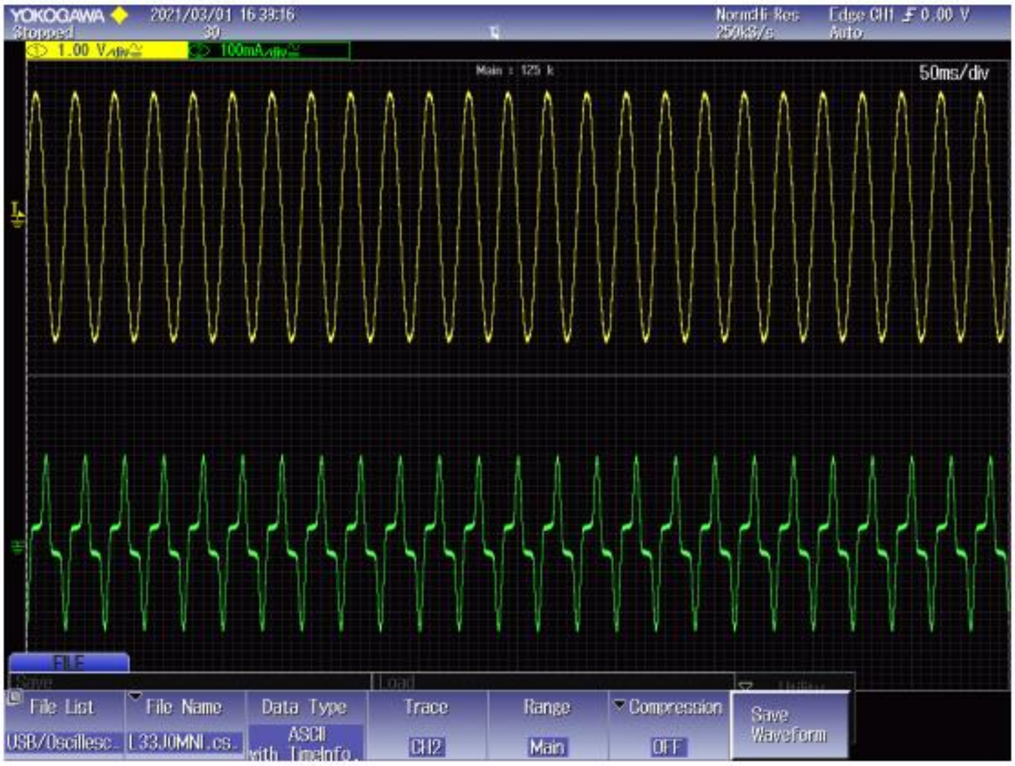


Figure 27: Three Phase Transformer Rated Voltage Waveform (Yellow), Integrated Voltage Waveform (Red) and Magnetizing Current of Leg 2 (Green)

Likewise voltage waveforms and magnetizing current waveforms were obtained for other three voltage levels using oscilloscope.

Using the test results, captured graphs for magnetizing characteristic of leg 2 for all four voltage levels are shown in Figure 28.

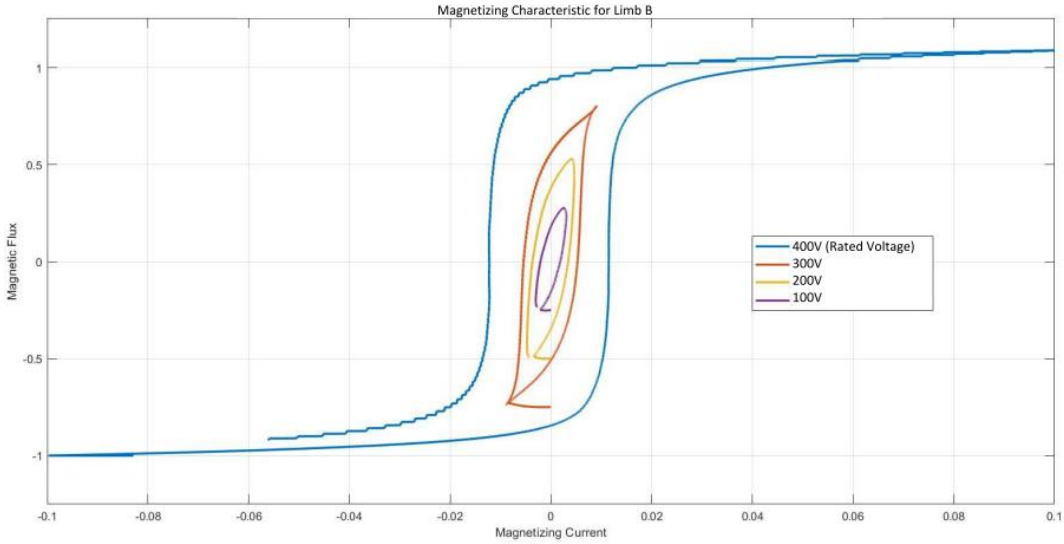


Figure 28: Magnetizing Characteristic of Leg 2

CHAPTER 4

4 DESIGNING AND SYSTEM MODELLING

4.1 Mathematical Modelling of Magnetic Hysteresis

Mathematical models for developing of magnetic hysteresis of transformers are discussed in this chapter. Two methods are proposed to model magnetic hysteresis for a given voltage based on the data availability.

4.1.1 Method 01 – When Magnetizing Characteristic Data Available Only for Rated Voltage

This model is developed by analyzing the data obtained from the single phase transformer which were shown in the Chapter 3.

4.1.1.1 Procedure

Steps carried out to plot the magnetizing curve.

- Obtain the magnetic hysteresis curve at the rated voltage
- Find positive and negative co-ordinates of six points, saturation, retentivity and coercivity as shown in the Figure 4-1.
- Find points for the required voltage level by using the method of, co-ordinates of six points multiply by the required voltage value and divide by the rated voltage value.
- Obtain co-ordinates for the required voltage level.
- Plot the curve using the curve fitting method in Matlab.

With refers to the magnetizing curve, following features can be identified.

- The coercivity points and the retentivity points are in the region where no saturation is present and these are axis crossing points, indicating that linearly related to the input voltage.
- Saturation points lies in the mean magnetization curve.

Retentivity and coercivity points can be identified from the Figure 29.

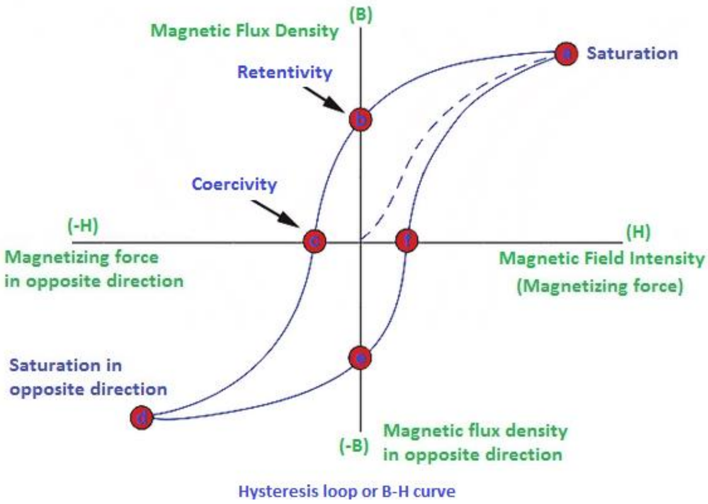


Figure 29: Magnetic Hysteresis Curve

4.1.1.2 Mathematical Modelling

Find co-ordinates of points, a_1, b_1, c_1, d_1, e_1 and f_1 which are identical to the points of rated voltage curve a, b, c, d, e, and f as shown in the Figure 30.

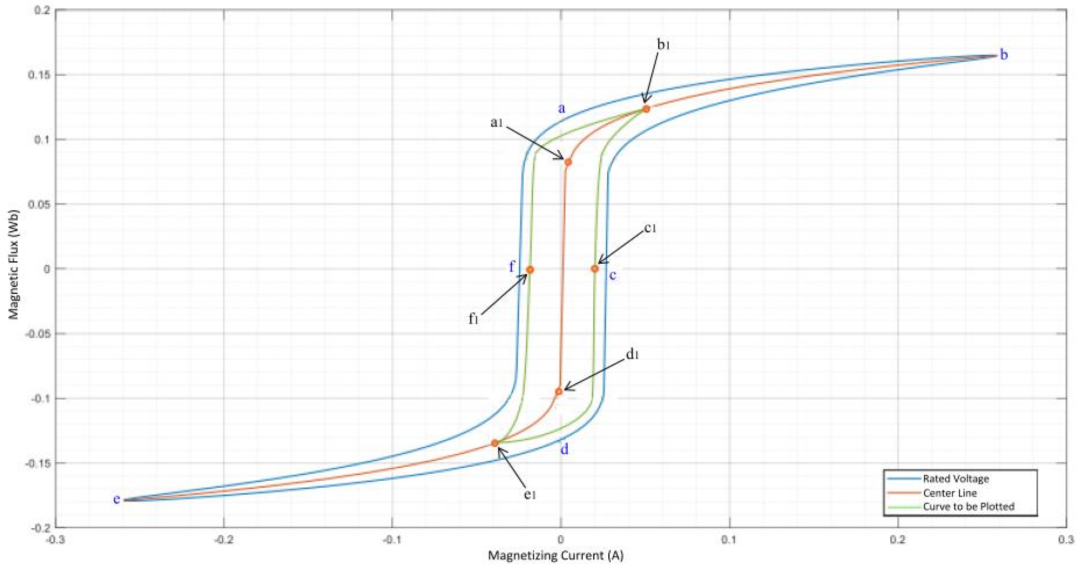


Figure 30: Required Points of Rated Voltage Hysteresis Curve

Finding of points a_1, c_1, d_1 and f_1 (axis crossing points).

- Point $a_1 = [a(x), a(y)*\frac{V}{Vrated}]$

- Point d_1 = [d(x), d(y)*$\frac{V}{Vrated}$]
- Point c_1 = [c(x)* $\frac{V}{Vrated}$, c(y)]
- Point f_1 = [f(x)* $\frac{V}{Vrated}$, f(y)]

Finding of points b_1 and e_1 (saturation points)

In order to find the co-ordinates of point b_1 and e_1, same procedure is followed to find the y co-ordinate. In the case of x co-ordinate, it is taken from the point which new y co-ordinate crossing the mean magnetizing curve which is parallel to x axis.

- Point b_1 = [$b(x_{new})$, $b(y_{new})$] = [$b(x_{new})$, {b(y)* $\frac{V}{Vrated}$}]
- Point e_1 = [$e(x_{new})$, $e(y_{new})$] = [$e(x_{new})$, {e(y)* $\frac{V}{Vrated}$}]

Modelling the magnetizing characteristic for 75% of rated voltage (300V)

Co-ordinates of required six points were extracted from the rated voltage (400V) curve as in the Figure 31.

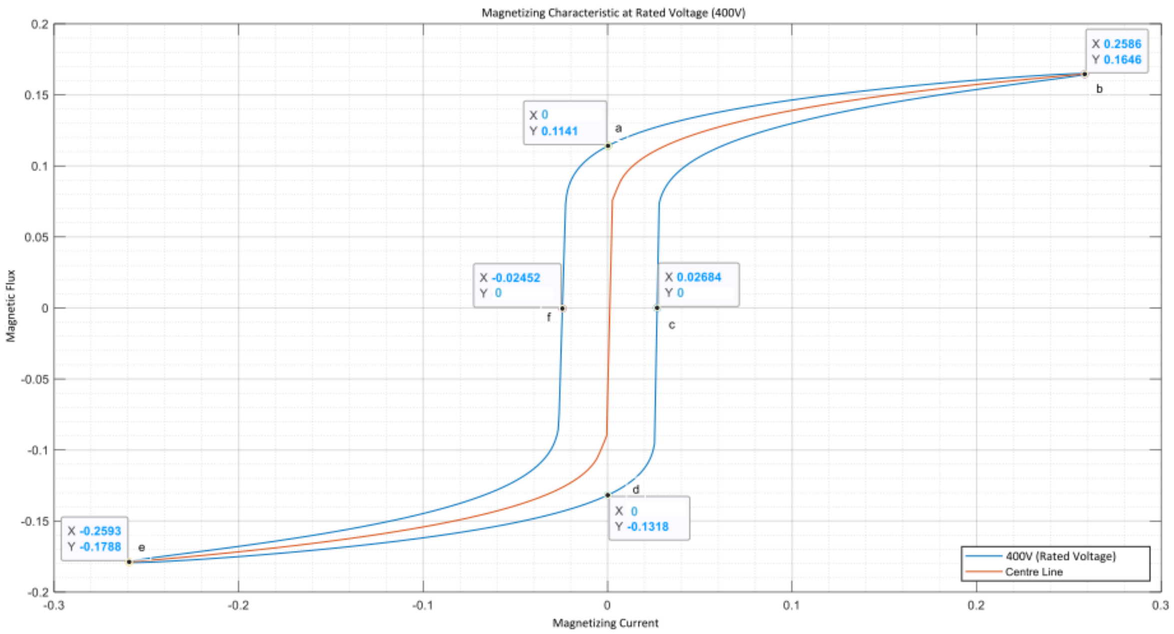


Figure 31: Magnetizing Characteristic Curve with Co-ordinates of Six Points

Finding of points a_1, c_1, d_1 and f_1 (axis crossing points).

- Point a_1

  a(x,y) = (0, 0.1141)

  $a_1(x,y) = [a(x), a(y)*\frac{V}{Vrated}] = [0, 0.1141*\frac{300}{400}] = (0, 0.08558)$

- Point d_1

  d(x,y) = (0, -0.1318)

  $d_1(x,y) = [d(x), d(y)*\frac{V}{Vrated}] = [0, -0.13138*\frac{300}{400}] = (0,- 0.09883)$

- Point c_1

  c(x,y) = (0.02684, 0)

  $c_1(x,y) = [c(x)* \frac{V}{Vrated}, d(y)] = [0.02684* \frac{300}{400}, 0] = (0.02013, 0)$

- Point f_1

  f(x,y) = (0.02684, 0)

  $f_1(x,y) = [f(x)* \frac{V}{Vrated}, f(y)] = [-0.02452* \frac{300}{400}, 0] = (-0.01839, 0)$

Finding of points b_1 and e_1 (saturation points).

- Point b_1

  b(x,y) = (0.2586, 0.1646)

  $b_1(y) = b(y)* \frac{V}{Vrated} = 0.1646*\frac{300}{400} = 0.1236$

  $b_1(x) = 0.05054$

  b1(x,y) = (0.05054, 0.1236)

- Point e_1

  e(x,y) = (-0.2593, -0.1788)

  $e_1(y) = e(y)* \frac{V}{Vrated} = -0.1788*\frac{300}{400} = -0.1341$

  $e_1(x) = -0.03907$

  $e_1(x,y) = (-0.03907, -0.1341)$

After obtained co-ordinates of six points, results were marked on the rated voltage curve as shown in the Figure 32.

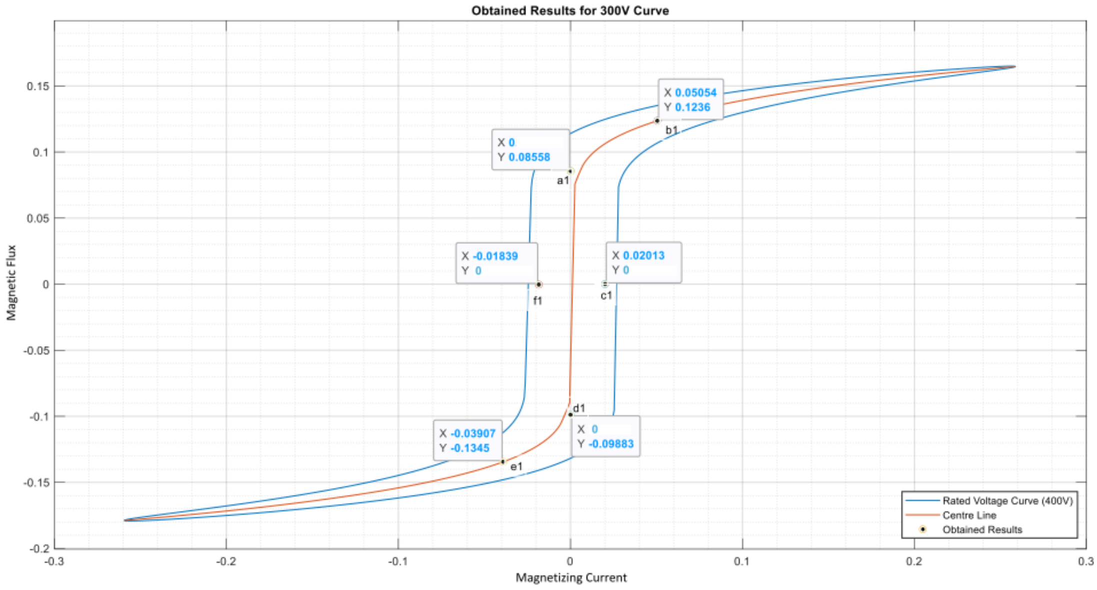


Figure 32: Obtained Values for 75% of Rated Voltage Curve

Magnetizing characteristic for 300V was obtained by connecting these points using Matab curve fitting method.

4.1.2 Method 2 – When Magnetizing Characteristic Data Available for Any Voltage

In order to identify a correlation among both single phase and three phase transformer magnetizing characteristic curves, all four voltage levels were taken into a single plot (Chapter 3 - Figure 24, Figure 25, and Figure 28).

4.1.2.1 Procedure

By observing the hysteresis curves in a single plot, it is found that coercivity points and saturation points (maximum loop points) affect to characterize the curve. Therefore these two parameters were selected to define the curves and variations which used to recreate hysteresis curves at any voltage level.

Maximum loop point and the coercivity point are shown in the Figure 33.

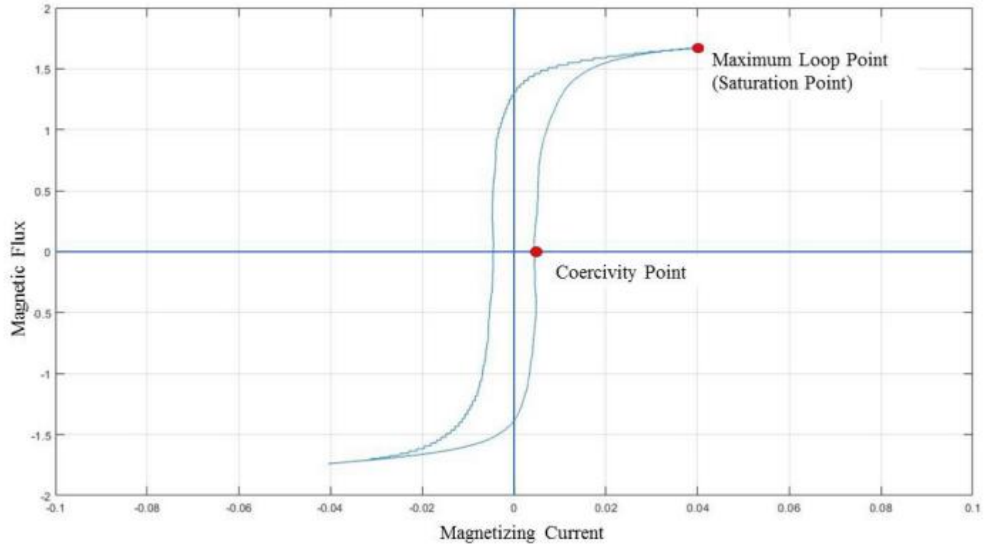


Figure 33: Maximum Loop Point and Coercivity Point of Hysteresis Curve

It was observed that double exponential curve can be used to represent the mean magnetizing curve (without hysteresis effect) of B-H curve by fitting the mean magnetizing curve to closely match with the experimental results.

Mean magnetizing curve is shown in the Figure 34.

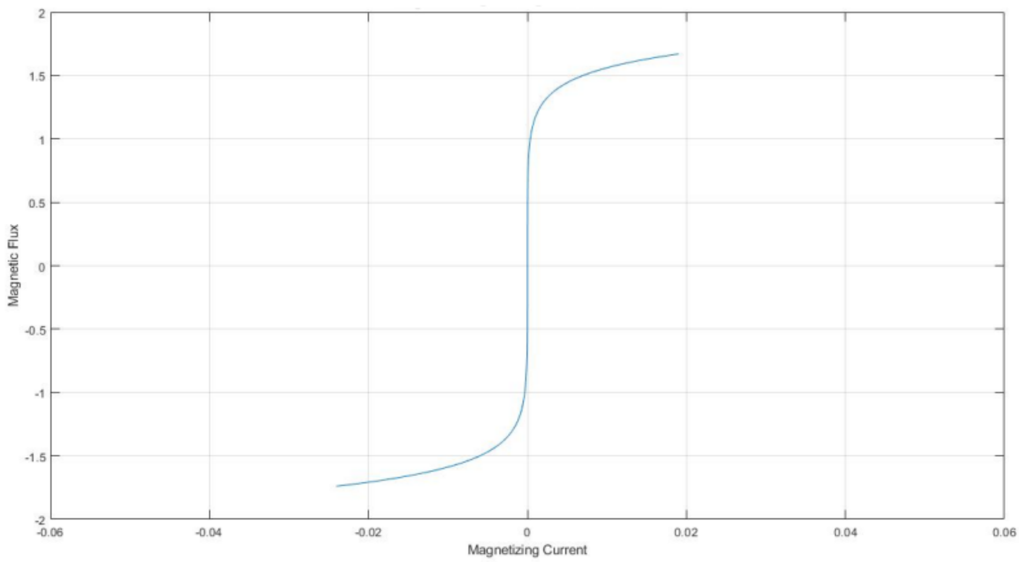


Figure 34: Mean Magnetizing Curve

4.1.2.2 Mathematical Modelling

General function for double exponential curve,

- $Y = K [(e^{ax} - e^{-bx})]$

Three parameters, K, a and b were identified in the function which change the shape of the double exponential curve. Therefore the individual effect of three parameters were studied by varying one parameter at once.

Changing of one parameter at once shows in the figures, Figure 35, Figure 36 and Figure 37.

- Changing of parameter K

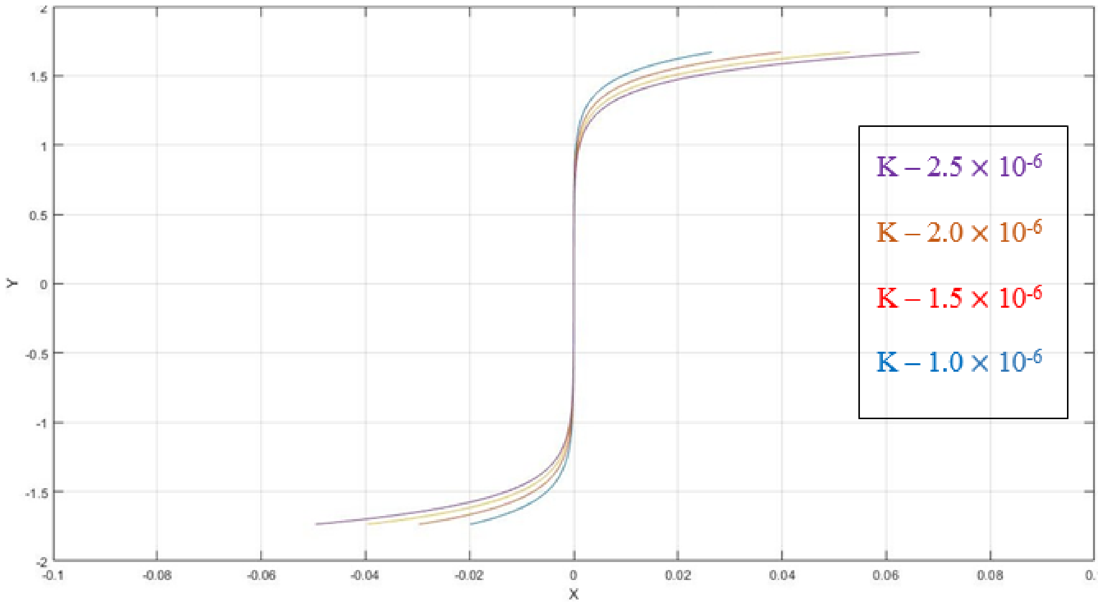


Figure 35: Double Exponential Curves with varying 'K' Values

- Changing of parameter a

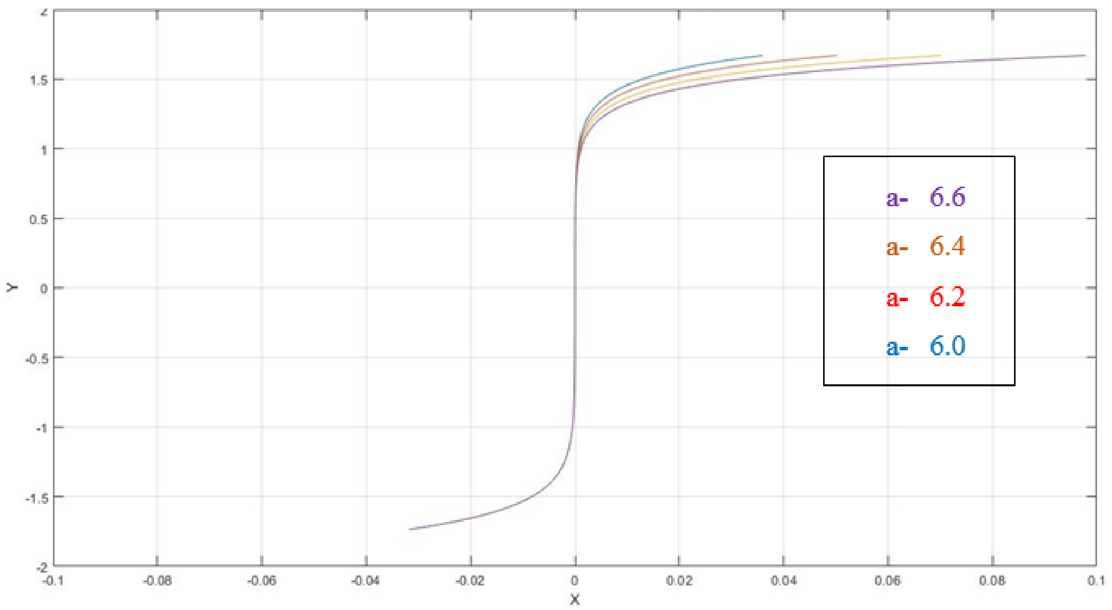


Figure 36: Double Exponential Curves with varying ‘a’ Values

- Changing of parameter b

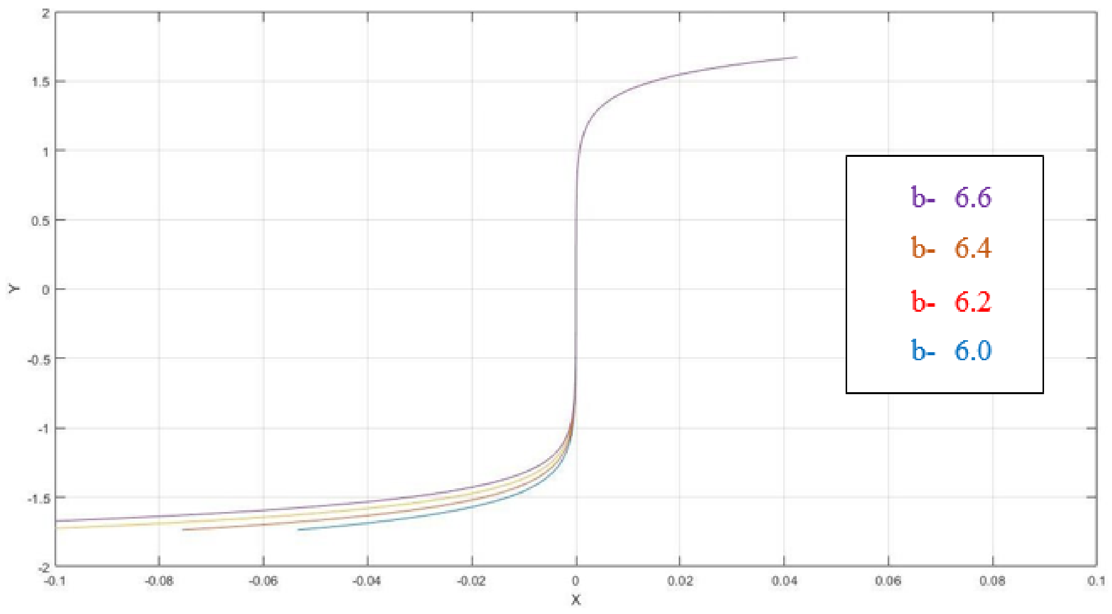


Figure 37: Double Exponential Curves with varying ‘b’ Values

In order to characterize the B-H curve,

As mentioned above, coercivity point and the maximum loop point of a particular B-H was selected. B-H curve was divided in to ascending and descending path for the clear representation. The hysteresis effect was represented by adjusting the mean magnetizing curve (Figure 34) using the selected parameters which represent the B-H curve. Coercivity point values were substituted to the mean magnetizing curve to produce the B-H loop width. The saturation point value provides the loop height. This procedure was repeated for both ascending and descending paths of the magnetizing curve.

Using this procedure, equations for ascending and descending paths were obtained.

Magnetic characteristic equation for ascending path,

- $\Phi = K\,[e^{a(Im-Ic)} - e^{-b(Im-Ic)}]$ ………… (1)

Magnetic characteristic equation for descending path,

- $\Phi = K\,[e^{c(Im+Ic)} - e^{-d(Im+Ic)}]$ ……….. (2)

I_m – Magnetizing current

I_c – Coercivity point

Φ – Magnetic flux

K – Scalar corresponding to saturation point

a, b, c, d – Constants

Equations (1) and (2) were adjusted by changing its parameters, K, a, b, c and d so that it matches the experimentally obtained curves as closely as possible.

In order to obtain the magnetizing characteristic curve at rated voltage;

Following this procedure, to obtain the magnetizing characteristic curves of three phase transformer at rated voltage, the initial mathematical model was derived.

Magnetic Characteristic equation for limb A and Limb C

Ascending path:

- $\Phi = 1.8851\ [e^{5.7\ (Im-0.0115)} - e^{-6.1\ (Im-0.0115)}]$ ………. (3)

Descending path:

- $\Phi = 1.8851\ [e^{5.7\ (Im+0.0115)} - e^{-6.1\ (Im+0.0115)}]$ ………. (4)

Magnetic Characteristic equation for limb B

Ascending path:

- $\Phi = 1.6702\ [e^{5.97\ (Im-0.0046)} - e^{-5.83\ (Im-0.0046)}]$ ……... (5)

Descending path:

- $\Phi = 1.6702\ [e^{5.97\ (Im+0.0046)} - e^{-5.83\ (Im+0.0046)}]$ …..… (6)

In order to obtain the magnetizing characteristic curve for any given voltage;

To obtain magnetizing characteristic for any given voltage, variation of above chosen parameters, K, a, b, c and d was analyzed and fitted into a function so that their values can be obtained at any voltage level. For this purpose, obtained experimental data for other voltage levels (75% of rated voltage, 50% of rated voltage and 25% of rated voltage) were well examined and obtained values for the six parameters which were best fit with experimental data. So that six separate third order polynomials were obtained using the Matlab curve fitting tool.

For Limb A and Limb C

$a = -4.373 \times 10^{-5} \times V^3 + 0.01152 \times V^2 - 1.029 \times V + 37.4$

$b = -6.187 \times 10^{-5} \times V^3 + 0.0148 \times V^2 - 1.175 \times V + 37.2$

$c = -5.013 \times 10^{-5} \times V^3 + 0.01264 \times V^2 - 1.053 \times V + 35$

$d = -5.12 \times 10^{-5} \times V^3 + 0.0128 \times V^2 - 1.1 \times V + 39$

$I_c = -2.134 \times 10^{-10} \times V^3 + 1.833 \times 10^{-7} \times V^2 + 3.254 \times 10^{-5} \times V - 3.217 \times 10^{-4}$

$K = -1.635 \times 10^{-6} \times V^3 + 2.447 \times 10^{-4} \times V^2 + 7.019 \times 10^{-3} \times V - 0.1563$

For Limb B

$a = -4.48 \times 10^{-5} \times V^3 + 0.01187 \times V^2 - 1.076 \times V + 39.7$

$b = -5.547 \times 10^{-5} \times V^3 + 0.0147 \times V^2 - 1.237 \times V + 41.3$

$c = -4.267 \times 10^{-5} \times V^3 + 0.0112 \times V^2 - 0.9733 \times V + 33.7$

$d = -5.76 \times 10^{-5} \times V^3 + 0.01472 \times V^2 - 1.292 \times V + 45.7$

$I_C = 4.369 \times 10^{-8} \times V^3 - 6.388 \times 10^{-6} \times V^2 + 3.503 \times 10^{-4} \times V - 3.359 \times 10^{-3}$

$K = -2.552 \times 10^{-7} \times V^3 + 6.702 \times 10^{-5} \times V^2 + 1.37 \times 10^{-2} \times V + 0.1001$

V – Applied voltage as a percentage (%) of rated voltage

For any given voltage, when the required voltage, V value (percentage value) is substituted to the above functions of six parameters, values for six parameters can be obtained. By substituting these six parameters in the equations (1) and (2), magnetizing characteristic for limb A, limb B and limb C can be obtained for the given voltage. So that, for any voltage level, magnetizing characteristic curves can be obtained using this mathematical model.

CHAPTER 5

5 RESULTS AND DISCUSSION

In this chapter, magnetizing characteristic curves are obtained by substituting values for two mathematical models under two methods discussed in the Chapter 4. And obtained results are validated with the experimental test results.

5.1 Mathematical Model – Method 01

Method 01 is used when magnetizing characteristic data available only for rated voltage. Using this method, obtained results for 75% percent of rated voltage curve is shown in the Figure 38.

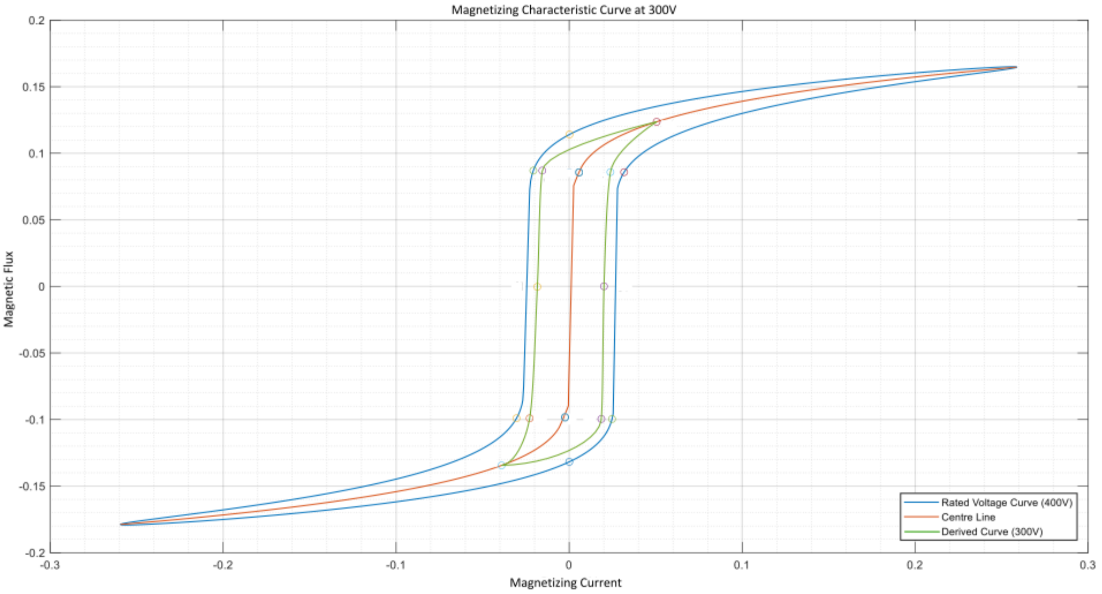


Figure 38: Magnetizing Characteristic Curve for 75% of Rated Voltage Using Method-01

Obtained results were validated with the experimental test results. Therefore comparison of curves was done for all voltage levels, 75% of rated voltage (300V), 50% of rated voltage (200V) and 25% of rated voltage (100V) of the single phase transformer. Captured curves are shown in the figures, Figure 39, Figure 40 and Figure 41.

5.1.1 Test Results and Modelled Results for 300V

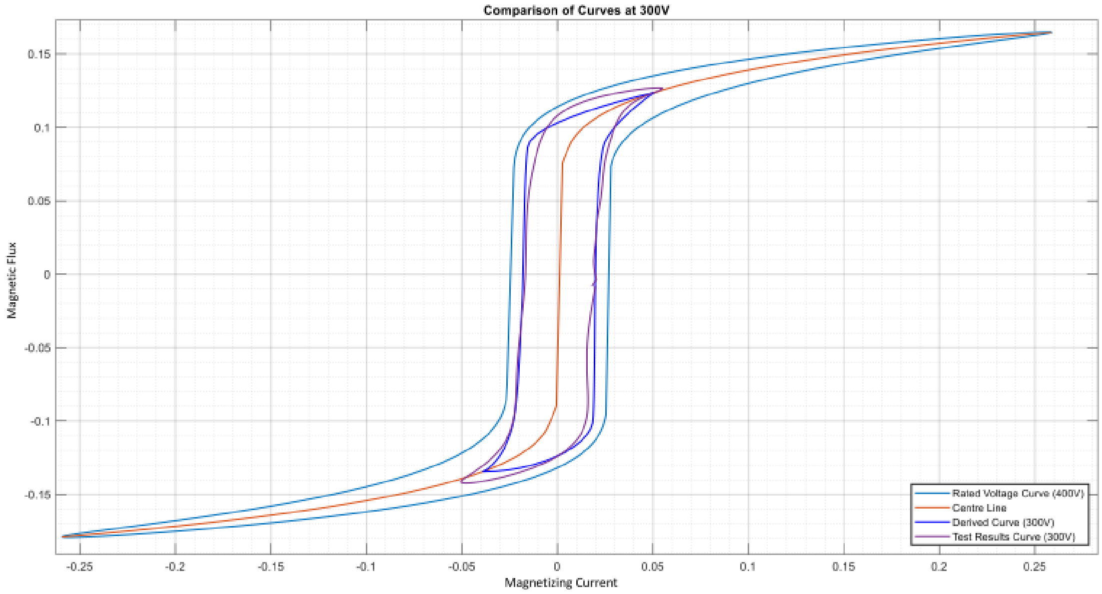


Figure 39: Comparison of Curves at 300V

5.1.2 Test Results and Modelled Results for 200V

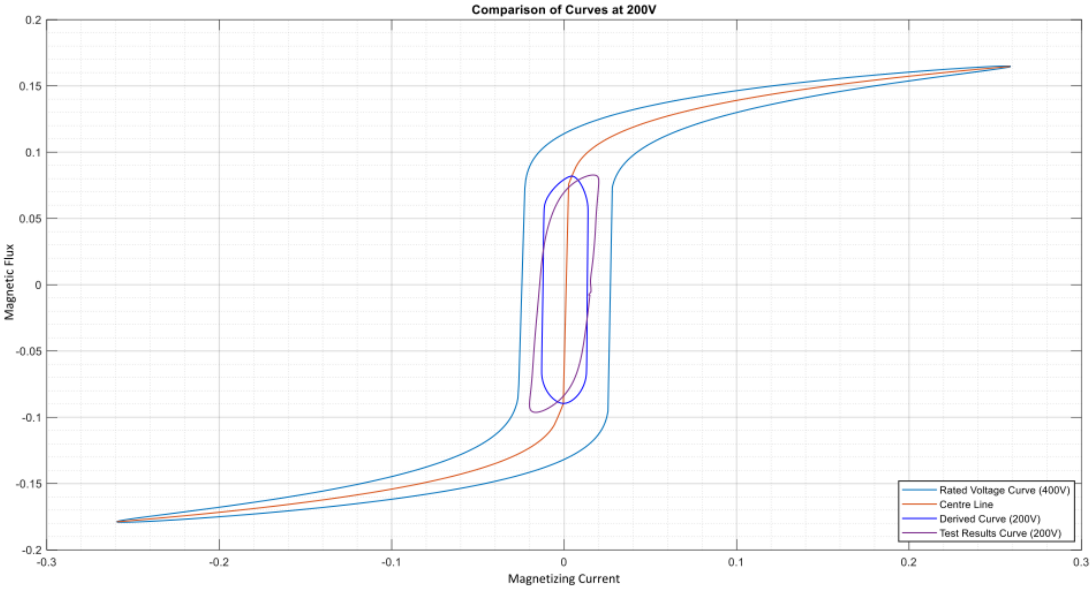


Figure 40: Comparison of Curves at 200V

5.1.3 Test Results and Modelled Results for 100V

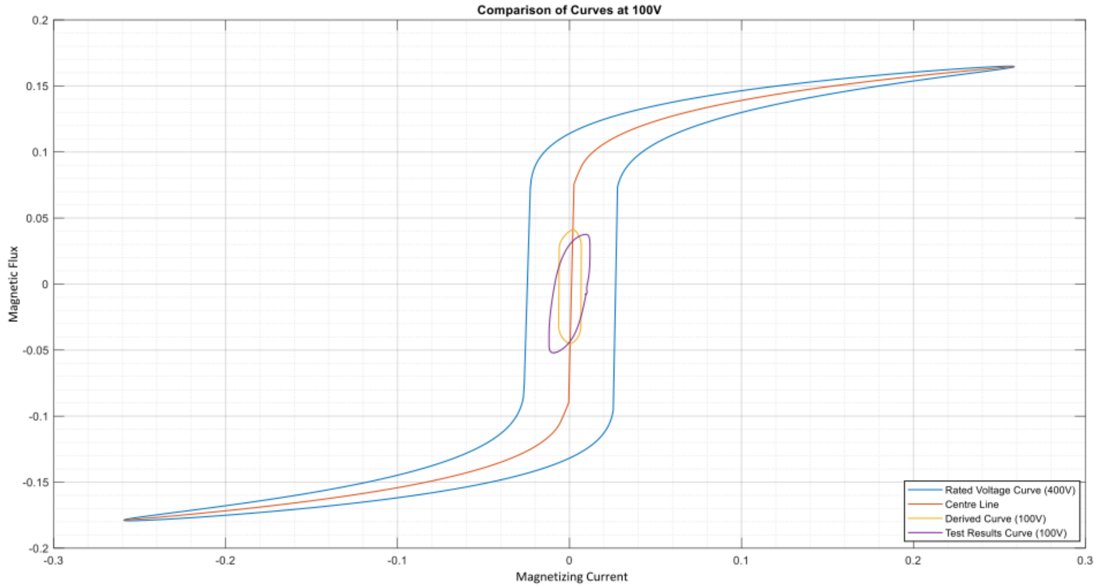


Figure 41: Comparison of Curves at 100V

5.2 Mathematical Model – Method 02

Method 02 is used when magnetizing characteristic data available for any voltage level. Using this method, results were obtained for 75% percent of rated voltage of three phase transformer, limb A, Limb B and limb C.

Obtained results were validated with the experimental test results. Therefore comparison of curves was done for 75% of rated voltage (300V) for three limbs of the transformer.

5.2.1 Test Results and Modelled Results for Limb A (400V)

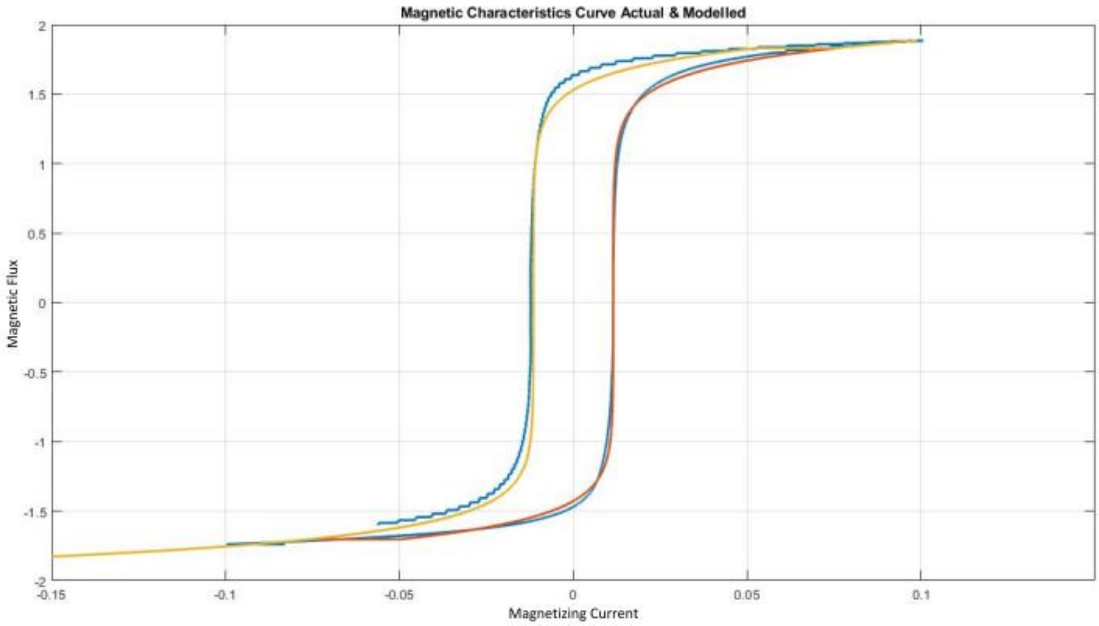


Figure 42: Magnetizing Characteristic of Limb A at Rated Voltage, Experimental Curve (Blue), Modelled Curve Ascending Path (Red) and Modelled Curve Descending Path (Yellow)

5.2.1.1 Test Results and Modelled Results for Limb A (300V)

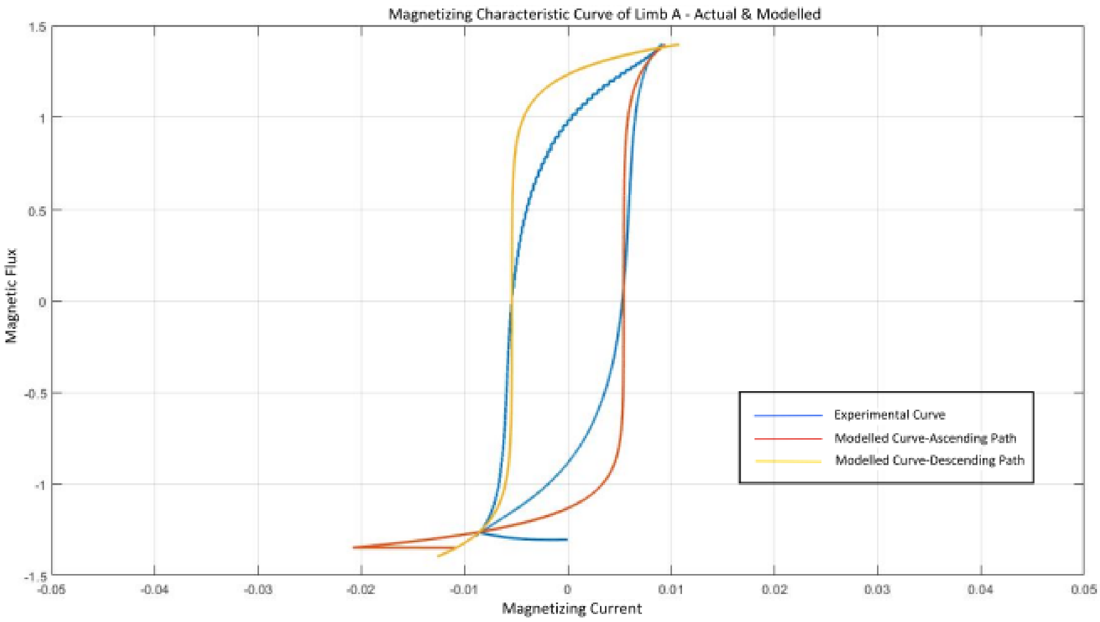


Figure 43: Magnetizing Characteristic of Limb A at 300V

5.2.2 Test Results and Modelled Results for Limb B (400V)

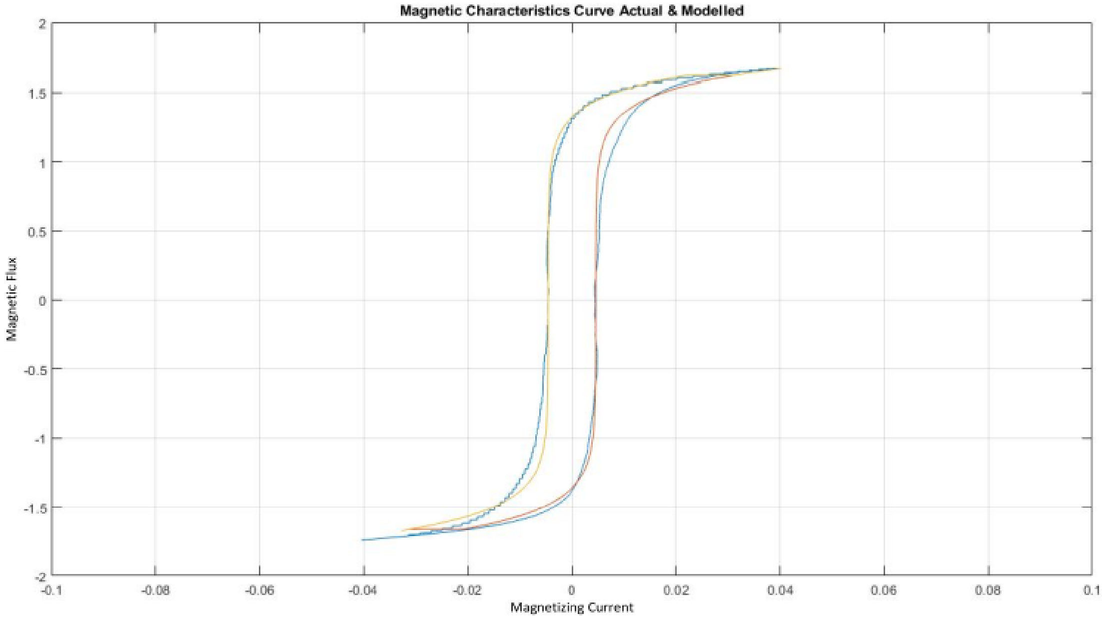


Figure 44: Magnetizing Characteristic of Limb B at Rated Voltage, Experimental Curve (Blue), Modelled Curve Ascending Path (Red) and Modelled Curve Descending Path (Yellow)

5.2.2.1 Test Results and Modelled Results for Limb B (300V)

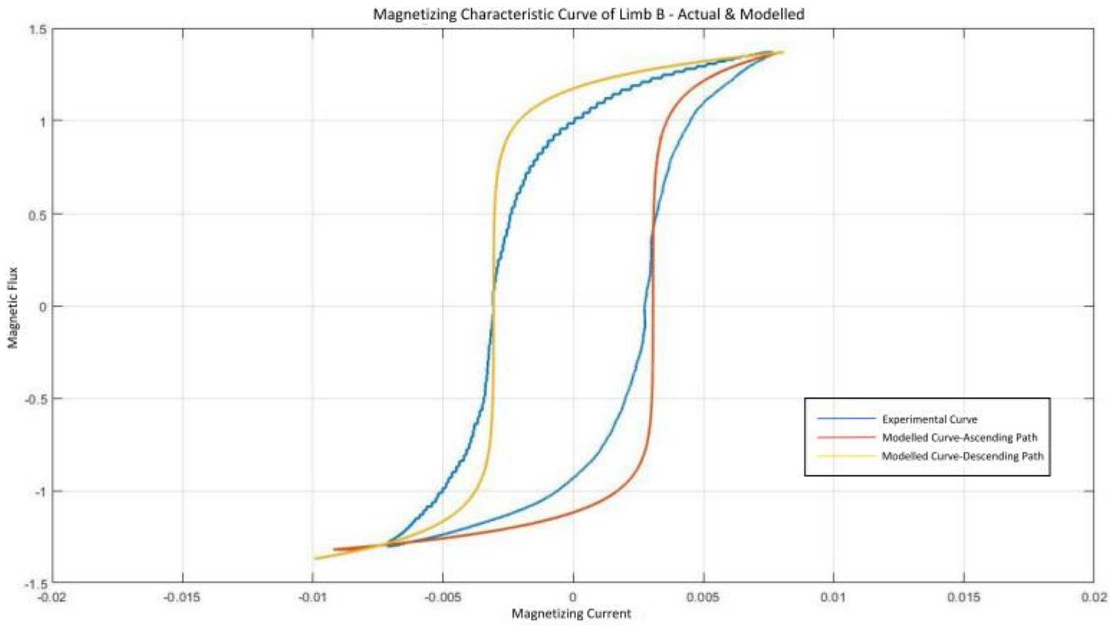


Figure 45: Magnetizing Characteristic of Limb B at 300V

5.2.3 Test Results and Modelled Results for Limb C (400V)

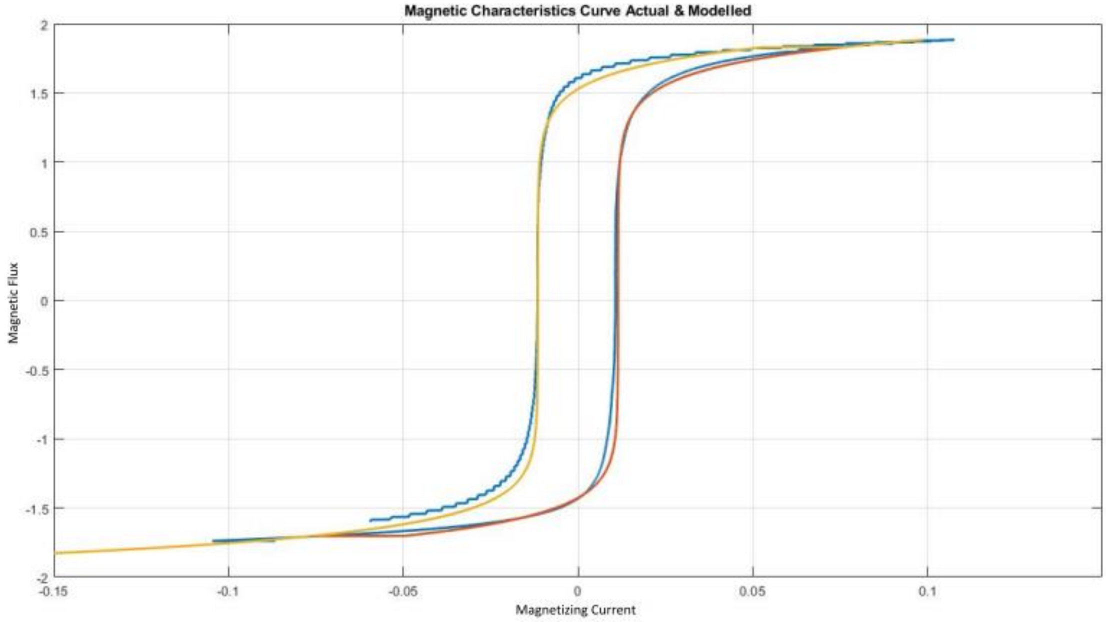


Figure 46: Magnetizing Characteristic of Limb B at Rated Voltage, Experimental Curve (Blue), Modelled Curve Ascending Path (Red) and Modelled Curve Descending Path (Yellow)

5.2.3.1 Test Results and Modelled Results for Limb C (300V)

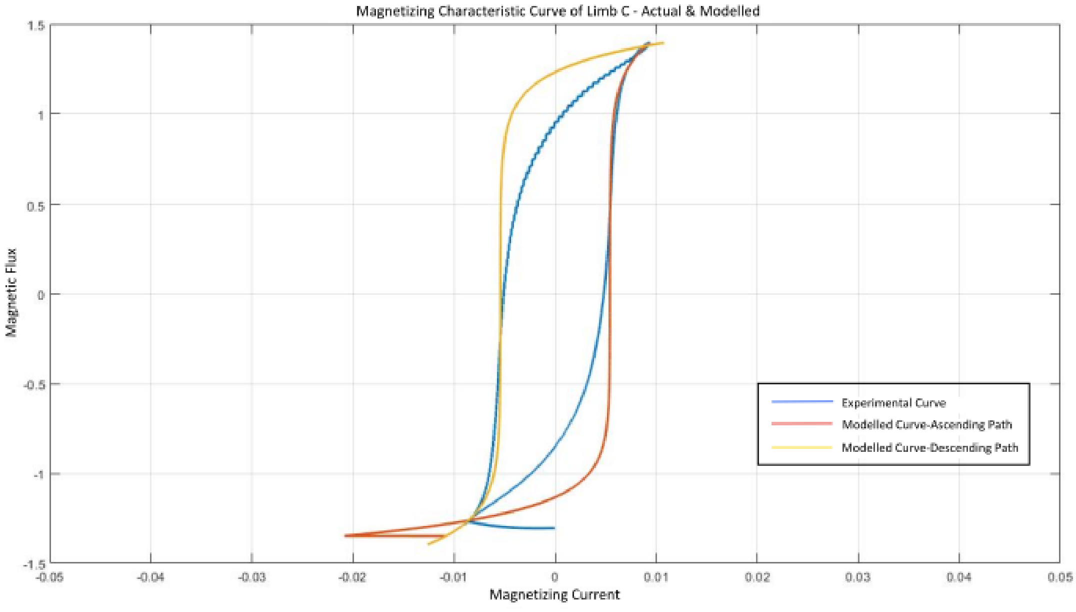


Figure 47: Magnetizing Characteristic of Limb C at 300V

5.3 Discussion

As evidence from the above comparison of magnetizing curves derived in both methods, Method 01 and Method 02, it can be seen that there is a slight difference in modeled curves compared with the experimental curves. But from proposed two methods, the shape of waveform obtained from theoretical model 2 (Method 02) is more accurate than the theoretical model 1 (Method 01).

These two options can be used according to the availability of data. Method 01 can be used when magnetizing characteristic data available only for rated voltage. From the magnetizing characteristic curve obtained for rated voltage, extract data for required voltage and can be plotted the curve. In this method, modelled curves show slight difference with experimental curves only when plotting curves for lower voltage levels with respect to rated voltage.

In the event of Method 02, when magnetizing characteristic data for distinct voltage levels are available, this theoretical method can be used. This leads to a more accurate result. Considering the practical application aspect, this model can be used to visualize and investigate the magnetic characteristics of a transformer limb wise at a required voltage level.

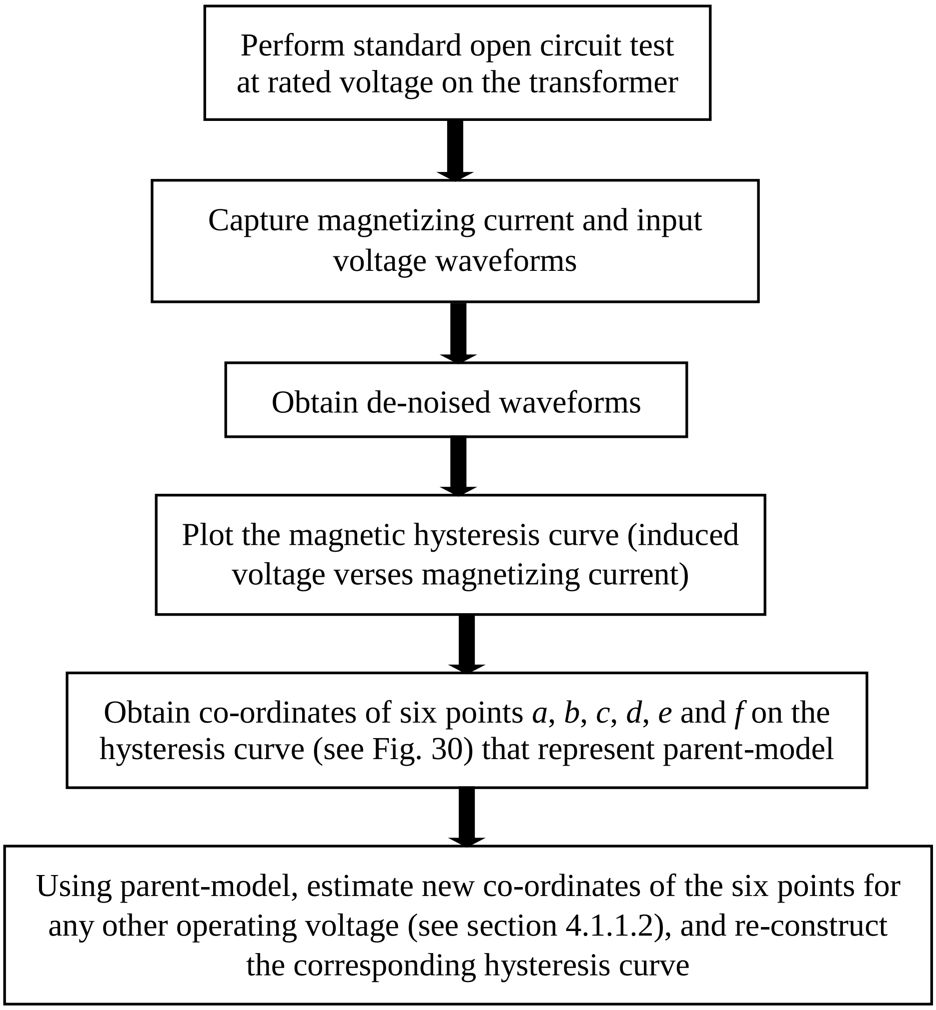
From both two methods, it can be seen that coercivity point and saturation point (maximum loop point) affect to characterize the magnetizing characteristic curve. Therefore these two parameters have been selected to recreate hysteresis curves for any amplitude of flux. Hence magnetic field strength, magnetizing current and magnetic flux are mainly responsible for the shape of the magnetizing characteristic curve. That is, application of a field strength causes the magnetic induction to increase in the field direction and if field strength is increased indefinitely, the magnetization eventually reaches saturation. Therefore magnetic saturation depends on the magnitude of the atomic magnetic moments and number of atoms per unit volume. With respect to the proposed models, when voltage varies it can be seen that magnetic characteristics vary non-linearly at boundaries and reaches saturation points as per the applied voltage.

When considering the frequency for this modelling, the tests were done for single phase transformer and three phase transformer at laboratory at the frequency of 50Hz.

Effects of frequency on the magnetic hysteresis were not investigated in this study. Therefore, the proposed models are well suited for transformers operating at power frequency and accompanying harmonics. On the other hand, the grades of iron chosen for transformers operating at higher frequencies are different (having narrow hysteresis loop) and the relative effects of frequency on hysteresis are minimal. Hence, the models are applicable for majority of real world applications without much of a loss of the accuracy.

5.4 Test Procedure and Outcome

1. Method 1 - When magnetizing characteristic data available for rated voltage
   - For a single phase transformer



- For a three phase transformer

Perform separate open circuit tests for Limbs 1, 2 and 3 of the transformer, all at rated voltage (see section 3.3.2)

↓

Capture magnetizing current and input voltage waveforms in each of the three tests

↓

Obtain de-noised waveforms

↓

Plot the magnetic hysteresis curves (induced voltage verses magnetizing current) for Limbs 1, 2 and 3

↓

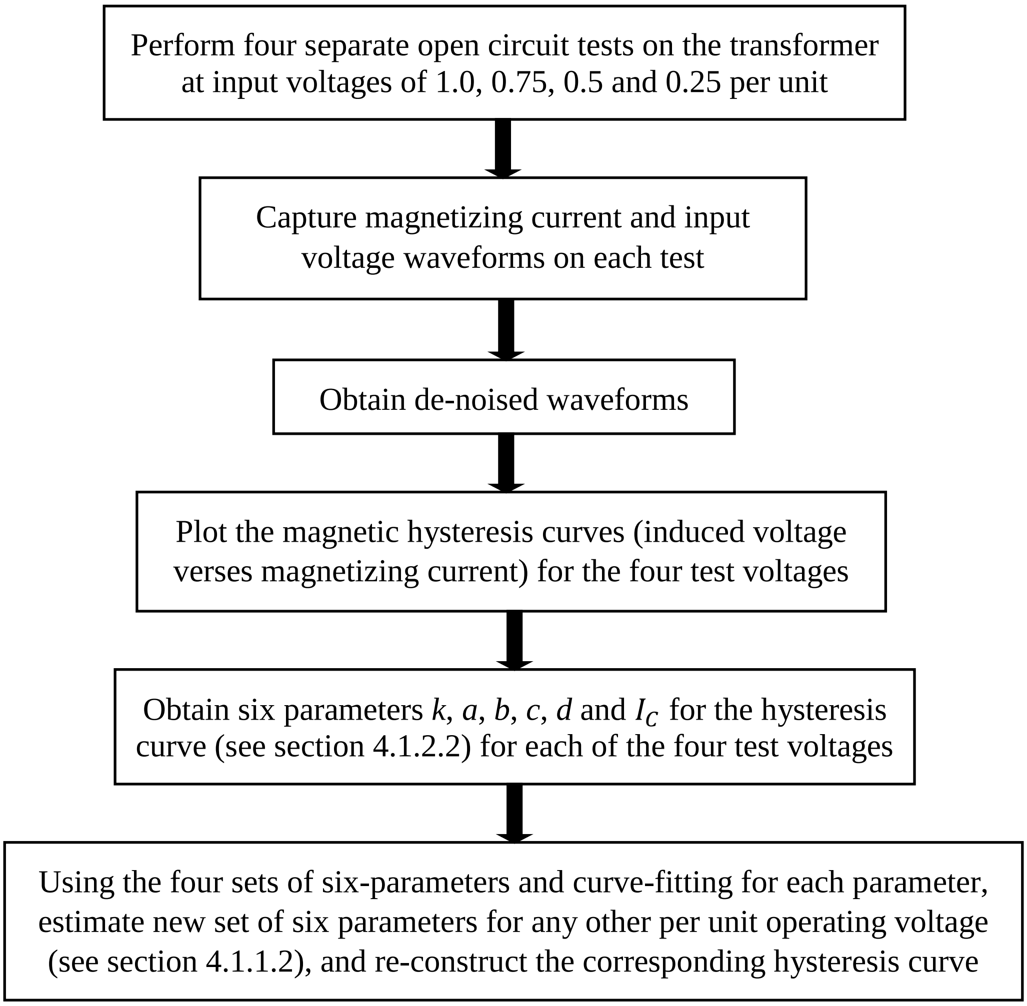
Obtain co-ordinates of six points *a*, *b*, *c*, *d*, *e* and *f* on each of the three hysteresis curves (see Fig. 30) that represent parent-models for Limbs 1, 2 and 3

↓

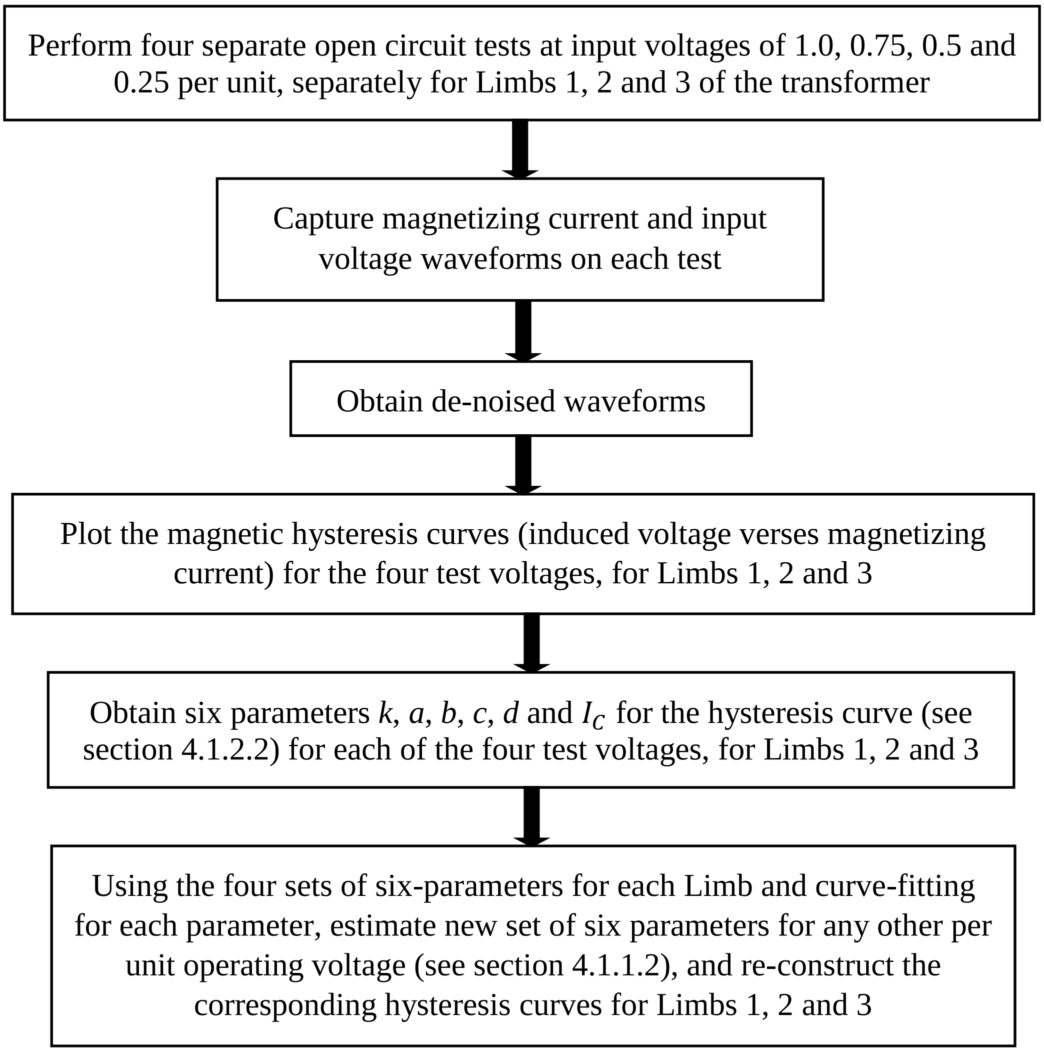
Using parent-models, estimate new co-ordinates of the six points for any other operating voltage (see section 4.1.1.2), and re-construct the corresponding hysteresis curves for Limbs 1, 2 and 3

2. Method 2 - When magnetizing characteristic data available for any voltage

- For a single phase transformer



- For a three phase transformer



CHAPTER 6

6 CONCLUSION AND RECOMMENDATIONS

The goal of this research was to create a comprehensive model for magnetic hysteresis behavior of a three phase transformer. In this study, two novel magnetizing characteristic models are introduced using magnetic circuit theory incorporate with a test based mathematical approach. To incorporate limb wise core magnetic characteristics depend on the core construction, without considering the core material specific data, two test were obtained which can be performed under normal laboratory conditions. Theoretically modelled hysteresis curves compared with the experimentally obtained curves. Results of the two methods are closely matched with some fewer variations.

Two mathematical models proposed in this study can be used based on the availability of data. Results obtained from both methods show expected resemblance. There are some small deviations at lower voltages and these are mainly due to the differences among the experimental set-ups at different voltage levels.

Mathematical modeling of hysteresis curve is not unique and this is an interesting area for further investigations and research, in particular to model the hysteresis curve for any voltage based on an experimental curve captured corresponding to rated voltage. Also, the influence on the hysteresis curve due to variation of frequency over a wider range is another area for further investigations.

The proposed method No. 1 needs only the open circuit test data at rated voltage from the transformer and hence it is easier to apply for any transformer, even those already in operation in respective applications. The method No. 2 needs open circuit test data for different other voltage levels too, in addition to rated voltage, so its applicability depends on whether such tests are possible with the given transformer.

By considering the outcomes of this research study, as a conclusion it can be stated that the proposed model will be a convenient method of obtaining magnetic characteristic effect of hysteresis behavior for a given transformer.

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