ESTIMATION OF APPARENT GROUND RESISTIVITY IN VERTICALLY LAYERED SOIL

B.V. Jayasundara

(149307 G)

Degree of Master of Science in Electrical Installations

Department of Electrical Engineering

University of Moratuwa
Sri Lanka

April 2019

ESTIMATION OF APPARENT GROUND RESISTIVITY IN VERTICALLY LAYERED SOIL

B.V. Jayasundara

(149307 G)

Dissertation submitted in partial fulfillment of the requirements for the

Degree Master of Science in Electrical Engineering

Department of Electrical Engineering

University of Moratuwa
Sri Lanka

April 2019

DECLARATION OF THE CANDIDATE AND SUPERVISOR

I declare that this is my own work and this dissertation does not incorporate without acknowledgement any material previously submitted for a Degree or Diploma in any other University or institute of higher learning and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text.

Also, I hereby grant to University of Moratuwa the non-exclusive right to reproduce and distribute my dissertation, in whole or in part in print, electronic or other medium. I retain the right to use this content in whole or part in future works (such as articles or books).

………………………… ……….…………….

Signature of the candidate Date:

(B.V.Jayasundara)

The above candidate has carried out research for the Masters Dissertation under my supervision.

………………………… ……...……………….

Signature of the supervisor Date:

(Dr. Asanka Rodrigo)

Abstract

Soil resistivity and grounding system resistance play key roles in designing earthing systems, safe operation of electric power systems and lighting protection systems. Also, it is beneficial in evaluating the degree of corrosion of underground pipelines.

Most of the researches contributed to this area are based on horizontal 2 layers and horizontal multi layers, but the results are still misinterpreted with the presence of vertical layers. Therefore, the knowledge of the ground structure and determination of soil resistivity with the vertically layered soil are critical in the overall design of the earthing systems in terms of safety, reliability and cost.

In this research, three vertically layered soil models were proposed including two vertical layer soil, three parallel vertical layer soil and three perpendicular vertical layer soil. Then a set of equations were derived to determine the apparent soil resistivity for each soil model using the method of images and Wenner four-point method. The results were further analyzed using MATLAB and compared with field measurements to check the accuracy and efficiency.

The results show that the apparent soil resistivity of vertically layered soil was affected by the direction of electrodes array and the distance between measuring electrodes and interface between the vertically layered soil.

Keywords: vertically layered soil, vertical layer soil apparent resistivity, method of images, grounding, earthing, wenner method, multi layered ground, multilayer soil

Acknowledgement

It is my great pleasure to express gratitude to those who were behind me in completing my research project.

First, I would like to express my sincere gratitude to my supervisor Dr. Asanka Rodrigo for providing this visionary concept to study and for his continuous guidance throughout the period.

My sincere thanks go to my immediate supervisor, Mr. Thusitha Gunasekara – Head of Design & Sales – KIK Lanka (Pvt) Ltd and all my colleagues at KIK Lanka (Pvt) Ltd who helped me in many ways during this period.

Last but not least, I would express my heartiest gratitude and love to my mother, my father and my girlfriend who took a lot of burden and patient, which helped me to complete this work in difficult circumstances.

Table of Contents

List of Figures

List of Tables

List of Abbreviations

Abbreviation	Description
IEEE	Institute of Electrical and Electronics Engineers
BS	British Standards

List of Appendices

CHAPTER 1

1. INRODUCTION

1.1. Background

An excellent grounding system guarantees the safe operation of electrical power systems, lightning protection systems and prevents dangers from extreme ground potential and ground fault.

When it comes to designing a high-performance grounding system soil resistivity plays an important role. There are various methods and models employed in measuring soil resistivity and most of them are based on uniform, horizontal two-layer and horizontal multi-layer soil models.

Uniform and horizontal two-layer model yields significant benefits in economy, accuracy and safety, [1] but it only identifies the surface layer to about 1 meter. The horizontal multi-layer model provides more accurate information with the presence of lower soil layers.

All these soil models are based on horizontal layers, but the real ground is a mix of horizontally and vertically layered soil and the results are still misinterpreted with the presence of vertical layers. Few examples for the vertical soil are,

1. Sites adjacent to the rivers and lakes
2. Areas with slopes
3. Sites with manmade boundary walls and large buildings
4. Filled lands with different types of soil

So, it is important to understand the characteristics of apparent soil resistivity of the vertically layered soil to minimize the errors in designing grounding systems.

1.2. Objectives of study

Analyze the apparent ground resistivity with the presence of vertical soil layers and propose an accurate method to calculate the apparent resistivity.

1.3. Motivation

This study would help in understanding the soil structure of an area and the characteristic of apparent soil resistivity with the presence of vertically layered soil. This will be helpful for the grounding system engineers to minimize the errors and work on more accurate earthing systems which improves the safety and reliability of the installation.

1.4. Methodology

Major activities of this research can be summarized as follows.

1. Understand the different soil models of the vertically layered of soil

Three different vertically layered soil models are studied in this research.

a. Two vertically layers
b. Three parallel vertical layers
c. Three perpendicular vertical layers

The research can be expanded to more complex structures, but in practical it is quite rare to find vertical soil models than these three cases.

2. Prepare a model/equation to calculate the apparent soil resistivity

By referring the international research papers, the method of images is identified as the most suitable way to build up an accurate and equivalent model to analyze the apparent soil resistivity of vertically layered soil.

3. Further analyze the behavior of the model using a simulation software.

It is important to analyze the models with a simulation software to understand the soil resistivity changes in one layer with respect to the other vertical layers. In this research MATLAB is used to analyze the mathematical model and have a clear picture of the changes of the soil resistivity.

4. Verify the model/equations with field measurements

The results of the mathematical model are compared with field measurements to check how accurate the model is.

CHAPTER 2

2. LITERATURE REVIEW

The international standards related to earth resistivity, grounding systems, earthing of electrical installations as well as many international research papers are based on uniform and horizontally layered soil. But the real ground is layered in horizontally, vertically or has a mix of horizontal and vertical layers.

Few examples for the vertical soil layers are lands adjacent to the large water volumes such as sea, rivers and lakes, manmade boundary walls, a base of a building or lands with slopes. Only a few studies were carried out to investigate the apparent resistivity of vertical soil layers, but most of them limited to two vertical soil layers and are not verified with field data.

IEEE standard, BS standard and a few other research papers are analyzed in this literature review to identify a proper method to analyze the apparent soil resistivity of vertically layered soil. The research papers are annexed to the Appendix - C

1. IEEE Guide for Measuring Earth Resistivity, Ground Impedance, and Earth Surface Potentials of a Grounding System IEEE Std 81™-2012 (Revision of IEEE Std 81-1983)

Werner four-point method has been suggested as a good method for measuring the apparent resistivity. Also, the standard discusses the apparent resistivity of homogeneous and horizontal two layered soil. But it doesn't include anything related to multi-layer horizonal or vertical layered soil.

2. BS 7430:2011 Code of practice for protective earthing of electrical installations

Werner four-point method has been suggested to measure the earth resistivity. But it doesn't mention anything about the soil layers. But the standard states "The resistance to earth of a given electrode depends upon the electrical resistivity of the soil. Most first approximation formulae are related to homogenous soil, which is rarely the case in practice, where the different layers of strata will affect the distribution of current passing through the electrode."[3]

3. Study of Soil Resistivity Measurements in Vertical Two-Layer Soil Model.

Author(s): Mohamed Nayel, Boyang Lu, Yu Tian, Yingzhen Zhao, Department of Electric and Electronic Engineering Xi'an Jiaotong-Liverpool University, Suzhou, China

An expression is proposed to estimate the soil resistivity of two vertically layered soil based on the method of images and Wenner four-point method. According to their analysis the apparent resistivity depends on the spacing between the electrodes, between the first electrode and the interface between the two layers and the direction of electrodes array.

But there is no suggestion on how to expand the analysis for more than 2 vertical layers. Also, they haven't checked the validity of the results against the practical data or the results of a software simulation.

4. A simple formula of grounding grid resistance in vertical two-layer soil.

Author(s): Xiaobin Cao, Guangning Wu, Shenglin Li, Weiming Zhou, RuiFang Li,

In this research an equation is developed based on the simulation done by CDEGS software package. The equation is not verified with practical data and only prepared based on the simulated model. Also, this equation can be applied only for the grounding grids.

5. Analysis of Linear Ground Electrodes Placed in vertical Three-Layer Earth
Author(s): Predrag D. Rancid (IEEE member), Zoran P. Stajic', Bojana S. ToSid Faculty of Electronic Eng. of NE, Srbija, SR Jugoslavija, Djordje R. Djordjevid, Faculty of Civil Eng. of NiS, Srbija, SR Jugoslavija

Three vertical layers of soil has been analyzed in this research, but the mathematical model is extremely hard to understand. The research paper further states that it can be used to get the grounding impedance including resistance, reactance and equivalent capacitance.

By considering the above standards and research papers, the method of images is the most suitable way to build up a mathematical model to calculate the apparent soil resistivity of vertical layers of soil.

CHAPTER 3

3. VERTICAL LAYER SOIL MODELS

The soil has a complex structure and is hardly uniform or homogeneous. Most of the time it is either horizontally or vertically layered or it is combination of both.

After analyzing several geographical areas three soil models were identified to continue the analysis of the vertically layered soil. These models can cover almost all the practical situations and the locations can be similar to these models or a combination of models.

1. Two vertical layers
2. Three parallel vertical layers
3. Three perpendicular vertical layers

Few examples are

1. Sites adjacent to the rivers and lakes
2. Areas with slopes
3. Sites with manmade boundary walls and large buildings
4. Filled lands with different types of soil

Figure 3.1: A site next to a large water volume

Figure 3.2: A land with slope

3.1. Identified soil models

3.1.1. Two vertical layers

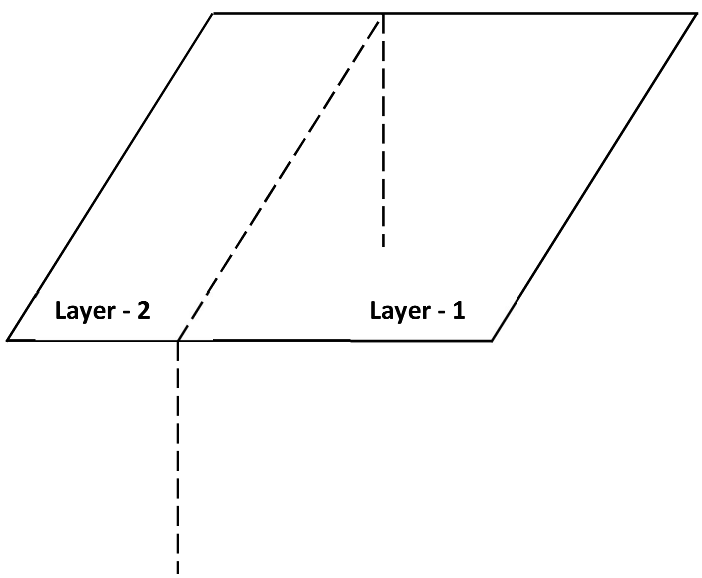

Figure 3.3: Two vertical layers

3.1.2. Three parallel vertical layers

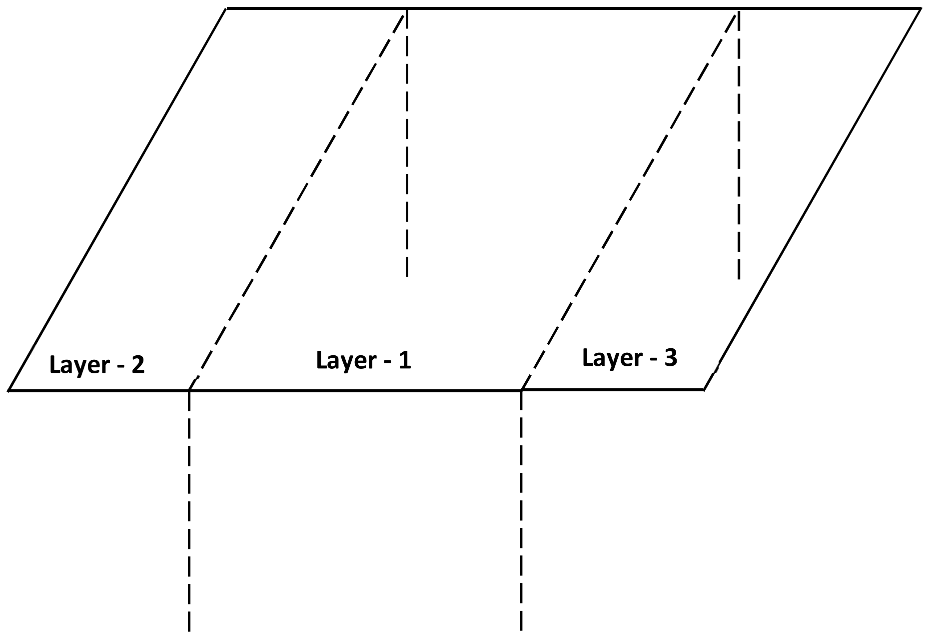

Figure 3.4: Three parallel vertical layers

3.1.3. Three perpendicular vertical layers

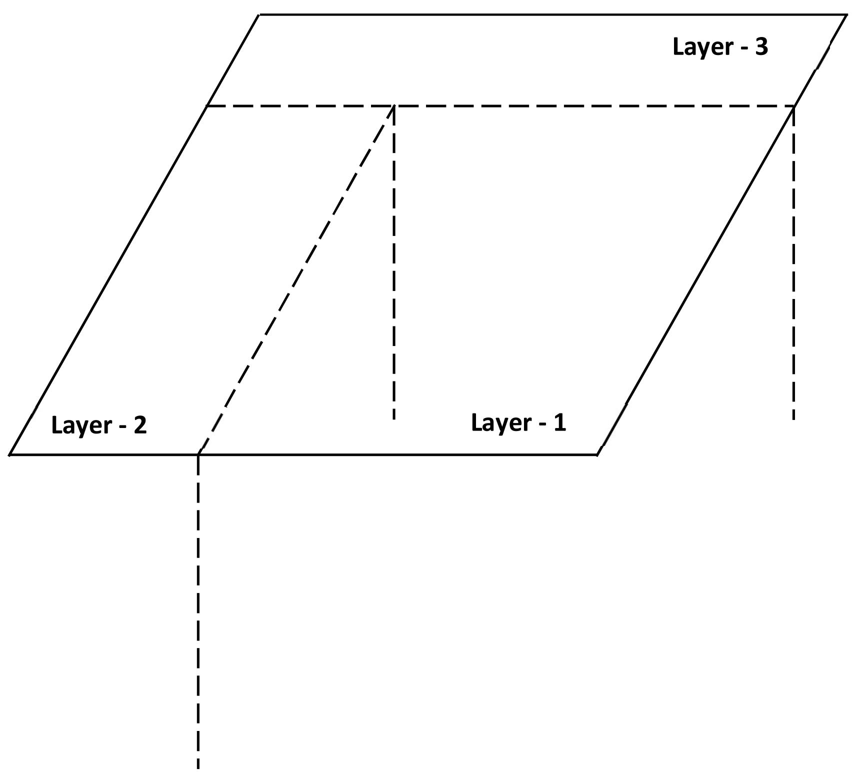

Figure 3.5: Three perpendicular vertical layers

CHAPTER 4

4. CALCULATIONS

"The method of images" is considered to propose an accurate and equivalent model for the vertically layered soil. Based on the model developed, Wenner four-point method is used to derive a set of equations to calculate the apparent resistivity.

4.1. Method of images

Method of images is a technique for solving problems related to electrostatics, magnetostatics and electromagnetic fields in cases where there are reflecting boundaries.

The image theory for electrostatics states that a given charge configuration above an infinite grounded perfect conducting plane can be replaced by the charge configuration itself, its image, and an equipotential surface in place of the conducting plane.

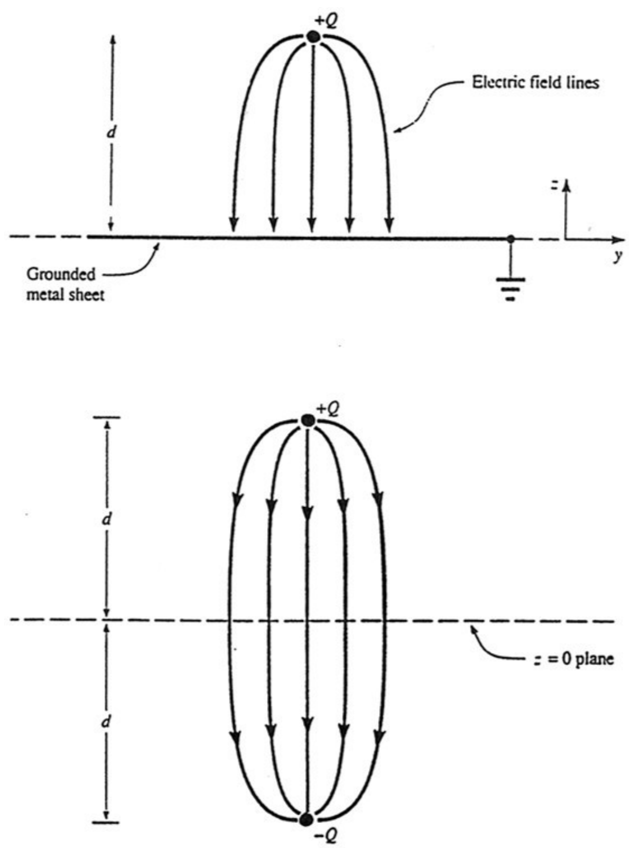

Figure 4.1: The image theory for electrostatics

The image theory for magnetostatics states that the field between a magnetic dipole over a superconducting surface can be replaced by the magnet and its symmetric one.

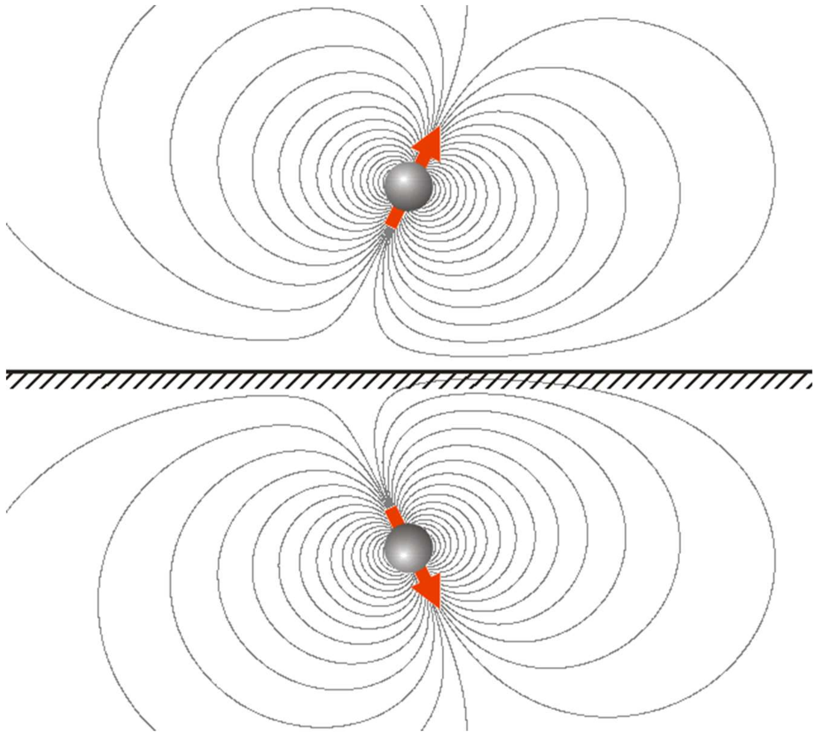

Figure 4.2: The image theory for magnetostatics

4.2. Wenner method

Wenner Four Pin Method is one of the best methods to measure the apparent resistivity of large volumes of undisturbed earth. [1] It is developed by Dr. Frank Wenner, US Bureau of Standards in 1915.

In this method four auxiliary probes are installed in the earth. The outer electrodes are current electrodes and inject the current to the earth. The inner electrodes measure the voltage drop due to resistance of soil when current passed between the outer electrodes.

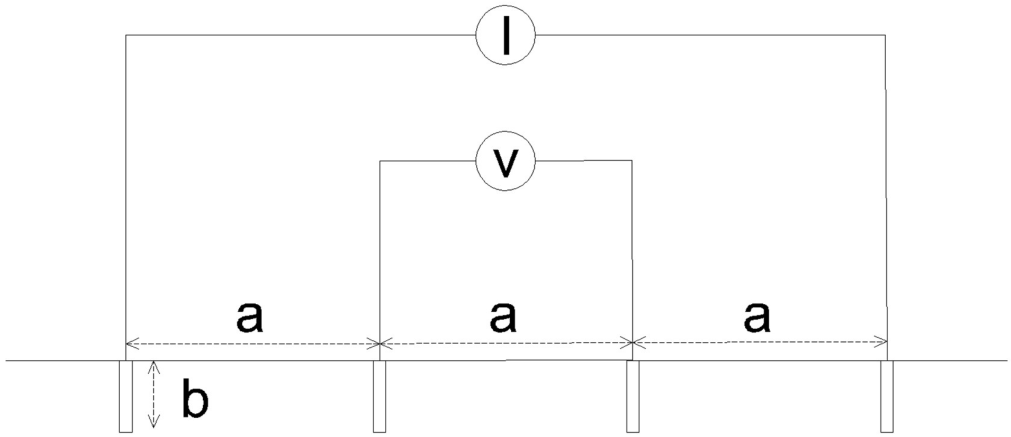

Figure 4.3: Wenner method with equally spaced test probes

A current I is passed between the outer probes, and the potential V between the inner probes is measured using a potentiometer or high-impedance voltmeter. Then, the apparent resistivity ρ in the terms of the length units in which a and b are measured is

$$\rho = \frac{4\pi a R}{1 + \frac{2a}{\sqrt{a^2 + 4b^2}} - \frac{a}{\sqrt{a^2 + b^2}}}$$

in practice, the electrodes are usually placed in a straight line at intervals a, driven to a depth not exceeding 0.1a. Then, we can assume b = 0 and the equation becomes

$$\rho = 2\pi a R$$

$$\rho = 2\pi a \frac{V}{I}$$

4.3. Calculation for two vertical layer soil

This soil model has two vertical layers with different soil resistivities. The probes are buried in the layer 1 as shown in the figure 4.4.

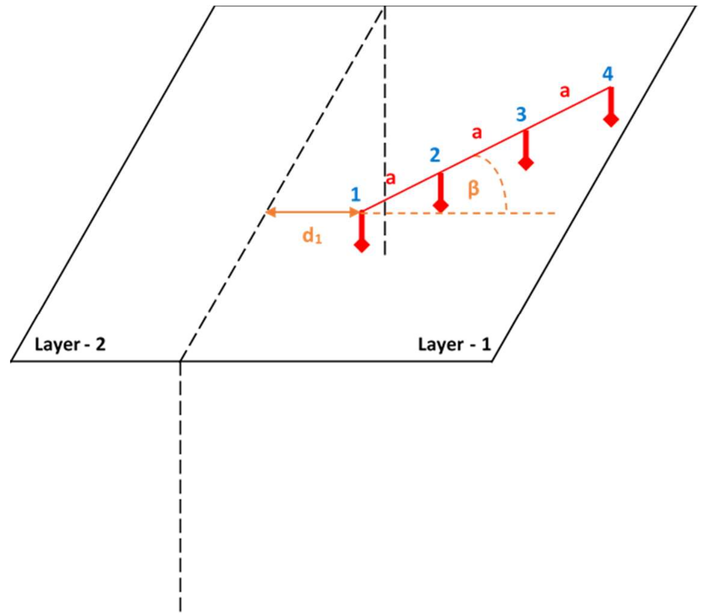

Figure 4.4: Electrode arrangement on two vertical layer soil

Image distribution of the two-vertically layered soil is shown in Figure 4.5.

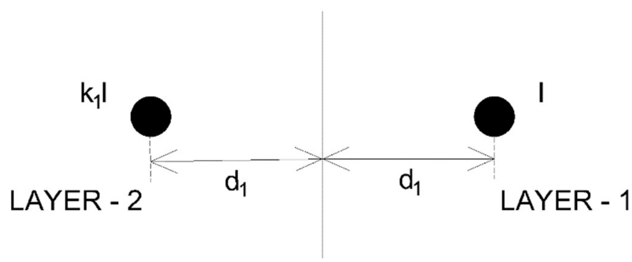

Figure 4.5: Image distribution of two vertical layer soil

a = Electrode spacing (Meters)

d_1= The distance between the 1st electrode and intersection (Meters)

β = The direction of probes array (Degree or Radian)

ρ_1= Soil resistivity of layer 1 (Ohm.meter)

ρ_2= Soil resistivity of layer 2 (Ohm.meter)

$k_1 = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$ = Reflection ratio between layer 1 and 2

The voltage of the probe 2 is equal to the sum of the potentials due to the injecting current at probe 1 and sinking current at probe 4. The same is apply for the voltage of the probe 3.

To simplify the calculation a voltage measuring probe which is located in x (Meters) distance from the current probe is considered.

Potential due to the injecting current at probe 1

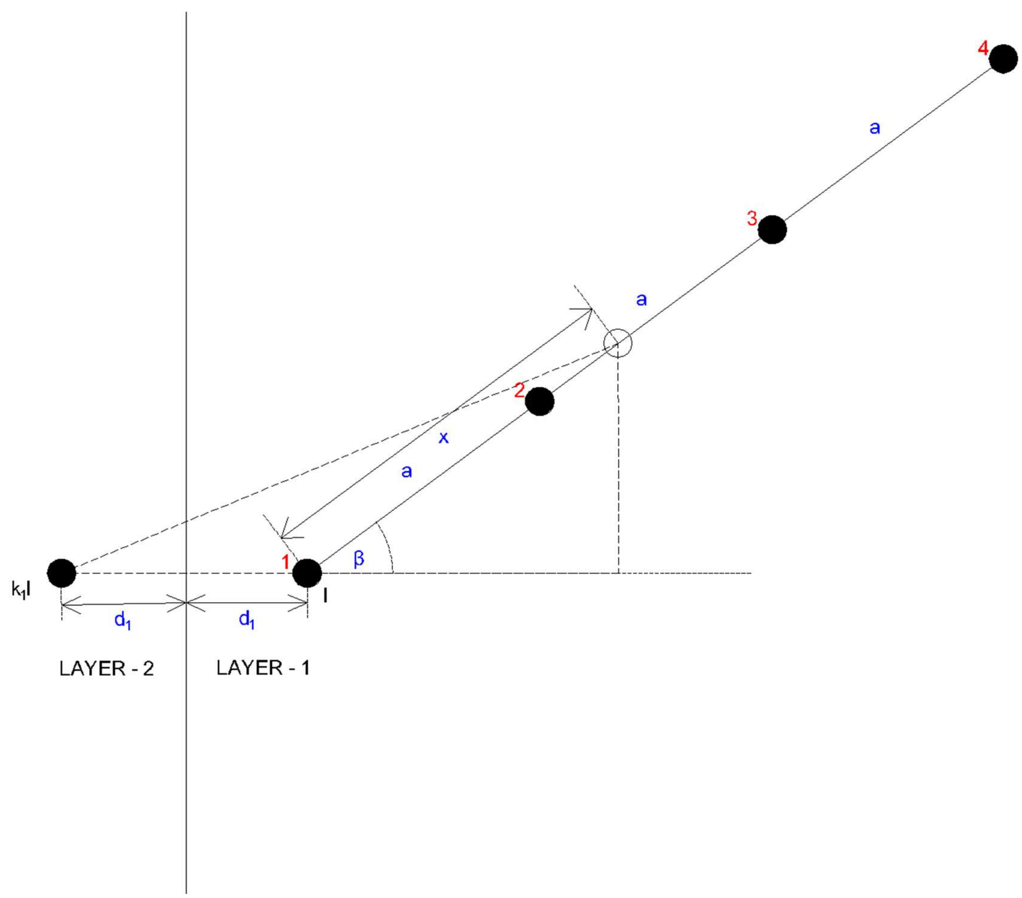

Figure 4.6: Image arrangement to calculate the potential due to the injecting current at probe 1

$$F_1(x) = \frac{I\rho_1}{2\pi x} + \frac{k_1 I \rho_1}{2\pi\sqrt{(2d_1 + x\cos\beta)^2 + (x\sin\beta)^2}}$$

Potential due to sinking current at probe 4

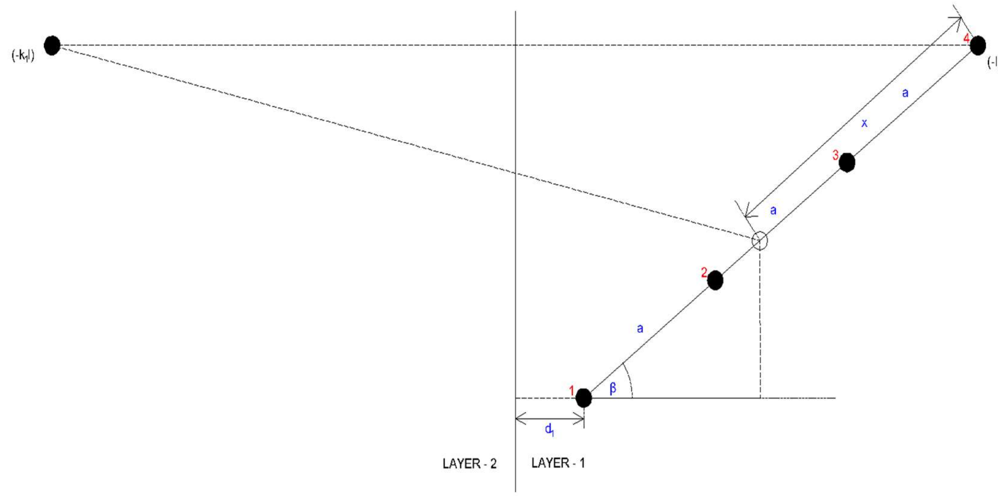

Figure 4.7: Image arrangement to calculate the potential due to the sinking current at probe 4

$$F_2(x) = \frac{(-I)\rho_1}{2\pi x} + \frac{k_1(-I)\rho_1}{2\pi\sqrt{(2d_1 + (6a - x)\cos\beta)^2 + (x\sin\beta)^2}}$$

Voltage at electrode 2 = Potential due to the injecting current in electrode 1 + Potential due to the sinking current in electrode 4

$V_2 = F_1(a) + F_2(2a)$

Voltage at electrode 3 = Potential due to the injecting current in electrode 1 + Potential due to the sinking current in electrode 4

$V_3 = F_1(2a) + F_2(a)$

Thus, the voltage difference between electrode 2 and 3

$V_2 - V_3 = [F_1(a) + F_2(2a) - \{(F_1(2a) + F_2(a)\}]$

$V_2 - V_3 = [F_1(a) + F_2(2a) - F_1(2a) - F_2(a)]$

$$V_2 - V_3 = [\frac{I\rho_1}{2\pi a} + \frac{k_1 I \rho_1}{2\pi\sqrt{(2d_1 + a\cos\quad)^2 + (a\sin\beta)^2}} + \frac{(-I)\rho_1}{4\pi} + \frac{k_1(-I)\rho_1}{2\pi\sqrt{(2d_1 + 4a\cos\beta)^2 + (2a\sin\quad)^2}} - \frac{I\rho_1}{4\pi a} - \frac{k_1 I \rho_1}{2\pi\sqrt{(2d_1 + 2a\cos\beta)^2 + (2a\sin\quad)^2}} - \frac{(-I)\rho_1}{2\pi a} - \frac{k_1(-I)\rho_1}{2\pi\sqrt{(2d_1 + 5a\,\mathrm{co}\quad)^2 + (a\sin\beta)^2}}]$$

$$V_2 - V_3 = \frac{\rho_1 I}{2\pi}[\frac{1}{a} + \frac{k_1}{\sqrt{(2d_1 + a\cos\quad)^2 + (a\sin\beta)^2}} - \frac{1}{2a} - \frac{k_1}{\sqrt{(2d_1 + 4a\,\mathrm{co}\quad)^2 + (2a\sin\quad)^2}} - \frac{1}{2a} - \frac{k_1}{\sqrt{(2d_1 + 2a\cos\beta)^2 + (2a\sin\quad)^2}} + \frac{1}{a} + \frac{k_1}{\sqrt{(2d_1 + 5a\cos\beta)^2 + (a\sin\beta)^2}}]$$

The apparent resistivity can be expressed as

$$\rho_a = 2\pi a R = 2\pi a\frac{V}{I}$$

Therefore, the apparent resistivity in two vertical layer soil can be expressed as:

$$\rho_a = 2\pi a R = \frac{2\pi a}{I} \times \frac{\rho_1 I}{2\pi}[\frac{1}{a} + \frac{k_1}{\sqrt{(2d_1 + a\cos\beta)^2 + (a\sin\beta)^2}} - \frac{1}{2a} - \frac{k_1}{\sqrt{(2d_1 + 4a\,\mathrm{co}\quad)^2 + (2a\sin\beta)^2}} - \frac{1}{2a} - \frac{k_1}{\sqrt{(2d_1 + 2a\cos\beta)^2 + (2a\sin\quad)^2}} + \frac{1}{a} + \frac{k_1}{\sqrt{(2d_1 + 5a\,\mathrm{co}\quad)^2 + (a\sin\beta)^2}}]$$

$$\rho_a = a\rho_1[\frac{1}{a} + \frac{k_1}{\sqrt{(2d_1 + a\cos\quad)^2 + (a\sin\beta)^2}} - \frac{k_1}{\sqrt{(2d_1 + 4a\cos\beta)^2 + (2a\sin\quad)^2}} - \frac{k_1}{\sqrt{(2d_1 + 2a\cos\beta)^2 + (2a\sin\quad)^2}} + \frac{k_1}{\sqrt{(2d_1 + 5a\cos\beta)^2 + (a\sin\beta)^2}}]$$

4.4. Calculation for three perpendicular vertical layer soil

This model has three perpendicular vertical layers with different soil resistivities. The probes are buried in the layer 1 as shown in the figure 4.8.

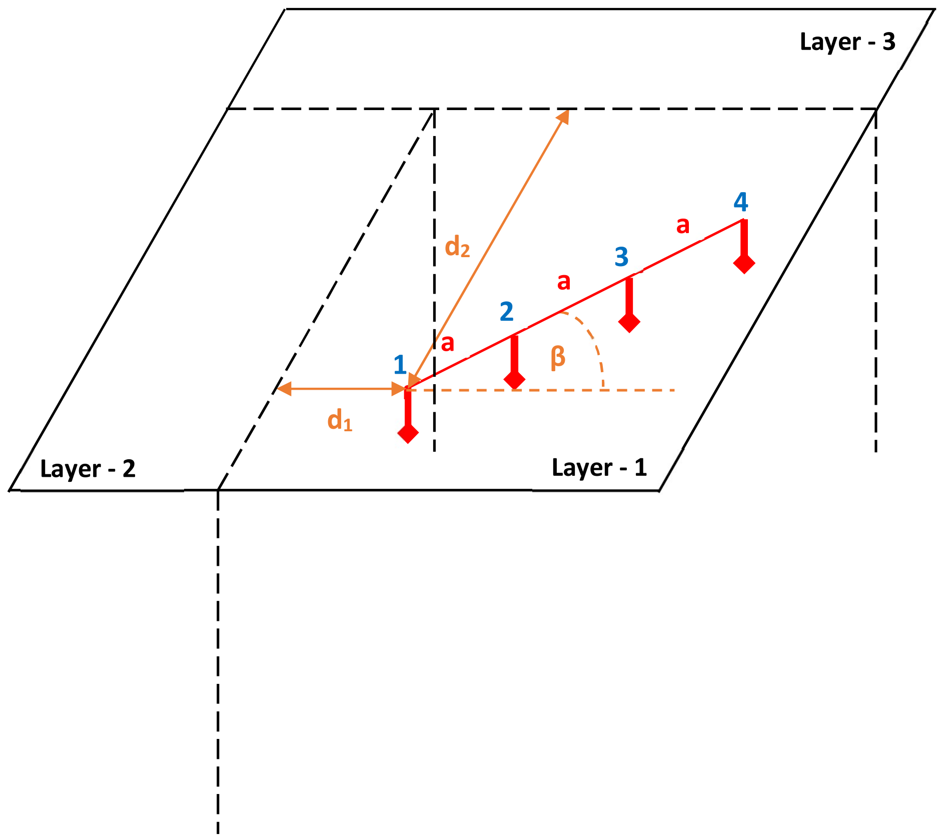

Figure 4.8: Electrode arrangement on three perpendicular vertical layer soil

Image distribution of the three-perpendicular vertical layer soil is shown in Figure 4.9.

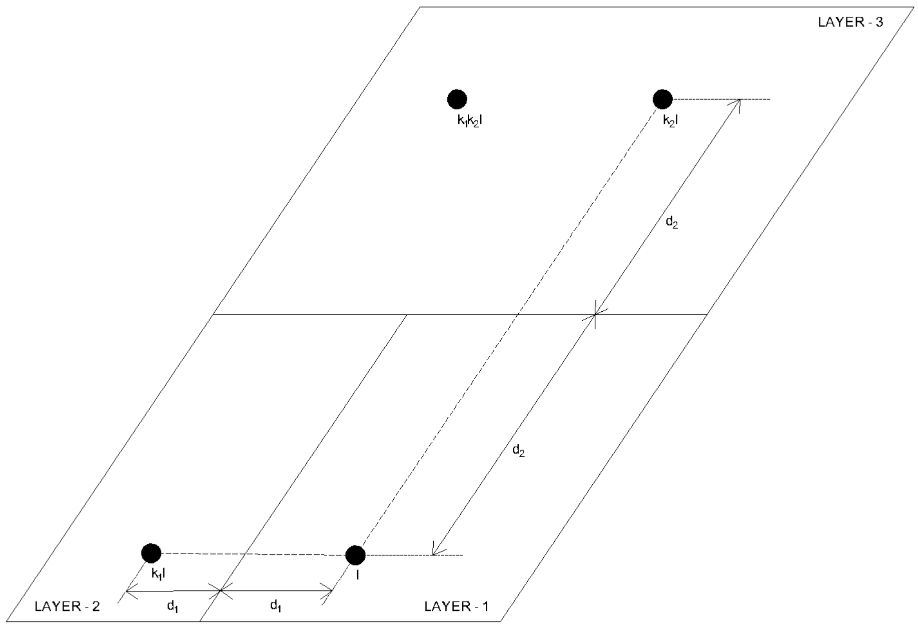

Figure 4.9: Image distribution of three-perpendicular vertical layer soil

a = Electrode spacing (Meters)

d_1= The distance between the 1st electrode and intersection between layer 1 and 2 (Meters)

d_2= The distance between the 1st electrode and intersection between layer 1 and 3 (Meters)

β = The direction of probes array (Degree or Radian)

x = The distance between current source and voltage measuring point(Meters)

ρ_1= Soil resistivity of layer 1 (Ohm.meter)

ρ_2= Soil resistivity of layer 2 (Ohm.meter)

ρ_3= Soil resistivity of layer 3 (Ohm.meter)

$k_1 = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$ = Reflection ratio between layer 1 and 2

$k_2 = \frac{\rho_3 - \rho_1}{\rho_3 + \rho_1}$ = Reflection ratio between layer 1 and 3

The voltage of the probe 2 is equal to the sum of the potentials due to the injecting current at probe 1 and sinking current at probe 4. The same is apply for the voltage of the probe 3.

To simplify the calculation a voltage measuring probe which is located in x (Meters) distance from the current probe is considered.

Potential due to the injection current at probe 1

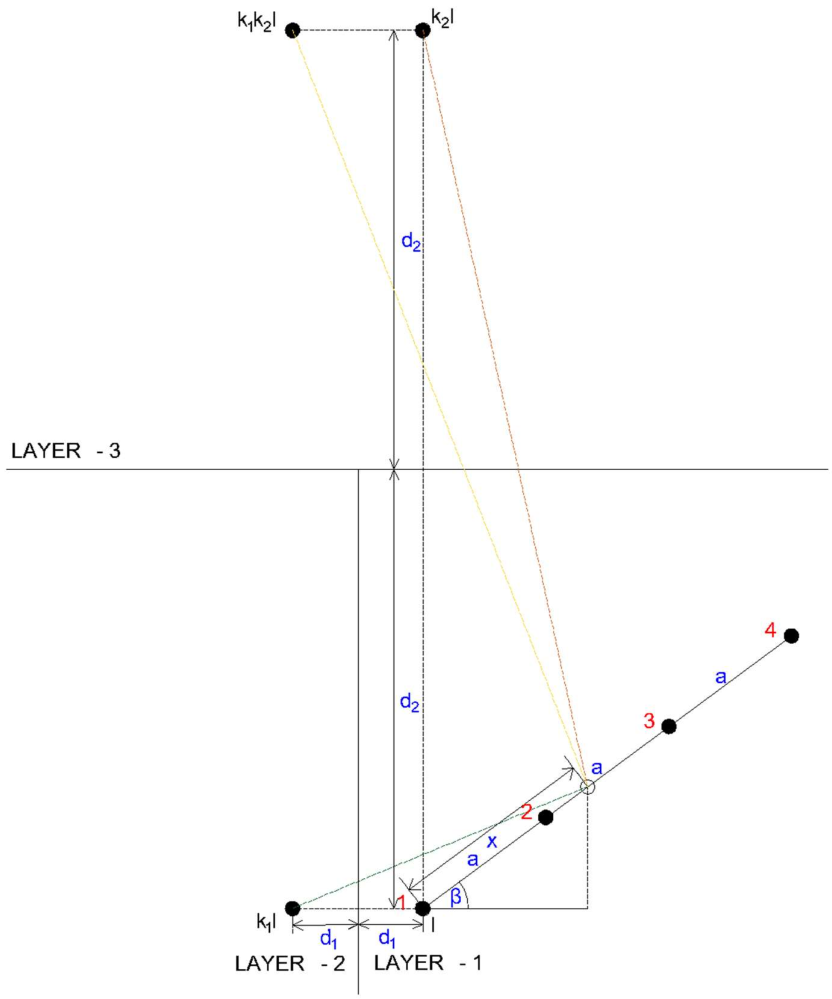

Figure 4.10: Image arrangement to calculate the potential due to the injection current at probe 1

$$F_1(x) = \frac{I\rho_1}{2\pi x} + \frac{k_1 I\rho_1}{2\pi\sqrt{(2d_1 + x\cos\beta)^2 + (x\sin\beta)^2}} + \frac{k_2 I\rho_1}{2\pi\sqrt{(2d_2 - x\sin\quad)^2 + (x\cos\beta)^2}} + \frac{k_1 k_2 I\rho_1}{2\pi\sqrt{(2d_2 - x\sin\beta)^2 + (2d_1 + x\cos\beta)^2}}$$

Potential due to the sinking current at probe 4

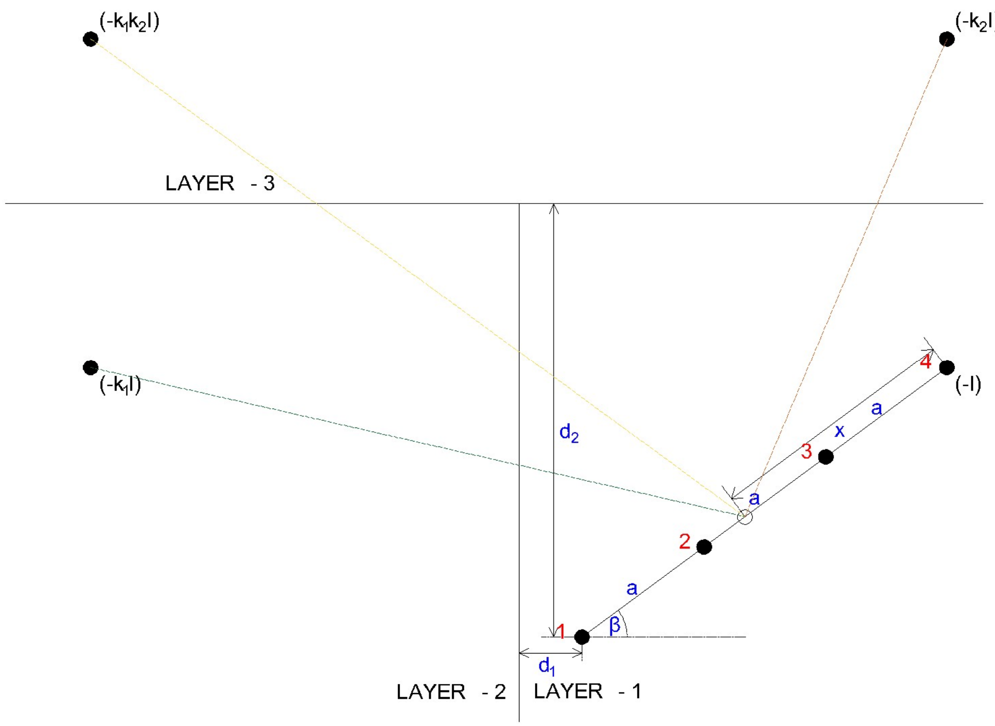

Figure 4.11: Image arrangement to calculate the potential due to the sinking current at probe 4

$$F_2(x) = \frac{(-I)\rho_1}{2\pi x} + \frac{k_1(-I)\rho_1}{2\pi\sqrt{(2d_1+6a\cos\beta-x\,\mathrm{co}\quad)^2+(x\sin\beta)^2}} +$$

$$\frac{k_2(-I)\rho_1}{2\pi\sqrt{(2d_2-6a\sin\beta+x\,\mathrm{si}\quad)^2+(x\cos\quad)^2}} + \frac{k_1k_2(-I)\rho_1}{2\pi\sqrt{(2d_1+6a\cos\beta-x\cos\beta)^2+(2d_2-6a\cos\qquad x\sin\beta)^2}}$$

Voltage at electrode 2 = Potential due to the injecting current in electrode 1 + Potential due to the sinking current in electrode 4

$$V_2 = F_1(a) + F_2(2a)$$

Voltage at electrode 3 = Potential due to the injecting current in electrode 1 + Potential due to the sinking current in electrode 4

$$V_3 = F_1(2a) + F_2(a)$$

Thus, the voltage difference between electrode 2 and 3

$$V_2 - V_3 = [F_1(a) + F_2(2a) - \{(F_1(2a) + F_2(a)\}]$$

$$V_2 - V_3 = [F_1(a) + F_2(2a) - F_1(2a) - F_2(a)]$$

$$V_2 - V_3 = [\frac{I\rho_1}{2\pi} + \frac{k_1 I\rho_1}{2\pi\sqrt{(2d_1+a\cos\beta)^2+(a\sin\beta)^2}} + \frac{k_2 I\rho_1}{2\pi\sqrt{(2d_2-a\sin\beta)^2+(a\cos\beta)^2}} +$$
$$\frac{k_1k_2 I\rho_1}{2\pi\sqrt{(2d_2-a\sin\beta)^2+(2d_1+a\cos\beta)^2}} + \frac{(-I)\rho_1}{4\pi a} + \frac{k_1(-I)\rho_1}{2\pi\sqrt{(2d_1+4a\,\mathrm{co}\quad)^2+(2a\sin\quad)^2}} +$$
$$\frac{k_2(-I)\rho_1}{2\pi\sqrt{(2d_2-4a\sin\beta)^2+(2a\cos\beta)^2}} + \frac{k_1k_2(-I)\rho_1}{2\pi\sqrt{(2d_1+4a\cos\beta)^2+(2d_2-6a\cos\quad 2a\sin\quad)^2}} -$$
$$\frac{I\rho_1}{4\pi a} - \frac{k_1 I\rho_1}{2\pi\sqrt{(2d_1+2a\cos\beta)^2+(2a\sin\beta)^2}} - \frac{k_2 I\rho_1}{2\pi\sqrt{(2d_2-2a\,\mathrm{si}\quad)^2+(2a\cos\quad)^2}} -$$
$$\frac{k_1k_2 I\rho_1}{2\pi\sqrt{(2d_2-2a\sin\beta)^2+(2d_1+2a\cos\beta)^2}} - \frac{(-I)\rho_1}{2\pi a} - \frac{k_1(-I)\rho_1}{2\pi\sqrt{(2d_1+5a\cos\beta)^2+(a\sin\beta)^2}} -$$
$$\frac{k_2(-I)\rho_1}{2\pi\sqrt{(2d_2-5a\sin\beta)^2+(a\cos\quad)^2}} - \frac{k_1k_2(-I)\rho_1}{2\pi\sqrt{(2d_1+5a\cos\beta)^2+(2d_2-6a\cos\quad a\sin\beta)^2}}]$$

$$V_2 - V_3 = \frac{\rho_1 I}{2\pi}[\frac{1}{a} + \frac{k_1}{\sqrt{(2d_1+a\cos\beta)^2+(a\sin\beta)^2}} + \frac{k_2}{\sqrt{(2d_2-a\sin\beta)^2+(a\cos\beta)^2}} +$$
$$\frac{k_1k_2}{\sqrt{(2d_2-a\sin\quad)^2+(2d_1+a\cos\beta)^2}} - \frac{1}{2a} - \frac{k_1}{\sqrt{(2d_1+4a\cos\beta)^2+(2a\sin\quad)^2}} -$$
$$\frac{k_2}{\sqrt{(2d_2-4a\sin\beta)^2+(2a\cos\quad)^2}} - \frac{k_1k_2}{\sqrt{(2d_1+4a\cos\beta)^2+(2d_2-6a\cos\quad 2a\sin\beta)^2}} -$$
$$\frac{1}{2a} - \frac{\rho_1}{\sqrt{(2d_1+2a\,\mathrm{co}\quad)^2+(2a\sin\beta)^2}} - \frac{k_2}{\sqrt{(2d_2-2a\sin\beta)^2+(2a\cos\quad)^2}} -$$
$$\frac{k_1k_2}{\sqrt{(2d_2-2a\sin\beta)^2+(2d_1+2a\cos\quad)^2}} + \frac{1}{a} + \frac{k_1}{\sqrt{(2d_1+5a\cos\beta)^2+(a\sin\quad)^2}} +$$
$$\frac{k_2}{\sqrt{(2d_2-5a\,\mathrm{si}\quad)^2+(a\cos\quad)^2}} + \frac{k_1k_2}{\sqrt{(2d_1+5a\cos\beta)^2+(2d_2-6a\cos\beta+a\sin\beta)^2}}]$$

The apparent resistivity can be expressed as

$$\rho_a = 2\pi aR = 2\pi a\frac{V}{I}$$

Therefore, the apparent resistivity in three perpendicular layers of soil can be expressed as:

$$\rho_a = 2\pi aR = \frac{2\pi a}{I} \times \frac{\rho_1 I}{2\pi}[\frac{1}{a} + \frac{k_1}{\sqrt{(2d_1 + a\cos\beta)^2 + (a\sin\beta)^2}} + \frac{k_2}{\sqrt{(2d_2 - a\sin\beta)^2 + (a\cos\beta)^2}} +$$

$$\frac{k_1 k_2}{\sqrt{(2d_2 - a\sin\beta)^2 + (2d_1 + a\cos\beta)^2}} - \frac{1}{2a} - \frac{k_1}{\sqrt{(2d_1 + 4a\cos\beta)^2 + (2a\sin\ \ )^2}} -$$

$$\frac{k_2}{\sqrt{(2d_2 - 4a\,\mathrm{si}\ \ \ )^2 + (2a\cos\ \ )^2}} - \frac{k_1 k_2}{\sqrt{(2d_1 + 4a\cos\beta)^2 + (2d_2 - 6a\cos\ \ \ \ 2a\sin\ \ )^2}} -$$

$$\frac{1}{2a} - \frac{\rho_1}{\sqrt{(2d_1 + 2a\cos\beta)^2 + (2a\sin\beta)^2}} - \frac{k_2}{\sqrt{(2d_2 - 2a\,\mathrm{si}\ \ \ \ )^2 + (2a\cos\ \ )^2}} -$$

$$\frac{k_1 k_2}{\sqrt{(2d_2 - 2a\sin\beta)^2 + (2d_1 + 2a\,\mathrm{co}\ \ \ \ )^2}} + \frac{1}{a} + \frac{k_1}{\sqrt{(2d_1 + 5a\cos\beta)^2 + (a\sin\beta)^2}} +$$

$$\frac{k_2}{\sqrt{(2d_2 - 5a\,\mathrm{si}\ \ \ \ )^2 + (a\cos\beta)^2}} + \frac{k_1 k_2}{\sqrt{(2d_1 + 5a\cos\beta)^2 + (2d_2 - 6a\cos\ \ \ \ \ a\sin\beta)^2}}]$$

$$\rho_a = a\rho_1 \left[\frac{1}{a} + \frac{k_1}{\sqrt{(2d_1 + a\cos\ \ \ )^2 + (a\sin\beta)^2}} + \frac{k_2}{\sqrt{(2d_2 - a\sin\ \ \ )^2 + (a\cos\beta)^2}} +\right.$$

$$\frac{k_1 k_2}{\sqrt{(2d_2 - a\sin\beta)^2 + (2d_1 + a\cos\ \ \ )^2}} - \frac{k_1}{\sqrt{(2d_1 + 4a\cos\beta)^2 + (2a\sin\ \ \ )^2}} -$$

$$\frac{k_2}{\sqrt{(2d_2 - 4a\sin\beta)^2 + (2a\cos\ \ \ )^2}} - \frac{k_1 k_2}{\sqrt{(2d_1 + 4a\cos\beta)^2 + (2d_2 - 6a\cos\beta + 2a\sin\beta)^2}} -$$

$$\frac{k_1}{\sqrt{(2d_1 + 2a\,\mathrm{co}\ \ \ \ )^2 + (2a\sin\beta)^2}} - \frac{k_2}{\sqrt{(2d_2 - 2a\,\mathrm{si}\ \ \ \ )^2 + (2a\cos\ \ )^2}} -$$

$$\frac{k_1 k_2}{\sqrt{(2d_2 - 2a\sin\beta)^2 + (2d_1 + 2a\,\mathrm{co}\ \ \ \ )^2}} + \frac{k_1}{\sqrt{(2d_1 + 5a\,\mathrm{co}\ \ \ \ )^2 + (a\sin\beta)^2}} +$$

$$\left.\frac{k_2}{\sqrt{(2d_2 - 5a\,\mathrm{si}\ \ \ \ )^2 + (a\cos\beta)^2}} + \frac{k_1 k_2}{\sqrt{(2d_1 + 5a\cos\beta)^2 + (2d_2 - 6a\cos\beta + a\sin\ \ \ )^2}}\right]$$

4.5. Calculation for three parallel vertical layer soil

This soil model has three parallel vertical layers with different soil resistivities. The probes are buried in the first layer as shown in the figure 4.12.

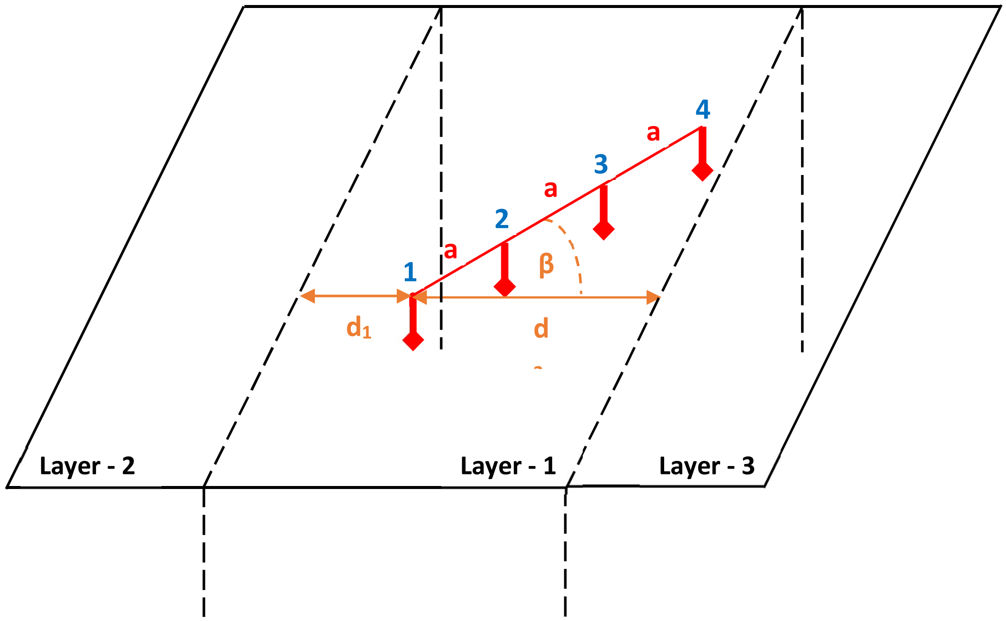

Figure 4.12: Electrode arrangement on three parallel vertical layer soil

Image distribution

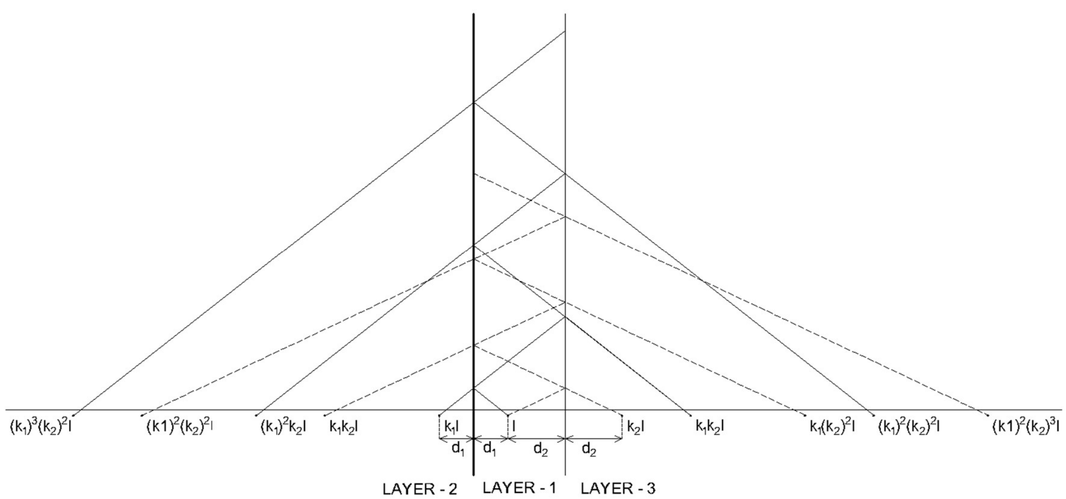

Figure 4.13: Image distribution of three-parallel vertical layer soil

a = Electrode spacing (Meters)

d_1= The distance between the 1st electrode and intersection between layer 1 and 2 (Meters)

d_2= The distance between the 1st electrode and intersection between layer 1 and 3 (Meters)

β = The direction of probes array (Degree or Radian)

x = The distance between current probe and voltage measuring point(Meters)

ρ_1= Soil resistivity of layer 1 (Ohm.meter)

ρ_2= Soil resistivity of layer 2 (Ohm.meter)

ρ_3= Soil resistivity of layer 3 (Ohm.meter)

$k_1 = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$ = Reflection ratio between layer 1 and 2

$k_2 = \frac{\rho_3 - \rho_1}{\rho_3 + \rho_1}$ = Reflection ratio between layer 1 and 3

The voltage of the probe 2 is equal to the sum of the potentials due to the injecting current at probe 1 and sinking current at probe 4. The same is apply for the voltage of the probe 3.

To simplify the calculation a voltage measuring probe which is located in x (Meters) distance from the current probe is considered.

Potential due to the injection current at probe 1

Refer Appendix – A for the image distribution of the injection current at probe 1

$$F_1(x) = \frac{I\rho_1}{2\pi x} + \sum_{n=1}^{\infty} \frac{K_1^n K_2^{n-1} I\rho_1}{2\pi\sqrt{(2nd_1+2(n-1)d_2+x\cos\beta)^2+(x\,\mathrm{si}\quad)^2}} +$$

$$\sum_{n=1}^{\infty} \frac{K_1^n K_2^n I\rho_1}{2\pi\sqrt{(2nd_1+2nd_2+x\cos\quad)^2+(x\sin\beta)^2}} +$$

$$\sum_{n=1}^{\infty} \frac{K_1^{n-1} K_2^n I\rho_1}{2\pi\sqrt{(2(n-1)d_1+2nd_2-x\,\mathrm{co}\quad)^2+(x\sin\beta)^2}} + \sum_{n=1}^{\infty} \frac{K_1^n K_2^n I\rho_1}{2\pi\sqrt{(2nd_1+2nd_2-x\cos\beta)^2+(x\sin\beta)^2}}$$

Potential due to sinking current at probe 4

Refer Appendix – B for the image distribution of the sinking current at probe 4

$$F_2(x) = \frac{(-I)\rho_1}{2\pi x} + \sum_{n=1}^{\infty} \frac{K_1^n K_2^{n-1}(-I)\rho_1}{2\pi\sqrt{(2nd_1+2(n-1)d_2+ 6a\cos\beta - x\cos\beta)^2+(x\sin\beta)^2}} +$$

$$\sum_{n=1}^{\infty} \frac{K_1^n K_2^n(-I)\rho_1}{2\pi\sqrt{(2nd_1+2nd_2- x\cos\beta)^2+(x\sin\beta)^2}} +$$

$$\sum_{n=1}^{\infty} \frac{K_1^{n-1} K_2^n(-I)\rho_1}{2\pi\sqrt{(2(n-1)d_1+2nd_2- 6a\cos \qquad x\cos\beta)^2+(x\sin\beta)^2}} +$$

$$\sum_{n=1}^{\infty} \frac{K_1^n K_2^n(-I)\rho_1}{2\pi\sqrt{(2nd_1+2nd_2+x\cos\beta)^2+(x\sin\beta)^2}}$$

Voltage at electrode 2 = Potential due to the injecting current in electrode 1 + Potential due to the sinking current in electrode 4

$$V_2 = F_1(a) + F_2(2a)$$

Voltage at electrode 3 = Potential due to the injecting current in electrode 1 + Potential due to the sinking current in electrode 4

$$V_3 = F_1(2a) + F_2(a)$$

Thus, the voltage difference between electrode 2 and 3

$$V_2 - V_3 = [F_1(a) + F_2(2a) - \{(F_1(2a) + F_2(a)\}]$$

$$V_2 - V_3 = [F_1(a) + F_2(2a) - F_1(2a) - F_2(a)]$$

$$V_2 - V_3 = [\frac{I\rho_1}{2\pi a} + \sum_{n=1}^{\infty} \frac{K_1^n K_2^{n-1} I\rho_1}{2\pi\sqrt{(2nd_1+2(n-1)d_2+ a\cos\beta)^2+(a\sin\beta)^2}} +$$

$$\sum_{n=1}^{\infty} \frac{K_1^n K_2^n I\rho_1}{2\pi\sqrt{(2nd_1+2nd_2+ a\cos\beta)^2+(a\sin\beta)^2}} +$$

$$\sum_{n=1}^{\infty} \frac{K_1^{n-1} K_2^n I\rho_1}{2\pi\sqrt{(2(n-1)d_1+2nd_2- a\cos\beta)^2+(a\sin \quad)^2}} +$$

$$\sum_{n=1}^{\infty} \frac{K_1^n K_2^n I\rho_1}{2\pi\sqrt{(2nd_1+2nd_2- a\cos\beta)^2+(a\sin\beta)^2}} + \frac{(-I)\rho_1}{4\pi a} +$$

$$\sum_{n=1}^{\infty} \frac{K_1^n K_2^{n-1}(-I)\rho_1}{2\pi\sqrt{(2nd_1+2(n-1)d_2+ 4a\cos \quad)^2+(2a\sin \quad)^2}} +$$

$$\sum_{n=1}^{\infty} \frac{K_1^n K_2^n(-I)\rho_1}{2\pi\sqrt{(2nd_1+2nd_2- 2a\cos\beta)^2+(2a\sin\beta)^2}} +$$

$$\sum_{n=1}^{\infty} \frac{K_1^{n-1} K_2^n(-I)\rho_1}{2\pi\sqrt{(2(n-1)d_1+2nd_2- 4a\cos\beta)^2+(2a\sin\beta)^2}} +$$

$$\sum_{n=1}^{\infty} \frac{K_1^n K_2^n(-I)\rho_1}{2\pi\sqrt{(2nd_1+2nd_2+2a\cos\beta)^2+(2a\sin \quad)^2}} - \frac{I\rho_1}{4\pi a} -$$

$$\sum_{n=1}^{\infty} \frac{K_1^n K_2^{n-1} I\rho_1}{2\pi\sqrt{(2nd_1+2(n-1)d_2+ 2a\cos \quad)^2+(2a\sin\beta)^2}} -$$

$$\sum_{n=1}^{\infty}\frac{K_1^nK_2^nI\rho_1}{2\pi\sqrt{(2nd_1+2nd_2+2a\cos\beta)^2+(2a\sin\beta)^2}}-$$

$$\sum_{n=1}^{\infty}\frac{K_1^{n-1}K_2^nI\rho_1}{2\pi\sqrt{(2(n-1)d_1+2nd_2-2a\cos\beta)^2+(2a\sin\beta)^2}}-$$

$$\sum_{n=1}^{\infty}\frac{K_1^nK_2^nI\rho_1}{2\pi\sqrt{(2nd_1+2nd_2-2a\cos\ )^2+(2a\sin\ )^2}}\quad]$$

$$V_2-V_3=\frac{I\rho_1}{2\pi}[\frac{1}{a}+\sum_{n=1}^{\infty}\frac{K_1^nK_2^{n-1}}{\sqrt{(2nd_1+2(n-1)d_2+a\cos\beta)^2+(a\sin\beta)^2}}+$$

$$\sum_{n=1}^{\infty}\frac{K_1^nK_2^n}{\sqrt{(2nd_1+2nd_2+a\cos\beta)^2+(a\sin\beta)^2}}+\sum_{n=1}^{\infty}\frac{K_1^{n-1}K_2^n}{\sqrt{(2(n-1)d_1+2nd_2-a\cos\beta)^2+(a\sin\beta)^2}}+$$

$$\sum_{n=1}^{\infty}\frac{K_1^nK_2^n}{\sqrt{(2nd_1+2nd_2-a\cos\beta)^2+(a\sin\beta)^2}}-\frac{1}{2a}-$$

$$\sum_{n=1}^{\infty}\frac{K_1^nK_2^{n-1}}{\sqrt{(2nd_1+2(n-1)d_2+4a\cos\ )^2+(2a\sin\ )^2}}-\sum_{n=1}^{\infty}\frac{K_1^nK_2^n}{\sqrt{(2nd_1+2nd_2-2a\cos\ )^2+(2a\sin\ )^2}}-$$

$$\sum_{n=1}^{\infty}\frac{K_1^{n-1}K_2^n}{\sqrt{(2(n-1)d_1+2nd_2-4a\cos\beta)^2+(2a\sin\ )^2}}-$$

$$\sum_{n=1}^{\infty}\frac{K_1^nK_2^n}{\sqrt{(2nd_1+2nd_2+2a\cos\beta)^2+(2a\sin\ )^2}}-\frac{1}{2a}-$$

$$\sum_{n=1}^{\infty}\frac{K_1^nK_2^{n-1}}{\sqrt{(2nd_1+2(n-1)d_2+2a\cos\beta)^2+(2a\sin\ )^2}}-$$

$$\sum_{n=1}^{\infty}\frac{K_1^nK_2^n}{\sqrt{(2nd_1+2nd_2+2a\cos\beta)^2+(2a\sin\beta)^2}}-$$

$$\sum_{n=1}^{\infty}\frac{K_1^{n-1}K_2^n}{\sqrt{(2(n-1)d_1+2nd_2-2a\cos\beta)^2+(2a\sin\beta)^2}}-$$

$$\sum_{n=1}^{\infty}\frac{K_1^nK_2^n}{\sqrt{(2nd_1+2nd_2-2a\cos\ )^2+(2a\sin\ )^2}}\quad+\frac{1}{a}+$$

$$\sum_{n=1}^{\infty}\frac{K_1^nK_2^{n-1}}{\sqrt{(2nd_1+2(n-1)d_2+5a\cos\beta)^2+(a\sin\beta)^2}}+\sum_{n=1}^{\infty}\frac{K_1^nK_2^n}{\sqrt{(2nd_1+2nd_2-a\cos\beta)^2+(a\sin\beta)^2}}+$$

$$\sum_{n=1}^{\infty}\frac{K_1^{n-1}K_2^n}{\sqrt{(2(n-1)d_1+2nd_2-5a\cos\ )^2+(a\sin\beta)^2}}+\sum_{n=1}^{\infty}\frac{K_1^nK_2^n}{\sqrt{(2nd_1+2nd_2+a\cos\ )^2+(a\sin\beta)^2}}\ ]$$

The apparent resistivity can be expressed as

$$\rho_a=2\pi aR=2\pi a\frac{V}{I}$$

Therefore, the apparent resistivity in three parallel layers of soil can be expressed as:

$$\rho_a = 2\pi aR = \frac{2\pi a}{I} \times \frac{I\rho_1}{2\pi}[\frac{1}{a} + \sum_{n=1}^{\infty} \frac{K_1^n K_2^{n-1}}{\sqrt{(2nd_1+2(n-1)d_2+ a\cos\beta)^2+(a\,\mathrm{si}\quad)^2}} +$$

$$2\sum_{n=1}^{\infty} \frac{K_1^n K_2^n}{\sqrt{(2nd_1+2nd_2+ a\cos\beta)^2+(a\sin\beta)^2}} + \sum_{n=1}^{\infty} \frac{K_1^{n-1} K_2^n}{\sqrt{(2(n-1)d_1+2nd_2- a\cos\beta)^2+(a\sin\beta)^2}} +$$

$$2\sum_{n=1}^{\infty} \frac{K_1^n K_2^n}{\sqrt{(2nd_1+2nd_2- a\cos\beta)^2+(a\sin\beta)^2}} - \sum_{n=1}^{\infty} \frac{K_1^n K_2^{n-1}}{\sqrt{(2nd_1+2(n-1)d_2+ 4a\cos\quad)^2+(2a\sin\beta)^2}} -$$

$$2\sum_{n=1}^{\infty} \frac{K_1^n K_2^n}{\sqrt{(2nd_1+2nd_2- 2a\cos\beta)^2+(2a\sin\beta)^2}} -$$

$$\sum_{n=1}^{\infty} \frac{K_1^{n-1} K_2^n}{\sqrt{(2(n-1)d_1+2nd_2- 4a\cos\quad)^2+(2a\sin\quad)^2}} -$$

$$2\sum_{n=1}^{\infty} \frac{K_1^n K_2^n}{\sqrt{(2nd_1+2nd_2+2a\cos\beta)^2+(2a\sin\quad)^2}} -$$

$$\sum_{n=1}^{\infty} \frac{K_1^n K_2^{n-1}}{\sqrt{(2nd_1+2(n-1)d_2+ 2a\cos\quad)^2+(2a\sin\beta)^2}} -$$

$$\sum_{n=1}^{\infty} \frac{K_1^{n-1} K_2^n}{\sqrt{(2(n-1)d_1+2nd_2- 2a\cos\beta)^2+(2a\sin\quad)^2}} +$$

$$\sum_{n=1}^{\infty} \frac{K_1^n K_2^{n-1}}{\sqrt{(2nd_1+2(n-1)d_2+ 5a\cos\beta)^2+(a\sin\beta)^2}} +$$

$$\sum_{n=1}^{\infty} \frac{K_1^{n-1} K_2^n}{\sqrt{(2(n-1)d_1+2nd_2- 5a\cos\quad)^2+(a\sin\beta)^2}}]$$

$$\rho_a = \rho_1 a[\frac{1}{a} + \sum_{n=1}^{\infty} \frac{K_1^n K_2^{n-1}}{\sqrt{(2nd_1+2(n-1)d_2+ a\cos\beta)^2+(a\sin\beta)^2}} +$$

$$2\sum_{n=1}^{\infty} \frac{K_1^n K_2^n}{\sqrt{(2nd_1+2nd_2+ a\cos\beta)^2+(a\sin\beta)^2}} + \sum_{n=1}^{\infty} \frac{K_1^{n-1} K_2^n}{\sqrt{(2(n-1)d_1+2nd_2- a\cos\beta)^2+(a\sin\beta)^2}} +$$

$$2\sum_{n=1}^{\infty} \frac{K_1^n K_2^n}{\sqrt{(2nd_1+2nd_2- a\cos\beta)^2+(a\sin\beta)^2}} - \sum_{n=1}^{\infty} \frac{K_1^n K_2^{n-1}}{\sqrt{(2nd_1+2(n-1)d_2+ 4a\cos\quad)^2+(2a\sin\beta)^2}} -$$

$$2\sum_{n=1}^{\infty} \frac{K_1^n K_2^n}{\sqrt{(2nd_1+2nd_2- 2a\cos\quad)^2+(2a\sin\quad)^2}} -$$

$$\sum_{n=1}^{\infty} \frac{K_1^{n-1} K_2^n}{\sqrt{(2(n-1)d_1+2nd_2- 4a\cos\beta)^2+(2a\sin\beta)^2}} -$$

$$2\sum_{n=1}^{\infty} \frac{K_1^n K_2^n}{\sqrt{(2nd_1+2nd_2+2a\cos\beta)^2+(2a\sin\beta)^2}} -$$

$$\sum_{n=1}^{\infty} \frac{K_1^n K_2^{n-1}}{\sqrt{(2nd_1+2(n-1)d_2+ 2a\cos\quad)^2+(2a\sin\beta)^2}} -$$

$$\sum_{n=1}^{\infty} \frac{K_1^{n-1} K_2^n}{\sqrt{(2(n-1)d_1+2nd_2- 2a\cos\beta)^2+(2a\sin\quad)^2}} +$$

$$\sum_{n=1}^{\infty} \frac{K_1^n K_2^{n-1}}{\sqrt{(2nd_1 + 2(n-1)d_2 + 5a\cos\quad)^2 + (a\sin\beta)^2}} +$$

$$\sum_{n=1}^{\infty} \frac{K_1^{n-1} K_2^{n}}{\sqrt{(2(n-1)d_1 + 2nd_2 - 5a\cos\quad)^2 + (a\sin\beta)^2}}]$$

CHAPTER 5

5. ANALYTICAL RESULTS

The formulas derived in the previous chapter are further analyzed by using the MATLAB. The study is helpful to understand the behavior of apparent soil resistivity with the existence of vertical soil layers and enable one to determine the how the soil is layered in a specific geographical area.

5.1. Two-vertical layer soil

Case - 1

Apparent resistivity (ρ_a) vs distance between the electrode 1 and the interface (d_1) when

ρ_1 (Resistivity of layer 1) = 100Ω.m

ρ_2 (Resistivity of layer 2) = 1000Ω.m

β (The direction of electrodes array) = 0 Radian

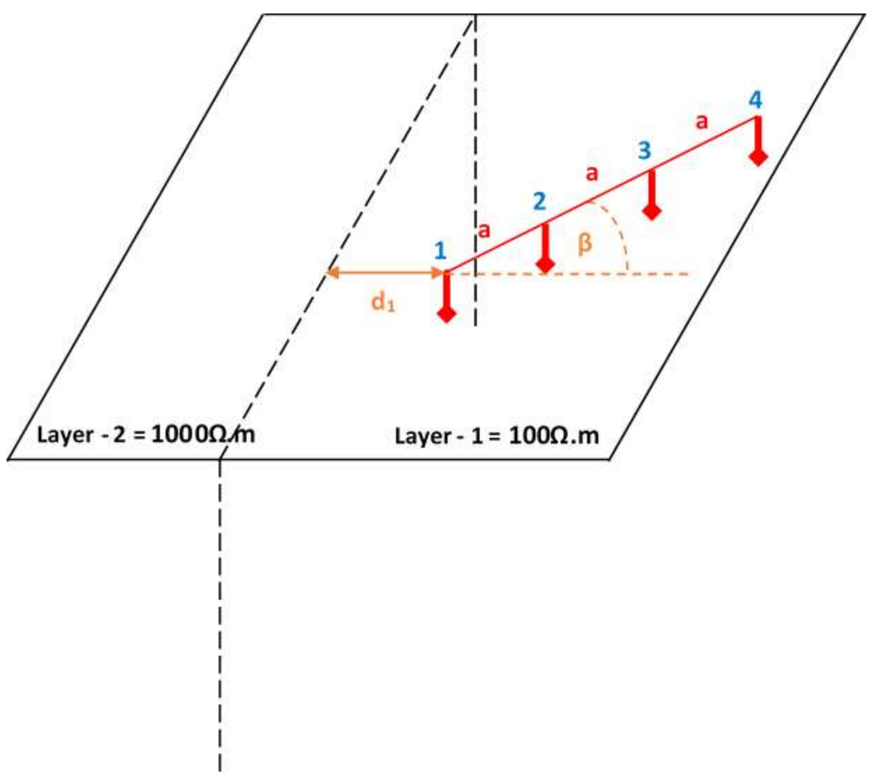

Figure 5.1: Electrode arrangement in two vertical layer soil

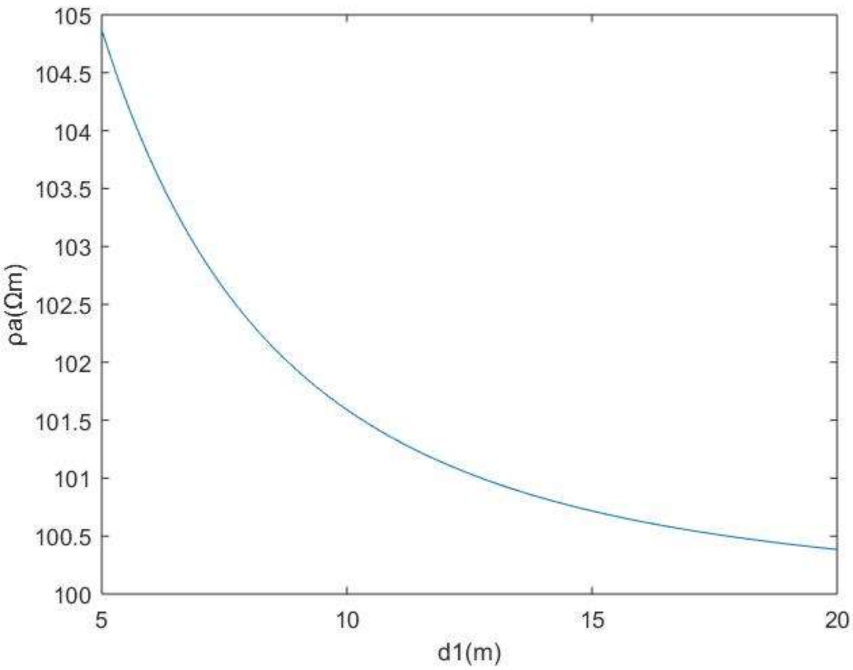

Figure 5.2: Apparent resistivity (ρ_a) vs distance between the probe 1 and the interface (d_1) when $\rho_1 = 100\Omega.m$, $\rho_2 = 1000\Omega.m$ and $\beta = 0$ Radian

The probes are located in the layer 1 with low resistivity as shown in the figure 5.1. With the increase of space between the first probe and the interface between the two layers the apparent resistivity gets decrease as shown in the figure 5.2. This indicates the decrease of the effect of the layer 2 of high resistivity.

Case - 2

Apparent resistivity (ρ_a) vs distance between the electrode 1 and the interface (d_1) when

ρ_1 (Resistivity of layer 1) = 1000Ω.m

ρ_2 (Resistivity of layer 2) = 100Ω.m

β (The direction of electrodes array) = 0 Radian

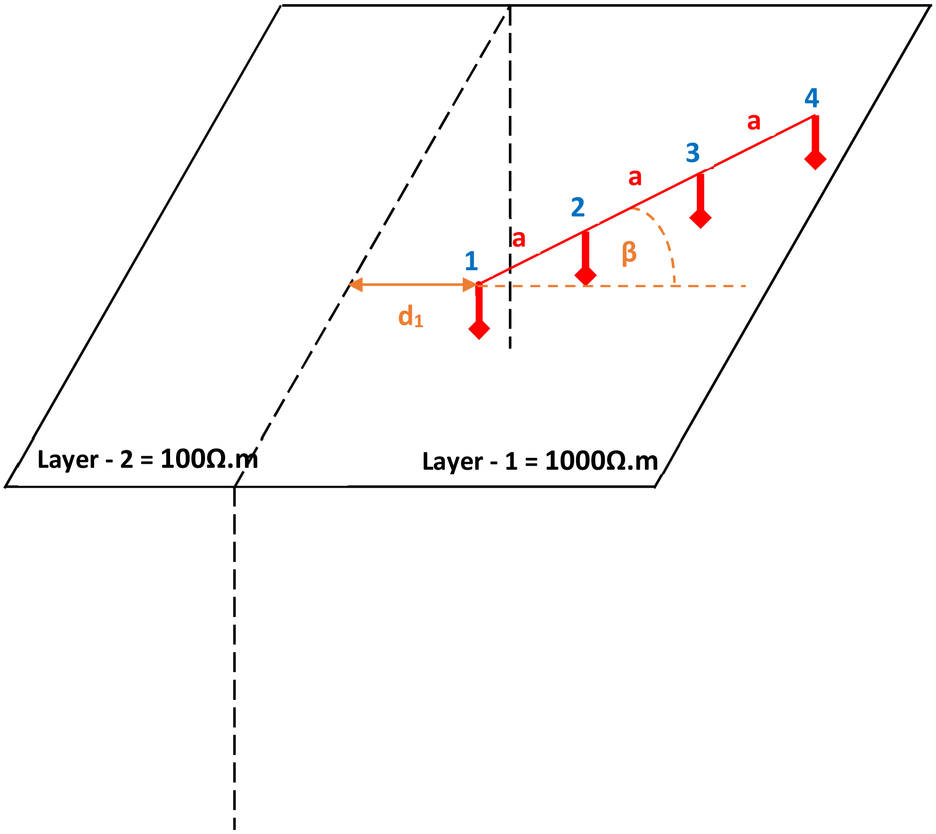

Figure 5.3: Electrode arrangement in two vertical layer soil

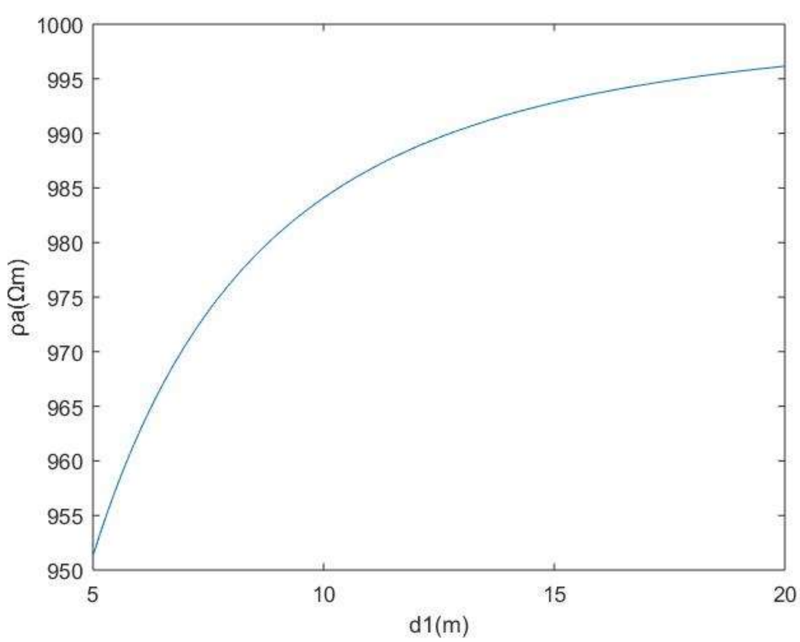

Figure 5.4: Apparent resistivity (ρ_a) vs distance between the probe 1 and the interface (d_1) when $\rho_1 = 1000\Omega.\text{m}$, ρ_2 = 100Ω.m and β = 0 Radian

The probes are located in the layer 1 with high resistivity as shown in the figure 5.3. With the increase of space between the first probe and the interface between the two layers the apparent resistivity gets increase as shown in the figure 5.4. This indicates the decrease of the effect of the layer 2 of low resistivity.

Case - 3

Apparent resistivity (ρ_a) vs the direction of probes array (β) when the electrodes are located in the low resistivity layer.

ρ_1 (Resistivity of layer 1) = 100Ω.m

ρ_2 (Resistivity of layer 2) = 1000Ω.m

d_1(The distance between the first probe and the intersection between layers) = 5m

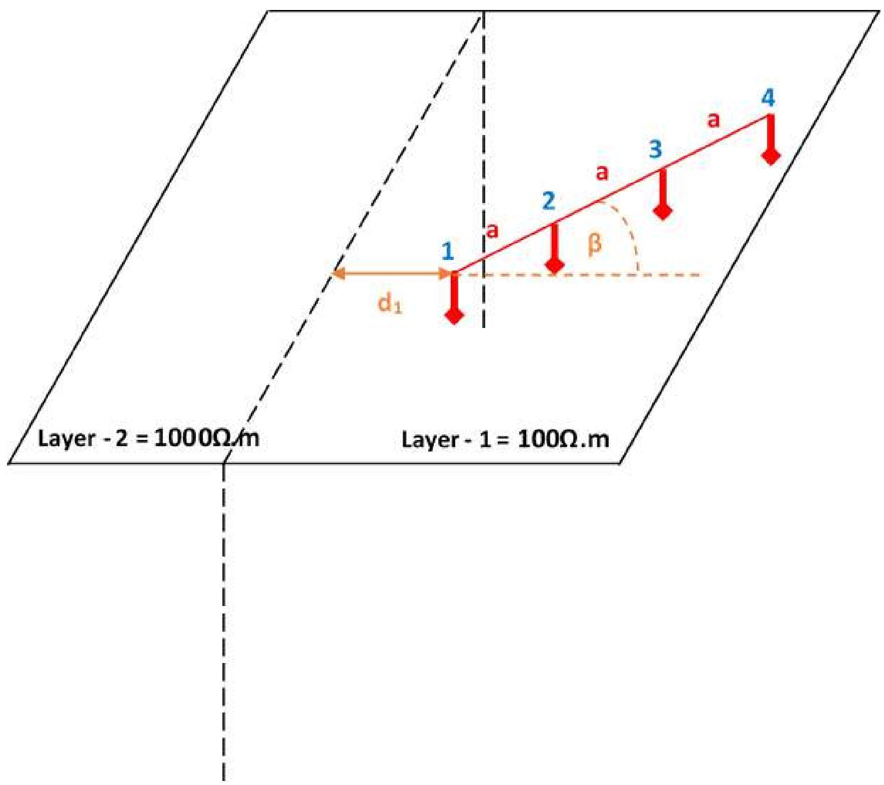

Figure 5.5: Electrode arrangement in two vertical layer soil

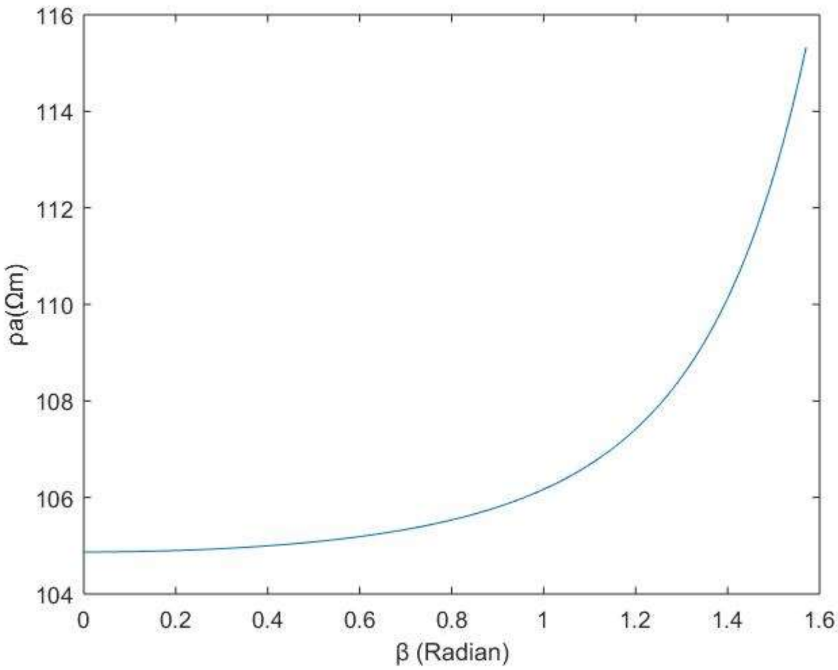

Figure 5.6: Apparent resistivity (ρ_a) vs the direction of electrodes array (β) when $\rho_1 = 100\Omega.\mathrm{m}$, ρ_2 = 1000Ω.m and d_1= 5m

The probes are located in the layer 1 with low resistivity as shown in the figure 5.5. With the increase of the direction of the probes array the apparent resistivity gets increase as shown in the figure 5.6. This indicates the increase of the effect of the layer 2 of high resistivity.

Case - 4

Apparent resistivity (ρ_a) vs the direction of electrodes array (β) when the electrodes are located in the high resistivity layer.

ρ_1 (Resistivity of layer 1) = 1000Ω.m

ρ_2 (Resistivity of layer 2) = 100Ω.m

d_1(The distance between the first probe and intersection between layers) = 5m

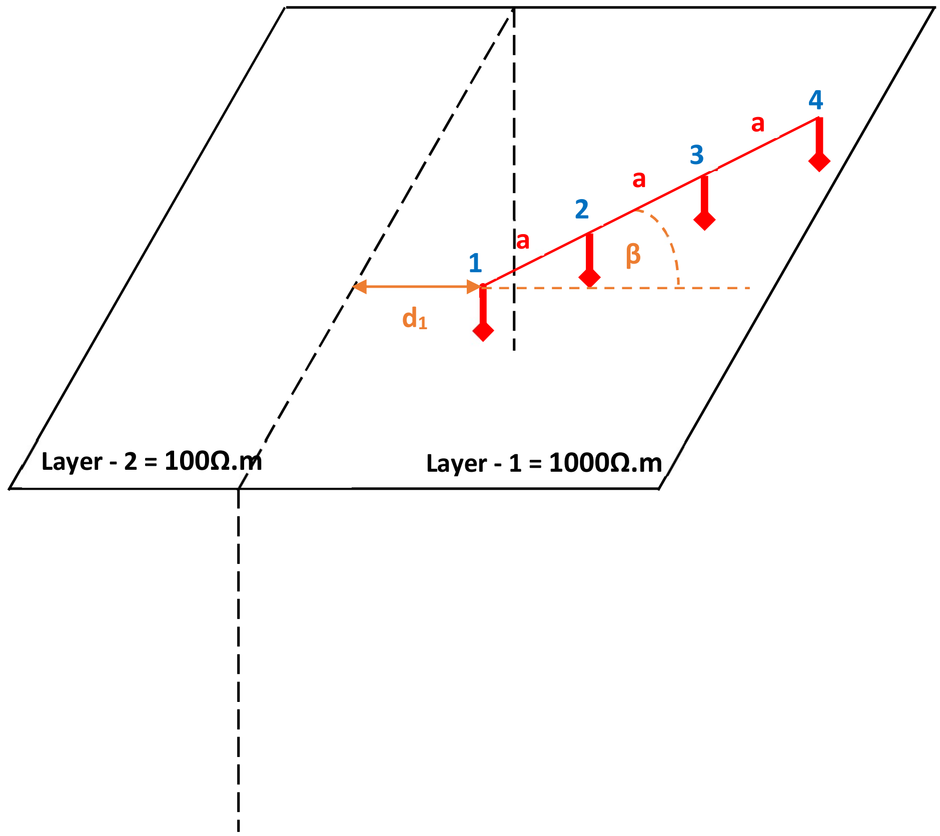

Figure 5.7: Electrode arrangement in two vertical layer soil

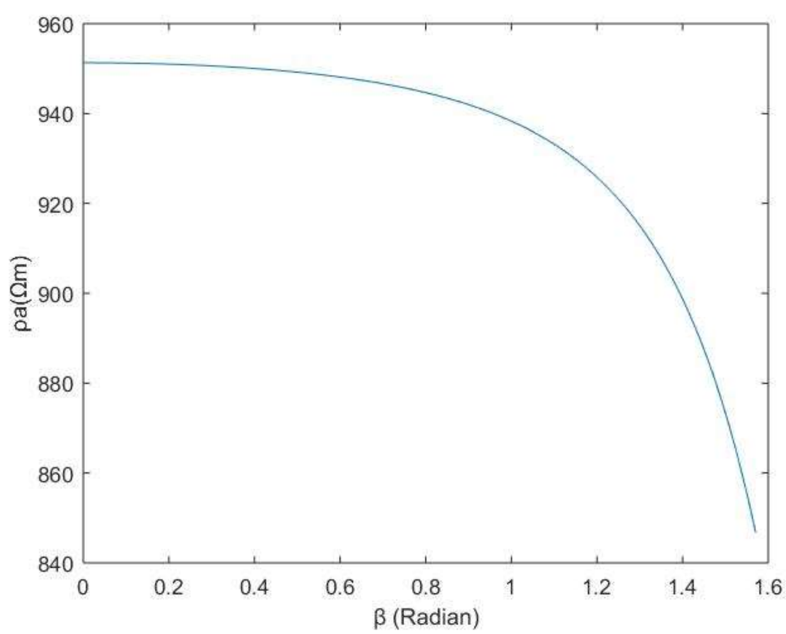

Figure 5.8: Apparent resistivity (ρ_a) vs the direction of electrodes array (β) when $\rho_1 = 1000\Omega.\mathrm{m}$, $\rho_2 = 100\Omega$.m and $d_1 = 5$m

The probes are located in the layer 1 with high resistivity as shown in the figure 5.7. With the increase of the direction of the probes array the apparent resistivity gets decrease as shown in the figure 5.8. This indicates the increase of the effect of the layer 2 of low resistivity.

5.2. Three perpendicular vertical layer soil

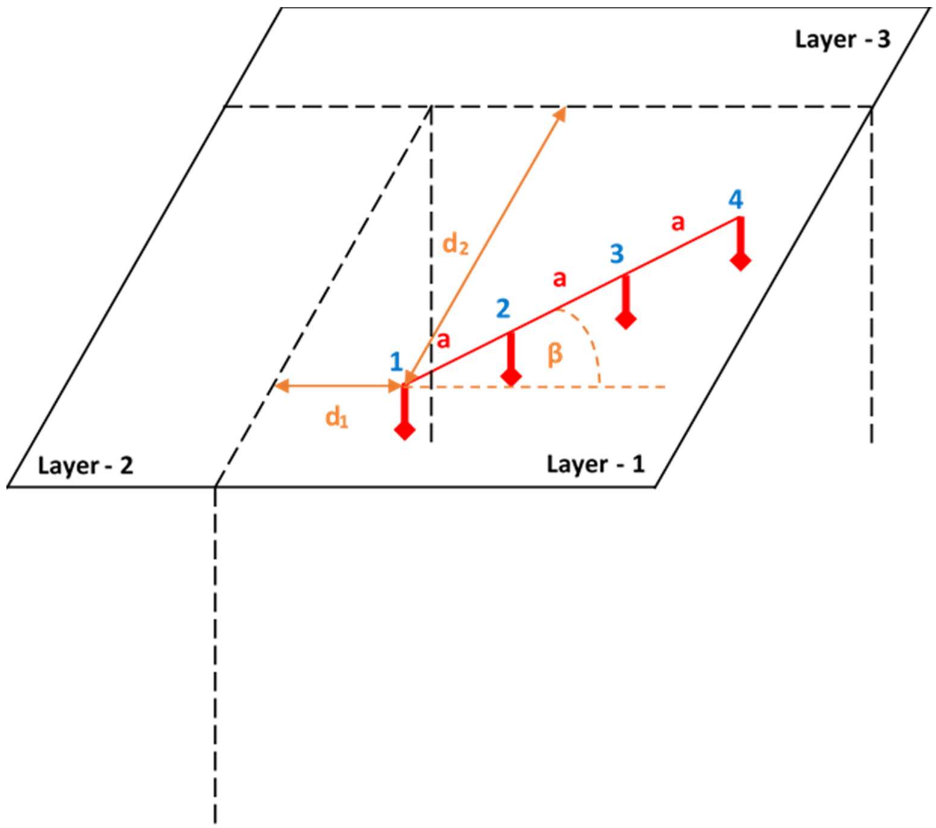

Figure 5.9: Electrode arrangement in three perpendicular vertical layer soil

Case – 1

Apparent resistivity (ρ_a) vs distances between the electrode 1 and the interfaces (d_1) and (d_2) when the direction of electrodes array (β) = 0 Degrees

Resistivity of soil layers are considered as follows.

ρ_1(Resistivity of layer 1) = 100Ω.m

ρ_2(Resistivity of layer 2) = 1000Ω.m

ρ_3(Resistivity of layer 3) = 20Ω.m

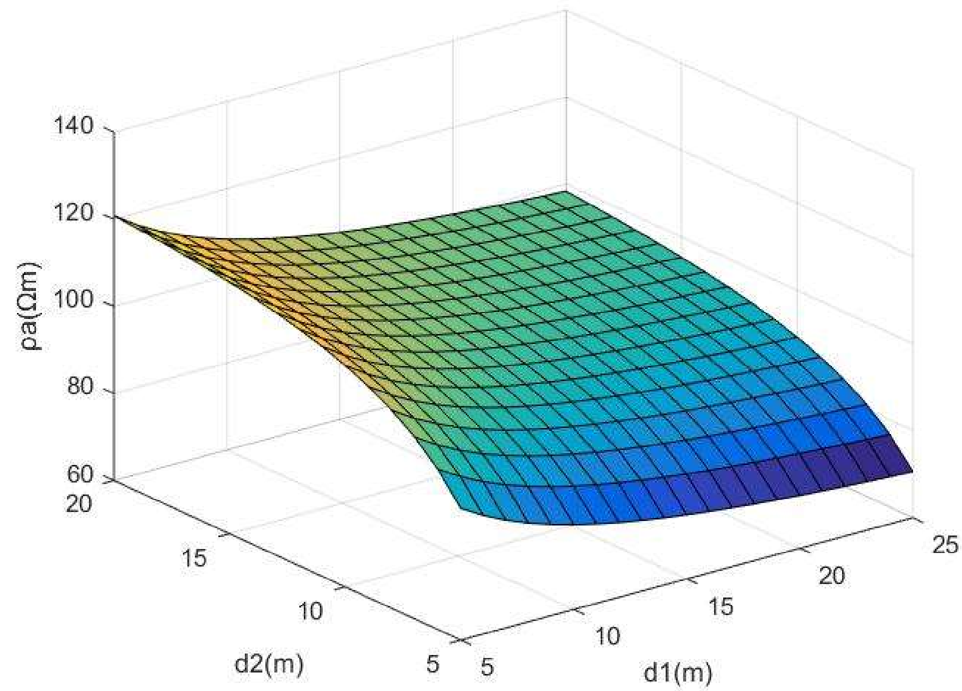

Figure 5.10: Apparent resistivity (ρ_a) vs distances between the electrode 1 and the interfaces (d_1) and (d_2) when ρ_1= 100Ω.m, ρ_2 = 1000Ω.m, ρ_3= 20Ω.m, (β) = 0 Degrees

Case – 2

Apparent resistivity (ρ_a) vs distances between the electrode 1 and the interfaces (d_1) and (d_2) when the direction of electrodes array (β) = 45 Degrees

Resistivity of soil layers are considered as follows.

ρ_1(Resistivity of layer 1) = 100Ω.m

ρ_2(Resistivity of layer 2) = 1000Ω.m

ρ_3(Resistivity of layer 3) = 20Ω.m

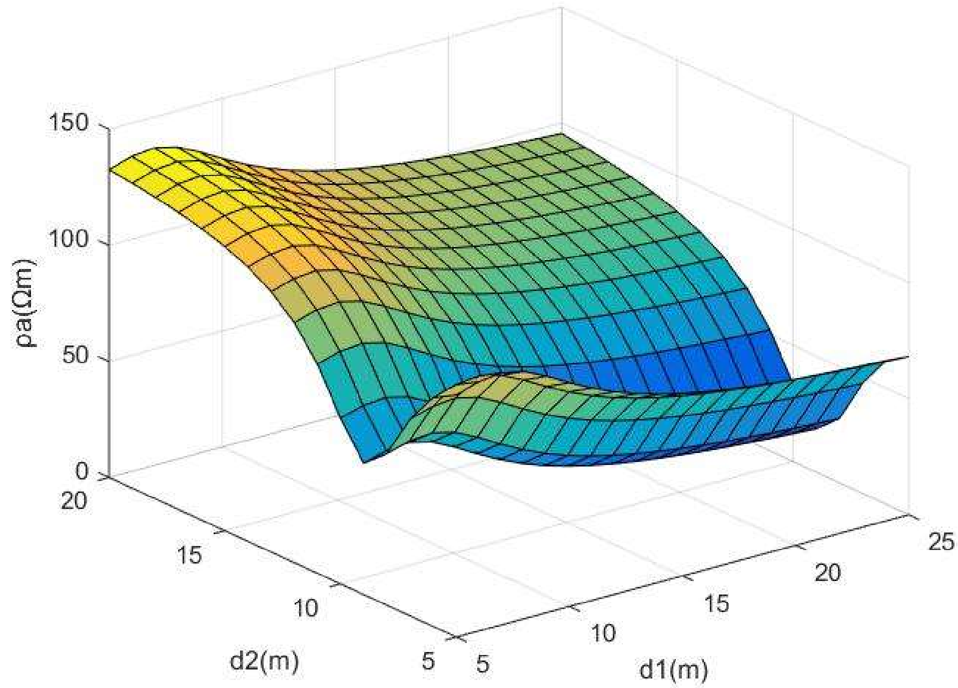

Figure 5.11: Apparent resistivity (ρ_a) vs distances between the electrode 1 and the interfaces (d_1) and (d_2) when ρ_1= 100Ω.m, ρ_2 = 1000Ω.m, ρ_3= 20Ω.m, (β) = 45 Degrees

Case – 3

Apparent resistivity (ρ_a) vs distances between the electrode 1 and the interfaces (d_1) and (d_2) when the direction of electrodes array (β) = 90 Degrees

ρ_1(Resistivity of layer 1) = 100Ω.m

ρ_2(Resistivity of layer 2) = 1000Ω.m

ρ_3(Resistivity of layer 3) = 20Ω.m

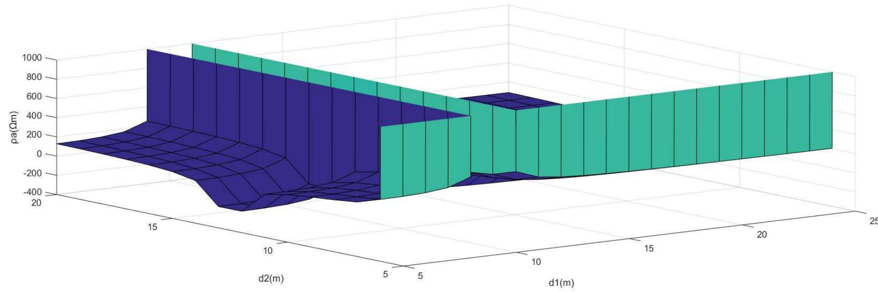

Figure 5.12: Apparent resistivity (ρ_a) vs distances between the electrode 1 and the interfaces (d_1) and (d_2) when ρ_1= 100Ω.m, ρ_2 = 1000Ω.m, ρ_3= 20Ω.m, (β) = 90 Degrees

5.3. Three parallel vertical layer soil

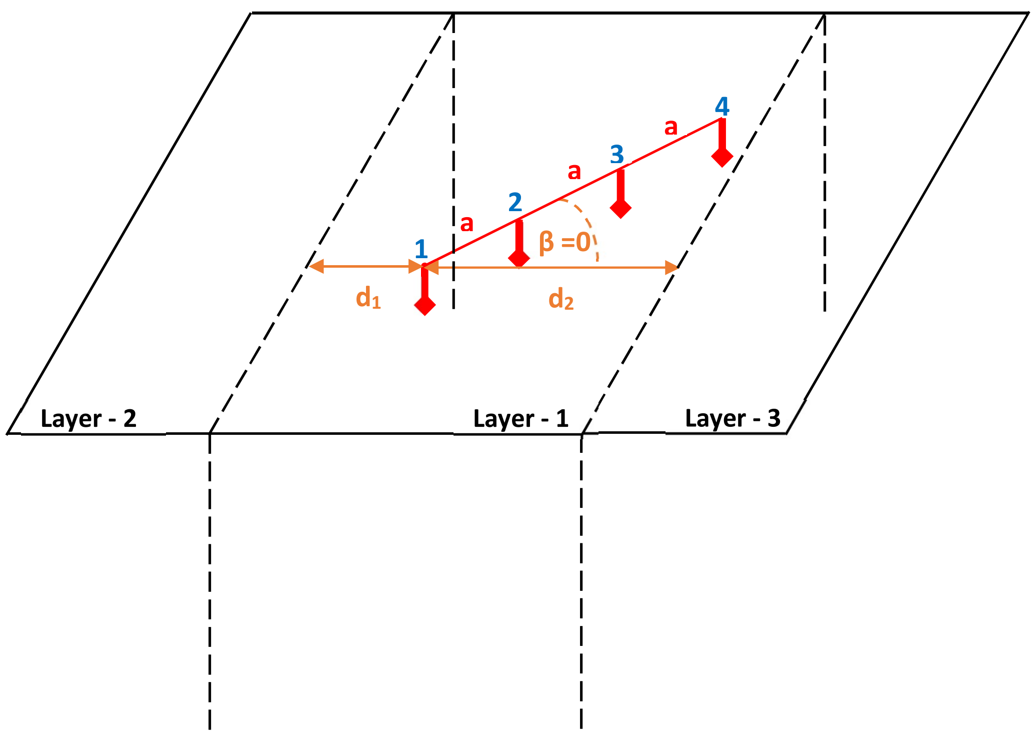

Figure 5.13: Electrode arrangement in three parallel vertical layer soil

Case – 1

Apparent resistivity (ρ_a) vs distance between the electrode 1 and the interface between layer 1 and layer 2 (d_1) when the direction of electrodes array (β) = 0 Degrees

ρ_1 (Resistivity of layer 1) = 4000Ω.m

ρ_2 (Resistivity of layer 2) = 20Ω.m

ρ_3 (Resistivity of layer 3) = 100000Ω.m

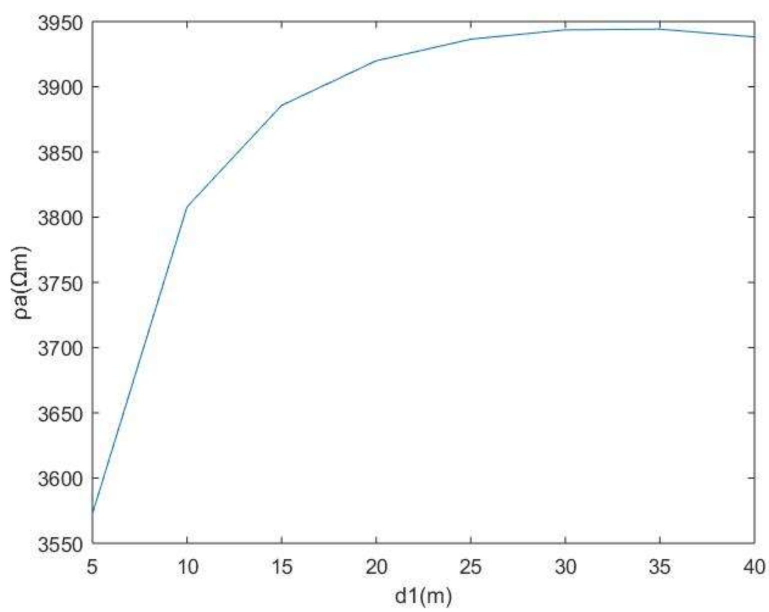

Figure 5.14: Apparent resistivity (ρ_a) vs distance between the electrode 1 and the interface between layer 1 and layer 2 (d_1) when ρ_1= 4000Ω.m, ρ_2 = 20Ω.m, ρ_3= 100000Ω.m, (β) = 0 Degrees

Case – 2

Apparent resistivity (ρ_a) vs distance between the electrode 1 and the interface between layer 1 and layer 2 (d_1) when the direction of electrodes array (β) = 45 Degrees

ρ_1 (Resistivity of layer 1) = 4000Ω.m

ρ_2 (Resistivity of layer 2) = 20Ω.m

ρ_3 (Resistivity of layer 3) = 100000Ω.m

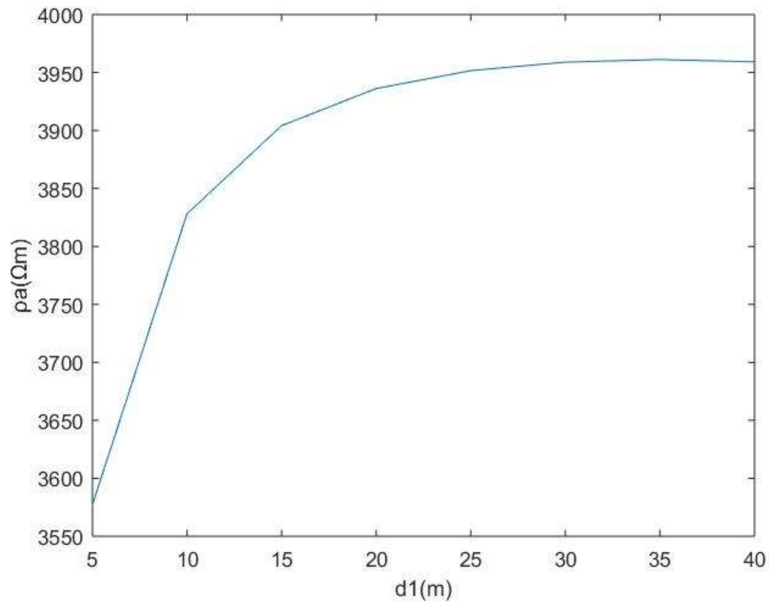

Figure 5.15: Apparent resistivity (ρ_a) vs distance between the electrode 1 and the interface between layer 1 and layer 2 (d_1) when ρ_1= 4000Ω.m, ρ_2 = 20Ω.m, ρ_3= 100000Ω.m, (β) = 45 Degrees

Case – 3

Apparent resistivity (ρ_a) vs distance between the electrode 1 and the interface between layer 1 and layer 2 (d_1) when the direction of electrodes array (β) = 90 Degrees

ρ_1 (Resistivity of layer 1) = 4000Ω.m

ρ_2 (Resistivity of layer 2) = 20Ω.m

ρ_3 (Resistivity of layer 3) = 100000Ω.m

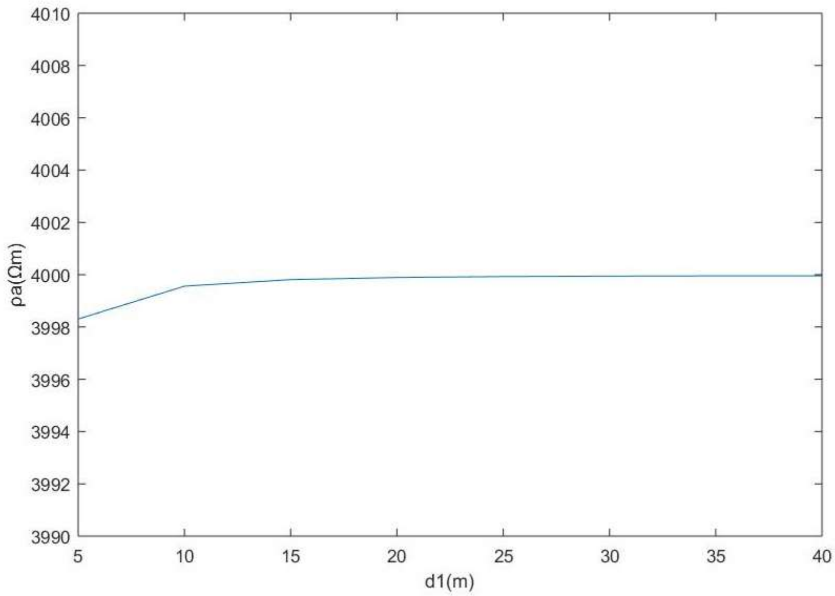

Figure 5.16: Apparent resistivity (ρ_a) vs distance between the electrode 1 and the interface between layer 1 and layer 2 (d_1) when ρ_1= 4000Ω.m, ρ_2 = 20Ω.m, ρ_3= 100000Ω.m, (β) = 90 Degrees

CHAPTER 6

6. FIELD MEASUREMENTS

Several field measurements had done to check the validity of the model and the formulas derived for vertically layered soil.

6.1. Field measurement – 1

A flat play ground near to the Bolgoda River is selected for the first field measurements. It has a large open space with nearly uniform soil surface and located next to the Bolgoda River.

Figure 6.1: Selected area for the field measurement - 1

Figure 6.2: Selected area on the map

Earth resistance measurements were taken by Megger® DET4TD2 digital earth resistance meter and it supports Wenner 4 probe method.

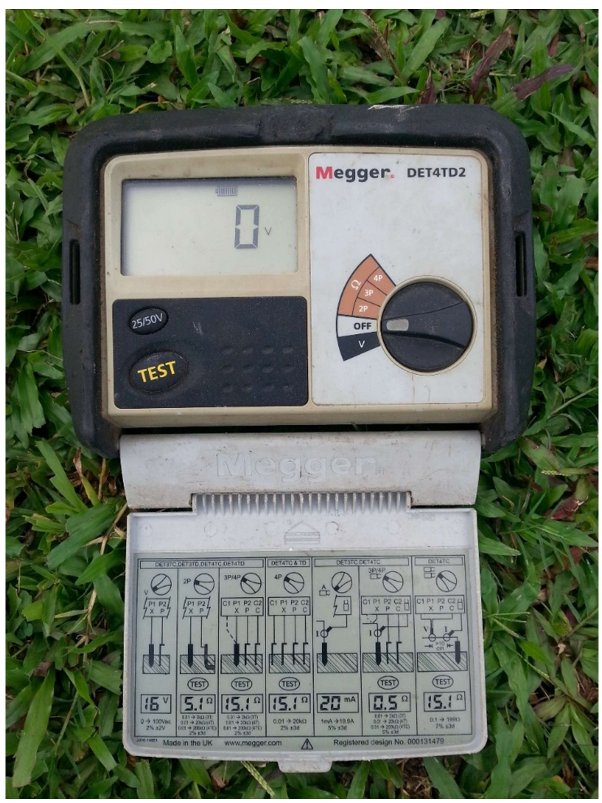

Figure 6.3: Digital earth resistance meter - Megger® DET4TD2

Case – 1

Measurements were taken by changing the distance between the probe 1 and the boundary between the ground and the river. The readings are shown in the table 6.1.

Table 6.1: Resistivity measurements

d_1 - Distance between the probe 1 and the boundary(m)	**Resistivity (Ω.m)**
5	4712.3
10	4932.2
15	5748.9
20	6220.2

As shown in the figure 6.4 the apparent resistivity gets increase with the increase of space between the first electrode and the interface. The ground has a higher resistivity than the river water and this is similar to the case 2 discussed in the subsection 5.1 of Chapter 5. The same graph is shown in figure 6.5 for easy comparison.

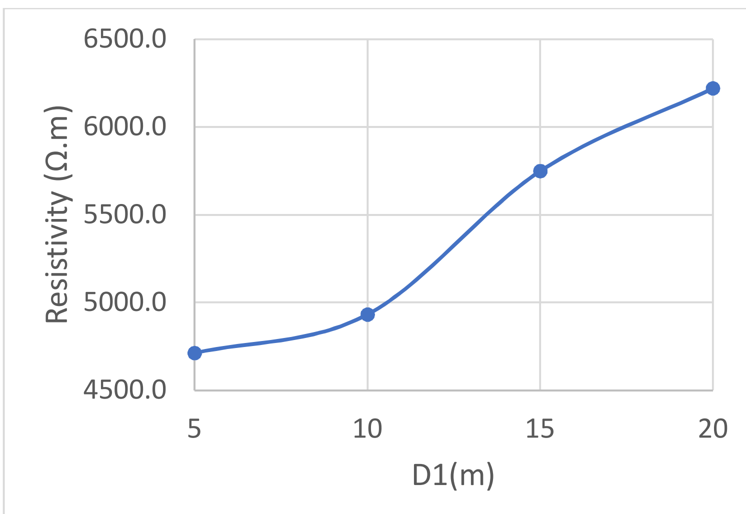

Figure 6.4: Apparent resistivity (ρ_a) vs distance between the electrode 1 and the interface (d_1)

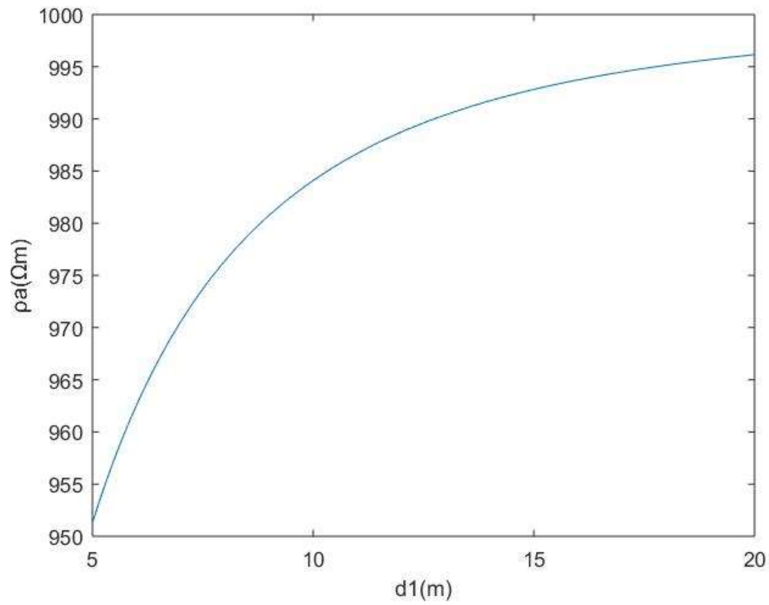

Figure 6.5: Apparent resistivity (ρ_a) vs distance between the electrode 1 and the interface (d1) when ρ_1= 1000Ω.m, ρ_2 = 100Ω.m and β = 0 Radian - MATLAB simulation

Case – 2

Measurements were taken by changing the angle of the electrode array. The readings are shown in the table 6.2.

Table 6.2: Resistivity measurements

Direction of the electrodes array (β) (Radian)	**Resistivity (Ω.m)**
0	4712.3
0.393	4021.1
0.785	4335.3
1.353	4335.3
1.571	3267.2

As shown in the figure 6.6 the apparent resistivity gets decrease with the increase of the direction of the electrode array. The ground has a higher resistivity than the river water and this is similar to the case 4 discussed in the subsection 5.1 of Chapter 5. The same graph is shown in figure 6.7 for easy comparison.

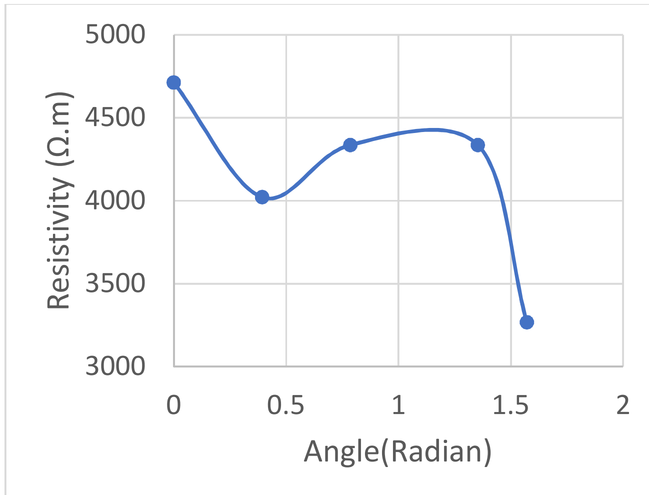

Figure 6.6: Apparent resistivity (ρ_a) vs the direction of electrodes array (β)

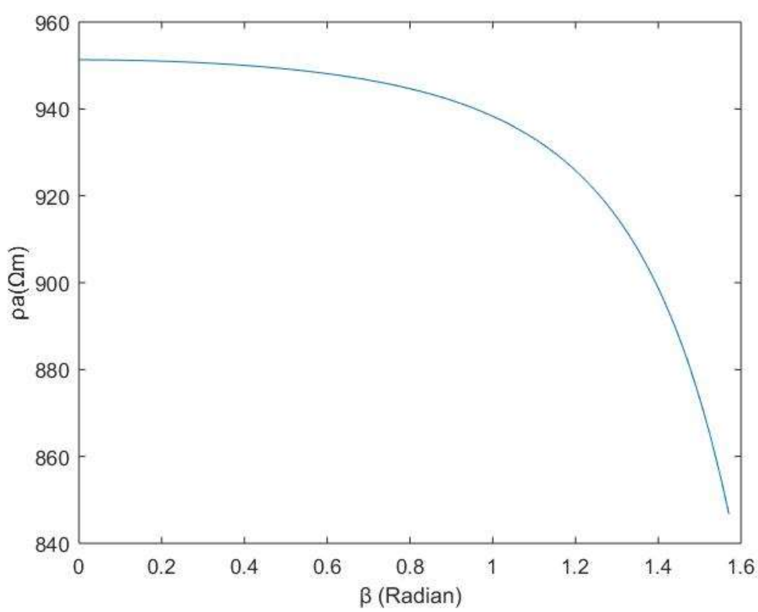

Figure 6.7: Apparent resistivity (ρ_a) vs the direction of electrodes array (β) when $\rho_1 = 1000\Omega.\mathrm{m}$, ρ_2 = 100Ω.m and d_1= 5m – MATLAB simulation

Case – 3

The electrode array moved parallel to the interface and the readings are shown in the table 6.3 and figure 6.8.

Table 6.3: Resistivity measurements

Distance(m)	**Resistivity parallel to the interface (Ω.m)**
0	3267.2
5	3298.6
10	3455.7
15	3392.8

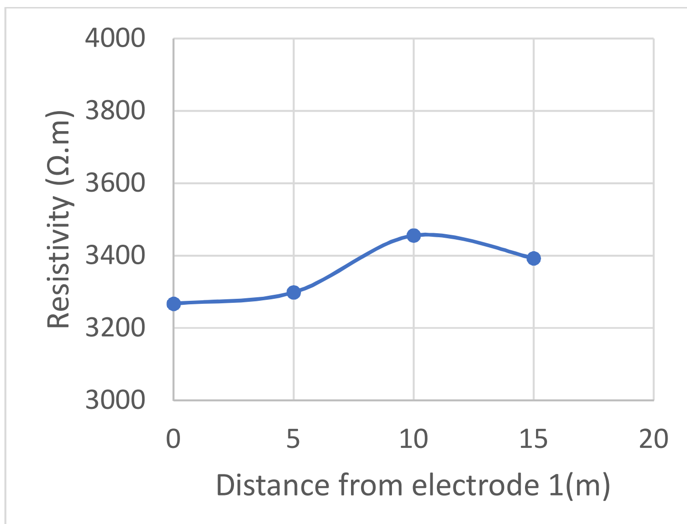

Figure 6.8: Apparent resistivity (ρ_a) parallel to the interface

6.2. Field measurement – 2

A flat ground above a boundary wall is considered for the second field measurements. The area has a uniform soil structure and due to the space limitation only a few measurements was taken by changing the angle of the electrode array.

Figure 6.9 : Selected area for the field measurement - 2

Earth resistance measurements were taken by Kyoritsu® 4105A digital earth resistance meter.

Figure 6.10: Digital earth resistance meter - Kyoritsu® 4105A

Measurements were taken by changing the angle of the electrode array. The readings are shown in the table 6.4.

Table 6.4: Resistivity measurements

Direction of the electrodes array (β) (Radian)	**Resistivity (Ω.m)**
-0.785 (-45)	304.1
-0.982 (-56.25)	308.2
-1.178 (-67.5)	315.1
-1.374 (-78.75)	313.0
-1.5708 (-90)	319.2

The instrument makes earth resistance measurement with fall of point method. Therefore, following equation is used to convert the resistance values to into resistivity values. This formula is developed by Professor H. R. Dwight of the Massachusetts Institute of Technology for single ground electrode systems. [2]

$$R = \frac{\rho}{2\pi L}\left[\left(ln\frac{4L}{r}\right) - 1\right]$$

R = Resistance in ohms of the ground rod to the earth (or soil)

L = Grounding electrode length

r = Grounding electrode radius

ρ = Average resistivity in ohms-cm

As shown in the figure 6.11 the apparent resistivity gets decrease with the increase of the direction of the electrode array. The sol has a lower resistivity than the air and this is similar to the case 3 which is discussed in the subsection 5.1 of Chapter 5. But the angle was measured in other direction, so another MATLAB simulation has done considering the average resistivity of the soil and shown in the Figure 6.12.

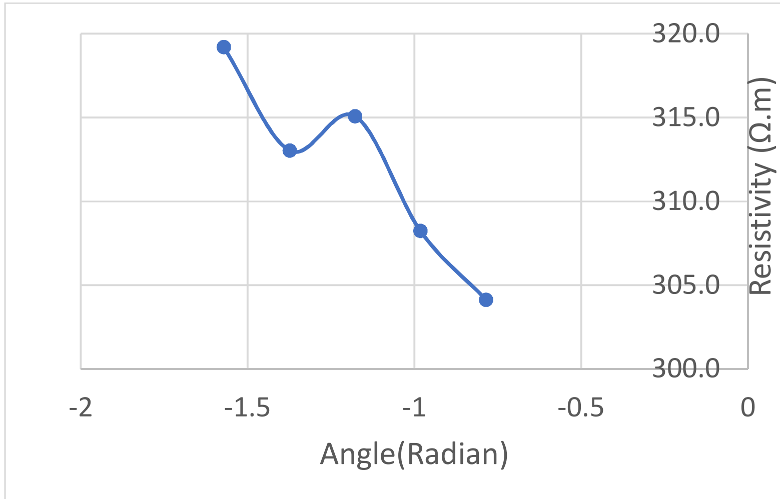

Figure 6.11: Apparent resistivity (ρ_a) vs the direction of electrodes array (β)

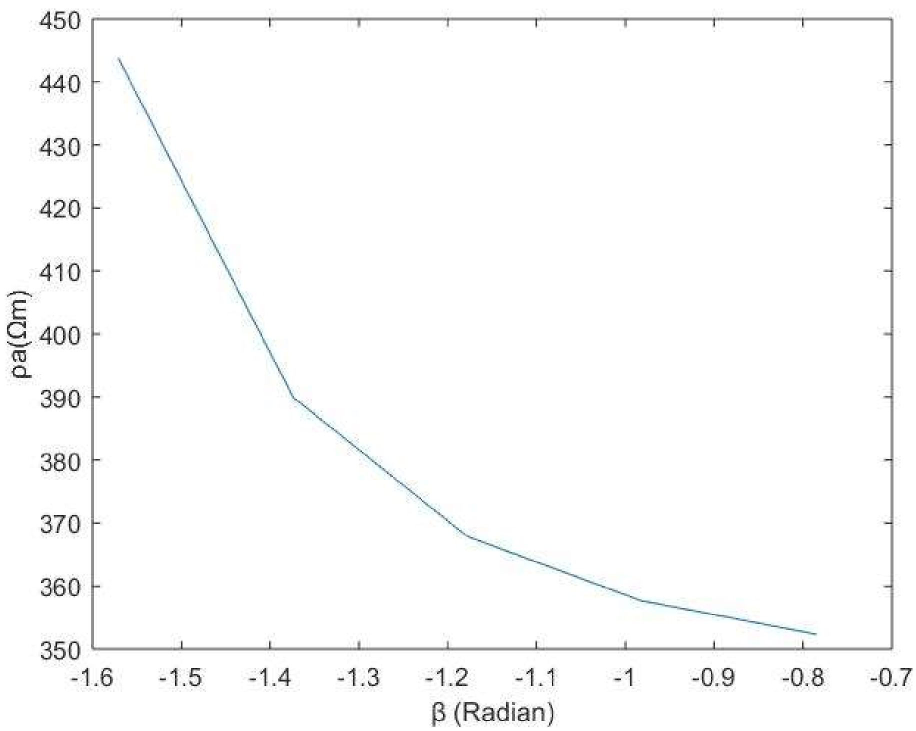

Figure 6.12: Apparent resistivity (ρ_a) vs the direction of electrodes array (β) – MATLAB simulation

CHAPTER 7

7. CONCLUSIONS AND RECOMMENDATIONS

7.1. Achievement of objective and research outcome

The objective of the research is “Analyze the apparent ground resistivity with the presence of vertically layered soil and propose an accurate method to calculate the apparent resistivity” The methodologies to achieve the objective were proposed and validated in the previous chapters.

Research outcomes can be summarized as follows:

1. Three different vertical layer soil models were identified as follows.
a. Two vertical layers
b. Three parallel vertical layers
c. Three perpendicular vertical layers
2. The model developed by using the method of images and Wenner four-point method satisfied to solve the apparent soil resistivity of vertically layered soil.
3. The apparent soil resistivity depends on the direction of the electrodes array and the space between the first probe and the interface between the layers.
4. Field measurements approved the results of the mathematical model.

7.2. Limitations of the study

Limitations of this study are listed below with the proposals for further studies.

1. Only a few field measurements were taken due to the non-availability of an earth resistance meter. But, more field measurements are required to check and improve the accuracy and efficiency of the model. It is easy to do the measurements with a digital earth resistance meter which supports Wenner 4 probe method.
2. The model was not being able to simulate using an Electromagnetic Simulation Software due to the complexities of the software packages, limitations of the trial versions and the cost related to purchase a license. It is suggested to simulate the model using an Electromagnetic Simulation software like Ansys Maxwell, CDEGS and compare the results with the mathematical model and field measurements to improve the model developed in this research.

3. This research only discusses about the vertically layered soil, but the real ground is a mix of horizontal and vertical layers. This study can be extended to get an accurate figure for the apparent resistivity by considering both horizontal and vertical layers.

7.3. Applications and recommendations

The study is helpful to understand the behavior of apparent soil resistivity with the existence of vertical soil layers and enable one to determine the how the soil is layered in a specific geographical area.

Also, it gives a good presentation of the soil to build more accurate grounding systems to increase the safety and reliability while minimizing the cost involve.

The designers can locate the earthing systems away from rivers, lakes and hills to minimize the effect of the resistivity changes due to the vertical layers.

References

[1] (81-2012 IEEE Guide for Measuring Earth Resistivity, Ground Impedance, and Earth Surface Potentials of a Grounding System, 2012)

[2] E&S Grounding. (2018). *Can you explain the ground resistance formula developed by Prof. H. R. Dwight? - E&S Grounding*. [online] Available at: http://www.esgroundingsolutions.com/can-you-explain-the-ground-resistance-formula-developed-by-prof-h-r-dwight/

[3] BS 7430:2011 Code of practice for protective earthing of electrical installations. (2011).

[4] Mohamed Nayel, Boyang Lu, Yu Tian, Yingzhen Zhao, Study of Soil Resistivity Measurements in Vertical Two-Layer Soil Model, Department of Electric and Electronic Engineering Xi'an Jiaotong-Liverpool University, Suzhou, China.

[5] Xiaobin Cao, Guangning Wu, Senior Member, IEEE, Shenglin Li, Weiming Zhou and RuiFang.Li, A simple formula of grounding grid resistance in vertical two-layer soil,

[6] Predrag D. Rancid (IEEE member), Zoran P. Stajic', Bojana S. ToSid Faculty of Electronic Eng. of NE, Srbija, SR Jugoslavija, Djordje R. Djordjevid, Faculty of Civil Eng. of NiS, Srbija, SR Jugoslavija, Analysis of Linear Ground Electrodes Placed in vertical Three-Layer Earth.

APPENDICES

Appendix A - Image distribution of the injection current at probe 1 in three parallel vertical layer soil

Appendix B - Image distribution of the sinking current at probe 4 in three parallel vertical layer soil

Appendix C – Research papers